Mathematisches Forschungsinstitut Oberwolfach

T a g u n g s b e r i c h t 42 /1984

Kombinatorische Geometrie

23.9. bis 29.9.1984

Die Tagung wurde von Andreas Dress (Bielefeld) und Jörg Wills (Siegen) organisiert. Sie diente vor allem der Absicht, gewisse in den letzten Jahren entwickelte systematische Ansätze und Methoden zur Bearbeitung kombinatorischer Probleme in der Geometrie zu konfrontieren mit der Fülle der in diesem Gebiet noch offenen und häufig herausfordernden Probleme und den in der Regel sehr einfallsreichen, aber häufig nicht systematisierbaren Ansätzen, die zur Bearbeitung gerade eben noch zugänglicher Spezialfälle solcher Probleme ersonnen worden sind. Durch die Hinzuziehung von Physikern und Chemikern wurde darüber hinaus der Versuch gemacht, die in den noch nicht zum Standard gehörigen Anwendungen von Mathematik in Physik und Chemie häufig auftretenden kombinatorisch-geometrischen Probleme in die Diskussionen der Tagungsteilnehmer (und vielleicht auch in deren künftige Arbeit) mit einzubringen.

Entsprechend dieser Konzeption kam es im Laufe der Tagung zu fruchtbaren Diskussionen, in denen die unterschiedlichen Ansätze einander gegenübergestellt und auf wechselseitige Anwendbarkeit abgeklopft wurden. Im Zentrum standen Fragen zur Struktur und Realisierung zwei- und höherdimensionaler geometrischer Zellenkomplexe mit bestimmten, jeweils unterschiedlich vorgegebenen Eigenschaften, darüber hinaus wurden aber auch Fragen aus der Überdeckungstheorie, der hyperbolischen Geometrie und der Theorie starrer und beweglicher Strukturen im Raum (Structural Topology im Sinne von H. CRAPO) angesprochen.

An der Konferenz nahmen Teilnehmer aus verschiedenen Ländern teil. Die gewohnte hervorragende Atmosphäre des Oberwolfacher Forschungsinstituts trug viel zu dem wohl für alle Teilnehmer anregenden Verlauf der Konferenz bei.

Vortragsauszüge

K. BEZDEK:

Symmetrien einiger Packungen und Überdeckungen

Im ersten Teil beschäftigen wir uns mit den dünnsten Überdeckungen eines Kreises, regulären Dreiecks bzw. Quadrats durch endlich viele kongruente Kreise. Die hier vorkommenden Symmetrien spielen auch in Beweisen der Sätze eine wichtige Rolle.

Ein öfter behandeltes Problem der diskreten Geometrie ist die dichteste Kugelpackung von endlich vielen kongruenten Kugeln in einem konvexen Körper. Der zweite Teil befaßt sich mit diesem Problem für Tetraeder, Oktaeder und Würfel im $\mathbb{R}^3$.

Im dritten Teil untersuchen wir ein von L. Fejes Tóth stammendes Verfahren, wann eine überall dichte Punktmenge oder eine reguläre Punktmenge in der Ebene entsteht.

Endlich im vierten Teil erwähnen wir eine "extremale" Aufgabe für ein reguläres Dreieck.

S. BILINSKI:

Die quasiregulären Polyeder zweiten Geschlechtes der Stufe größer als zwei

Wenn in dem Eckenzyklus $(\overbrace{m,n}^{1},\overbrace{m,n}^{2},\dots,\overbrace{m,n}^{s})$ eines quasiregulären Polyeders s alternierende Paare von zwei verschiedenen natürlichen Zahlen vorkommen, nennen wir diese Anzahl s die "Stufe" des betreffenden quasiregulären Polyeders. Die beiden klassischen elementaren quasireguläre Polyeder sind also zweiter Stufe. Von verallgemeinerten (topologischen und affinen) Polyedern des zweiten Geschlechtes sind alle quasiregulären Polyeder zweiter Stufe bekannt. Es werden fünf weitere quasireguläre Polyeder zweiten Geschlechtes konstruiert, von welchen vier von dritter Stufe sind und eines von vierter Stufe. Damit sind alle quasiregulären Polyeder zweiten Geschlechtes gegeben.

J. BOKOWSKI:

Grassmann-Plücker-Relationen 4. Grades

Eine algorithmische Bestimmung aller kombinatorischen Typen von (konvexen) d-Polytopen fester Eckenanzahl läßt sich auf das Problem zurückführen, bei vorgegebener kombinatorischer Sphäre algor. zu entscheiden, ob sie polytopal ist.

Ein Algorithmusansatz wird am Beispiel der Altshulersphäre M^{10}_{416} diskutiert. Dadurch wird ein Problem von M.A. Perles (Oberwolfach, Juli 1984) positiv beantwortet.

Satz (J.B.). Es gibt ein 2-benachbartes 4-Polytop, das keine universelle Kante besitzt. (M^{10}_{416} ist polytopal)

Damit ist das Shemersche Nähverfahren auf dieses Polytop nicht anzuwenden.

Satz (J.B./K. Garms) M^{10}_{425} (gleiche Eckenfigur) ist nicht polytopal.

Literatur: ALTSHULER, Can. J. Math. 29,2(1977), 400-420.

U. BREHM:

On weakly neighborly polyhedral maps

Definition: A weakly neighborly polyhedral map (wnp map) is a 2-dimensional topological cell-complex which decomposes a compact connected 2-manifold without boundary such that the 2-cells are closed topological discs and the meet of any two 2-cells is connected and for any two vertices there is a 2-cell containing them.

We give a complete list of all (combinatorial types of) wnp maps on 2-manifolds (orientable or non-orientable) of Euler characteristic χ with $|\chi| \leq 2$ and investigate which of these wnp maps can be realized in E^3 with convex planar facets.

We show that for each 2-manifold except the sphere there are only finitely many wnp maps and give an upper bound $\overline{V}(\chi)$ with

$$\lim_{|\chi|\to\infty} \frac{\overline{V}(\chi)}{|2\chi|^{2/3}} = 1.$$

We give an infinite list of non-orientable wnp maps and a construction principle which strongly suggest the following conjecture:

Conjecture: On every non-orientable 2-manifold there exists a wnp map.

Most of the mentioned results are contained in joint works of A. Altshuler and myself.

H. CRAPO:

Structural Topology - Topological Geometry

In scene analysis (the reconstruction of solid objects, given one single plane projection) and in mechanics (the rigidity of bar and joint structures, panel and hinge structures, etc.) we find problems which lead to a topological theory of objects which are more rigid, less pliable, than the "rubber sheets" studies by the topology of Henri Poincaré and his school. We present the first theorems of this new area of study.

In particular, we show how to extend the geometric rank function r, initially defined only for subsets $A \subseteq P$ of the set P of points in a geometric configuration G, to a valuation χ on the distributive lattice D of antichains of $2^P \cong$ lattice of order ideals of $2^P \cong$ lattice of simplicial complexes on P. To carry out this extension, we use the Möbius function μ_E on 2^P relative to an antichain E, defined recursively by

$$\sum_{A \subseteq B} \mu_E(B) = \mu_E(A) = \begin{cases} 1 \text{ if } A \subseteq C \text{ for some } C \in E \\ 0 \text{ otherwise} \end{cases}$$

($\mu_E(A) = 0$ unless A is an intersection of sets in the antichain E). Then the characteristic χ of the geometry G has the value on each antichain E given by the formula

$$\chi(E) = \sum_{A \in L_E} \mu_E(A)\, r(A),$$

where L_E is the semilattice of intersections of parts of the anti-chain E.

The characteristic χ of a geometry generalizes the classical Euler characteristic: simply take G to be the rank one geometry where all points P are coordinatized by scalar multiples of a single vector V, and resulting valuation is generated by a function on sets which has value 1 on nonempty sets, value 0 on the empty set.

Associated with this more general characteristic is a homology theory capable of measuring phenomena such as geometric special position, degrees of freedom of linkages, etc.

L. DANZER:

Über einige reguläre Inzidenzpolytope, deren Automorphismengruppe die PSL (3,2) als Normalteiler enthalten

Es wird das reguläre Inzidenzpolytop $P := \{3,8\}_8 \in \begin{pmatrix} 2 & 2 & 2 \\ & 6 & 16 \\ & & 672 \end{pmatrix}$ (der Dimension 3) näher untersucht. Für die Automorphismengruppe $A(P)$ gilt nach [1]

$$A(P) = G^{3,8,8} \cong C_2 \times PGL(2,7), \text{ also auch}$$
$$\cong C_2 \times (C_2 \cdot PSL(3,2)) \cong V_4 \cdot PSL(3,2),$$

wobei das letztere Produkt semidirekt ist (mit der einfachen Gruppe der Ordnung 168 als Normalteiler und der V_4 als komplementärer Untergruppe).

Durch Färbung der Dreiecke von P (56 weiße, sowie je 8 farbige in 7 Farben) wird ein Isomorphismus zwischen dem genannten Normalteiler von $A(P)$ und der Kollineationsgruppe der FANO-Ebene hergestellt. - Die Dreiecke jeder einzelnen Farbe bilden dieselbe Konfiguration wie die acht Dreiecke des Kuboktaeders. Demgemäß wird ein Modell von P im $\mathbb{E}^3$ konstruiert, das unter der Würfelgruppe invariant ist (es besteht aus einem archimedischen und sechs weiteren Kuboktaedern, wobei die letzteren zum ersten nur kombinatorisch isomorph aber noch konvex und untereinander kongruent sind).

Dualisierung, Anwendung der PETRIE-Operation und "Facetting" liefern eine Reihe mit P verwandter Inzidenzpolytope.

[1] COXETER, H.S.M.: The abstract groups $G^{m,n,p}$. Trans. AMS 45(1939), 73-150.

A. DRESS: (s. Anhang)

P. FLOERSHEIM:

A Computer-Implemented Method for the Description of Molecules and for the Perception of their Symmetry

A computer program called ONOMA will be explained and demonstrated. It is applicable to the description of mobile (non-rigid, quasi-rigid or rigid) molecular structures and to the perception of their symmetry.

ONOMA guides the user to obtain symmetry consistent descriptions in the form of finite relational systems. There the atoms of a molecule are represented by number labels and the structural relationships among the atoms by the tuples of several relations on the set of labels. These relations are collected in blocks, suggestively called identification (of atom types), connectivity, angularity, dihedrality and orientivity.

A canonization procedure (renumbering and ordering of the relational system) eliminates all arbitrariness of the description; the resulting unique description is taken as a structural name of the molecule. The canonization algorithm also finds classes of equivalent numberings (automorphisms and enantiomorphisms) which characterize a symmetry group of the described molecule.

P. GRITZMANN:

A geometric curiosity concerning polyhedral 2-manifolds

Let P be a polyhedral 2-manifold in E^3, i.e. a geometric cell-complex whose underlying point-set set P is a closed connected 2-manifold in E^3. A vertex v of P is called convex if, and only if, at least one of the two components into which set P divides a sufficiently small ball centred at v is convex. Let n(g) denote the minimal number of non-convex vertices of polyhedral 2-manifolds in E^3.

Solving a problem of Barnette, we have $n(g) = 5 \; (g \geq 1)$.

This result is a joint work with U. Betke.

H. HARBORTH:

Point sets with many unit circles

Consider n points in the plane. Each triple of points may determine a circle. What is the maximum number K(n) of congruent circles, say unit circles? For this problem of P. Erdös it is known

$cn \leq K(n) \leq \frac{n(n-1)}{3}$. The exact values K(n) = 1,4,4,8,11 for n = 3,4,5,6,7 are determined, which is tedious for n = 5 and n = 7.

T. HAVEL:

Distance Geometry and Macromolecular Structure

NMR is a new spectroscopic technique which yields imprecise but plentiful information on short distances (contacts) and dihedral angles about single bonds in biological marcomolecules. From such incomplete sets of local structural information it is then necessary to construct a complete global structure. This is done by the EMBED algorithm, in 3 stages: first, a complete set of bounds on all distances is extrapolated from those directly measured, using the triangle inequality (inconsistencies and/or redundancies in the data may be uncovered by this procedure); second, a random metric space is selected within these bounds, and approximate coordinates obtained from these by a projection technique; third, remaining deviations of the coordinates from the available distance and chirality information are removed by optimization vs. a penalty function. Applications of this algorithm to the design and interpretation of NMR experiments are described, including a de novo structure determination of a small protein, BUSI IIA, in solution (with Williamson and Wüthrich). Possible approaches to the problems of dealing directly with ensembles of solution conformations instead of the coordinates of exact single structures are proposed.

F. HOLROYD:

The Geometry of Tiling Hierarchies

Let α be an amalgamator on a tiling T. (See W. Wingate's abstract for a definition.) The tiling hierarchy $I(T,\alpha)$ generated by T and α is the sequence $\{T,\alpha(T),\alpha^2(T),\ldots,\alpha^k(T),\ldots\}$. If T is isohedral, it is interesting to ask how the symmetry group changes as we go up through the hierarchy. The simple isohedral sequence $\underset{\sim}{s} = \{s_o,s_1,s_2,\ldots\}$ is defined by: s_k is the Grünbaum-Shephard type of $\alpha^k(T)$ (= 0 if $\alpha^k(T)$ is not isohedral). It is difficult to say much about $\underset{\sim}{s}$ in general. Define a symmetry of $\alpha^k(T)$ to be hierarchical if it extends to a symmetry at each of levels $0,\ldots,k-1$. The behaviour of the hierarchical isohedral sequence $\underset{\sim}{\sigma} = \{\sigma_o,\sigma_1,\sigma_2,\ldots\}$ arising out of this definition seems easier to investigate, though we have no general results as yet. Even easier, define an automorphism of $\alpha^k(T)$ to be hierarchical if it extends to an automorphism at each lower level, and define the hierarchical algebraic sequence $\underset{\sim}{h} = \{h_o, h_1, h_2, \ldots\}$ analogously with $\underset{\sim}{\sigma}$. Our main theorem now is that $h_o \geq h_1 \geq h_2 \geq h_3 = h_k$ $(k \geq 3)$, and an example is given where $h_o > h_1 > h_2 > h_3$.

We also describe what is the maximum symmetry that can be preserved by a strict amalgamator on each of the maximally symmetric isohedral tilings (Laves nets).

H. IM HOF:

Hyperbolic Coxeter Groups

We shall present a family of polyhedra (truncated orthoschemes) giving rise to a class of hyperbolic Coxeter groups and related tessellations of hyperbolic spaces.

R. JAMISON:

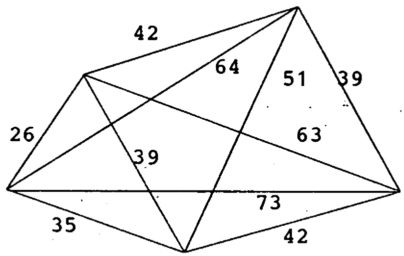
Greedy Sequences and Slope-Critical Configurations

P. Ungar (JCT, 1982) proved that n noncollinear points in the plane R^2 determine at most $n-1$ slopes. His proof was through the allowable sequences of Goodman & Pollack and hence was valid for all rank 3 oriented matroids and not just the realizable ones. For some time I have been attempting to classify - or at least limit - the possible structure of slope-critical configurations, for which the minimum number $n-1$ of slopes is attained. This report presents several structural limitations for slope-critical configurations and also gives some construction techniques for classes of (non-realizable) critical sequences for which these limitations do not apply.

A. KEMNITZ:

Punktmengen mit ganzzahligen Abständen

Eine n-elementige Punktmenge der euklidischen Ebene, für die gilt, daß je zwei Punkte ganzzahligen Abstand voneinander haben, wird als ganzzahliges n-Eck und der größte Abstand als Durchmesser des n-Ecks bezeichnet. Zu jedem vorgegebenen n gibt es ganzzahlige n-Ecke so, daß nicht alle Punkte kollinear sind, unendlich viele Punkte mit paarweise ganzzahligen Abständen liegen notwendig auf einer Geraden (Anning, Erdös 1945). Es werden Aussagen über minimale Durchmesser von ganzzahligen n-Ecken mit gewissen Eigenschaften gemacht. So ist 73 der kleinste Durchmesser eines Fünfecks mit den Eigenschaften, daß keine drei der fünf Punkte auf einer Geraden und keine vier auf einem Kreis liegen.



W. KÜHNEL:

Triangulated tori, the expanded simplex, and crystallographic groups

We call a combinatorial manifold 2-neighborly if for any pair of vertices the edge joining them belongs to the triangulation.

Theorem: For any dimension d there exists a 2-neighborly combinatorial d-torus ($\approx \mathbb{R}^d/\mathbb{Z}^d$) with $n := 2^{d+1} - 1$ vertices. Its automorphism group of order $2(d+1)(2^{d+1} - 1)$ acts transitively on the set of vertices.

Its universal covering is a subdivision of the tessellation of d-space by translated duals of the expanded simplex $2\alpha_4$ (Coxeter's notation), and its automorphism group appears as the quotient of two crystallographic groups preserving this tessellation. With a certain $\mathbb{Z}_n$-labelling of the n vertices of the d-torus this group is isomorphic to the group of affine $\mathbb{Z}_n$-transformations generated by $x \sim x+1$, $x \sim -x$, $x \sim 2x$. For d = 2 we just get the 7-vertex torus with its automorphism group of order 42.

C. LEE:

Triangulating the d-cube

Let I = [0,1] be the closed unit interval, and consider the unit d-cube $I^d \subseteq \mathbb{R}^d$. Suppose you have a triangulation of I^d into a simplicial d-complex, without introducing any new vertices. What is the minimum possible number T(d) of d-simplices that is achievable? We outline what is presently known about this problem, sketch a new proof that T(4) = 16, and discuss some general methods of triangulating convex d-polytopes, and their implications for this problem.

P. McMULLEN:

New regular polytopes and apeirotopes

In 1977, Grünbaum described how a suitable (very natural) generalization of the concept of regularity of polyhedra and apeirohedra led to a much expanded list (even in $\mathbb{E}^3$), which included the Coxeter-Petrie apeirohedra, as well as many other examples. The concept of regularity of polytopes and apeirotopes can be similarly generalized, but so far only a handful of new examples has appeared in the literature. It is the purpose of this talk to show how examples can be freely constructed from discrete reflexion groups and from previously known examples.

B. MOHAR:

Simplicial schemes

A simplicial scheme is a certain structure which can be defined on graphs. The purpose of this concept is a graph-theoretical description of simplicial complexes. It is shown that graphs and simplicial schemes give rise precisely to simplicial pseudocomplexes which are homogenous and in which the open star of every simplex is strongly connected and every simplex of codimension one is contained in at most two top-dimensional simplices. It is characterized when a complex arising from a graph and a simplicial scheme is orientable. Finally, a relation between graph maps and nondegenerate simplicial maps of assigned complexes is considered. Two applications are given.

E. MOLNÁR:

On geometrical presentation of discrete transformation groups in the sense of Poincaré

Poincaré proposed a method for describing isometry groups acting discontinuously on spaces of constant curvature. Such a group G is given by a fundamental polyhedron F whose faces are identified by isometries generating the group G. The so-called edge conditions for the equivalence classes of segments guarantee the discontinuous action of G on the space M and they serve a complete set of relations for the presentation of G. We can applicate the method in various directions, e.g. the minimal presentation of euclidean space groups by fundamental topological polyhedra (with minimal number of curved faces) can be interesting. There are intensive investigations in the hyperbolic space to guarantee the free action of G on M by criteria for F because then the factor space M/G will be a hyperbolic space form. Some examples will be presented, connections with well-known groups, e.g. Coxeter groups will be mentioned.

(Hopely, the discussions give some ideas to the further investigations.)

W. MOSER:

Research Problems in Discrete Geometry

At the Discrete Geometry week, Oberwolfach 1977 July, I presented 14 unsolved problems posed by my brother Leo Moser (1921-1970). Since then, this collection has grown substantially, has been frequently revised and has been widely distributed: the 1981 edition contained 68 problems and was mailed to 500 mathematicians.

At this time I present (in written form) problems 1 - 50 of the 1984 edition. Copies will be mailed to all participants at this meeting; copies of problems 51 - 100 will be mailed later this year. I welcome information on these problems, new problems and requests for copies of the 1984 edition.

R. POLLACK:

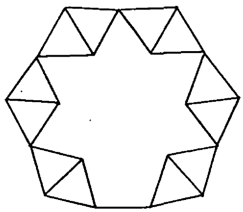
Increasing the Minimum Distance Among a Set of Points

P. Erdös has proposed the following problem. Let f(d,n) be the largest number such that among any n points in E^d with minimum distance 1 there exist f(d,n) points with minimum distance > 1. He asked for a proof or disproof of $f(d,n) > c\frac{n}{d}$. Clearly $f(d,n) \leq \frac{n}{d+1}$ by considering $\frac{n}{d+1}$ widely spaced simplices. Moreover, he had a proof that $f(2,n) > \frac{n}{5}$ and suggested that $f(2,n) = \frac{n}{3}$ might be easy.

Theorem: $f(2,n) \geq \frac{n}{4}$.

Proof: The unit distance graph on the given set of points is easily seen to be planar. Four color it, and choose the points colored with the most frequently used color.

R. Graham together with F. Chung and independantly J. Pach have observed that $f(2,n) \leq \frac{6}{19}n$ by constructing the following diagram.



(all edges have length 1)

G. PURDY:

Ubiquitous distances, angles and areas

Let $x_1, x_2, \ldots, x_n \in E_d$. The distance $d = \overline{x_i x_j}$ is not ubiquitous if it occurs only $o(n^2)$ times. The angle $\angle x_i x_j x_k$ of the area of the triangle $\Delta\, x_i x_j x_k$ is not ubiquitious if it occurs only $o(n^3)$ times. Distances are ubiquitous in E_6 but not E_5 (an old result of Paul Erdös) and areas are ubiquitous in E_6 but not E_5, also an old result of mine. A new result is that acute angles are ubiquitous in E_6 but not E_5, and right angles are not ubiquitous in E_k for any k. We discuss some generalizations to solid angles, giving some results and conjectures.

H. RASZILLIER:

Equivariant Tessellations of E^n ($n \leq 4$) in some Quantum Mechanical Problems

We start from the equivariant spectral problem for the Hodge laplacian in the L^2-space of differential forms over E^n, with the group of affine orthogonal transformations as symmetry group. We approximate the problem by one in the l^2-space defined over a tessellation of E^n. The possible choices will differ in the discrete symmetry left over, and in the number of degrees of freedom compared to the dimension $D = 2^n$ of the exterior algebra. We may ask the tessellation to admit as symmetry group the symmorphic space group corresponding to a maximal finite subgroup of $Gl(n,Z)$, in order to keep "as much as possible" of the original continuous symmetry. The degrees of freedom for the discrete Hodge laplacian are counted by the dimension of its space of forms, i.e. by the orbits of k-faces $(k = 0,1,\ldots,n)$ of the equivariant tessellation under lattice translation. Their number D_Γ depends on the group Γ and on the tessellation. Since $D_\Gamma \geq 2^n$, we would like to take, for a given Γ, those Γ-equivariant tessellations which give the smallest values of D_Γ.

We discuss the combinatorics of the construction and spectral analysis of discrete Hodge laplacians which meet the above requirements.

R. SCHARLAU:

Some applications of generalized Schläfli-symbols

As one application of the general theory of "chamber systems of tessellations", outlined by A. Dress in his talk, the following is proved.

Theorem: There exist precisely 37 combinatorial types of equivariant tilings (T,Γ) of the euclidean plane such that the group Γ has exactly two orbits on the vertices, edges and tiles, respectively.

All these tilings are listed explicitely. The theorem is a first step towards the classification of all (T,Γ) such that Γ has two orbits on the tiles. This is an open problem proposed by Grünbaum and Shephard.

E. SCHULTE:

Tiling space by isomorphic polytopes

We discuss the following problem of L. Danzer: Given a convex 3-polytope P, is there a locally finite (face-to-face) tiling of 3-space by convex polytopes all isomorphic to P ?

CH. SCHULZ:

Polyhedral 2-manifolds in E^3 with few vertices

The minimal number f_o of vertices of a polyhedral triangulation (i.e. one made up of plane triangles) $M \subset E^3$ of the closed orientable surface of genus g is only known for $g \leq 3$. Of special interest are those values of g for which a minimal polyhedral triangulation with complete 1-skeleton might exist, the next open case being $g = 6$, $f_o = 12$. One method for realizing these cases in 3-space is to embed M as a subcomplex into the boundary complex of some neighborly 4-polytope P, and then project it down into a Schlegel-diagram of P. Strong necessary conditions for P to admit $M \subset skel_2 P$ are given.

M. SENECHAL:

Tiling the torus and other space forms

The cylinder, the torus, the Möbius band and the Klein bottle are the four quotient manifolds E^2/F, where F is a fixed point free subgroup of E(2). Let Γ be a normal isohedral, isogonal or isotoxal tiling of E^2 with maximal symmetry for its net; we identify its symmetry group with the automorphism group of Γ considered as an edge graph. If $F \subseteq \mathrm{Aut}\,\Gamma$ then $\gamma = \Gamma/F$ is a graph on E^2/F which has the same local properties as Γ, but may not be transitive: $\mathrm{Aut}_M \gamma$ acts transitively on γ if and only if the normalizer of F in $\mathrm{Aut}\,\Gamma$ acts transitively on Γ. This enables us to classify and enumerate the transitive graphs Γ/F on E^2/F. Results include: Grünbaum's and Shephard's two families of isogonal toroidal polyhedra are (probably) the only ones; there are no transitive graphs on the Möbius band; there is a graph on the Klein bottle whose automorphism group acts transitively on its faces, edges, and vertices.

A. IVIĆ WEISS:

Polystroma of type {6,3,3}

A polystroma is a combinatorial structure that locally behaves like a polytope. We give a list of known "naturally generated" polystroma with toroidal cells of type $\{6,3\}_{b,c}$ and tetrahedral vertex figures. We discuss the conjecture of Branko Grünbaum that for every pair $(b,c) \neq$

(1,1) the polystroma are finite.

N. WHITE:

Motions in Frameworks

Joint work with Walter Whiteley on infinitesimal motions in bar-and-body frameworks is first surveyed. Then recent progress on hinge-and-body frameworks is described. In both cases, rigidity or non-rigidity of such frameworks is determined in terms of bracket algebra or Cayley algebra. A digression on a related topic of matroids satisfying certain counting conditions is then pursued. In these matroids we state a "pruning theorem", a generalization of well-known matching theorems, and apply it to frameworks.

W. WHITELEY:

Designing and Describing a Polyhedron in 3-space

We present the equivalent problems of (i) making a realizable sequence of choices in the design of a spatial polyhedron, and (ii) giving a minimal set of measurements of a polyhedron which will uniquely determine the object, at least locally. A number of classical geometric theorems give very partial results: Steinitz' Theorem (the realizability of 3-connected planar graphs), theorems of Cauchy and Alexandrov (the local uniqueness of 3-connected spherical polyhedra with a set of edge lengths triangulating the surface) etc. Other old and new results about the statics and mechanics of frameworks can also be applied: Maxwell's theorem connecting plane stresses and projected polyhedra gives results on the dependence of dihedral angles in polyhedra, the rigidity of 4-connected spherical polyhedra with one edge length replaced by another dihedral angle, etc.

We describe some of these partial results, and give some conjectures for further work. We also introduce the related problem of which sequences of data, or choices, give a direct construction of the polyhedron.

J. WILLS:

Platonic manifolds

We call a compact polyhedral 2-manifold in euclidean E^3 a Platonic manifold (PM) if it has a Platonic rotation group, if its vertices are q-valent, if its faces are (not necessarily convex) p-gons, and if a group isomorphic to a full Platonic symmetry group (or a subgroup of it) acts transitively on its vertices or faces. Notation: {p,q;g}, g : genus. As yet 13 PMs are known, namely {4,5;7}, {5,4;7} {3,7;g} and {7,3;g}, g = 3,5,11 (WILLS 1982/84) and {3,8;g}, g = 3,5,11 (B. GRÜNBAUM, G. SHEPHARD 1984) and {3,9;7}, {9,3;7} (Wills 84, still unpublished). The three {3,8;g}, g = 3,5,11 are of particular interest, because they only need the Platonic rotation group (i.e. isometries) for their vertex transitivity, so they are best possible.

There exists no analogue for face-transitivity as B. Grünbaum and G. Shephard prove in a forthcoming paper. So the {5,4;7} and {7,3;g}, g = 3,5,11 are best possible face-transitive PMs. The talk ends with U. Brehm's flat torus (1978) and realizations in E^3 of some of Coxeter's (and A. Boole-Stott's) finite skew polyhedra in E^3.

W. WINGATE:

Amalgamations of tilings

An amalgamation of a tiling T of the euclidean plane is a tiling each of whose tiles is a union of tiles of T, and an amalgamator of T is an isomorphism between T and an amalgamation of T.

Of particular interest are the amalgamations of isohedral tilings which are themselves isohedral. It is possible to generate all amalgamations of isohedral tilings which are "strongly isohedral" by examining subgroups of the symmetry group of the tiling which act effectively.

Anhang

A. DRESS:

The chamber system of a geometric cell complex: theory and applications

Let $\Sigma = \Sigma_n = < \sigma_o, \sigma_1, \ldots, \sigma_n \mid \sigma_i^2 = 1 >$ denote the free Coxeter group on $n+1$ generators $\sigma_o, \sigma_1, \ldots, \sigma_n$. A (thin) chamber system $\mathcal{C}$ of dimension n (without boundary) is a set $\mathcal{C}$ of "chambers" together with an action of Σ on $\mathcal{C}$ (from the right) such that each generator σ_i acts fixpointfree on $\mathcal{C}$. For any chamber system $\mathcal{C}$ the topological realization $\| \mathcal{C} \|_{top}$ is defined as the union of as many copies $\Delta(C), \Delta(C'), \ldots$ $(C, C', \ldots \in \mathcal{C})$ of the n-dimensional standard simplex $\Delta = \Delta_n \subseteq \mathbb{R}^{n+1}$ as we have chambers in $\mathcal{C}$ with the i-th face $\partial_i \Delta(C)$ of $\Delta(C)$ identified with $\partial_i \Delta(C\sigma_i)$ in the obvious way for all $C \in \mathcal{C}$ and $i = 0, 1, \ldots, n$.

A chamber system $\mathcal{C}$ is defined to be smooth, if $\| \mathcal{C} \|_{top}$ is a pl-manifold or - equivalently - if for all $i = 0, \ldots, n$ and all $C \in \mathcal{C}$ the topological realization of the orbits $C \cdot < \sigma_j \mid j \neq i >$ - considered as chamber-systems of dimension $n-1$ - are standard pl-spheres. So, in particular, $r_{ij}(C) := \inf(n > 0 \mid C(\sigma_i\sigma_j)^n = C)$ must be finite for all $i, j = 0, 1, \ldots, n$ and all C in a smooth chamber system.

For a smooth chamber system $\mathcal{C}$ one has (by standard algebraic topology)

$$\pi_1(\| \mathcal{C} \|, C) \simeq \Sigma_C / <\tau(\sigma_i\sigma_j)^{r_{ij}(C)}\tau^{-1} \mid \tau \in \Sigma,\ 0 \leq i < j \leq n>,$$

so for a "simply-connected" chamber system $\mathcal{C}$ one has $\Sigma_C =$ $<\tau(\sigma_i\sigma_j)^{r_{ij}(C)}\tau^{-1} \mid \tau \in \Sigma,\ 0 \leq i < j \leq n>$, in particular, if Γ is a group of automorphisms of $\mathcal{C}$ commuting with the Σ-action, if $\mathcal{D} = \Gamma \setminus \mathcal{C}$ denotes the Σ-set of Γ-orbits of $\mathcal{C}$ and if $\bar{r}_{ij} : \mathcal{D} \to \mathbb{N}$ denotes the induced map from $\mathcal{D}$ to $\mathbb{N}$, (well-!)defined by $\bar{r}_{ij}(\Gamma \cdot C) = r_{ij}(C)$, then $\mathcal{C} \simeq \Sigma_C \setminus \Sigma$ and Γ can be reconstructed from their "generalized Schläfli symbol" $(\mathcal{D}; \bar{r}_{ij}\ (0 \leq i<j \leq n))$. It is shown that this fact is especially useful in conjunction with (a) the observation that a smooth chamber $\mathcal{C}$ is (the bary-centric subdivision of) a geometric cell complex if and only if for all $C \in \mathcal{C}$ one has $r_{ij}(C) = 2$ for all $0 \leq i < j-1 < n$ and $r_{i-1,i}(C) \geq 3$ for all $i = 1, \ldots, n$ and (b) the theorem that a finite two-dimensional "Schläfli-symbol" $(\mathcal{D}; \bar{r}_{ij})$ stems from an equivariant chamber system $(\mathcal{C}, \Gamma)$ for which $\| \mathcal{C} \|_{top}$ can be identified with the euclidean plane in such a way that Γ becomes a (crystallographic) group of isometries if and only if

$$\sum_{D\in\mathcal{D}} \left(\frac{1}{r_{01}(D)} + \frac{1}{r_{02}(D)} + \frac{1}{r_{12}(D)}\right) - 1 = 0.$$

In addition, it is shown that the same analysis can be used to construct and/or to enumerate geometric cell complexes with certain prescribed properties (e.g. possessing a given group as a vertex-transitive automorphism group or - in the two-dimensional case - having prescribed genus and vertex - and/or face-structure).

Berichterstatter: Peter Gritzmann

Tagungsteilnehmer

Dr. K. Bezdek
Eötvös Lorant Universität
Abteilung Geometrie
Muzeum krt. 6-8

H - 1068 Budapest / Ungarn

Prof. Dr. S. Bilinski
Svibovac

Y - 41000 Zagreb

Jugoslawien

Prof. Dr. J. Bokowski
Fachbereich Mathematik
TH Darmstadt
Schloßgartenstr. 7

6100 Darmstadt

Dr. U. Brehm
Math. Inst. Techn. Univ. Berlin
Straße des 17.Juni 135

1000 Berlin

Prof. Dr. H. Crapo
62 Ave. de Lancry

F - 75010 Paris

Frankreich

Prof. Dr. L. Danzer
Math. Inst. der Univ. Dortmund
Postfach 50 05 00

4600 Dortmund 50

Prof. Dr. A. Dress
Fakultät für Mathematik
Universität Bielefeld
Universitätsstr.

4800 Bielefeld 1

Prof. Dr. G. Fejes Tóth
Math. Inst. of the Hungarian
Acad. of Sci.
Reâltanoda u. 13-15

H - 1053 Budapest / Ungarn

Dr. P. Floersheim
Organisch-Chemisches Institut
Universität Zürich-Irchel
Winterthurer Str. 190

CH - 8057 Zürich / Schweiz

Prof. Dr. J.F. Goodman
Dept. of Math.
City College of New York
310W 72nd Street

New York, N.Y. 10023

USA

Dr. P. Gritzmann
Fachbereich Mathematik
Universität Siegen
Hölderlinstr. 3

5900 Siegen

Prof. Dr. H. Harborth
Institut für Geometrie
Universität Braunschweig
Pockelsstr. 14

3300 Braunschweig

Dr. T. Havel
Inst. für Molekularbiologie
und Biophysik
ETH-Hönggerberg

CH - 8093 Zürich

Schweiz

Prof. Dr. W. Kühnel
Fachbereich Mathematik/FB 3
Technische Universität Berlin
Straße des 17.Juni 135

1000 Berlin

Prof. Dr. F. Hering
Abt. Statistik der
Universität Dortmund
Postfach 50 05 00

4600 Dortmund 50

Prof. Dr. C. Lee
Mathematisches Institut der
Universität Bochum
Universitätsstr. 150

4630 Bochum

Prof. Dr. F.C. Holroyd
Math. Dep., Open University

GB - Milton Keyns, MA7 6AA

England

Prof. Dr. P.McMullen
Department of Mathematics
University College London
Gower Street

GB - London WC1E 6BT

England

Prof. Dr. H.C. Im Hof
Mathematisches Institut der
Universität Basel
Rheinsprung 21

CH - Basel / Schweiz

Dr. B. Mohar
University of Ljubljana
Department of Mathematics
Jadranska 19

Y - 61111 Ljubljana/Jugoslawien

Prof. Dr. R.E. Jamison
Math. Inst. Univ. Freiburg
Albertstr. 23b

7800 Freiburg i.Br.

Prof. Dr. E. Molnar
Eötvös Lorand Universität
Muzeum krt. 6-8

H - 1088 Budapest / Ungarn

Dr. A. Kemnitz
Institut für Geometrie
Universität Braunschweig
Pockelsstr. 14

3300 Braunschweig

Prof. Dr. O.J. Moser
McGill University
Burnside Hall
805 Sherbrooke St.

Montreal H3A 2K6, Quebec

Kanada

Dr. P. Kleinschmidt
Mathematisches Institut
Universität Bochum
Universitätsstr. 150

4630 Bochum

Prof. Dr. R. Pollack
Courant Institute
251 Mercer St.

New York, N.Y. 10012

USA

Prof. Dr. G. Purdy
Mathematics Dept.
Texas A & M University
College Station

Texas 77843 / USA

Prof. Dr. Asia Ivič Weiss
Department of Mathematics
York University
4700 Keele St.

Downsview, Ont. M3T IP3

Kanada

Prof. Dr. H. Raszillier
Physikalisches Institut
Universität Bonn
Nussallee 12

5300 Bonn 1

Prof. Dr. N. White
Department of Mathematics
University of Florida

Gainsville, Florida 32166

USA

Dr. R. Scharlau
Fakultät für Mathematik
Universität Bielefeld
Universitätsstr.

4800 Bielefeld

Prof. Dr. W. Whiteley
Dept. of Mathematics
Champlain Regional College
900 Riverside Drive

St. Lambert, Que. 54P 3B8
Kanada

Dr. E. Schulte
Math. Inst. der Univ. Dortmund
Postfach 50 50 00

4600 Dortmund 50

Prof. Dr. J.M. Wills
Mathematisches Institut
Universität Siegen
Hölderlinstr. 3

5900 Siegen

Dr. Ch. Schulz
Fernuniversität Hagen
Zentrum für Fernstudienent-
wicklung
Postfach 940

5800 Hagen

Dr. W. Wingate
Math. Dep., Open University

GB - Milton Keyns, MA7 6AA

England

Prof. Dr. Marjorie Senechal
Smith Coll.

Northampton, MA 01063

USA

Gefördert durch
DFG Deutsche Forschungsgemeinschaft