MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Potentialtheorie

22.7. bis 28.7.1984

Die Tagung fand unter der Leitung von Prof. Dr. H. Bauer (Erlangen) und Prof. Dr. W. Hansen (Bielefeld) statt. Es war nach langer Zeit die zweite Tagung über Potentialtheorie in Oberwolfach; die erste fand 1974 statt. Die Teilnehmer waren einhellig der Meinung, daß die Tagung in jeder Hinsicht "harmonisch" verlaufen sei. Ihr Ziel, einen Überblick über neue Entwicklungen und Ergebnisse in der Potentialtheorie zu geben, wurde in vollem Umfang erreicht.

In den Vorträgen zeigte sich die Vielfalt der aktuellen Forschung auf diesem Gebiet. Die behandelten Themen reichten von den Grundlagen in der axiomatischen Potentialtheorie über probabilistische Potentialtheorie bis hin zu Anwendungen auf die Theorie der partiellen Differentialgleichungen. Wieder einmal wurde das fruchtbare Zusammenwirken von analytischen und wahrscheinlichkeitstheoretischen Methoden offenbar.

Die Vorträge lassen sich inhaltlich in zwei Gruppen unterteilen. Die erste Gruppe befaßte sich hauptsächlich mit Themen aus folgenden Bereichen:

(i) Markoffsche Prozesse (Konstruktion, Dualität)
(ii) "Feine" Potentialtheorie
(iii) Randverhalten
(iv) Martin-Rand
(v) Kerne, Resolventen

Bei der zweiten Gruppe stand die Anwendung auf die Theorie der partiellen Differentialgleichungen und die mathematische Physik im Mittelpunkt. Neben dem schon "klassischen" Anwendungsbereich "lineare partielle Differentialgleichungen der Ordnung 2" wurde über neue Entwicklungen zur Anwendung der Potentialtheorie auf folgenden Gebieten vorgetragen:

(i) Lineare partielle Differentialgleichungen höherer Ordnung

(ii) Nicht-lineare partielle Differentialgleichungen

(iii) Schrödinger Gleichung

(iv) Euklidsche Quantenfeldtheorie, statistische Mechanik.

Zudem gab es außerhalb der Vorträge wertvolle Diskussionen und Gespräche über spezielle Probleme, Querverbindungen zu anderen mathematischen Gebieten und Weiterentwicklungen der Potentialtheorie.

Es ist der übereinstimmende Wunsch aller Teilnehmer, nicht wieder 10 Jahre bis zur nächsten Tagung über Potentialtheorie in Oberwolfach vergehen zu lassen.

Vortragsauszüge

V. ANANDAM:

Bisubharmonic functions in $\mathbb{R}^n$

Let Ω be an open set in $\mathbb{R}^3$. A locally summable function $\omega \in L^1_{loc}(\Omega)$ is biharmonic (resp. bisubharmonic) if $\Delta^2\omega = o$ (resp. $\Delta^2\omega \geq o$).

Proposition: For $\omega \in L^1_{loc}(\Omega)$, the following assertions are equivalent:

1) ω is biharmonic.
2) $\omega = \omega_o$ a.e. where ω_o is a biharmonic function in the classical sense.
3) There exists a harmonic function h in Ω such that $A^a(r,\omega) - \omega(a) = r^2 h(a)$ for almost every $a \in \Omega$ and for all r sufficiently small; here $A^a(r,\omega)$ is the volume-mean of $\omega(x)$ in the ball $|x - a| < r$.

Remark: We can use this mean-value relation to obtain the basic properties of a biharmonic function.

Representation of a bisubharmonic function: A bisubharmonic function ω is the difference of two subharmonic functions. Consequently for the representation of ω in a relatively compact open set, we can use the Riesz theorem; and as for the global representation of ω in $\mathbb{R}^3$, we can use the Nevanlinna theory of meromorphic functions.

N. BOBOC and G. BUCUR:

Natural localization and natural sheaf property in standard H-cones of functions

The aim of this paper is to develop a theory of localization in a standard H-cone of functions S on a set X.

If G is a fine open subset of X we denote by $S'(G)$ the set of all positive functions f on G which are finite on a fine dense subset of G and such that there exists a sequence $(s_n)_n$ in S, $s_n < \infty$ for which the sequence $(s_n - B^{X \setminus G} s_n)$ increases to f. We show that $S'(G)$ is a standard H-cone of functions on G.

If G is an open subset of X and if f is a positive lower semicontinuous function on G which is finite on a dense subset of G, then $f \in S'(G)$ iff for any $x \in G$ there exists a fundamental system $\mathcal{V}_x$ of open neighbourhoods of x such that

$$\varepsilon_x^{X \setminus D}|_G(f) \leq f(x), \quad (\forall) D \in \mathcal{V}_x, \ \bar{D} \subset G$$

where ε_x^A is the unique measure on X defined by

$$\varepsilon_x^A(s) = B^A s(x) \quad (\forall)\ s \in S.$$

Many characterisations are given for each of the properties:

- $G \to S'(G)$ is a fine sheaf,
- $G \to S'(G)$ is a natural sheaf.

It is shown, for instance, that the fine sheaf property is equivalent with axiom D and also with the union between the natural sheaf property and the axiom of nearly continuity.

A. BOUKRICHA

Biharmonic spaces

We consider a biharmonic structure on a locally compact space X with countable base in the same way as E.P. Smyrnelis but without the hypothesis of the compatibility of the pairs of biharmonic functions. We give the following characterisation: $(X,\mathcal{H})$ is a biharmonic space if and only if there exist unique harmonic spaces $(X,\mathcal{H}_1)$ and $(X,\mathcal{H}_2)$ having a common base $\mathcal{U}$ of regular open sets, and a unique positive section of continuous and real potentials represented by a family $(p_U)_{U\in\mathcal{U}}$ of potentials on U such that for all $U \in \mathcal{U}$

$$H(U) := \{(h_1,h_2) \in C(U) \times C(U) \mid \text{for all } V \in \mathcal{U} \text{ with } \bar{V} \subset U \text{ we have } h_1 = H_V^1 h_1 + K_V h_1 \text{ and } h_2 = H_V^2 h_2\},$$

H_V^1 and H_V^2 are the harmonic measures corresponding to V in $(X,\mathcal{H}_1)$ and $(X,\mathcal{H}_2)$ respectively and K_V is the potential kernel associated to p_V.

This characterisation of biharmonic spaces simplifies many proofs of results of E.P. Smyrnelis and shows an intimate connection between biharmonic and harmonic structures.

K.L. CHUNG

Dirichlet and Neumann problems for Schrödinger's equation

For bounded domain D in R^d, and $q \in K_d$ (Kato class), the Gauge for (D,q) is defined to be

$$u(x) = E^x\{e^{\int_0^{\tau_D} q(X_s)ds}\},$$ where $\{X_s, s \geq o\}$ is the Brownian motion and τ_D the first exit time from D. If $u \not\equiv \infty$ in D, then u is bounded in $\bar{D}$ (Chung-Rao-Zhao). In this case the unique solution of $(\frac{\Delta}{2} + q)\varphi = 0$ in D, $\varphi = f$ (continuous on ∂D) is given by

$$\varphi(x) = E^x\{e^{\int_0^{\tau_D} q(X_s)ds} f(X_{\tau_D})\}.$$

Several equivalent conditions for $u \not\equiv \infty$ are given. There is a similar theorem for the Neumann problem. (thesis of P. Hsu).

I. CUCULESCU

Some constructions of Markov processes

The simplest exposition is: consider two transition semigroups (Q_t^1), (Q_t^2), the first having as state space $E^1 \oplus \Delta^1$, Δ^1 being a "cemetery" set, the second having E^2 as state space. Consider, on a probability space, a (Q_t^1)-process (x_t^1) (with right continuous sample paths until the first visit ζ^1 in Δ^1) and a (Q_t^2)-process (x_t^2), constituting altogether a Markov family, such that the conditional distribution of x_o^2 with respect to x_∞^1, on $(\zeta^1 < \infty)$, is a given one T^{12}, and construct $x_t^{12} = x_t^1$ for $t < \zeta^1$, $x_t^{12} = x_{t-\zeta^1}^2$ for $t \geq \zeta^1$; it is a (Q_t^{12})-process for a well determined (Q_t^{12}) on $E^1 \oplus E^2$. Then the case of a sequence of semigroups (Q_t^n) and transition probabilities $T^{n,n+1}$ is considered, and a cemetery set equal to the product of their cemetery sets is organized for the resulting semigroup. If all (Q_t^n) are the same, as well as all $T^{n,n+1}$, we can "group" the states in the resulting semigroup and obtain an analogue of the "minimal transition semigroups". Also the case where the second process has time set $(0,\infty)$, T^{12} is a mapping from Δ^1 to the set of entrance laws for (Q_t^2) etc., is considered, a case in which the hypothesis "the conditional distribution of ζ^1 with respect to x_o^1, x_∞^1 charges no singleton $\{t\}$" must be assumed.
As applications: recollement, processes with denumerable state space in which there is no totally ordered descending sequence of jumps "longer" than a sequence, Ito's construction (in Berkeley 1972) of a process having an instantaneous state and given excursions.

C. DELLACHERIE

Three applications of the analytic set theory to potential theory

1. Let V be a bounded (positive) measurable kernel from a Suslin measurable space $(E,\mathcal{E})$ into another $(F,\mathcal{F})$. Then V is basic (i.e. there exists a probability λ on $(F,\mathcal{F})$ such that $\varepsilon_x V$ is absolutely continuous w.r.t. λ for every $x \in E$) iff for every universally measurable positive function f on F, the function Vf is measurable on E.

2. Let E be a metrizable compact space and $E^{\#}$ the space of subprobability measures on E endowed with the vague topology. Let H be a Borel or, more

generally an analytic part of $E \times E^{\#}$ and set, for every positive Borel function f on E

$$Nf^x = \sup_{\mu \in H(x)} \mu(f)$$

where $H(x)$ is the section of H at x (if $H(x) = \emptyset$, set $Nf^x = 0$). For $\mu, \lambda \in E^{\#}$ define

$$\mu \leq \lambda N \text{ iff } \forall f \in \mathcal{B}^+ \quad \mu(f) \leq \lambda(Nf)$$

where $\mathcal{B}^+$ is the set of Borel positive functions. Since Nf is universally measurable (actually, it is an "analytic" function), it makes sense. Now we have the following extension of a well known Strassen's theorem: we have $\mu \leq \lambda N$ iff there exists a sequence (P_n) of Borel kernels s.t. $P_n \leq N$ for every n (i.e. $\varepsilon_x P_n \leq \varepsilon_x N \quad \forall x \in E$) and $\mu = \lim_n \lambda P_n$ where the limit is in norm.

3. Let $E, E^{\#}$ be as before and H a Borel subset of $E \times E^{\#}$ with countable section $H(x)$ (equivalent to: H is a countable union of graphs of Borel kernels). Define N as before and say that $f \in \mathcal{B}^+$ is a pure excessive function w.r.t. N if $Nf \leq f$ and for any $g \in \mathcal{B}^+$ ($g \leq f$ and $Ng = g \Rightarrow g = 0$). Define the potential operator G_N associated to N by: $G_N f$, for $f \in \mathcal{B}^+$, is the least solution of the Poisson equation $u = f + Nu$. Then we have: an excessive (finite) function u is pure iff

(i) For every kernel P with graph included in H, u is a pure excessive function w.r.t. P (or u is a potential w.r.t. P)

(ii) The set $\{u > o\}$ is proper for G_N, i.e. there exists a Borel function φ strictly positive on $\{u > o\}$ s.t. $G_N \varphi$ is finite everywhere.

K. DOPPEL

(Totally) partially harmonic functions

Contrary to the usual theory of polyharmonic functions we consider (totally) partially harmonic functions in $\mathbb{R}^n$, i.e. if $\mathbb{R}^n = \mathbb{R}^{n_1} \times \ldots \times \mathbb{R}^{n_k}$, $k > 1$ we consider Dirichlet-problems of the type

$$L(D)u := \frac{1}{2^k} \Delta_1 \ldots \Delta_k u = f \quad \text{in} \quad G$$

$$(*) \qquad D^\alpha u|_{\partial G} = g_\alpha \qquad |\alpha| \leq k-1 \quad ,$$

where the Δ_i are Laplacians in $\mathbb{R}^{n_i}$, $1 \leq i \leq k$ and the g_α are given functions "on the boundary ∂G" of the domain $G \subset \mathbb{R}^n$. The differential operator $L(D)$ is not hypoelliptic. In the homogeneous case ($f \equiv 0$) a solution of (*) is harmonic in each subspace $\mathbb{R}^{n_i}$. One can show that this

Dirichlet problem has a unique (weak) solution for all $f \in L^2(G)$.
If $L^\sim$ denotes the closure of the differential operator $L(D)$ in $L^2(G)$ and G is the cartesian product of open bounded sets $G_j \subset \mathbb{R}^{n_j}$

$$D(L^\sim) \text{ is dense in } L^2(G)$$

and

there exists a positive constant c, such that the estimate

$$\|(\lambda I - L^\sim)^{-1}\| \leq \frac{c}{1+|\lambda|}$$

holds for all $\lambda \in \mathbb{C}$ with $\operatorname{Re} \lambda \leq 0$. Thus, it is possible to solve a Cauchy problem of the first order evolution equation

$$u'(t) + L^\sim u(t) = f(t) \qquad (t > 0)$$

with the initial condition $u(0) = u_o$. If the operator valued function $V(.,.)$ is a fundamental solution of the Cauchy problem u is given by

$$u(t) = V(t,0)u_o + \int_o^t V(t,s)f(s)ds \qquad (t \geq o).$$

R.K. GETOOR

Weak duality and potential theory

Weak duality is a setting in which a good potential theory may be developed based on a pair of processes X and $\hat{X}$. One constructs a σ-finite measure on paths Z_t defined on a random time interval $]\alpha,\beta[$. In one direction X looks like X and in the other like $\hat{X}$. For example, if $\tau_K = \inf\{t : Z_t \in K\}$, $\lambda_K = \sup\{t : Z_t \in K\}$, where K is transient, then $P[\tau_K \in dt, Z_{\tau_K} \in dx] = dt\, \hat{\pi}_K(dx)$ and $P[\lambda_K \in dt, Z_{\lambda_K -} \in dx] = dt\, \pi_K(dx)$ where π_K and $\hat{\pi}_K$ are the capacitary and co-capacitary measures of K. In particular, $P[\tau_K \in dt] = C(K)dt = P[\lambda_K \in dt]$ where $C(K)$ is the capacity of K.

H. HUEBER

About the potential theory of the Laplace-Kohn operator

I consider the Laplace-Kohn operator Δ_K on $\mathbb{R}^3 = \{(x,y,t) | x,y,t \in \mathbb{R}\}$ which is given by $\Delta_K = X^2 + Y^2$, where

$$X = \frac{\partial}{\partial x} + 2y\frac{\partial}{\partial t} \quad , \quad Y = \frac{\partial}{\partial y} - 2x\frac{\partial}{\partial t} \; .$$

This operator is invariant under the group structure on $\mathbb{R}^3$ which is given by $(x,y,t)(x',y',t') = (x+x',y+y',t+t'+2(x'y-xy'))$. $\mathbb{R}^3$ with this group structure is the first Heisenberg group.
The harmonic space associated to Δ_K is a Brelot space. There are many analogies to classical potential theory, and I give a short introduction into results of Koranyi/Vagi, Folland, Gaveau, Greiner and Jerison. My own contributions to this subject are mainly the following:
Similar to the classical situation the potential theory of Δ_K on $H_+ = \{(x,y,t)|t>o\}$ is equivalent to the potential theory on a certain "ball"- the Koranyi ball. The Poisson space of this ball is homeomorphic to its topological boundary.

T. IKEGAMI

A Martin type compactification of harmonic spaces and its applications

We consider a compactification of a harmonic space on which the notion of minimal thinness is defined and is connected closely with the topology. For a harmonic morphism of harmonic spaces we construct a fine cluster set at a minimal boundary point. Then we develop theorems analogous to those of Riesz-Frostman-Nevanlinna and Fatou-Plessner which are obtained for analytic mappings of Riemann surfaces.

M. ITO

The convolution kernels of logarithmic type and the closure of Hunt convolution kernels

Let X be a locally compact, σ-compact abelian group. We define convolution kernels of logarithmic type on X in connection with semi-transient convolution semi-groups. Let N be a convolution kernel of logarithmic type and let $(\alpha_t)_{t\geq o}$ be its convolution semi-group. If N is not a Hunt convolution kernel, then the resolvent defined by $(\alpha_t)_{t\geq o}$ is not singular with respect to the Haar measure. Hence we can determine X on which convolution kernels of logarithmic type with the recurrent convolution semi-groups exist. By discussing some potential-theoretic properties of convolution kernels of logarithmic type, we consider the DENY PROBLEM concerning Hunt convolution kernels.

Let H(X) be the totality of Hunt convolution kernels and let D(X) be the totality of convolution kernels satisfying the domination principle. We shall show that in general, the equality $D(X) = \overline{H(X)}$ does not hold, where $\overline{H(X)}$ denotes the weak* closure of H(X).

K. JANSSEN

Resolvents on topological sums

Let X be a right process on a state space $(E,\mathcal{E})$. Let $E = \bigcup_{i\in I} E_i$ be a disjoint union of finely open nearly optional sets, where $(I,\mathcal{I})$ is a U-space such that the canonical projection $\Pi : E \to I$ is measurable. Then $T:=\inf\{t : \Pi\circ X_t \neq \Pi\circ X_0\}$ is a perfect, exact and terminal stopping time. If (U^α) is the resolvent of X, then the resolvent (V^α) of the process Y obtained by killing X at time T is exactly subordinated to (U^α) and $U^\alpha = V^\alpha + P_T^\alpha U^\alpha$ for the α-order hitting kernel P_T^α and each E_i is absorbing for the right process Y. Similar results can be obtained by purely analytic methods for a Ray resolvent (U^α) on a compact space E which is the direct topological sum of compact spaces. Using results on perturbations of Ray resolvents by BEN SAAD, one can prove that U is a weak coupling of Ray resolvents on the components. Details can be found in a joint paper with BEN SAAD.

J. KRAL

Removable singularities of solutions of the heat equation

Some extension results concerning solutions of the heat equation (=temperatures) are presented; related conjectures concerning subtemperatures are described.

I. LAINE

A quasi-linear potential theory

This talk presents a proposal for a non-linear axiomatic potential theory being not very far away from the usual linear theory. The background of this proposal comes from some phenomena in the theory of qc-mappings. These phenomena having been investigated by Martio, Granlund and Lindqvist remind us on some familiar properties in the usual linear theory. Our proposal for this axiomatics is in the spirit of Constantinescu and Cornea consisting of the usual axioms of resolutivity and convergence plus modified axioms for positivity and completeness. Finally, the axiom of quasilinearity tells us how close we are to the usual linear theory.

G. LEHA

Reversible measures for diffusion processes in Hilbert spaces.

One of the important problems in the study of evolution processes is to describe the equilibrium states.
We deal with a real separable Hilbert space and diffusion processes in this space arising as solutions to stochastic differential equations. Here equilibrium behaviour is given by means of probability measures on $\mathbb{H}$, which are invariant or reversible (i.e. symmetric with respect to time reversal) for the corresponding semigroups.
We characterize reversibility by the so called (and in finite dimensions well known) "principle of detailed balance" for a suitable differential operator L. Furthermore we give sufficient conditions on the (generally unbounded and non-linear) drift part of the stochastic differential equation [the diffusions part is identity] in order to prove existence of reversible measures. The theory covers example of "continuous spin models" of statistical mechanics which occur in the "lattice approximation" to Euclidean quantum field theory.

H. LEUTWILER

On a distance invariant under Möbius transformations on $\mathbb{R}^n$

Let (X,H) be a Brelot space. We study the metric

$$\rho(x,y) = \log \sup \left\{\frac{h(x)}{h(y)} : h \in H^+(X)\right\} - \log \inf \left\{\frac{h(x)}{h(y)} : h \in H^+(X)\right\},$$

assuming hereby that the set $H^+(X)$ of positive harmonic functions separates the points of X. The metric ρ has the following properties:
(1) It is complete, (2) on the unit ball B_n of $\mathbb{R}^n$ $(n\geq 2)$ it agrees (up to a constant factor) with the Poincaré distance, and (3) it is invariant under Möbius transformations of $\mathbb{R}^n$. In case (X,g) is a Riemannian manifold and $H(X)$ denotes the solutions of the Laplace-Beltrami operator, we also study the corresponding differential metric.

P. A. LOEB

A measure-theoretic boundary limit theorem

Our main result is a purely measure-theoretic limit theorem for which the proof is quite simple. From this result, the fine limit theorem of Fatou-Naïm-Doob and its extensions to general potential theories follow immediately by using the order isomorphism between finite measures and positive harmonic functions and also employing some simple facts about reduced functions.

J. LUKEŠ

The fine Dirichlet problem

The existence and the uniqueness of the solution of the Dirichlet problem in the fine potential theory without axiom D is investigated. Special attention is payed to Wiener's type solution and to the quasi-solution using fine topology methods.

T. LYONS

An example of instability for the potential theoretic structure of quasi-isometric Riemannian manifolds and reversible Markov chains

Two metrics $g, \hat{g}$ on a manifold M are quasi-isometrically equivalent if $g_x(u,u)/\tilde{g}_x(u,u) \in [\frac{1}{c}, c] \subset \mathbb{R}_+$ for every $u \in TM_x$ and $x \in M$.
A countable set X and a symmetric function $\underline{a} : X \times X \to \mathbb{R}_+$ determine a reversible Markov chain on X providing $\pi_x = \sum_{y \in X} a_{xy}$ is strictly positive and finite for each x by putting $p_{xy} = a_{xy}/\pi_x$ and using p as a transition kernel. Two such Markov chains on X are quasi equivalent if $a_{xy}/b_{xy} \in [\frac{1}{c}, c] \subset \mathbb{R}_+$ for all $x, y \in X$.
We consider the following question: If (M,g) admits no nonconstant bounded harmonic functions then can $(M,\tilde{g})$; and also the analogous question for Markov chains. We give the following negative answer:
There is a Riemann surface M and a second one $\tilde{M}$ quasi-conformally (or isometrically) equivalent to the first such that M has trivial Martin boundary and $\tilde{M}$ admits a two point boundary - and each extremal positive harmonic function is bounded. Analysis of a rather complicated counter example to the analogous Markov chain problem plays a vital role.

F.Y. MAEDA

Boundary value problems with respect to a non-linearly perturbed structure of a harmonic space

Let (X,H) be a connected self-adjoint P-harmonic space with $1 \in H(X)$, G be a symmetric Green function on X, σ be the canonical measure representation associated to G, $D[f,g]$ be the mutual Dirichlet integral of f and g defined in terms of σ and $D[f] = D[f,f]$. Let X^* be a resolutive compactification of X and ω be the harmonic measure on $\Gamma = X^* \setminus X$. Let $\Phi_{BD} = \{\phi \in L^\infty(\omega) | D[H_\phi] < \infty\}$ and let Ψ be a subspace of Φ_{BD} closed w.r.t. the BD-topology and max-min operations. Let M_F be the space of finite Radon measures μ on X such that $G|\mu|$ is bounded continuous and let $R_F = \{H_\phi + G\mu | \phi \in \Phi_{BD}, \mu \in M_F\}$. Given (non-linear) mappings $F : R_F \to M_F$ and $\beta : \Phi_{BD} \to L^1(\omega)$ which satisfy (local) Lipschitz conditions, and given $\phi_o \in \Phi_{BD}$, we discuss the following boundary value problem: Find $u = H_\phi + G\mu \in R_F$ satisfying

(1) $\sigma(u) + F(u) = 0$ on X ;

(2) $\phi - \phi_o \in \Psi$;

(3) $D[u,H_\psi] - \int_X H_\psi d\sigma(u) + \int_\Gamma \psi\beta(\phi)d\omega = 0$ for alle $\psi \in \Psi$.

B. MAIR

Fine and Parabolic Limits

In this talk we consider the parabolic differential operator L, given by

$$Lu = \sum_{i,j=1}^{n} \frac{\partial}{\partial x_j}\left(a_{ij}(x,t)\frac{\partial u}{\partial x_i} + a_j(x,t)u\right) + \sum_{j=1}^{n} b_j(x,t)\frac{\partial u}{\partial x_j} + c(x,t)u - \frac{\partial u}{\partial t} ,$$

whose coefficients are measurable and satisfy certain weak conditions on $X = \mathbb{R}^n \times (0,T)$. These conditions include (as a special case) the classical case of uniformly parabolic L with bounded, Hölder-continuous coefficients.

It is shown that the weak solutions of $Lu=0$ form a P-harmonic space on X and the notion of semi-thinness at points of $B = \mathbb{R}^n \times \{0\}$ is introduced.

The relationships between fine, semi-fine and parabolic convergence at points of B are examined and the fine limit theorem is used to deduce the Fatou theorem that every positive weak solution of $Lu=0$ has finite parabolic limits Lebesgue almost-everywhere on B.

Furthermore, it is shown that if u is a weak solution defined only by a union of parabolic regions and u is either upper or lower bounded on each parabolic region, then u has finite parabolic limits almost everywhere on the set of vertices of the parabolic regions.

J. MALÝ

Finely hyperharmonic functions

The following theorem is presented: Let $f \geq 0$ be a finely l.s.c. function on a finely open subset U of a standard balayage space X. Assume f is quasi-l.s.c. in the natural topology on U. If for every open set $G \subset X$ and $x \in G$ there is a finely open set V such that $x \in V \subset \bar{V} \subset G \cap U$ and

$$\int_U^* f \, d\, \varepsilon_x^{CV} \leq f(x) \quad ,$$

then the same inequality holds for every finely open set V with $x \in V \subset \bar{V} \subset U$. This result clarifies some relations between various definitions of finely hyperharmonic functions due to B. Fuglede (1972), J. Malý and J. Lukeš (1982), and N. Boboc, Gh. Bucur and A. Cornea (1981). Both the natural topology and the quasi-notions are with respect to the localized cone.

G. MOKOBODZKI

Opérateurs de subordination des familles résolvantes

On étudie des opérateurs définis sur les cônes de fonctions excessives et qui, à la manière des opérateurs de réduction sur les ensembles, permettent de passer d'une famille résolvante à une famille résolvante subordonnée.
Ces opérateurs sont caractérisés par des propriétés de troncature et de recollement. On montre qu'ils transforment des familles résolvantes équivalentes en famille résolvantes équivalentes, l'équivalence consistant à engendrer le même cône de fonctions excessives.

I. NETUKA

1. Ninomiya operators for the generalized Dirichlet problem
2. Fine strict maxima (joint work with J. Kràl)

1. Let U be a relatively compact open subset of a harmonic space and A an

operator sending continuous functions on ∂U into real functions on U. Such an operator A is said to be a Ninomiya operator, if A is linear and positive, $A(p_{|\partial U}) = p_{|U}$ for every continuous potential harmonic on U, and if there is a strict continuous potential q such that $A(q_{|\partial U})$ is subharmonic. A necessary and sufficient condition for uniqueness of a Ninomiya operator is established. The relation to the Keldyš type theorem is also discussed.

2. Main result: If $f : \mathbb{R}^n \to \mathbb{R}$ is a Borel function, then the set $M(f)$ of fine strict maxima is polar. (Here $M(f)$ is the set of all x such that $\{y; f(y) \geq f(x)\}$ is thin at x.) The case of arbitrary functions is also investigated. Various results from real analysis concerning the size of the set of suitably defined strict maxima are mentioned.

M. RAO

Report on the thesis of Ming Liao done at Stanford under Prof. K.L. Chung

K.L. Chung introduced an important condition on the potential density guaranteeing the validity of some well-known results of classical potential theory. Ming Liao in his dissertation weakens the conditions of Chung. At the same time he proves that the process admits a strong Markov dual albeit some branch points exist.

G. RITTER

One-sided growth conditions for the coefficients of stochastic differential equations

We consider the stochastic differential equation

$$(*)\ d\xi_t = A(\xi_t)\, d\beta_t + b(\xi_t)\, dt$$

in a separable Hilbert space $\mathbb{H}$. Here $(\beta_t)_{t\geq o}$ is Brownian motion with covariance Q (positive, nuclear operator on $\mathbb{H}$) and mean 0, A is a "locally" Lipschitz function $\mathbb{H} \to L(\mathbb{H}, \mathbb{H})$ (bounded linear operators on $\mathbb{H}$), and b is "locally" Lipschitz $\mathbb{H} \to \mathbb{H}$.

Theorem. a) If (i) $\|A(y)\| = O(\|y\|)$ and

(ii) $(y, b(y)) \leq K(1+\|y\|^2)$

then for each $x \in \mathbb{H}$ there exists exactly one solution $(\xi_{x,t})$ to $(*)$ with infinite lifetime.

b) $P_t f(x) := E[f \circ \xi_{x,t}]$ is a semigroup whose generator is an extension of the differential operator

$$Lf(x) = \mathrm{tr}\, f''(x)\, A(x)\, Q\, A^*(x) + f'(x)\, b(x)$$

(f sufficiently regular).

Under the stronger conditions

(iii) $||A(y)|| \leq K$

(iv) $|(y,b(y))| \leq K(1+||y||^2)$

$(P_t)_{t\geq 0}$ induces a Feller semigroup, i.e. a strongly continuous semigroup on the functions that are small outside bounded sets and uniformly continuous on bounded sets.

If $A(y)$ = identity the case of discontinuous drift terms b as arising in Euclidean quantum field theory was also considered.

M. RÖCKNER

A Dirichlet problem for distributions and application to the prediction problem for Gaussian generalized random fields

We explicitly construct consistent conditional distributions for a large class of Gaussian measures defined on the space of (tempered) distributions on a domain D in $\mathbb{R}^d$. The conditional distributions are with respect to an (uncountable) family of σ-fields associated with the complements of the (relatively compact) open subsets of D. The construction involves solving a Dirichlet problem whose "boundary data" is given by a distribution. Furthermore, the associated set of Gibbs states is studied. We characterize the extreme Gibbs states, prove that they have the global Markov property and, using the Dirichlet solution for distributions, we can represent any Gibbs state in terms of extreme Gibbs states.

U. SCHIRMEIER

Measure representations in harmonic spaces

In classical potential theory the sheaf of harmonic functions is defined by the Laplacian operator Δ:

$$h \text{ harmonic} \Leftrightarrow \Delta h = 0.$$

The abstract theory of harmonic spaces starts with a sheaf of functions without the intervention of a defining operator. As a substitute F.-Y. Maeda introduced the notion of a measure representation and developed his theory of Dirichlet integrals for those harmonic spaces admitting a measure representation σ. By definition, σ is a homomorphism of the sheaf $\mathcal{R}$ of local differences of continuous superharmonic functions into the sheaf $\mathcal{M}$ of signed Radon measures such that

$$f \text{ superharmonic} \Leftrightarrow \sigma(f) \geq 0$$

In this lecture the following result is presented:

Every harmonic space with a countable base admits a measure representation satisfying additional continuity properties.

B.W. SCHULZE

The asymptotics of solutions of elliptic boundary value problems in non-smooth domains or for conditions with jumps

The solutions of boundary value problems in domains with wedges or of mixed boundary value problems admit asymptotic expansions (or have a conormal singularity) of the form

$$u(y,t) \sim \sum_{j=0}^{\infty} \sum_{k=0}^{m_j(y)} \zeta_{jk}(y)\, t^{p_j(y)} (\log t)^k$$

for t near the wedge Y or the jump of the boundary conditions. Here $p_j(y) \in \mathbb{C}$, $m_j(y)$ may have branchings and jumps under varying $y \in Y$. A part of the solvability theory is a description of classes of spaces of functions being C^∞ in the interior, where the conormal singularity is reflected in an adequate way. In this context analytic functionals in the complex plane play a role and some balayage with respect to a fundamental solution of the Cauchy-Riemann operator.

M. SIEVEKING

Comparison of Green functions for parabolic operators

Let G_Ω^E be the Greens function of $\frac{\partial}{\partial t} - E$ on a domain $\Omega \subset \mathbb{R} \times \mathbb{R}^n$ where E is some linear second order elliptic partial differential operator. Inequalitys of the type

$$\frac{1}{c}\, G_\Omega^{\frac{1}{\gamma}\Delta} \leq G_\Omega^E \leq c\, G_\Omega^{\gamma\Delta}$$

were established in 1968 by Aronson for a large class of operators E (in divergence form) and bands $\Omega =]0,T[\times \mathbb{R}^n$ (c,γ constants $> o$). Recently they have been shown to hold for $n = 1$ and Ω's which are unions of rect-angles $]T,S[\times]a,b[$. They imply similar inequalities for the Poisson kernels resp. Naïm-kernels and hence for the Dirichlet resp. Neumann problems.

J. VESELÝ

Keldyš type operators and continuability of the Riesz α-harmonic functions

The Riesz α-harmonic functions represent the third "Standard-Beispiel" of non-local axiomatic potential theories. Continuability properties for α-harmonic functions are studied and Keldyš type theorems for α-Dirichlet problem are investigated. As a by-product the simpliciality of a cone of α-harmonic functions is obtained.

G. WILDENHAIN

Potentialtheoretische Methoden bei der Approximation von Lösungen elliptischer Gleichungen höherer Ordnung

Es sei L ein linearer, eigentlich elliptischer Differentialoperator im R^n mit reellen, beliebig oft differenzierbaren Koeffizienten. Der adjungierte Operator $\overset{*}{L}$ besitze die eindeutige Fortsetzungseigenschaft. $\Omega \subset R^n$ sei ein beschränktes Gebiet mit dem glatten Rand $\partial\Omega$ und auf $\partial\Omega$ sei ein System $B_1,\dots,B_m$ von Randoperatoren der Ordnung $\leq 2m-1$ mit glatten Koeffizienten gegeben, welches normal ist und den Operator L auf $\partial\Omega$ überdeckt. $\Gamma \subset \Omega$ sei eine glatte, k-dimensionale ($1 \leq k \leq n-1$) Fläche, die Ω nicht zerlegt, $V \subset \partial\Omega$ sei eine vorgegebene offene Teilmenge des Randes. Wir betrachten

$$L_V(\Omega) = \{u \in C^\infty(\bar{\Omega}) : Lu = 0 \text{ in } \Omega,\ B_j u|_{\partial\Omega\setminus V} = 0\ (j=1,\dots,m)\}$$

sowie $L_V(\Gamma) = L_V(\Omega)|_\Gamma$. Man kann dann zeigen, daß $L_V(\Gamma)$ in den Sobolev-Slobodeckij-Räumen $W_p^s(\Gamma)$ ($1 < p < \infty, s \geq o$ reell) dicht liegt. Ist Γ eine hinreichend reguläre, geschlossene Fläche, die innerhalb Ω ein Teilgebiet Ω_i begrenzt, für welches das Dirichlet-Problem der homogenen Gleichung $Lu = 0$ eindeutig lösbar ist, so läßt sich ferner die Dichtheit von $L_V(\Gamma)$ im Raum $W^{m-1}(\Gamma)$ der Whitney-Taylorfelder der Ordnung $m-1$ zeigen, falls man noch zusätzlich die Existenz einer globalen Fundamentallösung voraussetzt.

R. WITTMANN

The Martin boundary of subdomains (of a harmonic space) satisfying corkscrew conditions

Let $(X,\mathcal{H})$ be a Brelot harmonic space endowed with a metric d such that $z,y \in B(x,r) \subset B(x,2r)$, $h \in \mathcal{H}_+(B(x,2r))$ implies $h(y) \leq C\, h(z)$, where $C > 0$ is a constant independent of x,r,h. On such a harmonic space we consider NTA (= non-tangential accessible) domains which were introduced into classical potential theory by D.S. Jerison and C.E. Kenig. For these domains we can show that the Martin boundary coincides with the topological boundary. This result is a wide generalisation of a result of R. Hunt, R. Wheeden (1970). Further applications to Fatou type theorems are indicated.

Berichterstatter: M.Röckner (Bielefeld)

Tagungsteilenehmer

Dr. Victor Anandam
6 av. Omar Ibn
Khatab Appt. 8
Agdal Rabat

Maroc

Prof. Kai Lai Chung
Stanford University
Dept. of Mathematics
Stanford CA-94305

USA

Prof. Alano Ancona
Equipe d'Analyse
Université Paris VI
4, Place Jussieu
Tour 46, 4e étage

F-75230 Paris - Cedex 05

Prof. Dr. Aurel Cornea
Kath. Universität Eichstätt
Mathematik
Ostenstr. 26-28

8078 Eichstätt

Prof. Dr. Heinz Bauer
Mathematisches Institut
Universität Erlangen-Nürnberg
Bismarckstraße 1 1/2

D-8520 Erlangen
W.-Germany

Prof. Ion Cuculescu
Institute of Mathematics
Strada Academiei 14
Bucuresti 1

Romania

Prof. N. Boboc
The National Institute for
Scientific and Technical Creation
INCREST
Bdul Pacii 220

R-77538 Bucharest Romania

Prof. Dellacherie
Equipe d'Analyse
Université Paris VI
4, Place Jussieu
Tour 46, 4e étage

F-75230 Paris - Cedex 05

Prof. Abderahman Boukricha
Département de Mathématiques
Faculté des Sciences
Campus Universitaire
Belvedère - Tunis

TUNESIEN

Prof. Jacques Deny
37, Parc d'Ardenay

D-91120 Palaiseau

Prof. Marcel Brelot
13, Avenue franco-russe

F-75007 Paris

Prof. Dr. K. Doppel
Fachbereich Mathematik
Freie Universität Berlin
Arnimallee 2-6

1000 Berlin 33

Monsieur Feyel
Equipe d'Analyse
Université Paris VI
4, Place Jussieu
Tour 46, 4^e étage

F-75230 Paris - Cedex 05

Prof. Teruo Ikegami
Dept. of Mathematics
Osaka City University
Sugimoto-cho, Sumiyoshi-ku
Osaka 558

Japan

Prof. Ronald K. Getoor
Dept. of Mathematics, UCSD
La Jolla, CA 92093

USA

Prof. Masayuki Itô
Dept. of Mathematics
Nagoya University
Chikusa-ku
Nagoya 464

Japan

Prof. Dr. W. Hansen
Universität Bielefeld
Fakultät für Mathematik
Universitätsstr. 25

4800 Bielefeld 1

Prof. Dr. K. Janssen
Universität Düsseldorf
Institut für Statistik und
Dokumentation
Universitätsstr. 1

4000 Düsseldorf

Mme Rose-Marie Hervé
6, Rue du Général Bertrand

F - 75007 Paris

Prof. Josef Král
Matematicky ústav CSAV
Zitná ulice 25

CSSR-11567 Praha 1

Prof. F. Hirsch
Ecole Normale Superieure de
l'enseignement technique
61, Avenue due Président Wilson

F-94230 - Cachan

Dipl math. P. Kröger
Mathematisches Institut
Universität Erlangen-Nürnberg
Bismarckstr. 1 1/2

D-8520 Erlangen

Dr. Hermann Hueber
Fakultät für Mathematik
Universität Bielefeld
Universitätsstr. 1

4800 Bielefeld 1

Prof. Ü. Kuran
Dept. of Pure Mathematics
University of Liverpool
P.O.Box 147
Liverpool L69 3BX

GREAT BRITAIN

Prof. Ilpo Laine
University of Joensuu
Dept. of Mathematics
Box 111

SF-80101 Joensuu 10
Finnland

Prof. Linda Lumer-Naim
3, Rue des Mélèzes

1050 Bruxelles

Dr. G. Leha
Mathem. Inst. der Universität
Erlangen-Nürnberg
Bismarckstr. 1 1/2

8520 Erlangen

Prof. Terry J. Lyons
Mathematical Institute
Jesus College
Oxford OX1 3DW

GREAT BRITAIN

Prof. Dr. H. Leutwiler
Mathem. Inst. der Universität
Erlangen-Nürnber
Bismarckstr. 1 1/2

8520 Erlangen

Prof. Fumi-Yuki Maeda
Dept. of Mathematics
Faculty of Science
Hiroshima University
Higashi-senda-machi 1-1-89
Hiroshima 730
JAPAN

Prof. Dr. P.A. Loeb
Department of Mathematics
University of Illinois at
Urbana Champaign
Urbana, IL 61801

USA

Dr. Bernard Mair
Dept. of Mathematics
University of the West Inidies
Mora, Kinston 7

Jamaica

Dr. Jaroslav Lukeš
Matematicko-fyzikálni fakulta
univerzity karlovy
katedra matematické analýzy
Sokolovská 83

CSSR-186 00 Praha 8

Dr. Jan Malý
Matematicko-fyzikálni fakulta
univerzity karlovy
katedra matematické analýzy
Sokolovská 83

CSSR-186 00 Praha 8

Prof. Dr. Gunter Lumer
3, Rue des Mélèzes

1050 Bruxelles

Prof. G. Mokobodzki
Equipe d'Analyse
Université Paris VI
4, Place Jussieu
Tour 46, 4e étage

F-75230 Paris - Cedex 05

Dr. Ivan Netuka
Matematicko-fyzikálni fakulta
univerzity karlovy
katedra matematické analýzy
Sokolovská 83

CSSR-186 00 Praha 8

Monsieur D. Pinchon
Laboratoire de Probabilité
Université P. et M. Curie
4, Place Jussieu

F-75230 Paris - Cedex 05

Prof. Dr. Murali Rao
Matematisk Institut
Aarhus Universitet
Ny Munkegade
8000 Aarhus

Dänemark

Prof. Dr. Gunter Ritter
Fachbereich Informatik
Universität Passau
Graf Salm Straße 7

8390 Passau

Dr. Michael Röckner
Universität Bielefeld
Fakultät für Mathematik
Universitätsstr. 25

4800 Bielefeld 1

Dr. Ursula Schirmeier
Kath. Universität Eichstätt
Mathematik
Ostenstr. 26 - 28

8078 Eichstätt

Prof. Dr. B.W. Schulze
Akademie der Wissenschaften der DDR
Zentralinstitut für Mathematik
und Mechanik
DDR-108 Berlin
Mohrenstraße 39

Prof. Dr. M. Sieveking
Universität Frankfurt
Angewandte Mathematik
Robert-Mayer-Str. 10

6000 Frankfurt 1

Frau
Dr. J. Steffens
Institut für Statistik und
Dokumentation
Universitätsstraße 1

4000 Düsseldorf

Dr. Jiri Veselý
Matematicko-fyzikálni fakulta
univerzity karlovy
katedra matematické analýzy
Sokolovský 83

CSSR-186 00 Praha 8

Prof. Dr. G. Wildenhain
Wilhelm-Pieck-Universität
Sektion Mathematik
Universitätsplatz 1

DDR-25 Rostock

Dr. Rainer Wittmann
Kath. Universität Eichstätt
Math.-Geographische Fakultät
Ostenstr. 26 - 28

8078 Eichstätt