MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 29/1984

Konstruktive Methoden zur praktischen Behandlung von Integralgleichungen

24.6. bis 30.6.1984

Die Tagung fand unter der Leitung von G. Hämmerlin (München) und K.-H. Hoffmann (Augsburg) statt. 30 Vorträge gaben den Teilnehmern aus 12 Ländern reichlich Gelegenheit zur Diskussion und zum wissenschaftlichen Austausch.

Unter den vielen Fragestellungen, die sich zum Thema konstruktiver Methoden zur praktischen Behandlung von Integralgleichungen ergeben, lassen sich Schwerpunkte erkennen, die auch im Vortragsprogramm deutlich wurden. Dazu ist zunächst die numerische Behandlung von Volterraschen Integralgleichungen zu nennen, wo die Behandlung schwachsingulärer Kerne und Stabilitätsfragen im Vordergrund stehen. An schwachsingulären Integralgleichungen vom Fredholmschen Typ wird nach wie vor intensiv gearbeitet. Daneben sind Kollokations- und Galerkin-Methoden von großem Interesse, insbesondere die Verwendung von Splines zur Gewinnung von Näherungslösungen. Mehrere Vorträge beschäftigten sich mit speziellen nichtlinearen Integralgleichungen und ihren Anwendungen, z.B. in der Streutheorie. Weiterhin sind Beiträge über Randintegralmethoden zu erwähnen. Integralgleichungen erster Art und die Untersuchung von Regularisierungen stellen die Verbindung zu den inkorrekt gestellten Problemen her. Sie verdienen wegen ihrer praktischen Bedeutung gegenwärtig starke Beachtung. In der Reihe ISNM des Birkhäuser Verlags wird ein Band erscheinen, in dem Ausarbeitungen der meisten Vorträge zu finden sind, die auf der Tagung gehalten wurden. Dieser Band wird auch einen Abschnitt enthalten, in dem offene Probleme dargestellt werden, die im Laufe einer sehr anregenden Diskussion zur Sprache kamen.

Die angenehme Atmosphäre der Tagung, die nicht zuletzt der guten Betreuung durch die Mitarbeiter des Instituts zu verdanken ist, soll besonders erwähnt werden. Im Namen der Tagungsteilnehmer danken wir allen Angehörigen des Instituts herzlich dafür.

Vortragsauszüge

G. AKRIVIS:

The Norm of the Error Functional of Certain Gaussian Quadrature Formulae

We consider Gaussian quadrature formulae Q_n, $n \in \mathbb{N}$, approximating the integral $I(f) := \int_{-1}^{1} w(x)f(x)dx$. In certain spaces of holomorphic functions the error functional $R_n := I-Q_n$ is continuous. For the special weight functions

$$w(x) = \frac{1}{k-x^2}(1-x)^{\alpha}(1+x)^{\beta},\ k>1,\ \alpha,\beta = \pm\frac{1}{2},$$

we characterize the abscissae of Q_n and calculate the norm of R_n explicitly.

J. ALBRECHT:

Die Monotonie der Temple'schen Quotienten

In einem Hilbert-Raum $(H,\langle\cdot,\cdot\rangle)$ sei eine EWA $M\phi=\lambda N\phi$, $\phi\in D(M)$ mit $M:D(M)\to H$ sym., pos.def., $N:D(N)\to H$ sym.,pos.def., $D(M)\subset D(N)\subset H$ gegeben. Beim Verfahren der schrittweisen Näherungen sind Folgen $\{F_p\},\{a_p\},\{\mu_p\}$ (Schwarz'sche Quotienten) und - unter der Voraussetzung $\mu_1<L<\lambda_2$ - $\{\tau_p\}$ (Temple'sche Quotienten) gemäß

$F_o\in D(N)$, $MF_p=NF_{p-1}$, $F_p\in D(M)$ $(p\in\mathbb{N})$; $a_p=\langle F_o,NF_p\rangle$ $(p\in N_o)$;

$\mu_p=a_{p-1}/a_p$ $(p\in\mathbb{N})$; $\tau_p=(La_p-a_{p-1})/(La_{p+1}-a_p)$ $(p\in\mathbb{N})$

zu berechnen. Es wird bewiesen, daß (neben der Einschließung $\tau_p<\lambda_1<\mu_p$ $(p\in\mathbb{N})$ und der Monotonie der Schwarz'schen Quotienten $\mu_1<\mu_2$ ...) auch die Monotonie der Temple'schen Quotienten $\tau_1<\tau_2$ ... gilt.

Arbeitet man mit <u>mehreren</u> Folgen $\{F_p^{[\nu]}\}(\nu=1,\ldots,n)$, so ergeben sich in analoger Weise Intervallschachtelungen für die ersten s Eigenwerte $(1\leq s\leq n)$.

Lit. L.Collatz: Eigenwertaufgaben..., Leipzig 1963(2.Aufl.);
F.Goerisch,J.Albrecht: Die Monotonie...,ZAMM 64(1984)T278-T279.

Gefördert durch
DFG Deutsche Forschungsgemeinschaft

H. ARNDT:

An Adaptive Stepsize Control for Volterra Integral Equations

Stepsize control strategies normally rely on an error expansion of the local and/or global error. For Volterra integral equations of the second kind such an error expansion exists e.g. in the case of extended Runge Kutta methods (Hairer, Lubich, Nørsett). We develop an adaptive stepsize control which differs from the strategies known in ordinary differential equations. The main point is that we always look whether integrations that have to be performed at a later time will be carried out according to the prescribed precision.

K.E. ATKINSON:

Solving Integral Equations on Surfaces in Space

A method is described for solving integral equations defined on piecewise smooth surfaces in three dimensions. The surface is triangulated and approximated with quadratic isoparametric elements. A collocation method is described and analysed for the solution of integral equations of the second kind. We then discuss several questions that are important for the practical implementation of the method, especially when applying the results to equations from potential theory. First, how is the surface to be described and how is the triangulation to be carried out, including how to store the triangular elements. Next, how should we numerically evaluate the weakly singular collocation integrals that arise in applications to potential theory. Finally, we consider iterative methods of solution for the resulting large linear systems.

C.T.H. BAKER

Evolutionary Problems of Volterra Type

Amongst the competitors as numerical methods for Volterra integral equations of the second kind are quadrature methods, extended

Runge-Kutta methods, and mixed quadrature-Runge-Kutta methods. Various considerations enter into a comparison of such methods: We remark that reducible quadrature methods which are A-stable are of low order, whilst A-stable extended methods (whether or not of high order) require considerable work per step. Against this background, mixed quadrature-Runge-Kutta methods appear attractive but seem not to have received support because of their apparent stability properties. We show that A-, A(α)- and A(o)-(implicit) quadrature-Runge-Kutta methods do indeed exist. The extension of the stability analysis to convolution equations is indicated, and the analysis can also be extended to cover the use of mixed methods for Volterra integro-differential equations.

M. BRANNIGAN:

Constrained Approximation Methods for Integral Equations

Classical techniques for solving integral equations use an approximation to the unknown function, and obtain the unknown parameters of this approximation by interpolation. Thus collocation interpolates on a given set of points, and the Galerkin method finds a 'good' set of interpolation points. We suggest that the complete continuous approximation problem is solved, which invariably will give a better set of parameters for the approximation than an interpolation scheme. If this approach is made then constraints on the function and/or the parameters are easily incorporated into the method. Also the theory developed in the proposed method is extendable to problems with more than one variable.

M. BROKATE:

Optimal Control of a Volterra Process Involving Hysteresis

We consider the problem of optimal control

$$\begin{aligned} &\text{minimize} && L_T(x(T),y(T)) \\ &\text{subject to} && \dot{x} = f(t,x,y,u)\ ,\ x(0) = x_o \\ & && y = W[Sx,y_o] \\ & && u(t) \in \Omega \end{aligned}$$

where W is a hysteron of first resp. second kind in the sense of Krasnoselski. For some special situations we prove theorems on existence of optimal controls and on necessary optimality conditions.

H. BRUNNER:

On the Numerical Solution by Collocation of Volterra Integral and Integro-Differential Equations with Weakly Singular Kernels

It is well known that Volterra integral equations of the second kind and Volterra integro-differential equations with weakly singular kernels $(t-s)^{-\alpha}K(t,s)$ $(0<\alpha<1;$ K smooth) possess solutions which, near the left endpoint of the interval of integration $[0,T]$, behave like $y(t)\sim t^{1-\alpha}$ and $y(t)\sim t^{2-\alpha}$, respectively. Consequently, if such equations are solved by collocation in polynomial spline spaces with respect to quasi-uniform mesh sequences $\{\pi_N\}$, the resulting convergence order is given by $O(N^{-(1-\alpha)})$, regardless of the choice of the degree of the approximating spline. The optimal convergence order (with respect to the given spline space) can be restored if one employs suitably graded meshes,

$$t_n := \left(\frac{n}{N}\right)^r \cdot T \qquad (n = 0,\ldots,N) ,$$

with grading exponent $r := (m+1)/(1-\alpha)$, or $r := (m+1)/(2-\alpha)$, respectively, where m is the degree of the spline. We discuss convergence proofs, applications and also certain limitations of these collocation methods on graded meshes.

L. COLLATZ:

Inclusion of Regular and Singular Solutions of Certain Types of Integral Equations

Let us consider a functional equation u=Tu for a function $u(x)=u(x_1,\ldots,x_n)$. Let T be "monotonically decomposable" in the sense of J. Schröder. An iteration procedure, starting with two functions v_o,w_o may give the functions v_1,w_1 with $v_o \leqq v_1 \leqq w_1 \leqq w_o$.

Then Schauder's fixed point theorem gives the existence of at least one solution u in the interval $[v_1, w_1]$. This is in many cases the only (easily calculable) possibility of an inclusion for u. This method is applicable for Hammerstein's integral equation and some further types of nonlinear integral equations. If the solution has singularities, one has to discern whether the location of the singularities is known or unknown ("hidden singularities"). Numerical examples for both cases are given. Recently three-dimensional singularities became more important. The inclusion of solutions in the case of distributed singularities in the three-dimensional space $\mathbb{R}^3$ is described.

D. COLTON:

The Solution of Nonlinear Integral Equations in Acoustic Scattering Theory

We consider the problem of determining the shape of an obstacle from a knowledge of the far field pattern of the scattered acoustic wave. This problem is complicated by the fact that it is nonlinear and improperly posed. Two methods of solution are presented, both of which require the minimization of a nonlinear functional subject to constraints. The second approach, which has yet to be numerically tested, has the advantage of a simple Frechêt derivative and avoids the need to solve an integral equation at each step of the iterative procedure for obtaining the solution.

P. EGGERMONT:

Beyond Superconvergence of Collocation Methods for Volterra Integral Equations of the First Kind

We discuss superconvergence aspects of the numerical solution of Volterra integral equations of the first kind (VIE 1) by collocation methods with piecewise polynomials of degree $\leq p$ on a uniform mesh. It is well-known that (i) under "normal" conditions convergence of order $O(h^{p+1})$ is achieved (De Hoog & Weiss (1973)), (ii) under slightly more special conditions, $O(h^{p+2})$ convergence is achieved at special points inside each subinterval (Brunner (1978),E.(1982)), (iii) higher order convergence is impossible (Brunner (1978)). We show here that it is possible to do some postprocessing on the "superconvergence" collocation solution to obtain $O(h^{p+3})$ convergence at yet another set of special points. This possibility is based on the oscillating behavior of the error in the collocation solution inside each subinterval. For the "pure" differentiation case of VIE 1 this is easily shown (applying Lagrange interpolation). For the general case it follows from this special case and from the closeness of the projectors associated with the collocation method under compact perturbations. Some numerical illustrations are presented. There appears to be a connection between the above and the convergence analysis of certain Runge-Kutta methods for VIE 1 (Keech, 1978) but the details are not yet clear.

H.W. ENGL:

Discrepancy Principles for the Choice of the Regularization Parameter for Solving Integral Equations of the First Kind by Tikhonov-Regularization

Since the solution of an integral equation of the first kind is in general an ill-posed problem, one has to use regularization methods, e.g. Tikhonov regularization. There the problem of a choice of the regularization parameter that leads to the optimal rate of convergence arises. We are interested in such parameter choices that are "a-posteriori" choices in the sense that the parameter is calculated from quantities that appear during the calculations. A class of such methods are the so-called "discrepancy principles" due to Morozov and Arcangeli. It is known that these methods do not yield the optimal convergence rate.
We present a variant of the discrepancy method that yields the optimal convergence rates. This variant is applicable to different versions of Tikhonov regularization, namely classical Tikhonov regularization, Tikhonov regularization with differential operators (cf.Locker-Prenter), and Tikhonov regularization in Hilbert scales (cf.Natterer). Finally we present some preliminary results

on constrained Tikhonov regularization and numerical results.

D. EYRE:

Spline-Galerkin Method for Solving some Quantum Mechanic Integral Equations

An investigation is made of the Galerkin technique with cubic B-spline approximants to solve some quantum mechanic integral equations. The problem is to find the numerical solution of a linear integral equation of the second kind. This equation has a singular kernel, and a non-smooth (cusp) behaviour in the solution function. The discontinuity in the solution function is built into the spline approximation by combining knots (multiple knot) at the point of discontinuity. A one-dimensional example is used to test the performance of both the Galerkin and iterated Galerkin methods.

T. FAWZY:

Integral Treatment of O.D.E. with Spline Functions

A new method for approximating the solution of the initial value problem:

$$y''=f(x,y,y') \;,\; y(x_0)=y_0 \;,\; y'(x_0)=y'_0$$

with spline functions is presented. The spline function approximating the solution is not necessarily a polynomial spline. It has been shown that if $f \in Lip_M \alpha$ and $f \in C^r([a,b]\times R^2)$, then the error is $0(h^{r+2+\alpha})$ in $y^{(i)}(x)$ for all $i=0,1,\dots,r+2$ where $0<\alpha<1$.

K.-H. HOFFMANN:

Towards the Identification of Ordinary Differential Equations from Measurements

The identification problem of estimating certain functions in a system of linear ordinary differential equations from measured data of its state is considered. The approach consists in an imbedding of the problem into a family of parameter-dependent problems which can be solved at least numerically. The corresponding solutions are proved to converge to the unknown functions as the parameters tend to infinity. Stability results with respect to disturbances in the measurements and the initial data are developed as well. The method is applied to determine mass exchange rates in a compartmental system of pharmaco kinetic models.

P.J. VAN DER HOUWEN:

Stability Results for Discrete Volterra Equations

Firstly, stability results for a general class of linear multistep methods for Volterra integral equations (VLM methods) are presented. These results are obtained by deriving recurrence relations of finite length approximating the discrete Volterra equations.
Secondly, it is shown that a special VLM method for second kind equations with decomposable kernel is algebraicly equivalent with a linear multistep method applied to a certain system of ODEs. Finally, the theoretical results will be illustrated by numerical examples.

D. KERSHAW:

The Design of Acoustic Torpedoes

The homing mechanism of an acoustic torpedo requires that its nose should contain a circular disc. The design problem is that of connecting this vertical flat disc with the main body which is a right circular cylinder.

The pressure distribution of the flow around the torpedo needs to be kept as high as possible to prevent separation, and so by Bernoulli's equation this means that the maximum speed of the flow should be kept as low as possible.
A Fredholm integral equation of the second kind whose kernel has a logarithmic singularity can be derived which is satisfied by the speed of flow. This was obtained first by F. Vandrey but published only as an internal report for the British Admiralty, the derivation depended on hydrodynamic considerations. An alternative proof based on Green's third theorem was outlined here.
In order to find a numerical solution of the integral equation a novel type of Gaussian quadrature was developed and used to calculate the flow for a variety of shapes. It was found that within a restricted class the best shape was when the generating curve of the torpedo consisted of the flat nose and flat back being connected by the quadrant of an ellipse chosen so that the curve was smooth.

R. KRESS:

On the Condition of Boundary Integral Equations in Scattering Theory

The question of non-uniqueness in boundary integral equation formulations of exterior boundary value problems in time-harmonic acoustic and electromagnetic scattering can be resolved by seeking the solutions in the form of a combined single- and double-layer potential in acoustics or a combined electric- and magnetic-dipole field in electromagnetics. We present an analysis of the appropriate choice of the coupling parameters which is optimal in the sense of minimizing the condition number of the boundary integral operators.

F. KUHNERT:

Numerische Methoden für singuläre Integralgleichungen

Betrachtet wird die singuläre Integralgleichung

$$(1) \qquad W\phi \equiv \int_0^\infty \frac{\phi(y)}{y-x}\,dy - \int_0^\infty \phi(y)\,\frac{x-y}{(y-x)^2+b^2}\,dy = R(x)\,, \qquad 0 < x < \infty\,.$$

Der Operator W ist in allen Räumen $L_p(0,\infty)$, $1 < p < \infty$, beschränkt und nicht normal auflösbar. Die Gleichung (1) entsteht u.a. bei der Modellierung umströmter Profile mit zusätzlichem Ausblasstrahl an der Hinterkante des Profils, wobei ϕ die Wirbeldichte entlang Profil und Blasstrahl bezeichnet und die rechte Seite $R(x)$ bei $x = 1$ einen endlichen Sprung aufweist und sonst stetig ist. Aus physikalischer Sicht ist außerdem von der Lösung $\phi(y)$ der Gleichung (1) $\phi(y) \to \infty$ bei $y \to 0$ und $y \to 1$ zu fordern.
Die Lösung $\phi(y)$ wird in der Form $\phi(y) = F(y) + S(y)$ gesucht, wobei der Anteil $S(y)$ so konstruiert wird, daß er den Sprung der rechten Seite $R(x)$ aufnimmt. Der Anteil $F(y)$ ist dann aus der Gleichung $WF = r(x)$ mit stetiger rechter Seite $r(x)$ zu ermitteln. Nach Reduktion des Integrationsgebiets auf das endliche Intervall $(0,G)$ ergibt sich eine Gleichung für die Funktion $f(y) = \sqrt{\frac{G-y}{y}}\,g(y)$ mit einer hölderstetigen Funktion $g(y)$ in der Form

$$W_G g \equiv \int_0^G g(y)\sqrt{\frac{G-y}{y}}\,\frac{dy}{y-x} - \int_0^G g(y)\,\frac{y-x}{(y-x)^2+b^2}\sqrt{\frac{G-y}{y}}\,dy = r(x)\,, \quad 0 < x < G\,,$$

die näherungsweise mit Gauß-Quadraturformeln mit dem Jacobischen Gewicht $(G-y)^{1/2}\,y^{-1/2}$ gelöst wird. Als Interpolationsknoten werden die Wurzeln der zugehörigen orthogonalen Polynome $P_n(y)$ und als Kollokationsstellen die Wurzeln der "Funktionen zweiter Art"

$$Q_n(x) = -\int_0^G P_n(y)\sqrt{\frac{G-y}{y}}\,\frac{dy}{y-x}$$

verwendet.
Um eine größere Dichte der Quadraturknoten im Intervall (0,1) zu erreichen, wird die Gleichung $WF = r(x)$ (betrachtet im Intervall $(0,\infty)$) durch

$$F(y) = \frac{H(y)}{y^{\beta}(1+y)^{\alpha}} \qquad \alpha > -\beta > -1 ,$$

mit anschließender Substitution

$$x = \frac{1+\xi}{1-\xi} , \quad y = \frac{1+\eta}{1-\eta} ,$$

auf die Form

$$H_1(\xi) \int_{-1}^{1} H_2(\xi,\eta)(1-\eta)^{\alpha+\beta-1}(1+\eta)^{-\beta} \frac{d\eta}{\eta-\xi} = \rho(\xi)$$

gebracht. Diese Gleichung wird wiederum näherungsweise durch Gauß-Quadraturformeln gelöst.
Es werden Restglieddarstellungen und Fehlerabschätzungen angegeben. Weiterhin werden Vor- und Nachteile der verwendeten numerischen Strategien diskutiert und Anwendungen auf konkrete physikalische Beispiele beschrieben.

A. LOUIS:

Thikonov Regularization of the Radon Transform

When the Tikhonov regularization is applied to an ill-posed problem two questions arise. First the regularization norm has to be selected which can be done with the help of available information on the solution. The difficult task is then the selection of an optimal regularization parameter. For the Radon transform this problem is treated via an explicit representation of the solution with the help of the complete singular system of the Radon transform. The effect of the parameter on the solution is studied. Also the limited angle problem is discussed in this framework.

J.T. MARTI:

On the Numerical Solution of Ill-Posed Problems with Weakly Singular Integral Operators

Regularization methods for the numerical solution of ill-posed problems based on the finite element method are very sensitive to the computation of the corresponding matrix elements. Several ideas are presented to overcome these difficulties for the case of Fredholm integral equations with weakly singular kernels.

VU QUOC PHONG:

Integro-Differential Inequalities

We obtain necessary and sufficient conditions for validity of inequalities

$$\| A^m f \| \leq C_{n,m}(A) \| A^n f \|^{\frac{n-m}{n}} \| f \|^{\frac{m}{n}} \qquad \forall f \in H ,$$

where A is an arbitrary symmetric operator in Hilbert space. Our method also enables us to compute best possible constants. When the operator A is a differential operator in Hilbert space we get integro-differential inequalities.

H.J.J. TE RIELE:

Numerical Solution of a First Kind Fredholm Integral Equation Arising in Atomic Physics

In a study of dispersion relations for electron-atomic scattering, the following first kind Fredholm integral equation arises:

$$\Delta(x) = \frac{1}{\pi} \int_{x_0}^{\infty} \frac{\rho(y)\,dy}{x+y} .$$

Here, the function $\Delta(x)$ is given in 23 points of the interval [1,500] with an accuracy of about 3 percent. For $x > 500$, Δ may be assumed to vanish. The unknown function $\rho(y)$ may be assumed to tend to zero, as y tends to ∞, at least as fast as 1/sqrt(y). Experiments with the regularization method of Phillips and Tikhonov will be reported. The results obtained are acceptable to the physicist, at least in a qualitative sense.

Literature

[1] R. Wagenaar, Small angle elastic scattering of electrons by noble gas atoms, Doctor's Thesis, Amsterdam, 1984.

C. SCHNEIDER:

Product Integration for Two Dimensional Weakly Singular Integral Equations

The lecture concerns the numerical solution of the second kind Fredholm integral equation

$$(*) \qquad y(t) = f(t) + \lambda \int_{\overline{\Omega}} \phi_\alpha(\|t-s\|_2) g(t,s) y(s) ds, \quad t \in \overline{\Omega} \subset \mathbb{R}^m .$$

where $\lambda \in \mathbb{C}$, f and g are given continuous functions, $\phi_\alpha (0<\alpha\leq m)$ is a function with a weak singularity, e.g. $\phi_\alpha(r) = r^{-\alpha} (0<\alpha<m)$, $\phi_m(r) = \log(r)$.

Choosing a mesh $\{\Omega_1,\ldots,\Omega_N\} : \overline{\Omega} = \bigcup_{i=1}^{N} \overline{\Omega}_i$, Ω_i open simply connected pairwise disjoint subsets of Ω with the property that each Ω_i contains its centroid t_i, we approximate the solution of (*) by its product integration solution y_n which is defined by the equation

$$y_n(t) = f(t) + \lambda \sum_{i=1}^{N} g(t,t_i) y_n(t_i) \int_{\overline{\Omega}_i} \phi_\alpha(\|t-s\|_2) ds, \quad t \in \overline{\Omega} .$$

Order of convergence results and numerical results are given.

E. SCHOCK:

Arbitrarily Slow Convergence, Uniform Convergence and Superconvergence of Galerkin-like Methods

Let $M : X \to (X_n)$ be a method for the approximate solution of an equation $x - Tx = y$, where T is a compact linear operator in a Banach space X, which assigns to each $x = (I-T)^{-1}y$ an $x_n^M \in X_n$, in an n-dimensional subspace of X. The method M is said to be converging, if for all $x \in X$ hold

$$\lim R_n^M x = \lim x - x_n^M = 0 .$$

A converging method is said to be arbitrarily slow converging if for each monotone decreasing null sequence (ω_n) there is an $x \in X$

$$\lim\sup \frac{\|x - x_n^M\|}{\omega_n} = \infty$$

Otherwise the method is said to be uniformly converging. It is shown, that the Galerkin method converges arbitrarily slow. In Hilbert spaces the iterated Galerkin method and the Kantorowich method are equivalent and converge uniformly. In the space of continuous functions the iterated Galerkin method converges arbitrarily slow, if it uses interpolating projections, while the Kantorowich method converges uniformly in any case. Furthermore, it is a unified theorem on superconvergence of the three mentioned methods given.

J.A. SCOTT:

A Unified Analysis of Discretization Methods for Volterra Type Equations

This talk is a brief outline of the work contained in my D. Phil. thesis, Oxford University (submitted 1984).
A unified analysis of discretization methods for Volterra type equations is presented. The concept of analytic and discrete fundamental forms is introduced. Prolongation and restriction operators reduce the problem of comparing the exact solution to that of using new Gronwall inequalities and their discrete analogues to consider the effect of perturbations in the fundamental forms. A concept of optimal consistency permits two-sided error bounds to be presented.
An extensive class of m-block methods for the numerical solution of ordinary differential equations is shown to be convergent using the general convergence results. The m-block methods are employed to obtain convergent discretization methods for second kind Volterra integral equations and integro-differential equations.
A general class of quadrature methods for first kind Volterra integral equations is introduced and sufficient conditions for convergence derived. The possibility of using methods for ordinary differential equations to give methods for first kind Volterra integral equations is discussed.

I.H. SLOAN:

Wiener-Hopf Integral Equations and their Finite Section Approximation

This talk is concerned with Wiener-Hopf integral equations of the form

$$x(s) - \frac{1}{\lambda}\int_0^\infty \kappa(s-t)x(t)dt = y(s) , \quad s \in \mathbb{R}^+ ,$$

and their finite-section approximation

$$x_\beta(s) - \frac{1}{\lambda}\int_0^\beta \kappa(s-t)x_\beta(t)dt = y(s) , \quad s \in \mathbb{R}^+ ,$$

where $\beta \in \mathbb{R}^+$. It is assumed that $\kappa \in L_1(\mathbb{R})$, and that y, x and x_β belong to X^+, the space of bounded continuous functions on $\mathbb{R}^+$ with the uniform norm. With the finite-section equation written as $(I - \frac{1}{\lambda} K_\beta)x_\beta = y$, recent joint work with P.M. Anselone has established that $(I - \frac{1}{\lambda} K_\beta)^{-1}$ exists and is uniformly bounded as an operator on X^+ for all β sufficiently large. The key is a 'sliding' variant of the Arzelà-Ascoli theorem. It follows from earlier results of K.E. Atkinson that $x_\beta(s) \to x(s)$ as $\beta \to \infty$, uniformly for s in finite intervals.

A. SPENCE:

Collocation Methods for Integral Equations on the Half-Line

Convergence results are proved for projection methods for integral equations of the form $y(t) = f(t) + \int_0^\infty k(t,s)y(s)ds$. The conditions on k(t,s) are such that the Wiener-Hopf integral equations are included in our analysis. The convergence results indicate that the iterated collocation solution may exhibit superconvergence. The case of collocation using piecewise-constant basis functions applied to an integral equation with kernel $e^{-|t-s|}$ is discussed in detail and numerical results are given.

S. VESSELLA:

Stability Results for Abel Equation

We study the Abel equation:

$$(1) \qquad \frac{1}{\Gamma(\alpha)} \int_0^x \frac{K(x,t)u(t)}{(x-t)^{1-\alpha}}\, dt = f(x) \qquad 0 < x < 1 ,$$

where $0 < \alpha < 1$, f and K are known. u is the unknown. K satisfies the properties:

i) $K(x,x) = 1 \quad x \in (0,1)$, ii) $K \in C^{\circ}$, $\frac{\partial K}{\partial x} \in L^{\infty}$ on $\{(x,t) : 0 \leq t \leq x \leq 1\}$

The problem of solving (1) is ill-posed in L^p-spaces because there is not a continuous dependence of u on f. We find the estimates:

$$(2) \qquad \|u\|_p \leq \text{const}\ \|u'\|_p^{\frac{\alpha}{1+\alpha}} \|f\|_p^{\frac{1}{1+\alpha}}$$

that assures the stability of (1) with the a-priori bound $\|u'\|_p \leq E$.

W.L. WENDLAND:

Spline Collocation for Singular Integral Equations and Integro-differential Equations

Spline collocation is the most frequently used approximation of boundary integral equations in the boundary element method. Here the singular integral equations with Cauchy kernel form a special subclass but also the most important model problem. Let the plane boundary curves all be given by smooth 1-periodic parameter representations and all functions be 1-periodic. Then we use $S_d(\Delta)$, the 1-periodic splines of degree d in C^{d-1} subordinate to the partition $\Delta = \{0 = t_o < t_1 < \ldots < t_N = 1\}$ and consider the naive collocation method: Find $u_\Delta \in (S_d(\Delta))^p$ and $\omega_\Delta \in \mathbb{R}^q$ satisfying

$$Au_\Delta(\tau_k) + B\omega_\Delta(\tau_k) = f(\tau_k) , \quad k = 1,\dots,N,$$
$$\int_0^1 u_\Delta dt = \beta$$

where A denotes an integro differential operator or pseudodifferential operator or, in the simplest case, a Cauchy singular integral operator

$$A_u(\tau) = a(\tau)u(\tau) + \frac{b(\tau)}{\pi i}\int_0^1 \frac{u(t)d\zeta}{\zeta - e^{2\pi i\tau}} + \int_0^1 L(\tau,t)u(t)dt ,$$

$$\zeta = e^{2\pi it} \quad \text{and} \quad \tau_k = \begin{cases} t_k & \text{for d odd ,} \\ t_k + \frac{1}{2}(t_{k+1} - t_k) & \text{for d even .} \end{cases}$$

In the Sobolev spaces $H^\sigma(\Gamma)$ we obtain optimal order asymptotic error estimates

$$|\omega - \omega_\Delta| + \|u - u_\Delta\|_\sigma \leq c\, h^{\tau-\sigma} \|u\|_\tau$$

with $0 \leq \sigma \leq \frac{d+1}{2} \leq \tau \leq d+1$ for d odd and arbitrary families of meshes and with $0 \leq \sigma \leq \tau \leq d+1$, $\frac{1}{2} < \tau$, $\sigma < d + \frac{1}{2}$ for both cases of d and uniformly grades meshes Δ, if (and only if) A is strongly elliptic, i.e. in the above case

$$\det(a(\tau) + \eta b(\tau)) \neq 0 \quad \text{for all } \eta \in [-1,1] \text{ and all } \tau .$$

The results presented arose from joint work with D.N. Arnold (Math. Comp. 41 (1983) 349-381) and J. Saranen (Math. Comp., to appear).

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