MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 44/1984

Analytische Zahlentheorie

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Die Tagung fand unter der Leitung von Herrn Professor Dr. H.-E.Richert, Herrn Professor Dr.W.Schwarz und Herrn Professor Dr.E.Wirsing statt. Im Mittelpunkt des Interesses standen Fragen aus der Zahlentheorie, die vorwiegend mit analytischen Methoden behandelt wurden.

40 Teilnehmer aus 13 Ländern trugen zum Erfolg dieser Tagung bei. Im relativ kleinen Teilnehmerkreis war es möglich nicht nur ausführlich über die z.T. überdurchschnittlich guten Vorträge zu diskutieren sondern auch in kleinen Gruppen mathematische Probleme zu erörtern. Unter anderem wurde über die Lösungen zweier berühmter Preisfragen von P.Erdös vorgetragen (Wert $600, $500; H.Maier-G.Tenenbaum , H.L.Montgomery-R.C.Vaughan). Der Vortrag von E.Fouvry handelte von der Unlösbarkeit des Fermat'schen Problems (Fall 1).

Einen wesentlichen Anteil am Gelingen dieser Tagung hatte wiederum das Oberwolfacher Institut unter der Leitung von Herrn Professor Dr.M.Barner. Die Atmosphäre, die das Institut ausstrahlt, ist einmalig. Abgerundet wurde der angenehme Aufenthalt durch die hervorragende Unterstützung, die wir stets durch das Personal des Instituts hatten.

Vortragsauszüge

A.BALOG:

p+a without large prime factors

Let $a \neq 0$ be a fixed integer, and

$$\pi(x,y) = \pi_a(x,y) = \sum_{\substack{p \leq x \\ P(p+a) \leq y}} 1 ,$$

where $P(n)$ denotes the greatest prime factor of $n \neq 1$. It is shown that

THEOREM: If $y \geq x^{0.35}$ then

$$\pi(x,y) \gg \frac{x}{\log^2 x} .$$

In other words, for a great many primes p all prime factors of $p+a$ are at most $p^{0.35}$.

The result has the following

COROLLARY: For infinitely many n the number of solutions of the equation

$$\varphi(m) = n \quad (\varphi \text{ is Euler's function})$$

is at least $n^{0.65}$.

J.-M. DE KONINCK:

On the distance betweeen consecutive divisors of an integer

Let $\omega(n)$ denote the number of distinct prime divisors of a positive integer n. Then we define $h: \mathbb{N} \to \mathbb{R}$ by $h(n)=0$ if $\omega(n) \leq 1$ and $h(n) = \sum_{i=2}^{r} 1/(q_i - q_{i-1})$ if $n = q_1^{\alpha_1} \cdots q_r^{\alpha_r}$, where $q_1 < q_2 < \ldots < q_r$ are primes and $r \geq 2$. Similarly denote by $\tau(n)$ the number of divisors of n and let $H: \mathbb{N} \to \mathbb{R}$ be defined by $H(n) = \sum_{i=2}^{\tau(n)} 1/(d_i - d_{i-1})$, where $1 = d_1 < d_2 \ldots < d_{\tau(n)} = n$ are the divisors of n. We prove that there exists constants A,B such that

$$\sum_{n \leq x} h(n) = Ax + O(x(\log\log x)(\log x)^{-1}) \quad \text{and}$$

$$\sum_{n \leq x} H(n) = Bx + O(x(\log x)^{-1/3}) .$$

H.DELANGE:

Moments of additive functions

Recently K. Alladi developped a method to estimate sums of the form $$\sum_{\substack{n\in S\\ n\leq x}} a_n (f(n)-A(x))^q$$
where f is a strongly additive function and S is an infinite set of positive integers satisfying some general conditions (involving the a_n's), q is a positive integer. He uses the combinatorial sieve. We give a method which is both simpler (no sieve technique is needed) and more powerful (the considered function need not be non-negative).

G.DUFNER:

An analytical approach to the prime number theorem of Pjatecki-Shapiro

Der Primzahlsatz von Pjatecki-Shapiro besagt, daß für $\gamma\in(\frac{11}{12},1)$ die Asymptotik $$\sum_{\substack{m\leq x\\ [m^{1/\gamma}]\text{ prim}}} 1 \sim \gamma\frac{x}{\log x}$$
gilt. Dieses Resultat - und ebenso das mehrfache Ausweiten des γ- Bereichs auf bislang $(\frac{34}{39},1)$ - beruht im wesentlichen auf der Verwendung ausgefeilter Methoden im Umgang mit Exponentialsummen.

Hier soll ein neuer Zugang angedeutet werden: Zur Abschätzung des Restgliedes werden Dirichlet'sche L-Reihen ins Spiel gebracht und jeweils die expliziten Formeln eingesetzt, so daß dasselbe als Nullstellensumme behandelt werden kann.

Das numerische Ergebnis ist -trotz Verwendung der verallgemeinerten Riemann'schen Vermutung - schwach: Asymptotik für $\gamma\in(\frac{21}{22},1)$, Existenz ∞-vieler γ-Primzahlen für $\gamma\in(\frac{8}{9},1)$.

P.D.T.A.ELLIOTT:

Arithmetic functions and integer products

Let Q^* be the multiplicative group of positive rationals. Let $R(x)$ be a rational function $P_1(x)/P_2(x)$ with $P_i(x) \in \mathbb{Z}[x]$,positive leading coefficient . Let $\Gamma(R(x))$ be the subgroup of Q^* generated by the $R(n)$ with $n>k$, and form the quotient group $G=Q^*/\Gamma(R(n))$. For

$$R(x)=\prod_{i=1}^{s}(x-a_i)^{b_i}$$

$a_i,b_i \in \mathbb{Z}$, $(b_1,\ldots b_s)=1$; $R(x)=x^2+bx+c$, $b^2 \neq 4c$; $R(x)=x(x^2+c)$, $c \neq 0$ $R(x)=(ax+b)/(cx+d)$, $ad \neq bc$; $R(x)=x^{-1}(bx^2+c)$, $bc \neq 0$, G has the form Free group $\oplus$ Finite group. CONJECTURE 2: This is true whenever $R(x)$ is irreducible or ,more generally , does not have a power factor . In examples 1,3,5 G does not depend on k . CONJECTURE 1: This is always true .

THEOREM 1 . Let f , defined on the positive integers , take values $\mathbb{R}/\mathbb{Z}$ and satisfy $f(ab)=f(a)+f(b)$ when $(a,b)=1$. Suppose that f is constant on a sequence of integers of positive upper density d . Then there is a positive integer $m \leq 1/d$ for which the series

$$\sum_{\|mf(p)\| \neq 0} \frac{1}{p}$$

converges to a sum bounded in terms of d . Here $\|y\|$ denotes the distance of y to the nearest integer . Using this we obtain

THEOREM 2 . Let the integers $a_1,a_2,\ldots$ form a sequence of upper density $d>0$. Let $\Gamma(a_n)$ be the subgroup of Q^* they generate , and let $G=Q^*/\Gamma(a_n)$. Then there is a sequence of primes q_j for which $\sum 1/q_j$ does not exceed a constant depending on d , so that those $n(\bmod \Gamma)$, with n which are not divisible by any q_j generate a subgroup of G of order $\leq 1/d$.

COROLLARY . Every such n has a representation

$$n^s=\prod_{j=1}^{k} a_{i_j}^{\varepsilon_j} \quad , \quad \varepsilon_j=\pm 1 \quad , \quad 1 \leq s \leq 1/d \ .$$

P. ERDÖS:

Solved and unsolved problems in combinatorial and analytic number theory

Several of my old problems have been solved in the last 2-3 years . Merrhy and I conjectured that $d(n)=d(n+1)$ has infinitely many solutions . Using a previous weaker result of Claudia Spiro this was proved by Heath-Brown .

I conjectured and Montgomery and Vaughan proved that if $1=a_1<a_2 \ldots < a_{\varphi(n)}=n-1$ are the integers relatively prime to n then

$$\sum_{i=1}^{\varphi(n)-1} (a_{i+1}-a_i)^r < c_r \frac{n^r}{(\varphi(n))^{r-1}} .$$

I offered 500 dollars for this .

One of my oldest conjectures stated that almost all integers n have two divisors $d_1<d_2<2d_1$. This and much more was proved by Maier and Tenenbaum . This cost me more than 600 dollars .

I conjectured that if $n_{k+1}/n_k>l>1$ then $n_1<n_2<\ldots$ can not be an essential component . This and much more was recently proved by Rusza.

Here are three very old conjectures:

Let $1=a_1<a_2 \ldots <a_h=n$, all sums $\sum_i \varepsilon_i a_i$ are distinct , $\varepsilon_i=0$ or 1 , max $h<\log n/\log 2 + C$?(500 dollars).

Let $a_1 \ldots$; $f(n)$ the number of solutions of $n=a_i+a_j$. Turán and I conjectured that if $f(n)>0$ then $\limsup f(n) = \infty$ (500 dollars).

$a_i \pmod{n_i}$ $1<n_1<n_2 \ldots <n_k$ is a covering congruence if every integer satisfies at least one of these congruences. Can n_1 be as large as we please? (1000 dollars). Schinzel and I believe that the fact that $n-2^u-2^v$ is not a prime for infinitely many n can not be proved by covering congruences , i.e. there is no finite set of primes $p_1,p_2,\ldots p_k$ so that for infinitely many n ,$n=2^u-2^v$ is always a multiple of one of these primes .

G.FREIMAN:

On the measure of large trigonometric sums

Let K be a set of k integers $K=\{a_0<a_1 \ldots <a_{k-1}\}$, $S_K(x)=\sum_{j=0}^{k-1} e(xa_j)$,

E_u-set of all those values of x for which $|S_K(x)| \geq k-u$, $0\leq x<1$,
$\mu_K(u)=\text{mes } E_u$, $\mu^*(u)=\sup_{|K|=k} \mu_K(u)$.

THEOREM 1 (Freiman 1968). Let $a_0=0$, $a_{k-1}<0.05k^{1.5}$, then

$$\mu^*(1)=\frac{2\sqrt{6}}{\pi} k^{-1.5} + O(k^{-2}).$$

THEOREM 2 (Yudin 1968). If $u=o(k)$, then

$$\mu^*(u)=\frac{2\sqrt{6}}{\pi k}(u/k)^{0.5}(1+o(1)).$$

Theorem 3 (Besser,Freiman,Lev 1984) $\exists\, \varepsilon>0$ such that for $u<\varepsilon k$

$$\mu^*(u)=\frac{2\sqrt{6}}{\pi k}(u/k)^{0.5}\Gamma(k,u) , \quad \Gamma(k,u)=\frac{c_0}{\frac{\sqrt{6}}{\pi k}(u/k)^{0.5}} ;$$

c_0 can be found from the equation $\sin(\pi c_0 k)/\sin(\pi c_0)=k-u$ .We have
$\lim_{\frac{u}{k}\to 0+}\Gamma(k,u)=1$, $\Gamma(k,u)=1-\frac{3}{10}u/k+\ldots$.

E.M.FOUVRY:

Brun-Titchmarsh theorem-application to Fermat last theorem

We prove new upper bounds for the function $\pi(x;q,a)$, valid for almost all q between $x^{1/2}$ and $x^{1-\varepsilon}$. We give an application to the greatest prime factor of p-1 , and also to the first case of Fermat last theorem (via a criterion due to Adleman and Heath-Brown).

The improvements are based on results coming from dispersion method and Kloosterman sums .

G.GREAVES:

The weighted linear sieve and Selberg's λ^2-method

As usual in the theory of sieves , write $\#\{a\in\mathcal{A}_X,\ a\equiv 0 \bmod d\} = X\rho(d)/d+R(\mathcal{A},d)$, where ρ is multiplicative and

$$-L<\sum_{w\le p<z}(\varrho(p)-1)\log p/p<A_1\quad,\quad \varrho(p)/p<1-1/A_2\ .$$

Introduce a 'level of distribution' $y=y(x)$ (e.g. such that $\sum_{d<y} 3^{\omega(d)}\ R(\mathcal{A},d)<x/\log^2 y\ ,x>x_o$) and a degree g such that $a\in\mathcal{A}_X \Rightarrow a<y^g$. We aim for results like : if $g<R-\delta_R=\Lambda_R$ then some a in $\mathcal{A}_X$ has $\le R$ prime factors . Numerical work based on the theorem below leads to the following improved values : $\delta_2=0.047$, $\delta_3=0.076$, $\delta_4=0.105$, although improvements on earlier results do not appear to follow for large R .

In the theorem there is a weight function $w(p)=W(\log p/\log y)$. Let $V<1/2<U<1$ with $V+RU\ge g$.We require $W(1)=U-V$ and

$$0\le W(t)\le t-V \quad \text{if} \quad V\le t\le U \text{ and } t>0 \qquad (1)$$

$$0\le W(t)\le t-\max(V,(1-U)/2) \quad \text{if} \quad \max(V,(1-U)/2)<t<1/3 \qquad (2)$$

$$W(t)\le 9(U-1/3)t^2 \quad \text{if} \quad 0<t<U. \qquad (3)$$

Let $p(a)$ denote the least prime factor of a.

THEOREM: Under the stated conditions

$$\sum_{\substack{a\in\mathcal{A}_X\\ \omega(a)\le R}} w(p(a))=2Xe^{\delta}\prod_{p<y}(1-\varrho(p)/p)\,(M(W)+O(L/\log^{1/3}y))+O(\sum_{d<y}3^{\omega(d)}\ R(\mathcal{A},d))\ ,$$

where

$$M(W)=-\int_{1/2}^{1}\frac{W(1)-W(t)}{1-t}\frac{dt}{t}+\int_0^{1/2}\frac{W(t)}{t}(1/(1-t)\ -h(t))dt,$$

for a certain function satisfying $h(t)<1/5$.

(We also suppose $1<L<\log^{1/3}y$.)

The function h is smaller than that in the author's article in Acta Arith.40 , but the requirements (2),(3) impose additional restrictions on the function w .

F.GRUPP:

<u>On the sieving limit in the Rosser-Iwaniec-Sieve</u>

Let $\kappa>1/2$ and $q(s)$ a special solution of the difference-differential-equation

$$(sq(s))' = \kappa q(s)+\kappa q(s+1)\ .$$

If $\varrho=\beta-1$ is the largest zero of $q(s)$, then β is the sieving limit of the Rosser-Ivaniec-Sieve with dimension κ . (cf. Iwaniec , Rosser's-Sieve,AA 36(1980)).

A good lower bound estimate for this largest zero ϱ is of importance for an improvement of Iwaniec's result . Iwaniec proved

$$\varrho > K + K\sqrt{1-1/K} \quad , K>3/2 .$$

For $K>2$ better results can be proved , for example

$$\varrho > K(2+\log 2) + \sqrt{K/2} - 3.5 \quad , \quad K>3,$$

holds true.

A.HILDEBRAND:

On integers free of large prime divisors

In 1938, in a well-known paper on large differences between consecutive primes, R.A.Rankin derived an upper bound for $\Psi(x,y)$, the number of positive integers $\leq x$ and free of prime factors $>y$, by means of a simple but very effective trick: For every $\alpha>0$

$$\Psi(x,y) = \sum_{\substack{n\geq 1\\ p/n \Rightarrow p\leq y}} (x/n)^{\alpha} = x^{\alpha} \prod_{p\leq y} (1-p^{-\alpha})^{-1} .$$

The optimal value α is given by the equation (*) $\log x = \sum_{p\leq y} \log p/(p^{\alpha}-1)$.

Using analytic tools, Rankin's upper bound can be improved to a fairly sharp approximation for $\Psi(x,y)$:

THEOREM (Hildebrand, Tenenbaum). Uniformly for $x\geq y\geq 2$, we have

$$\Psi(x,y) = c(x,y)x^{\alpha} \prod_{p\leq y} (1-p^{-\alpha})^{-1} (1+O(\log y/\log x)+O(\log y/y)) ,$$

where $c(x,y) = \alpha^{-1}(2\pi\sum_{p\leq y} p^{\alpha}\log^2 p/ (p^{\alpha}-1)^2)^{-1/2}$ and $\alpha=\alpha(x,y)$ is defined by (*).

The factor $c(x,y)$, which measures the dicrepancy between $\Psi(x,y)$ and Rankin's bound for $\Psi(x,y)$, is easy to estimate; it satisfies $(y/\log y)^{-1/2} \ll c(x,y) \ll (\log y)^{-1}$ uniformly for $x\geq y\geq 2$, and is asymptotically equal to $(2\pi y/\log y)^{-1/2}$, if y and $(\log x)/y$ tend to infinity .

K.-H. INDLEKOFER:

Some properties of multiplicative functions

Let f be multiplicative and $L^q=\{f:\mathbb{N}\to\mathbb{C},\ \|f\|_q=(\limsup x^{-1}\sum_{n\le x}|f(n)|^q)^{1/q}<\infty\}$ $(q\ge 1)$. Then the following result holds: Let $f\in L^q$, $\varepsilon>0, q>1$ and α be irrational. Then there exists a $x_0=x_0(\|f\|_q, q, \alpha, \varepsilon)$ such that $|x^{-1}\sum_{n\le x} f(n)e(n\alpha)|\le\varepsilon$ for all $x\ge x_0$. Further, some results on the characterisation of multiplicative functions $f\in L^1$ are given. In the last part of the talk several theorems on uniformly summable multiplicative functions are formulated. As an application it is shown that, if $f(n)=\tau^2(n)n^{-11}$, where τ denotes Ramanujan's τ-function, the mean-values of $|f|^{1-\varepsilon}$ exist and are zero for all $0<\varepsilon<1$, whereas the mean-value of f is different from zero (result of R.A.Rankin).

A.IVIĆ:

Some asymptotic formulas involving the largest prime factor of an integer

The distribution of prime factors of an integer is reflected in the asymptotic behaviour of sums with the functions $P(n)$, $\omega(n)$ and $\Omega(n)$. In the usual notations these functions denote the largest prime factor of $n\ge 2$, the number of distinct prime factors of n and the total number of prime factors of n respectively. A survey of recent results in this field is presented. These include the asymptotic formulas (obtained jointly with P.Erdös and C.Pomerance)

$$\sum_{2\le n\le x} (\Omega(n)-\omega(n))/P(n) = (C+o(1)) \sum_{2\le n\le x} 1/P(n) ,$$

$$\sum_{2\le n\le x} 1/P(n) = x\,\delta(x)\left(1+O\left(\left(\frac{\log\log x}{\log x}\right)^{1/2}\right)\right) .$$

Here $C>0$ is an absolute constant, and $\delta(x)$ is a precisely defined function for which we have, as $x\to\infty$,

$$\delta(x) = \exp(-(2\log x\cdot\log\log x)^{1/2}(1+O(\frac{\log\log\log x}{\log\log x}))).$$

M.JUTILA:

Applications of Voronoi's summation formula to Riemann's zeta-function

The following two topics are discussed:

(1) It is shown how the approximate functional equation for $\zeta^2(s)$ can be proved by Voronoi's summation formula ; this method gives an error term which is in general smaller than in the classical version of Hardy-Littlewood and Titchmarsh.

(2) I found recently an analogue of Voronoi's summation formula for sums of the type $\sum d(n)e(nh/k)f(n)$. By this formula it is possible to transform 'short' Dirichlet polynomials $\sum d(n)n^{-1/2-it}$, which gives a new method for the estimation of the zeta-function on the critical line .

I.KÁTAI:

Distribution of q-additive functions on the set of primes

Let $q\geq 2, q\in\mathbb{N}, \mathcal{A}_q=\{0,1,2\ldots q-1\}$. Then $n\in\mathbb{N}$ can be witten as $n=\sum_{j=0}^{\infty} a_j(n)q^j$, $a_j(n)\in\mathcal{A}_q$. Let $N=N(x)\in\mathbb{N}$ be defined by $q^N\leq x<q^{N+1}$. Let $(0\leq)j_1<\ldots<j_r\leq N$, $b_1,\ldots,b_r\in\mathcal{A}_q$; $\varrho=\begin{bmatrix} j_1,\ldots,j_r\\ b_1,\ldots,b_r\end{bmatrix}$; $\bar\varrho=\begin{bmatrix} 0 & \\ b & \varrho\end{bmatrix}$. Let

$$A(x|\varrho)=\#\{0\leq n<x: a_{j_l}(n)=b_l \quad (l=1,\ldots,r)\},$$

$$\pi(x|\varrho)=\#\{p<x, p \text{ prime}: \quad a_{j_l}(p)=b_l \quad (l=1,\ldots,r)\}.$$

The following assertions are proved:

THEOREM 1: If $0\leq r\leq\sqrt{N}$, $(b,q)=1$, then

$$\pi(x|\bar\varrho) = \frac{\operatorname{li} x}{q^r\varphi(q)} + O\left(\frac{\operatorname{li} x}{q^r}\exp(-dN^{1/2})\right) + O\left(\frac{\operatorname{li} x}{q^r} N^3 (q^{j_r}/x)^{1/2}\right).$$

THEOREM 2: If $2^r<N^{1/5}$, then

$$\frac{\pi(x|\bar\varrho)\log x}{A(x|\bar\varrho)} = \frac{q}{\varphi(q)} + O((\log x)^{9/20-1}) .$$

Several other results are stated.

H.W.LENSTRA:

Recent advances in primality testing

In the past few years notable progress has been booked in the area of primality testing. In this lecture we shall present the underlying ideas of the new methods, which are due to Adleman and Rumely. It will also be shown how these methods can be combined with the older methods, due to Lucas and Lehmer and developped further by Brillhart, Selfridge and Williams.

H.MAIER:

Incomplete convolutions of the Moebius function

A fundamental relation for the Moebius function is $\sum_{d/n}\mu(d)=\begin{cases}1 & \text{, if } n=1\\ 0 & \text{otherwise.}\end{cases}$

In this talk we consider $M(n)=\sup_{z\leq n}\left|\sum_{\substack{d/n\\ d\leq z}}\mu(d)\right|$. We line out the proof of two inequalities, which are true for all integers n from a sequence of asymptotic density 1:

$(\log\log n)^{\gamma}\leq M(n)\leq\psi(n)\log\log n$, $\gamma<-\dfrac{\log 2}{\log(1-1/\log 3)}=0.28754\ldots$, $\psi(n)\to\infty$ arbitrarily slowly.

The methods for the lower bound are an extension of those applied in the paper of G.Tenenbaum and the author.(On the set of divisors of an integer, Invent.Math.76, 121-128 (1984).)

H.L.MONTGOMERY:

The distribution of reduced residues (mod q)

In 1940, Erdös posed the problem of showing that

$$\sum_{i=1}^{\varphi(q)}(a_{i+1}-a_i)^2\ll q^2/\varphi(q) .$$

where $1=a_1<a_2 \ldots$ are the numbers relatively prime to q. Recently R.C.Vaughan and I received \$250 apiece for establishing this estimate. In fact we have shown that if $\gamma>0$ is a fixed number then

$$\sum_{i=1}^{\varphi(q)}(a_{i+1}-a_i)^{\gamma}\ll\varphi(q)\,(q/\varphi(q))^{\gamma} .$$

(C.Hooley,F.R.S., received no money for showing this for $\gamma<2$.) This follows easily from the following more fundamental result: Let k be a fixed positive integer. Then

$$\sum_{n=1}^{q}\Big(\sum_{\substack{m=1\\(m+n,q)=1}}^{h} 1 - Ph\Big)^{2k} \ll q(Ph)^{2k} + qPh$$

where $P=\varphi(q)/q$ is the 'probability' that a randomly chosen integer is coprime to q. This estimate can be seen to be sharp if

$$\prod_{\substack{p/q\\p>h}}(1-1/p) \leq 1/10 \quad .$$

Our proof depends on combining estimates obtained by means of finite Fourier transform with estimates based on combinatorial considerations.

W.G.NOVAK:

Some contributions to lattice point theory

At first we consider a compact domain $D\subset R^2$ the boundary $\partial D=C$ of which is a Jordan curve defined by $\phi(u,v)=0$, where ϕ is analytic on C and $\operatorname{grad}\phi\neq(0,0)$. For a large parameter t, we consider the 'lattice rest' $P(t)=|\sqrt{t}D\cap Z^2|-Vt$ (V the area of D). It is a classical result of Van der Corput that $P(t)\ll t^{\theta}$ for a $\theta<1/3$, provided that the curvature k of C vanishes nowhere. Colin and Verdière proved in 1977 that $P(t)\ll t^{(1-1/n)/2}$,if k has zeros of order $\leq n-2$. We obtain

THEOREM 1: Suppose that the slope of C is rational in every point P_i where $k=0$ and let n_i-2 denote the order of the zero of k in P_i. Then

$$P(t)=\sum_{P_i:k=0}\ \sum_{j\geq 1} F_{j,n_i}(t)\ t^{(1-j/n_i)/2} + O(t^{\theta}) \quad (\theta<1/3)$$

where the functions $F_{j,n_i}(t)$ are both $O(1)$ and(in general) $\Omega_{\pm}(1)$ as $t\to\infty$.

Furthermore we can give several Ω-estimates.

THEOREM 2: Let $D\subset R^2$ be compact,convex,$\partial D\in C^{\infty}$,$\underline{0}\in D^{0}$ and suppose that $k\neq 0$ throughout. Then $P(t)=\Omega_{-}(t^{1/4}(\log t)^{1/4})$.

THEOREM 3: For $s\geq 2$,$D\subset R^{s}$ suppose the above assumptions to be satisfied. Then $P(t)=\Omega(t^{(s-1)/4}(\log t)^{1/4})$.

J.PINTZ:

Oszillatory properties of arithmetical functions

Let $f(x)$ be a real function, $F(s)=\int_0^\infty f(x)x^{-s-1}dx$ be regular for $\sigma>\Theta$, but not for any $\sigma>\Theta-\varepsilon$. Let $F(s)$ has its 'lowest' singularity on the $\sigma=\Theta$ at the point $s=\Theta+i\gamma$, $\gamma>0$; and let us denote the number of sign changes of $f(x)$ in the interval $[0,Y]$ by $V(f,Y)$. J.Kaczorowski and the author have shown that if there are at most denumerable many singularities of $F(s)$ in some in some half-plane $\sigma>\Theta-c_0$ and for these singularities $F(s)=g_\nu(s)(s-\varrho_\nu)^{a_\nu}\log^{k_\nu}(s-\varrho_\nu)+F_\nu(s)$ (for $s\to\varrho_\nu$) where $g_\nu(s)$ is regular, $F_\nu(s)$ is meromorphic in $s=\varrho_\nu$, a_ν is an arbitrary complex, k_ν an arbitrary natural number or zero then $\liminf_{Y\to\infty} V(f,Y)/\log Y \geq \gamma/\pi$. A further thoerem, proved by the speaker asserts that if $\int_1^\infty \Delta(x)x^{-s-1}dx = F(s)/G(s)$ where $F(s),G(s)$ are regular and both of $O(|s+2|^{K-2})$ for $\sigma\geq 1$, further $F(s)/G(s)$ has a pole of order l at $s=\varrho_0=\beta_0+i\gamma_0$ $(\beta_0\geq 0)$ then $Y^{-1}\int_1^Y|\Delta(x)|dx \gg Y^{\beta_0}\log^{l-1}Y$, where the implied constant depends in an effective way on the functions $F(s),G(s)$ and ϱ_0. The two theorems have many number theoretical applications and are essentially best possible in the stated general form.

J.PITMAN:

Diophantine problems in many variables

Let F_i $(i=1,\dots,R)$ be R diagonal forms of degree n in s variables $x_1,\dots,x_s$. Davenport and Lewis studied in depth the problem of non-trivial solvability in integers of the system of equations $F_i=0$ when the forms F_i are integral. If the forms have real coefficients one seeks non-trivial solvability of the Diophantine inequalities $|F_i|<\varepsilon$, for arbitrarily small $\varepsilon>0$. The talk covered work on this and related problems by Cook, Tolliver, Lloyd, Nadesalingam and the speaker.

In order to attack the problem by the Hardy-Littlewood method, one needs to consider a mixed system of r inequalities $|F_i|<1$ and R-t equations $F_i=0$ with integral F_i's and to give bounds for the smallest non-trivial solution of such a system. This approach has been used for R=2 by the speaker and for $R\geq 2$ and odd n by Nadesalingam. Two of the main difficulties in extending this work to even n for R>2 have been the need for a suitable condition for the existence of a non-singular solution and the need for bounds on such a solution. Two lemmas overcoming these difficulties were discussed.

A. SCHINZEL:

The fundamental lemma of Brun's sieve in a new setting

The following theorem is presented: Let A be a finite set and $\{T_p\}$ a family of sets indexed by primes from a certain set P. Assume that for a certain multiplicative $\omega(d)$ defined on all squarefree positive integers d and for suitable real numbers $X>0, A_1\geq 1, A_2\geq 1, A_3, k\geq 1, \kappa$ we have

$$0\leq \omega(p)/p\leq 1-1/A_1 \quad \text{for all primes } p,$$

$$\sum_{w\leq p<z} \omega(p)\log p/p \leq \kappa \log(z/w) + A_2 \quad \text{for all } w,z \text{ with } 2\leq w<z,$$

$$\left|\left|A \cap \bigcap_{\substack{p\in P\\ p/d}} T_p\right| - \frac{\omega(d)}{d} X\right| \leq A_3 X^{1-1/K} d^{K-1} \omega(d) .$$

Then for all $z\leq X$ the number $S(A,P,z)=\left|A-\bigcup_{\substack{p\in P\\ p\leq z}} T_p\right|$ satisfies the relation

$$S(A,P,z)=X\prod_{p<z}(1-\omega(p)/p)(1+O(\exp(-(\frac{u}{K^2}(\log u - \log\log 3u-\log\kappa -2)))) +$$

$$O(\exp(-K\sqrt{\log X})) \quad , \text{ where } u=\log X/\log z.$$

W. SCHWARZ:

On a special partition function connected with the number of finite abelian groups of order n

In order to study the distribution of values of the function $a(n)$ (the number of non-isomorphic abelian groups of order n) A. Ivić (J. of Number Theory 16 (1983), 119-137) defined the function $E(x)=\sum_{n\leq x} b(n)$ where $b(n)$ denotes the number of essentially different solutions of the equation $n=a(s)$ in full-square s. $E(x)$ may be interpreted as a special partition function , and by applying a general result on partitions by W. Schwarz (1968) it is proved , that

$$\log E(x) = B\cdot\log^{2/3}x + B^*\cdot\log^{1/3}x \log\log x + O(\log^{1/3}x\sqrt{\log\log x})$$

(joint with J. Herzog).

The same method is applied to prove the Tauberian part of a result of J.L. Geluk (Proc. AMS 82 (1981), 571-575).

Finally J. Herzog showed, again applying a Tauberian result of the speaker, that there is an asymptotic formula for $E(x)$ itself.

G.TENENBAUM:

New methods and results in the study of some concentration functions

The arithmetical function $\Delta(n)=\max_u \operatorname{card}\{d:d/n,e^u<d\leq e^{u+1}\}$ can be easily interpreted in terms of the concentration function of the random variable D_n taking the values log d, as d runs through all divisors of n, with uniform probability $1/\tau(n)$. The best known results on the normal and average order of $\Delta(n)$ are

$$(1) \qquad (\log\log n)^{\gamma} < \Delta(n) < \Psi(n)\cdot\log\log n$$

for almost all n, where γ is any constant $<-\log 2/\log(1-1/\log 3))=0.28754\ldots$, and $\Psi(n)$ is any function tending to infinity.

$$(2) \qquad x\cdot\log\log x \ll \sum_{n\leq x} \Delta(n) < x\cdot L(\log x)$$

where L(u) is the slowly increasing function defined by $L(u)=\exp(c\sqrt{\log u\cdot\log\log u})$ for a suitable absolute constant c.

Result (1) is due to Maier and Tenenbaum (Inv.Math.76(1984)and J.L.M.S.,to appear).In particular,the lower bound solves an ancient conjecture of Erdös (1934).

Result (2) was proved in a joint work with R.R.Hall (J.L.M.S. (2)25(1982)) for the lower bound,and the upper bound ,proved by the author,is submitted for publication. It improves on previous works by Hooley and Hall-Tenenbaum.As shown by Hooley,it has applications to different branches in number theory,as Diophantine approximation and Waring's problem.

V.SÓS:

Additive properties of sequences

Let A: $1\leq a_1$ ... be a sequence of integers, $R_1(n)$ be the number of solutions $a_i+a_j=n$, $R_2(n)$ respectively $R_3(n)$ be the number of solutions $a_i+a_j=n$, $a_i<a_j$,respectively $a_i\leq a_j$. Different behaviours of $R_1(n)$ are investigated (Erdös-Sárközi,Erdös-Sárközy-Sós).E.g.,if $R_1(n)$ is non-increasing for $n>n_o$,then $n\in A$ for $n>n_1$,but there exists an A such that $R_2(n)$ is non-increasing and $\sum_{a_i\leq n} 1 < n-cn^{1/3}$. A is called Sidon-sequence if $R_3(n)\leq 1$.Analogously we call $S\subset G$ a Sidon-set of the group G,if for all $x,y,z,w\in S$ of which at least three are different $xy\neq zw$ (resp. $xy^{-1}\neq zw^{-1}$).

THEOREM:Every group G of order n contains a Sidon-set of size $>cn^{1/3}$. For some abelian groups $n^{1/2}$ (as best possible result) can be proved. These and some further results have applications in combinatorial grouptheory.(Results with L.Babai,Europian J.of Comb.)

R.C.VAUGHAN:

On Warings problem for smaller exponents

Let R(n) denote the number of natural numbers not exceeding N which are the sum of three cubes of natural numbers, let G(k) denote the smallest s such that every sufficiently large natural number is the sum of at most s kth powers of natural numbers and let $G_1(4)$ denote the smallest s such that whenever $1 \leq r \leq s$ every sufficiently large natural number n with $n \equiv r \pmod{16}$ is the sum of at most s biquadrates. Then Davenport showed that $R(N) \gg N^{13/15-\varepsilon}$ (1939), $R(N) \gg N^{47/54-\varepsilon}$ (1951), $G_1(4) \leq 14$ (1939), $G(5) \leq 23$, $G(6) \leq 36$ (1942). His methods can be used to show that $G(7) \leq 53$, $G(8) \leq 73$ and Thanigasalam(1977) has shown that $G(9) \leq 90$. Very recently Thanigasalam has shown that $G(5) \leq 22$, $G(6) \leq 34$, $G(7) \leq 50$, $G(8) \leq 68$, $G(9) \leq 87$.

We introduce a new method which enables us to establish the following theorems

THEOREM 1. $R(N) \gg N^{8/9-\varepsilon}$.

THEOREM 2. $G_1(4) \leq 13$.

THEOREM 3. $G(5) \leq 21$, $G(6) \leq 32$, $G(7) \leq 45$, $G(8) \leq 62$, $G(9) \leq 82$.

R.WARLIMONT:

Covering sets by subsets

Let M be a finite non-empty set and a mapping M→ collection of all subsets of M , $x \mapsto M(x)$ such that $M(x) \neq \emptyset$ for all $x \in M$ and $x \in M(y) \Rightarrow y \in M(x)$ for all $x,y \in M$. Let $\mathfrak{X}$ be the collection of all subsets $X \subset M$ such that $\bigcup_{x \in X} M(x) = M$. Since $M \in \mathfrak{X}$ the collection $\mathfrak{X}$ is $\neq \emptyset$. Put

$$m = \min_{X \in \mathfrak{X}} |X| \quad \text{and} \quad h = \min_{x \in M} |M(x)| .$$

Then the following upper estimates of m are stated:

(1) $m \leq |M| (1+\log h)/h$

(2) $m \leq 1 + [(\log |M| / \log(1/(1-h/|M|)))]$.

Applications to Abbotts latice point problems and additive situations in number theory are given.

E.WIRSING:

Multiplicative functions with $\Delta f \to 0$

The following theorem is proved:
Let G^* be the set of non-zero Gauss-integers (or integers in an other imaginary quadratic field). If $f: G^* \to R/Z$ is additive and $\Delta f \to 0$ in the cononical metric of this group, then

$$f(a) = \tau \log|a| / Z + k \arg a / 2\pi, \quad \tau \in R, \quad k \in Z.$$

This sharpens a theorem of Kátai and Amer, where the condition

$$|\Delta f(a)| \leq \delta(|a|), \quad \sum_r \delta(2^r) < \infty$$

is needed.

It applies to multiplicative $F: N \to C$ as follows: If again $\Delta F(a) \to 0$ as $|a| \to \infty$, then

$$F(a) \to 0 \quad \text{or} \quad F(a) = a^{\sigma + i\tau} e^{ik \cdot \arg a}.$$

Concerning the analog theorem with N instead of G^* see the lecture during the meeting on Diophantine Approximation, this april .

D.WOLKE:

Some applications of zero density theorems for L-functions

By means of zero density results the following mean value theorem (which is similar to the Bombieri-Vinogradov theorem) is proved

THEOREM 1. Let $S(N,\alpha) = \sum_{n \leq N} \Lambda(n) e(n\alpha)$, $\alpha \in R$, $B > 0$, $Q = N^{1/3-1/100}$, $\mu = Q^{-3}(\ln N)^{-2(B+7)}$. Then

$$\sum_{q \leq Q} \max_{(a,q)=1} \max_{y \leq N} \max_{|\beta| \leq \mu} \left| S(y, a/q+\beta) - \frac{\mu(q)}{\varphi(q)} \sum_{n \leq y} e(n\beta) \right| \ll_B \frac{N}{\log^B N}.$$

There are no immediate applications of this theorem, but by a slight modification of the proof one gets the following version of the Goldbach-Vinogradov theorem.

THEOREM 2. There is a subset $\mathbb{P}_1 \subset \mathbb{P}$ with the properties :

(1) $P_1(x) = |\{p_1 \leq x, p_1 \in \mathbb{P}_1\}| \ll x^{9/10+\varepsilon}$.

(2) For all $N \geq N_0$, $N \equiv 1 (2)$: $N = p_1 + p_2 + p_3$, $p_i \in \mathbb{P}_1$.

(3) $\mathbb{P}_1$ is the union of sets $\{p \equiv 1 (q_i), q_i \text{ prime}, M_i < p \leq 2M_i\}$.

On the minor arcs a recent result of Balog and Perelli is used.

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