MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 37/1984

Algebraic K-Theory of Spaces and Pseudoisotopy Theory

19. 8. bis 25. 8. 1984

Die Tagung fand unter der Leitung von Herrn Waldhausen (Bielefeld) und Herrn Burghelea (Columbus) statt.

Es handelte sich um eine Spezialtagung, die sich mit der algebraischen K-Theorie topologischer Räume und deren Anwendungen beschäftigte.

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Vortragsauszüge

Amir H. ASSADI: Transfer in Whitehead Theory and G-actions

Let $\tilde{X} \to X$ be a G-covering, where G is finite and $|G| = q$, X is finitely dominated and $\tilde{X}$ is homotopy equivalent to a finite complex Y (fixed). Then the obstructions to choosing a finite complex X' homotopy equivalent to X such that $\tilde{X}'$ is homotopy equivalent to Y via a π-simple homotopy equivalence lie in an abelian group $Wh_1^T(\pi_1 Y \to \pi_1 X)$. In general, for an extension $1 \to \pi \to \Gamma \to G \to 1$, $|G| = q < \infty$, one has a long exact sequence $Wh_1(\Gamma) \xrightarrow{Tr} Wh_1(\pi) \xrightarrow{\beta} Wh_1^T(\pi \to \Gamma) \xrightarrow{\alpha} Wh_0(\Gamma) \longrightarrow Wh_0(\pi)$ and a commutative diagram, when $\Gamma = \pi \times G$

$$\begin{array}{ccc} Wh_1(\pi) & \xrightarrow{\beta} & Wh_1^T(\pi \to \Gamma) \\ & \searrow \quad \nearrow_{\gamma} & \\ & Wh_1(\pi;\mathbb{Z}/_q) & \end{array} .$$

This exact sequence can be shown to be the lower portion of the exact homotopy sequence associated to the fibration obtained by delooping a geometric transfer between $\mathbf{Wh}(B\Gamma) \xrightarrow{\tau} \mathbf{Wh}(B\pi)$, where $\mathbf{Wh} = \Omega^{-1}Wh$, Wh = Hatcher's Whitehead theory and $\tau = \Omega^{-1}T$, where T is the transfer constructed using Burghelea-Lashof type arguments on their geometric transfer between concordance spaces. There are geometric applications for Wh_1^T to transformation groups, which show that Wh_1^T is the analogue of the $\tilde{K}_0$-functor in the case of G-actions on non-simply-connected spaces.

Marcel BÖKSTEDT: K-theory and stable K-theory

Using étale homotopy theory one constructs a commutative diagram of 2-complete spaces

$$\begin{array}{ccc} K(\mathbb{Z})^\wedge_{(2)} & \longrightarrow & (\mathbb{Z} \times BO)^\wedge_2 \\ \downarrow & & \downarrow \\ K(\mathbb{F}_3) & \longrightarrow & (\mathbb{Z} \times BU)^\wedge_{(2)} \end{array} .$$

This defines a map of $K(\mathbb{Z})^\wedge_{(2)}$ to the pullback of the other spaces in the diagram. After taking a connected cover $JK(\mathbb{Z})$ of the pullback this maps the subgroups of $\pi_* K(\mathbb{Z})^\wedge_{(2)}$ generated by étale K-theory and the Borel classes (i.e. all known homotopy in $K(\mathbb{Z})^\wedge_{(2)}$) isomorphically to the homotopy groups $\pi_*(JK(\mathbb{Z}))$.

Theorem 1: $K(\mathbb{Z}) \longrightarrow JK(\mathbb{Z})$ is not a 7-connected map.

The proof uses the Hochschild homology $H_{QS^0}(\mathbb{Z},\mathbb{Z})$ (see Waldhausen's lecture).

Theorem 2: $H_{QS^0}(\mathbb{Z}) = \mathbb{Z} \times \prod_r \mathbb{Z}/r\,[2r-1]$, where $\mathbb{Z}/r[i]$ denotes the i-dimensional Eilenberg-MacLane space.

Assume Theorem 1 false. Then the space $JK(\mathbb{Z})$ gives a concrete model for $K(\mathbb{Z})$ (at least in a dimension range). Direct computation using this model gives two conflicting results about the maps

$$\begin{cases} H^7(K(\mathbb{Z})_1;\mathbb{Z}/4) \longrightarrow H^7(H_{QS^0}(\mathbb{Z})_1;\mathbb{Z}/4) \\ H^7(K(\mathbb{Z})_o;\mathbb{Z}/4) \longrightarrow H^7(H_{QS^0}(\mathbb{Z})_o;\mathbb{Z}/4). \end{cases}$$

D. BURGHELEA: <u>Calculation of the rational K-Theory of spaces via cyclic homology</u>

In this lecture $HH_*(X)$ and $HC_*(X)$ denote the Hochschild resp. cyclic homology with rational coefficients of X.

<u>Proposition:</u> Given two spaces X and Y one has the following exact sequence

$$0 \leftarrow HC_*(X) \underset{HC_*(*)}{\square} HC_*(Y) \leftarrow HC_*(X \times Y) \leftarrow \Sigma(\underset{HC_*(*)}{\mathrm{Cotor}}\, HC_*(X), HC_*(Y)) \leftarrow 0$$

and if $HC_*(Y)$ is a quasifree $HC_*(k)$-comodule of the form $HC_*(Y) = HC_*(p) \otimes_{\mathbb{Q}} W_* + V_*$ (with $HC_*(pt) \otimes_{\mathbb{Q}} W_*$ the free part and V_* the trivial part), then $HC_*(X \times Y) = HC_*(X) \otimes_{\mathbb{Q}} W_* + HH_*(X) \otimes_{\mathbb{Q}} V_*$. If Y is a suspension or $K(Z,n)$ then $HC_*(Y)$ is quasifree and explicit formulas for both $HH_*(Y)$ and $HC_*(Y)$ are given.

If X has $(\Lambda[x_\alpha],d)$ as a Sullivan minimal model $(\deg x_\alpha \geq 2)$ and $\Lambda[x_\alpha,\bar{x}_\alpha,u]$, $\mathcal{D}$ denotes the commutative differential graded algebra with $\deg \bar{x}_\alpha = \deg x_\alpha - 1$, $\deg u = 2$ and $\mathcal{D}x_\alpha = dx_\alpha + \bar{x}_\alpha u$, $\mathcal{D}u = o$ and $\mathcal{D}\bar{x}_\alpha = \beta(dx_\alpha)$ $(\beta : \Lambda[x_\alpha] \to [x_\alpha,\bar{x}_\alpha]$ the unique derivation with $\beta(x_\alpha) = \bar{x}_\alpha$,) then:

<u>Theorem</u> (joint work with M. Vigné-Poissier). $HC^*(X) = H^*(\Lambda[x_\alpha,\bar{x}_\alpha,u],\mathcal{D})$ with $HC^n(X) = \mathrm{Hom}(HC_n(X),\mathbb{Q})$

<u>Corollary:</u> If X in CP^n or QP^n (quaternionic projective spaces) $HC_*(X)$ is quasi free and explicit calculations are provided for $HC_*(X)$ and $HH_*(X)$ (similarly for complex Grassmannians).

Combined with the known relationship between $A(X) \otimes Q$ and $HC(X)$ these results recover all known computations of $A(X) \otimes Q$ and permit a few other.

Z. FIEDOROWICZ: Cyclic Homology, Monads and Group Actions

Connes' notion of a cyclic set is analyzed. It is shown that for a cyclic set $X_* : \Lambda^{op} \longrightarrow Sets$, the Connes-Gysin sequence relating cyclic homology to simplicial homology can be obtained from a fibration of the form $|X_*| \simeq \text{hocolim}_{\Delta^{op}} X_* \longrightarrow \text{hocolim}_{\Delta^{op}} X_* \longrightarrow B\Lambda^{op} \simeq \mathbb{C}P^\infty$. It is then shown that there is a natural S^1-action on the geometric realization of a cyclic set and that the usual adjunction between simplicial sets and topological spaces extends to give a combinatorial description of S^1-actions. This combinatorial description is then generalized to describe actions by a certain limited class of Lie groups. For these groups G_* one can define a similar category $\Lambda[G_*]$ and for combinatorial G_* actions on simplicial sets described by functors $X_* : \Lambda[G_*]^{op} \longrightarrow Sets$ one has a similar fibration sequence $\text{hocolim}_{\Delta^{op}} X_* \longrightarrow \text{hocolim}_{\Lambda[G_*]^{op}} X_* \longrightarrow B\Lambda[G_*]^{op}$ and that this fibration sequence can be naturally identified up to homotopy with $|X_*| \longrightarrow |X_*| \times_{|G_*|} E|G_*| \longrightarrow B|G_*|$. This result can be used to give a conceptual proof of the isomorphism

$$HC_*(k[\Omega_* X]) \cong H_*(X^{S^1} \times_{S^1} ES^1).$$

Thomas GOODWILLIE: K-Theory and cyclic homology

Theorem: For any one-connected map $f : A \to B$ of simplicial rings there is an isomorphism

$$K_n(f) \otimes \mathbb{Q} \cong HC_{n-1}(f) \otimes \mathbb{Q} \quad \forall n .$$

Explanation: Simplicial rings are simplicial objects in the category of associative rings with 1. "One-connected" means that f induces an isomorphism $\pi_0 A \to \pi_0 B$ and a surjection $\pi_1 A \to \pi_1 B$.

$K_n(f)$ is a relative algebraic K-group: The K-groups of a simplicial ring are defined (à la Waldhausen) by $K_n(A) = \pi_n B\hat{GL}(A)^+$, and relative K-groups are defined as relative homotopy groups, so that there is a long exact sequence

$$(*) \qquad \ldots \longrightarrow K_n A \longrightarrow K_n B \longrightarrow K_n f \longrightarrow K_{n-1} A \longrightarrow \ldots$$

$HC_{n-1}(f)$ is a relative cyclic homology group:
The cyclic homology groups $HC_n(A)$ of a simplicial ring A are defined by a straightforward generalization of the definition of cyclic homology of a discrete ring. (For example, if you like to define cyclic homology as the total homology of a certain double chain complex, then a simplicial ring gives you a triple complex instead...). Relative HC_n for a map of simplicial rings

is defined by an algebraic mapping cone so as to yield a long exact sequence formally analogous to (*).

Thomas GUNNARSSON: Some generalities on continuous functors, monads and rings up to homotopy

Functors $F : TOP_* \longrightarrow TOP_*$ (which commute with directed colimits, are continuous and have $F(pt) \simeq pt$) are models for abelian groups up to homotopy. $F(-)$ codifies structures on $F(S^0)$. If F is such a functor then $F^S(-) = \operatorname{colim} \Omega^n(F(S^n \wedge -))$ is a reduced homology theory. Composition of functors gives a monoidal structure. This leads to the notion of A_∞-monads and a theory for homotopy invariance of such structures. Multiplicative structures are preserved under stabilization. In the stable case A_∞-monads can be changed to monads. K-theory is defined for monads as in classical theory (ring := monad), in particular constructions used in the analysis of the algebraic K-theory of spaces can be performed using monads (as demonstrated by F. Waldhausen).

Björn JAHREN: Comparison of Involutions on $A(X)$ (joint with W.-C. Hsiang)

W. Vogell constructs involutions τ_ξ on $A(X)$, corresponding (up to sign) to the involution on pseudoisotopy theory for manifolds X with tangent sphere fibration $\simeq \xi$.
On the other hand, R. Steiner has proved that the operation $A \longrightarrow \bar{A}^t$ on matrices over $Q(G_+)$ also gives rise to an involution on $A(X)$, defined as $\mathbf{Z} \times B\widehat{GL}(Q(G_+))^+$ (Here $G = \Omega X$, and conjugation induced by $g \longrightarrow g^{-1}$ on G).

Theorem 1: This involution corresponds to Vogell's τ_ε, where ε is the trivial spherical fibration.

The proof of this uses a geometric ("~manifold") version of $\widehat{GL}(Q(G_+))$.

For computations one would also like to define the more general τ_ξ on $B\widehat{GL}(Q(G_+))^+$. In view of Vogell's work, it suffices to identify the maps $\xi\cdot : A(X) \longrightarrow A(X)$ using the GL-definition. Given ξ, there is a homomorphism $\alpha : G \longrightarrow \Omega^d S^d$ - the loops on the classifying map. If $f : S^n \longrightarrow S^n(G_+)$ represents an element of $Q(G_+)$, let $f^\alpha : S^d \wedge S^n \longrightarrow S^d \wedge S^n \wedge (G_+)$ be defined by $f^\alpha(u,x) = (\alpha(g)u,y,g)$, where $f(x) = (y,g)$.

Theorem 2: $f \longmapsto f^\alpha : QG \longrightarrow QG$ induces a map $B\widehat{GL}(Q(G_+))^+ \longrightarrow B\widehat{GL}(Q(G_+))^+$, which corresponds to Vogell's $\xi\cdot : A(X) \longrightarrow A(X)$.

Christian KASSEL: Hochschild homology outside algebraic K-Theory

The fact that the Hochschild homology of the algebra $\mathcal{D}_n = \mathbb{C} <x_1,\dots,x_n, \frac{\partial}{\partial x_1},\dots,\frac{\partial}{\partial x_n}>$ of differential operators in the affine space of dimension n is given by

$$H_i(\mathcal{D}_n,\mathcal{D}_n) = \begin{cases} \mathbb{C} & \text{if } i = 2n \\ 0 & \text{otherwise} \end{cases}$$

is explained in connection with Feigin-Tsigan's work on the cohomology of certain Lie algebras of matrices and with the notion of semi-tensor product invented in the early sixties by Massey-Peterson for topological purposes. This semi-tensor product allows to construct numerous non-commutative algebras used in various fields (algebra, analysis, topology, Kac-Moody Lie algebras) and to compute their Hochschild homology by means of a spectral sequence.

Wolfgang LÜCK: An algebraic description of the transfer induced by a fibration on K_o and K_1

For certain fibrations $F \longrightarrow E \xrightarrow{p} B$ there is a geometrically defined homomorphism $p^! : K_i(\mathbb{Z}[\pi_1(B)]) \rightarrow K_i(\mathbb{Z}[\pi_1(E)])$ using the pull-back construction. Using chain-complexes with a twist one can define pairings $K_i(\mathbb{Z}[\pi_1(B)]) \otimes K_o(\mathbb{Z}[\Delta] - \pi_1(E)) \xrightarrow{\otimes t} K_i(\mathbb{Z}[\pi_1(E)]$ where $K_o(\mathbb{Z}[\Delta] - \pi_1(E))$ is the Grothendieck group of $\mathbb{Z}[\Delta]$-chain complexes with a $\pi_1(E)$-twist. A $\pi_1(E)$-twist is a homotopy extension of the Δ-action to an $\pi_1(E)$-action. Δ denotes the kernel of $p_*:\pi_1(E) \longrightarrow \pi_1(B)$. This gives an element $L(p) \in K_o(\mathbb{Z}[\Delta] - \pi_1(E))$ and $p^!$ is just $?\otimes_t L(p)$. If $\pi_1(E)$ acts trivially (up to homotopy) on the pointed fibre $p_* \circ p^!$ and $p^! \circ p_*$ vanish. If Δ is contained in center $(\pi_1(E))$ and is free as abelian group and $G_1(F) = \pi_1(F)$, then $p^!$ is trivial.

Ib MADSEN: The equivariant Top/PL (joint work with M. Rothenberg)

Let G be a finite group of odd order. If V is an $\mathbb{R}G$-module, write $Top_G(V)$ resp. $PL_G(V)$ for the groups of equivariant homeomorphisms (resp. PL-isomorphisms) of V. Let

$$Top_G = \lim_{\substack{\rightarrow \\ V}} Top_G(V) , \quad PL_G = \lim_{\substack{\rightarrow \\ V}} PL_G(V).$$

Theorem: $\pi_k(Top_G/PL_G) = \sum^{\oplus} L_{k+1}^{<-\infty>}(\mathbb{Z}[NH/H])/L_{k+1}^{s}(\mathbb{Z}[NH/H]), \quad k \neq 3,$

$$= \sum^{\oplus} L_{k+1}^{<-\infty>}(\mathbb{Z}[NH/H])/L_{k+1}^{s}(\mathbb{Z}[NH/H]) \oplus A(G) \oplus \mathbb{Z}/2, \quad k = 3.$$

Here $A(G)$ is the Burnside ring, L^s_{k+1} are the simple surgery groups of Wall and $L^{<-\infty>}_{k+1}(\mathbb{Z}\Gamma) = L^s_{k+1+j}(\mathbb{Z}[\Gamma \times \mathbb{Z}^j])^{INV}$, j large. The groups $L^{<-\infty>}_k(\mathbb{Z}G)/L^s_k(\mathbb{Z}G)$ are easy to tabulate. If k is odd then $L^{<-\infty>}_k(\mathbb{Z}G)/L^s_k(\mathbb{Z}G) = \hat{H}^1(\mathbb{Z}/2;K_{-1}(\mathbb{Z}G))$, where $\tilde{K}_o(\mathbb{Q}G) \oplus \tilde{K}_o(\hat{\mathbb{Z}}_{|G|}G) \longrightarrow \tilde{K}_o(\mathbb{Q}_{|G|}G) \longrightarrow K_{-1}(\mathbb{Z}G) \longrightarrow 0$ is exact. If K is odd, $L^{<-\infty>}_k(\mathbb{Z}G)/L^s_k(\mathbb{Z}G) \subset \hat{H}^1(\mathbb{Z}/2;K_1(\mathbb{R}G))$. (Its rank is $\mathrm{rk}R_{\mathbb{C}}G - \mathrm{rk}R_{\mathbb{Q}}G$). The fibration sequence $F_G/PL_G \longrightarrow F_G/Top_G \longrightarrow B(Top_G/PL_G)$ gives on homotopy groups

$0 \longrightarrow L^s_k(\mathbb{Z}G)_{(2)} \longrightarrow L^{<-\infty>}_k(\mathbb{Z}G)_{(2)} \longrightarrow L^{<-\infty>}_k(\mathbb{Z}G)/L^s_k(\mathbb{Z}G) \longrightarrow 0$, when $k \neq 3$ (and G is abelian).

H.J. MUNKHOLM: Lower K-theory and parametrized spaces with bounded control
(joint work with D.R. Anderson)

Let (Z,ρ) be a metric space. The category $\underline{Top}^c/Z$ has as objects all maps $p : X \longrightarrow Z$; a morphism $f : (X,p) \longrightarrow (Y,q)$ is a map $f : X \longrightarrow Y$ with $\rho(px,qfx)$ bounded. We develop "an algebraic topology" for $\underline{Top}^c/Z$ including chain-, homology-, and homotopy "groups" that take values in an abelian category $\underline{\underline{A}}(X,p)$ (analogous to the category of $\mathbb{Z}[\pi_1 X]$-modules in the classical case (Z=pt.)). We prove a Hurevicz- and a Whitehead theorem in this context. The results are applied to study simply homotopy theory with bounded control over Z. There result obstructions in a group $Wh(\underline{\underline{A}}(X,p))$ constructed from the category of "boundedly finitely generated" projectives in $\underline{\underline{A}}(X,p)$ in the standard way. If $Z = \mathbb{R}^k$ and (X,p) has "uniformly boundedly defined" $\pi_1(X)$ then $Wh(\underline{\underline{A}}(X,p)) \cong \tilde{K}_{1-k}(\mathbb{Z}\pi_1 X)$.

Nguyen H.V. HUNG: Dickson-Huỳnh Mùi's invariants and the homology coalgebras of loop spaces $\Omega^q S^q X$

This talk announces some current researches of our seminar in Hanoi, particularly of Huỳnh Mùi and the author, on applications of modular invariants to Algebraic Topology.
We introduce the mod p Dickson characteristic classes for finite coverings over paracompact spaces derived from the Dickson-Huỳnh Mùi's invariants of $GL(n,\mathbb{Z}/p)$. These Dickson classes are closely related to the classical Stiefel-Whitney or Chern classes.
Cohomology algebras of the (universal) loop spaces $\Omega^q S^q$ are determined using

the isomorphisms $H^*(\Omega_0^q S^q;\mathbb{Z}/p) \cong H^*(F(R^q,\infty)/\Sigma_\infty;\mathbb{Z}/p)$ and the Dickson classes for the Σ_∞-principal covering over $F(\mathbb{R}^q,\infty)/\Sigma_\infty$. The action of Steenrod operations on $H^*(\Omega_0^q S^q;\mathbb{Z}/p)$ are computed by reducing them to those on the $GL(n,\mathbb{Z}/p)$-invariants.

Generalizing these results, Huỳnh Mùi describes the coalgebra structures of $H_*(\Omega^q S^q X)$ by introducing the homology operations derived from the modular invariants, which are certain linear combinations of iterated Dyer-Lashof operations, on the loop spaces $\Omega^q S^q X$. The invariants led us to overcome the Adem phenomenon occurring in the Dyer-Lashof approach.

Crickton OGLE: Two Questions in Integral Algebraic K-Theory

We discuss two conjectures in K-Theory which are integral analogues of rational constructions. The first involves a configuration space model for K-theory. The space

$$CGL(R) = \coprod_{n\geq 0} C(n;\mathbb{R}^\infty) \times_{\Sigma_n} BGL_n(R)/\sim$$

is analyzed in analogy to $C(X) = \coprod_{n\geq 0} C(n;\mathbb{R}^\infty) \times_{\Sigma_n} X^n/\sim \simeq Q(X)$.

We show that there is a map $BGL(R)^+ \longrightarrow CGL(R)$ which is a rational homotopy equivalence. The motivation for the construction of $CGL(R)$ is that there is a map

$$CGL(R) \longrightarrow NGL(R) = \coprod_{n\geq 0} * \times_{\Sigma_n} BGL_n(R)/\sim \simeq \prod_{n\geq 1} K(\pi_*,n)$$

which is a rational homotopy equivalence, e.g., together with a map $Sp_\infty(\Sigma_\infty \smallsetminus B(R)) \longrightarrow NGL(R)$ which induces a map $HC_*(D_*^0(R) \longrightarrow \pi_*(\Sigma_\infty \smallsetminus BGL(R)) = \pi_*(NGL(R)) \longrightarrow K_*(\mathbb{C})$ rationally; $D_*^0(R)$ a certain cyclic subcomplex of $C_*(R)$ = Connes complex. It is conjectured that the ∞-loop space $CGL(R)$ is either algebraic K-theory, or algebraic K-theory "away" from $Q(S^0)$. In particular, one can construct $C\hat{G}L(R)$ for the ring up to homotopy $R = Q(\Omega X_+)$, and it is conjectured that $CGL(Q(S^0)$ is either $A(*)$ or $Wh^{Diff}(*)$ integrally (this is true rationally).

The second question involves the integral K-theory of a square-zero ideal I. We construct maps $I^{\otimes n}/\sim \xrightarrow{L} K_*(I) \xrightarrow{\bar{D}} I^{\otimes n}/\sim$ (where $\sim$ is the cyclic relation in cyclic homology) whose composition is multiplied by n. By Staffeldt, the map $I^{\otimes n}/\sim \longrightarrow K_*(I)$ is a rational isomorphism, but it is shown not to be an integral one. However, we conjecture that im(L) generates $K_*(I)$ as an ideal in the graded ring $K_*(\mathbb{Z} \oplus I)$ integrally.

Richard STEINER: A non-connective delooping of the algebraic K-theory of spaces

Let Y be an A_∞ ring space which is ringlike ($\pi_0 Y$ is a ring). Its K-theory KY is $K_0(\pi_0 Y) \times (Bgl\ Y)^+$, where $(Bgl\ Y)^+$ is the plus-construction on the classifying space of the telescope of the invertible components $gl_d Y$ in $m_d Y \simeq Y^{d^2}$. In particular, if X is a based space and $Y = \Omega^\infty \Sigma^\infty((\Omega X)_+)$ ($(\ )_+$ denotes the addition of a base-point), then KY is a possible definition for the algebraic K-theory of X.

Imitating Wagoner, one can deloop KY non-connectively as follows, perhaps more informatively than the usual way. There is a sequence Y, sY, s^2Y, ... of ringlike A_∞ ring spaces such that $KY \simeq \Omega KsY$, $KsY \simeq \Omega Ks^2Y, \dots$. Here sY is such that $\pi_q sY \simeq S\pi_q Y$ (locally finite matrices over $\pi_q Y$ module finite ones), etc. It is got from a bar construction, using the general principle that a construction on semirings extends to A_∞ ring spaces provided one never adds two equal terms.

Pierre VOGEL: A commutativity formula for Nil-groups

Let A and B be two rings and ${}_AS_B$ and ${}_BT_A$ be two bimodules. Consider the category of objects (P,Q,p,q) where $P \in \mathcal{P}_A$ and $Q \in \mathcal{P}_B$ are finitely generated projective right modules and $p : P \to Q \otimes_B T$, $q : Q \to P \otimes_A T$ are maps. The category $Nil(A,B;S,T)$ of such objects which are nilpotent in an obvious sense is an exact category and we have a K-theoretical spectrum $K\ Nil(A,B;S,T)$ which splits: $K\ Nil(A,B;S,T) \simeq K(A) \times K(B) \times K\ \widetilde{Nil}(A,B;S,T)$.

Theorem: If S and T are free on each side, the rule $(P,Q,p,q) \mapsto (P,q \cdot p)$ gives a homotopy equivalence of spectra:

$$K\ \widetilde{Nil}(A,B;S,T) \xrightarrow{\sim} K\ \widetilde{Nil}(A,S \otimes_B T).$$

Corollary: Under the same conditions we have a homotopy equivalence:

$$K\ Nil(A,S \otimes_B T) \simeq K\ Nil(B,T \otimes_A S).$$

As a consequence of this result we see that the Nil functors defined by Waldhausen in the computation of Mayer-Vietoris exact sequences in algebraic K-theory associated to push-out of groups, are of the form $K\ \widetilde{Nil}(A,S)$ and are contractible in many cases. For example we have the following:

Theorem: Let $\begin{array}{ccc} H & \hookrightarrow & G \\ \downarrow & & \downarrow \\ G' & \to & \pi \end{array}$ be a push-out of groups. Suppose that $H \cap \alpha H \alpha^{-1} \cap \beta H \beta^{-1}$ is regular coherent for every $\alpha \in G-H$ and $\beta \in G'-H$. Then we have a cartesian square of spectra:

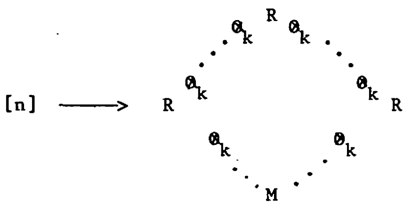
$$\begin{array}{ccc} Wh(H) & \longrightarrow & Wh(G) \\ \downarrow & & \downarrow \\ Wh(G') & \longrightarrow & Wh(\pi) \end{array} .$$

Wolrad VOGELL: <u>Involutions on A(X)</u>

Various models of the algebraic K-theory of spaces functor $A(X)$ were described and it was shown how to put natural involutions on these. There is a notion of equivariant Spanier-Whitehead duality underlying the construction of these involutions. It turns out that the correct notion of equivalent duality to use is a generalization of Ranicki duality where the group acting on the spaces under consideration is no longer discrete but is allowed to be any simplicial group. The involutions constructed depend on a chosen spherical fibration ξ over X. If X is a manifold these involutions on $A(X)$ are shown to correspond to the natural involution on the stable concordance space $\mathcal{C}(X)$, where ξ is the (fibrewise one-point-compactification of the) tangent bundle of X.

Friedhelm WALDHAUSEN: <u>Hochschild homology and stable K-theory</u>

Let R be a ring and M a bimodule over it. Supposing that R is an algebra over a ground ring k one defines the <u>Hochschild homology</u> $H_k(R,M)$ as the simplicial object



(n factors R – the circular display is to indicate that the j-th face map is given by the collapsing of the j-th tensor product sign). It turns out that the construction can be extended to a framework of "rings up to homotopy" (one uses monads, and algebras over monads, to carry this out technically). The interest of the extended construction is in its use to compute the <u>stable</u> K-theory $K^S(R,M)$. The assertion (whose proof is fairly difficult) is that the natural map $K^S(R,M) \longrightarrow H_k(R,M)$ is a homotopy equivalence provided that for the ground ring k one takes the "universal" ring up to homotopy, QS^o, whose homotopy groups are the stable homotopy groups of spheres.
(Note, if $M = R = k$ then $H_k(R,M) \simeq k$, and if $M = R = QS^o$ then $K^S(R,M) = A^S(*)$, so this generalizes the assertion that $A^S(*) \longrightarrow QS^o$ is a homotopy equivalence.)

Chuck WEIBEL: Delooping K-theory by parametrized modules
(joint with E.K. Pederson)

Given an additive category A and a metric space X, one can construct a category $C_X(A)$ of A-objects parametrized by X (in a locally finite way), the morphisms being given by "bounded matrices". The point of the lecture was that the K-theory spaces of $A = C_o$, $C_{\mathbb{R}}(A)$, $C_{\mathbb{R}^2}(A), \ldots$ form a non-connective infinite loop spectrum, at least when all short exact sequences split in A. This allows us to define the negative K-theory of A. In fact, we recover the definition given by Karoubi in LNM vol. 36 (1968), naturally in a different form. The hope is that this machinery works in case short exact sequences in A do not split, but the problem at present is defining an exact structure on the category $C_X(A)$. Assuming all s.e.s. split in A, we can define the K-theory of $C_X(A)$ by allowing only split monomorphisms. For convenience, assume A idempotent complete. Then we prove:

$$K(C_{X\times[o,\infty)}) \simeq *$$

$$K(C_Y) \longrightarrow K(C_X) \longrightarrow K(C_X/C_Y) \quad \text{is a fibration}$$

$$\begin{array}{ccc} K(\hat{C}_X) & \longrightarrow & K(C_{X\times[o,\infty)}) \simeq * \\ \downarrow & & \downarrow \\ * \simeq K(C_{X\times(-\infty,o]}) & \longrightarrow & K(C_{X\times\mathbb{R}}). \end{array} \quad \text{is homotopy cartesian}$$

From this it follows that $\Omega^n K(C_{\mathbb{R}^n}) \simeq K(A)$, and that $K(C_{\mathbb{R}^n})$ is a covering space of $\Omega K(C_{\mathbb{R}^{n+1}})$.

Bruce WILLIAMS: Surgery theory, automorphisms of manifolds, and higher algebraic K-theory (joint work with Bill Dwyer)

For a spectrum A with $\mathbb{Z}/2$ action we construct a "Tate cohomology" fibration, $H.(\mathbb{Z}/2,A) \xrightarrow{N} H^{\cdot}(\mathbb{Z}/2,A) \to \hat{H}(A)$, e.g. A = Waldhausen's $A(X)$ with Vogell's involution. If $A^{(n)} = A$ twisted by n copies of the flip representation, then $\Omega^n\hat{H}(A) = \hat{H}(A^{(n)})$.

For M^n a topological manifold, let $H(M)$ = (simple) homotopy automorphisms of M and $TOP(M)$ = homeomorphisms of M.

Conjecture: There exists a commutative diagram of natural transformations

$$\begin{array}{ccccc} M_+ \wedge L(\mathbb{Z}) & \longrightarrow & L(\mathbb{Z}\pi) & \longrightarrow & S(M) \\ \downarrow & & \downarrow & & \downarrow \theta \\ \hat{H}(M_+ \wedge A(*)) & \longrightarrow & \hat{H}(A(M)) & \longrightarrow & \hat{H}(Wh(M)) \end{array} \quad (*)$$

such that $\frac{H(M)}{TOP(M)}$ is the homotopy fiber of the map

$$\Omega^{n+1}S(X) \xrightarrow{\Omega^{n+1}\Theta} \Omega^{n+1} H(Wh(X)) \xrightarrow{\sim} \Omega H(Wh(X)^{(n)} \xrightarrow{\partial} H.(\mathbb{Z}/2, Wh(X)^{(n)}).$$

Thus (*) would be the "glueing data" between surgery theory and the algebraic K-theory of spaces.

Evidence for the conjecture comes from the work of Hatcher, Hsiang-Sharpe, and Burghelea-Lashof.

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