MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 45/1984

Nichtlineare Evolutionsgleichungen

21.10. bis 27.10.1984

Diese Tagung wurde organisiert durch die Herren Klainerman (Courant Institute for Mathematical Sciences in New York University) und von Wahl (Universität Bayreuth). In insgesamt 34 Vorträgen wurde ein Überblick über die neuesten Entwicklungen auf diesem Teilgebiet der Mathematik gegeben, wobei sowohl konkrete Anwendungen auf nichtlineare partielle Differentialgleichungen als auch abstrakte Fragestellungen aus dem Gebiet der nichtlinearen Evolutionsgleichungen behandelt wurden.

Unter anderem wurden Resultate aus den folgenden Spezialgebieten vorgestellt: Nichtlineare Wellengleichungen und nichtlineare Schrödinger-Gleichungen, nichtlineare parabolische Gleichungen, Vlasov-Gleichungen, Navier-Stokes-Gleichungen, Verzweigung bei Evolutionsgleichungen, abstrakte nichtlineare Evolutionsgleichungen.

Das Vortragsprogramm wurde durch anregende Diskussionen und persönliche Gespräche in harmonischer Atmosphäre ergänzt. Die vorbildliche Organisation des Instituts trug wesentlich zum guten Gelingen der Tagung bei.

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Vortragsauszüge

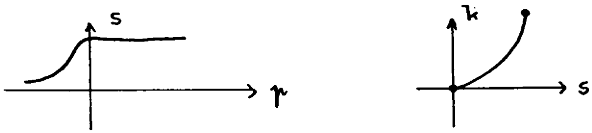
H. W. ALT:

Non-steady fluid flow through porous media

(Joint work with S. Luckhaus). The flow of water through an earth dam is described by the elliptic-parabolic equation

$$\partial_t s(p) - \nabla \cdot (k(s(p))(\nabla p + e_n)) = 0 \text{ in } \Omega \subset \mathbb{R}^n,$$

where p is the pressure, and the saturation s and the conductivity k are nonlinear functions like



We impose Dirichlet, Neumann, and overflow condition on $\partial\Omega$, and initial condition on s. Under regularity assumptions on the data there is a unique solution.

For certain applications one is interested in the solution p_ε for the saturation functions $s_\varepsilon(\xi) := s(\xi/\varepsilon)$ as $\varepsilon \to 0$. In the limit one gets the elliptic-hyperbolic free boundary problem

$$\partial_t s - \nabla \cdot (k(s)(\nabla p + e)) = 0 \text{ in } \Omega$$

with $s \in \chi_{\{p>0\}}$. We prove the strong convergence of a subsequence p_ε and $s_\varepsilon(p_\varepsilon)$ to a solution p,s satisfying an entropy condition.

H. AMANN:

Quasilinear parabolic systems

We consider general quasilinear parabolic systems of order 2m, acting on N-vector-valued functions under Dirichlet boundary conditions on a bounded domain. Given only regularity hypotheses, but neither compatibility nor growth conditions, we show that there exists a unique maximal classical solution depending continuously upon the initial data. The proof is based upon a general theorem about quasilinear parabolic evolution equations in Banach spaces.

W. ARENDT:

Kato's inequality. A characterization of generators of positive semigroups

Let A be the generator of a (linear) C_o-semigroup on $X = L^p(\Omega,\Sigma,\mu)$, where $1 \le p < \infty$ and (X,Σ,μ) is σ-finite. Then the semigroup consists of positive (= positivity preserving) operators if and only if A satisfies the following abstract version of Kato's inequality

$$\langle \operatorname{sign} f \; Af, \varphi \rangle \le \langle |f|, A'\varphi \rangle$$
$$f \in D(A), \quad 0 \le \varphi \in D(A')$$

and there exists $\varphi \in D(A')$ such that $\varphi(x) > 0$ a.e. and $A'\varphi \le \lambda\varphi$, where A' denotes the adjoint of A.

C. BARDOS:

Regularity properties for the Vlasov equation

This talk is devoted to the study of different aspects of the Vlasov Poisson and Vlasov Maxwell equations. In particular one shows the existence of a dispersion relation for the charge. This is used to prove the existence, in the large of a smooth solution in 3 space variables with small initial data.

N. BAZLEY:

A class of explicitly resolvable evolution equations

With R. Weinacht we consider a real Hilbert space $\mathcal{H}$ with scalar product (u,v). Let A be a linear, self-adjoint operator in $\mathcal{H}$ having a purely point spectrum. Let g(z) be a continuous real valued function of the real variable z.

We consider the nonlinear evolution equation

$$\frac{dv}{dt} + g\{\tfrac{1}{2}(Av,v)\}Av - \lambda v = h(t),$$

where λ is a real parameter and h(t) is a given vector. We reduce its solution to that of an explicitly known nonlinear functional differential equation. A stability analysis of the steady state solution is given and domains of attraction are discussed.

J. BEMELMANS:

A non-stationary free boundary problem for the Navier-Stokes equations

We consider the free boundary problem for the Navier-Stokes system:

$$(1)\quad \begin{aligned} v_t - \nu\Delta v + Dp + (v\cdot D)v &= f \\ \operatorname{div} v &= 0 \end{aligned} \qquad \text{in } Q_T = \bigcup_{0<t<T} \Omega(t)$$

$$(2)\quad v\cdot n = v_\Sigma,\; t_k\cdot T(v,p)\cdot n = 0 \qquad \text{on } \Gamma_T = \bigcup_{0<t<T} \partial\Omega(t)$$

$$(3)\quad n\cdot T(v,p)\cdot n = p_o \qquad \text{on } \Gamma_T$$

$$(4)\quad v(x,0) = v_o(x) \qquad \text{in } \Omega(0).$$

It describes the motion inside a fluid body $\Omega(t)$ as well as the shape of its free boundary $\partial\Omega(t)$ under the influence of an exterior force density f and a "athmospheric" pressure p_o. The problem can be regarded as an extension of the problem of classical equilibrium figures of rotating liquids. The basic existence theorem will be established via hard implicit function theorems.

E. DI BENEDETTO:

Hölder estimates for non-linear degenerate evolution systems

We establish Hölder estimates for the spatial gradient ∇u of local weak solutions of degenerate parabolic systems of the form

$$\frac{\partial}{\partial t}u^i - \mathrm{div}(|\nabla u|^{p-2}\nabla u^i) = F_i(x,t,\nabla u) \qquad (i = 1,2,\dots,m)$$

where $u = (u^1,\dots,u^m)$, $x \in \Omega \subset \mathbb{R}^N$, $t > 0$, $\max\{1,\frac{2N}{N+2}\} < p < \infty$ and

$$|F_i(x,t,\nabla u)| \leq C|\nabla u|^{p-1} + f_i(x,t),$$

$$f_i \in L^q(\Omega_T), \; q > \frac{pN}{p-1}:$$

here Ω is any bounded open set, $0 < T < \infty$, and $\Omega_T = \Omega \times (0,T)$.

We also establish Hölder continuity of nonnegative solutions of the degenerate parabolic equation

$$\frac{\partial}{\partial t}u - \frac{\partial}{\partial x_k}(a^{k,l}(x,t,\nabla u)\frac{\partial}{\partial x_l}u^m) = f(x,t,u,\nabla u), \; m > 1.$$

P. BRENNER:

Everywhere defined scattering operators for nonlinear Klein-Gordon equations

Space-time estimates for the linear Klein-Gordon equation are derived, and then applied together with previous results by the author to prove similar estimates for the nonlinear Klein-Gordon equation. Applying an argument originally due to W. A. Strauss in the small data case, we then use these estimates to prove the existence of everywhere defined scattering operators in the case of nonlinearities of the form $f(u) = |u|^{\rho-1}u$ for $1+4/n < \rho \leq 1+4/(n-1)$ for $n \geq 3$.

T. CAZENAVE:

Uniform estimates for solutions of some nonlinear evolution equations

We consider semilinear Klein-Gordon equations of the type

(K.G) $u_{tt} - \Delta u + ku = g(u)$ in $\mathbb{R}^N$

and semilinear heat equations of the type

(H) $u_t - \Delta u = g(u)$ in $\Omega \subset \mathbb{R}^N$, $u = 0$ in $\partial\Omega$.

On applying a method based upon differential inequalities, we prove that if g satisfies some growth and superlinearity conditions, then any global solution of (K.G) or (H) (that is any solution that exists for all $t \geq 0$) is uniformly bounded (with respect to t) in some appropriate norms.

P. CONSTANTIN:

Blow up for nonlocal evolution equations

We show that the solution to the vorticity equation

$$\begin{cases} \frac{\partial}{\partial t}\omega(t,x) = \omega(t,x)(H\omega)(t,x) \\ \omega(0,x) = \omega_0(x) \end{cases} \qquad t \geq 0,\ x \in \mathbb{R},\ H = \text{the Hilbert transform}$$

can be explicitly computed (S. Klainerman, A. Majda, P. Lax, P. Constantin). In particular, the blowing up solutions can be analyzed.

We prove blow up for the Fake-Euler equation (semi-lagrangian Euler eq.)

$$\begin{cases} \frac{\partial}{\partial t}U(t,x) + U^2(t,x) = -R(TrU^2)(t,x) \\ U(0,x) = U_0(x) \end{cases}$$

$U = (U_{ij})_{i,j=1,..n}$; $R = (R_i R_j)_{i,j=1,..n}$, R_i the Riesz transforms, $x \in \mathbb{R}^n$ on Π^n.

G. DA PRATO:

Maximal regularity for abstract parabolic and elliptic systems

Consider the equation

(1) $u'(t) = A(t)u(t) + g(t)$

with periodic solution $u(0) = u(2\pi)$. We prove the existence of regular solutions of problem (1) under suitable hypotheses for $A(t)$. We use the linear results to study a nonlinear problem

(2) $u' = g(\lambda,t,u)$

by linearization.

Y. EBIHARA:

Classical solutions of $U_{tt} = (U^m)_{xx}$ $(m > 1)$

We establish the local existence and uniqueness of classical solutions to the following problem:

$$\begin{cases} u_{tt}(x,t) = \{(u(x,t))^m\}_{xx}, & a < x < b,\ t > 0 \\ u(x,0) = u_0(x),\ u_t(x,0) = u_1(x), & a < x < b \\ u(a,t) = u(b,t) = 0,\ t \geq 0, \end{cases}$$

where m is an integer with $m \geq 5$, and $-\infty < a < b < \infty$.

We apply Galerkin approximate procedure with a certain penalty scheme.

H. ENGLER:

Gradient estimates for first order quasilinear evolution equations in bounded domains

We study initial-boundary value problems for Hamilton-Jacobi equations

$$\partial_t u + H(x,u,\nabla_x u) = 0$$

in bounded cylindrical smooth domains $\Omega \times (0,T)$. We show under "natural" conditions for H and the boundary data

(i) existence of Lipschitz-continuous solutions for Lipschitz data

(ii) existence and regularity properties of generalized (viscosity-) solutions for continuous data

(iii) corresponding results for the stationary problem.

The proofs are based on parabolic regularization and (i) a version of Bernstein's technique, (ii) interpolation and approximation, (iii) fixed point arguments.

A. FRIEDMAN:

Blow-up of solutions of nonlinear parabolic equations

Consider

$$u_t - \Delta u = f(u) \text{ in } \Omega \times (0,\infty),$$
$$u = 0 \text{ on } \partial\Omega\ (0,\infty),$$
$$u(x,0) = \varphi(x) \text{ on } \Omega$$

where Ω is a bounded convex domain in $\mathbb{R}^n$, $\varphi(x) \geq 0$ and $f(u) > 0$ if $u > 0$,

$$\int^{\infty} \frac{ds}{f(s)} < \infty.$$

Under some general conditions on f,φ, the solution blows up in finite time T. We are interested in the following questions: (i) What is the size of the blow-up set? (ii) Estimate $\max_{x\in\Omega} u(x,t)$ as $t \to T$. (iii) Find the smallest value q^* such that $\|u(t)\|_q \to \infty$ if $q > q^*$. It is proved that, for a large class of functions f (including $f = (u+\lambda)^p$, $\lambda \geq 0$, $p > 1$ or $f(u) = e^u$) the blow up set stays away from $\partial\Omega$; if $n = 1$ and $\varphi'(x)$ changes sign just once then the blow up set consists of a single point. If $f(u) = (u+\lambda)^p$,

$$u(x,t) \leq C(T-t)^{-1/(p-1)}$$

and $q^* = (p-1)n/2$.

Y. GIGA:

An application of L^p-theory for regularity of weak solutions of the Navier-Stokes system

We construct a local strong solution in $C([0,T],L^n(\Omega))$ and $L^q(0,T;L^p(\Omega))$ $(\frac{n}{p}+\frac{2}{q} = 1,\ p > n)$ for the Navier-Stokes initial value problem in a smoothly bounded domain Ω in $\mathbb{R}^n$ with L^n initial data. As an application we show that the time singularity set of Leray-Hopf weak solutions for $n=4$ has Lebesque measure zero.

J. GINIBRE:

Scattering theory in the energy space for the nonlinear Schrödinger (NLS) equation

We present a general theory of scattering for the NLS equation $i\dot{\varphi} = -(\frac{1}{2})\Delta\varphi + f(\varphi)$ for general initial data and/or asymptotic states in the energy space H^1. Here φ is a complex function defined in space time $\mathbb{R}^{n+1}$ $(n \geq 2)$ and f is a nonlinear interaction, typically

$$f(\varphi) = (\lambda_1|\varphi|^{p_1-1} + \lambda_2|\varphi|^{p_2-1})\varphi \qquad (*)$$

with $1 \leq p_1 \leq p_2$. The existence of the wave operators is proved by solving the Cauchy problem at infinity by a contraction method in a space of functions satisfying suitable space time integrability properties. That argument yields asymptotic completeness for small data in H^1 as a byproduct. Finally for $n \geq 3$ asymptotic completeness in H^1 for repulsive interactions is proved by an extension of the method of Lin and Strauss. In particular, all solutions with initial data in H^1 are proved to satisfy suitable space time integrability properties under assumptions on f that reduce to $\lambda_i > 0$ and $1+4/n < p_1 \leq p_2 < 1+4(n-2)$ in the special case (*).

H. KIELHÖFER:

Some new aspects of Hopf bifurcation for evolution equations

We consider parameterdependent evolution equations

$$\frac{du}{dt} + A(\lambda)u + F(\lambda,u) = 0$$

in some real Hilbert space E. The classical Hopf bifurcation theorem was considerably generalized by Alexander-Yorke, Ize, Chow-Mallet-Paret-Yorke. Their condition simply says that at a critical value λ_o of the parameter λ a nonzero number of eigenvalues of $A(\lambda)$ crosses the imaginary axis apart from zero. We think that this nonzero crossing number is not the essential condition for Hopf bifurcation. After fixing the phase and eliminating the unknown period we end up with a system of codimension 1. It is a nonzero crossing number of this reduced system which entails bifurcation. Our condition allows a zero crossing number of the original system as well as an eigenvalue zero of $A(\lambda_o)$.

S. KLAINERMAN:

Nonlinear wave equations

We consider equations of the form,

(NWE) $\Box u = F(u,u',u'')$

(NKGE) $\Box u+u = F(u,u',u'')$

where $u = u(t,x)$, $x \in \mathbb{R}^n$, u',u'' the first and second derivatives of u, F a smooth function of u,u',u'' vanishing of first order for $u,u',u'' = 0$. Consider the initial value problem

(I.V.P.) $u = \varepsilon f(x)$, $u_t = \varepsilon g(x)$ at $t = 0$

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and $T_*(\varepsilon)$ the life span of solutions of the nonlinear equations subject to the I.V.P. with $f,g \in C_0^\infty(\mathbb{R}^n)$. We prove, for ε sufficiently small,

Theorem 1. For (NWE) if F does not depend on u, or can be written in conservation form we have $T_*(\varepsilon) = \infty$ for $n > 3$ and $T_*(\varepsilon) \geq \exp(A\frac{1}{\varepsilon})$ of $n = 3$.

Theorem 2. For (NKGE) in $n = 3$, $T_*(\varepsilon) = \infty$.

H. LANGE:

Blow-up for a class of nonlinear Schrödinger equations

We consider nonexistence of classical solutions to nonlinear Schrödinger equations of the form

$$(1)\quad \begin{cases} iu_t = -u_{xx} + f(|u|^2)u + \kappa\partial_x^2 h(|u|^2)h'(|u|^2)u \\ u(x,0) = u_0(x) \end{cases}$$

(f,h given real functions, $\kappa = \text{const.}$) arising in various physical situations (plasma physics, Heisenberg ferromagnet, superfluid films). If certain assumptions are valid, namely

$$f(s)\cdot s \leq cg(s) \quad (\forall s \geq 0),\ g(s) = \int_0^s f(t)\,dt,$$

$$\kappa\frac{d}{ds}(h'(s))^2 \geq 0 \quad (\forall s \geq 0),$$

$$c \geq 3\ (\kappa = 0),\ 3 \leq c \leq 4\ (\kappa > 0),\ c \geq 4\ (\kappa < 0)$$

and either $E_1(0) < 0$ or $E_1(0) \leq 0$, $2G(0)^2 > (c-1)E_1(0)F(0)$ [where $E_1(0), G(0), F(0)$ are defined by $F(t) = \int_{\mathbb{R}} x^2|u(x,t)|^2dx$, $G(t) = \operatorname{Im}\int_{\mathbb{R}} xu^*u_x\,dx$, $E_1(t) = \int\{|u_x|^2 + g(|u|^2) - \kappa/2[\partial_x h(|u|^2)]^2\}\,dx$], then every classical solution to (1) has a finite life span.

V. LOVICAR:

Periodic solutions to some nonlinear evolution equations

The problem (P) given by

$$(P)\quad \begin{cases} c(-1)^{m+1}\dfrac{\partial^{2m}u}{\partial t^{2m}}(t,x)+(-1)^n\dfrac{\partial^{2n}u}{\partial x^{2n}}(t,x)=F(u(t,x)),\ 0<x<\pi, \\ \dfrac{\partial^{2j}u}{\partial x^{2j}}(t,0)=\dfrac{\partial^{2j}u}{\partial x^{2j}}(t,\pi)=0,\ t\in R,\ j=0,\dots,n-1, \\ u(t+2\pi,x)=u(t,x),\ t\in R,\ 0<x<\pi \end{cases}$$

is considered, where $m,n \in N$, $c>0$ and F is an odd sublinear or superlinear function. The assumptions on c,m,n,F are stated, under which the problem (P) has a sequence of generalized non-zero solutions, whose (suitable) norms converge to 0 (if F is sublinear) or to ∞ (if F is superlinear).

S. LUCKHAUS:

Global boundedness for a differential-delay equation

We consider the equation

(1) $\partial_t u(x,t) - \Delta u(x,t) = u(x,t)(1-u(x,t-\tau))$

and the corresponding inequality. The question of boundedness is reduced to the corresponding question for

(2) $\partial_t u - \Delta u \leq u,\ u \geq 0,\ u(t,x)\cdot u(t-\tau,x) \equiv 0.$

The result is that

I If $\Omega = [a.b]$ and the b.c. are Neumann or Dirichlet then $\exists\ K(|b-a|)$ s.th. for $t > t(u_o)$, $u(t) < K$ for u sol. of (1).

II If $\tau < \tau(n)$ (resp. for Neumann cond. $\tau < \tau(n, \mathrm{curv}(\partial\Omega))$ the same results holds for any Ω.

III For large delays and large more-dimensional domains as well as for $\Omega = \mathbb{R}$, there is an exponentially growing solution of (2).

K. MASUDA:

Analyticity of solutions of some nonlinear evolution equations

We shall show, by a new method, that solutions of KdV equations are analytic in x if the initial functions are analytic. Using the same method, we can also treat, for instance, Euler equations on Ω (Ω: domain of R^n, $n = 2,3$), Navier-Stokes equations on R^n, degenerate parabolic equations of porous medium.

H. PECHER:

Low energy scattering for nonlinear Klein-Gordon equations

We study the scattering operator which belongs to the pair of equations

(NLKG) $u_{tt} - \Delta u + m^2 u + f(u) = 0$

and

(KG) $u_{tt} - \Delta u + m^2 u = 0.$

Here f denotes a C^1-function with $f(0) = 0$ and $|f'(s)| \leq c|s|^{\rho-1}$ for any $s \in \mathbb{R}$, $m \neq 0$.

The scattering operator which describes solutions of the Cauchy problem for (NLKG) asymptotically as $t \to \pm\infty$ can be shown to exist on a whole neighbourhood of the origin in energy space provided $1+\frac{4}{n-1} < \rho < 1+\frac{4}{n-2}$, where the space dimension $n \geq 2$ is arbitrary. This generalizes previous results of W. Strauss, Tsutsumi-Hayashi, and the author.

J. PRÜSS:

Wellposedness of some quasilinear equations from population biology

Let $\Omega \subset \mathbb{R}^N$ be a bounded smooth domain and consider the problem

$$(1)\quad \begin{cases} u_t + u_a + \delta u = \mathrm{div}_x[u \cdot \int_0^\infty \kappa(a,a',x,u)\nabla_x u(t,a',x)da'] & t,a \geq 0,\ x \in \Omega \\ u(t,0,x) = \int_0^\infty \beta(a,x,u)u(t,a,x)\ da, & t \geq 0,\ x \in \overline{\Omega} \\ u \cdot \int_0^\infty \kappa(a,a',x,u)\frac{\partial u}{\partial n}(t,a',x)da' = 0, & t,a \geq 0,\ x \in \partial\Omega \\ u(0,a,x) = u_o(a,x), & a \geq 0,\ x \in \Omega \end{cases}$$

where δ,β,κ,u_o are given functions, and $n(x)$ denotes the outer normal at $\partial\Omega$. We show how this problem arises in population biology and present several wellposedness results in $L^1_+(\mathbb{R}_+ \times \Omega)$, the natural space for this problem. In particular, we prove the existence of a nonlinear semigroup $T(t)$ in $L^1_+(\mathbb{R}_+ \times \Omega)$ associated with (1), assuming $\beta = \beta(a,x)$, $\delta = \delta(a,x)$ and $\kappa = \kappa(a,x)\delta_o(a-a')$, i.e. for the case when the right hand side of eq. (1) is a local operator.

M. SCHATZMAN:

Continuous Glimm functionals

For a system of hyperbolic conservation laws

(1) $u_t + f(u)_x = 0,\ x \in \mathbb{R},\ r \geq 0,\ u \in U \subset \mathbb{R}^n$

there exists a functional F such that

(2) $F(u) \sim TV(u)$

(3) if u is piecewise Lipschitz continuous and is an entropic solution of (1) with small enough initial data and values in an adequate bounded set

$$\frac{d}{dt}F(u(t)) \leq 0.$$

The uniqueness of the solution of Riemann problem can be deduced from this result.

T. SIDERIS:

Formation of singularities in three-dimensional compressible fluids

Sufficient conditions on the initial data are given in order that smooth solutions of the compressible Euler equation in three space dimensions develop singularities in finite time. Roughly speaking, the initial data must be localized and, on average, compressed and outgoing.

E. SINESTRARI:

Totally non-linear integrodifferential equations

Some recent investigations by the author and A. Lunardi (+) refer to the integrodifferential equation

$$(P)\ \begin{cases} u'(t) = f(t,u(t)) + \int_0^t g(t,s,u(s))\,ds, & t \geq 0 \\ u(0) = u_0 \end{cases}$$

where $f:[0,T]\times D \to X$, $g:\Delta_T \times D \to X$ are nonlinear functions, D and X Banach spaces with D X and $\Delta_T = \{(t,s) \in R^2,\ 0 \leq s \leq t\}$. Problem (P) has been studied under the main assumption that the Fréchet derivative $f_u(0,u_o) = A:D \subset X \to X$ generates an analytic semigroup. The local existence and uniqueness of a differentiable solution of (P) is proved by supposing also that $f(.,u)$ is α-hölder continuous, $f(t,.)$ is lipschitz differentiable near u_o, $g(.,s,u)$ is α-hölder continuous, $g(t,.,u)$ is in $L^{1/1-\alpha}$ and $g(t,s,.)$ is continuously differentiable near u_o. Moreover it is shown that u' is α-hölder continuous up to $t = 0$ if a compatibility condition is satisfied. In the autonomous and convolution case some results are given concerning the existence of a solution for large t if u_o is suitably small and the Lyapunov stability of the zero solution.

(+) -C^α-regularity for non autonomous linear integrodifferential equations of parabolic type. Preprint Univ. Pisa, June 1984

-Fully non linear integrodifferential equations in general Banach space. Preprint Univ. Pisa, October 1984

-Existence in the large and stability for nonlinear Volterra equations. Preprint Univ. Roma, October 1984.

H. SOHR:

Regularity problems for the equations of Navier Stokes

We prove the regularity of weak solutions of the equations of Navier Stokes if the space variables are sufficiently large. The result is contained in a joint paper with Prof. von Wahl. It holds the following

Theorem: Let $\Omega \subseteq \mathbb{R}^3$ be a smooth exterior domain, $u_o \in \overset{\circ}{H}{}^{1,2}(\Omega) \cap H^{2,2}(\Omega) \cap L^q(\Omega)$ with $\frac{10}{7} \leq q < \frac{3}{2}$ and let u be a weak solution of the equations of Navier Stokes

$$u_t - \Delta u + (u\cdot\nabla)u + \nabla p = 0, \; \operatorname{div} u = 0, \; u|_{\partial\Omega} = 0, \; u(0) = u_o$$

such that additionally the generalized energy inequality holds. Then there exist constants $K > 0$, $L > 0$ with

$$|u(x,t)| \leq K \text{ a.e.}$$

for $|x| \geq L$, $t \geq 0$. It follows the C^∞-regularity of u and p in space for $t > 0$ and large $|x|$.

H. TANABE:

Nonlinear integro-differential equations of Volterra type

This talk is concerned with the initial-boundary value problem for the integro-differential equation of Volterra type:

$$\frac{\partial u}{\partial t}(x,t) - \Delta u(x,t) = \int_0^t a(t-s) \sum_{i=1}^n \frac{\partial}{\partial x_i}\sigma_i(\nabla u(x,s))\, ds, \qquad x \in \Omega,\; 0 < t < \infty,$$

$$u(x,t) = 0, \qquad x \in \partial\Omega,\; 0 < t < \infty,$$

$$u(x,0) = u_o(x), \qquad x \in \Omega,$$

where σ_i, $i = 1,\dots,n$, are real valued functions with bounded continuous first order derivatives in R^n. For any initial value $u_o \in W_o^{1,p}(\Omega)$, $1 < p < \infty$, an L^p solution of this problem exists. The uniqueness follows from the fact that if we consider the above problem in the space $W^{-1,p}(\Omega)$, then the nonlinear part satisfies a uniform Lipschitz condition. That Δ with the Dirichlet boundary condition generates an analytic semigroup in $W^{-1,p}(\Omega)$ can be shown with the aid of R. Seeley's interpolation theorem.

M. TSUTSUMI:

Global existence of classical solutions of nonlinear Klein-Gordon equations and of nonlinear Schrödinger equations in the exterior domain

Recently Y. Shibata and Y. Tsutsumi have studied intensively the global existence of sufficiently smooth solutions to nonlinear wave equations and nonlinear Schrödinger equations in an exterior domain with homogeneous Dirichlet boundary condition for small initial data. Here, in semi-linear case, their results are improved and also extended to the variable coefficients case. Morewetz estimates are found to be useful.

G. VELO:

The Cauchy problem in the energy space for the nonlinear Schrödinger (NLS) and the nonlinear Klein-Gordon (NLKG) equations

We review the existence and uniqueness results for the solutions of the Cauchy problem for the NLS equation $i\dot{\varphi} = -(\frac{1}{2})\Delta\varphi + f(\varphi)$ and the NLKG equation $\Box\varphi + f(\varphi) = 0$ for general initial data in the energy space. Here φ is a complex function defined in space time $\mathbb{R}^{n+1}$ and f is a nonlinear interaction, typically

$$f(\varphi) = (\lambda_1|\varphi|^{p_1-1} + \lambda_2|\varphi|^{p_2-1})\varphi \qquad (*)$$

with $1 \leq p_1 \leq p_2$ and $\lambda_2 > 0$. We compare the results obtained by the method of contraction for the local (in time) Cauchy problem and a priori estimates for the global problem with the method of compactness for the existence of global (weak) solutions and partial contraction for uniqueness. The best results are obtained by the second method, whereby global existence and uniqueness are proved, in the special case (*), for $1 \leq p_1 \leq p_2 < 1+4/(n-2)$

for the NLKG equation, and under a similar, but slightly stronger condition for the NLS equation.

P. WEIDEMAIER:

Local classical solvability of a quasilinear hyperbolic initial-boundary-value problem with 3rd. boundary condition

Existence of a local classical solution for the initial-boundary-value problem

$$u'' - \partial_i(a_{ij}(t,u)\partial_j u) = f(t,u,\dot{u}) \text{ in } \Omega \subset \mathbb{R}^3$$

$$a_{ij}(t,u)\nu_i\partial_j u + \sigma(t)u\big|_{\partial\Omega} = 0$$

$$u(0) = u_0,\ u'(0) = u_1$$

is proved.

R. WEINACHT:

A special class of quasi-linear evolution equations

We consider the nonlinear evolution equation

$$(*)\quad \frac{du}{dt} + g\{\tfrac{1}{2}(Au,u)\}Au + \lambda u = h(t)$$

in a Hilbert space H where g is a real valued function and A is a self-adjoint operator on H. The nonlinear term in (*) is a "scalar nonlinearity" which was introduced independently by Medeiros and by Bazley and Küpper.

The equation (*) can be solved explicitly in terms of a solution of a scalar ordinary differential equation. We also examine equilibrium solutions of (*) and present some results on their stability.

F. WEISSLER:

Single point blow-up for a semilinear heat equation

We consider the finite time blow-up behaviour of solutions to the semilinear heat equation

$$(*)\quad \begin{cases} u_t(t,x) = \Delta u(t,x) + F(u(t,x)), & t > 0,\ x \in \Omega \\ u(t,y) = 0 & t > 0,\ y \in \partial\Omega \\ u(0,x) = f(x) \geq 0, & x \in \overline{\Omega} \end{cases}$$

which is formally equivalent to the integral equation

$$(**)\quad u(t) = e^{t\Delta} f + \int_0^t e^{(t-s)\Delta} F(u(s))\, ds.$$

Theorem. Suppose $\Omega = \{x \in \mathbb{R}^n : |x| < R\}$ or $\mathbb{R}^n$. Assume $f \geq 0$ in $C_o(\Omega)$ is radially symmetric and radially non-increasing. Suppose further that $F:[0,\infty) \to [0,\infty)$ is convex, $F(0) = 0$, $F(x) > 0$ for $x > 0$, and $\liminf_{x\to\infty} xF'(x)/F(x) > 1$. Let $u(t) = u(t,.)$ be the corresponding maximal solution of (**) and (*), and suppose its existence time T is finite. Then for all $x \neq 0$ in Ω

$$\limsup_{t\to T} u(t,x) < \infty.$$

Berichterstatter: W. von Wahl

Tagungsteilnehmer

Prof. Dr. H. W. Alt
Institut für Mathematik
der Universität
Wegelerstr. 6

5300 B o n n

Dr. E. Di Benedetto
Department of Mathematics
Indiana University

Bloomington, Indiana

U S A

Prof. Dr. H. Amann
Mathematisches Institut
der Universität
Rämistraße 74

CH 8001 Z ü r i c h

Prof. P. Brenner
Department of Mathematics
Chalmers University of Technology and University of
Göteborg

S 412 96 G ö t e b o r g

Dr. W. Arendt
Mathematisches Institut
der Universität
Auf der Morgenstelle 10

7400 T ü b i n g e n

Dr. L. Brüll
Mathematisches Institut
der Universität
Weyertal 86-90

5000 K ö l n

Prof. Claude Bardos
Lab. Anal. Numerique 55-65
Université de Paris 6
4 Place Jussieu

F 75230 P a r i s Cedex 05

Prof. T. Cazenave
Laboratoire Analyse Numerique
Université P. et M. Curie
4 Place Jussieu

F 75005 P a r i s 5

Prof. N. Bazley
Mathematisches Institut
der Universität
Weyertal 86-90

5000 K ö l n

Prof. P. Constantin
Department of Mathematics
Indiana University

Bloomington, Indiana

U S A

Dr. J. Bemelmans
Mathematisches Institut
der Universität
Wegelerstraße 10

5300 B o n n

Prof. G. Da Prato
Scuola Normale Sup. di Pisa

I - P i s a

Prof. Y. Ebihara
Department for Appl. Mathematics
Faculty of Science
Fukuoka University

Fukuoka 814

J a p a n

Dr. B. Kawohl
Mathematisches Institut
der Universität
Bismarckstr. 1 1/2

8520 E r l a n g e n

Dr. H. Engler
Department of Mathematics
University of Texas at Austin

Austin, Texas 78712

U S A

Prof. H. Kielhöfer
Institut für Angew. Mathematik
der Universität
Am Hubland

8700 W ü r z b u r g

Prof. A. Friedman
Department of Mathematics
Northwestern University

Evanston, Illinois

U S A

Prof. S. Klainerman
Courant Institute of
Mathematical Sciences
New York University
251 Mercer Street
New York, New York 10012
U S A

Dr. Y. Giga
Institute for Mathematics
and its Applications
University of Minnesota
514 Vincent Hall

Minneapolis, Minnesota 55455
U S A

Dr. R. Landes
The University of Chicago
Department of Mathematics
5734 University Ave.

Chicago, Illinois 60637

U S A

Prof. J. Ginibre
Laboratoire de Physique
Théorique et Hautes Energies
Université de Paris-Sud

F 91405 O r s a y

Prof. H. Lange
Mathematisches Institut
der Universität
Weyertal 86-90

5000 K ö l n

Prof. F. John
Courant Institute of
Mathematical Sciences
New York University
251 Mercer Street
New York, New York 10012

U S A

Dr. V. Lovicar
Ceskoslovenská akademie véd
Matematický Ustav
Nové Město, Zitná 25

P r a h a 1

C S S R

Dr. S. Luckhaus
Institut für Angew. Mathematik
der Universität
Im Neuenheimer Feld 294

6900 Heidelberg

Dr. Michelle Schatzman
Centre de Mathematiques
Appliquees
Ecole Polytechnique

F 91128 Palaiseau Cedex

Prof. K. Masuda
Mathematical Institute
Tohoko University

Sendai 980

Japan

Dr. Tom Sideris
Department of Mathematics
Purdue University

West Lafayette, Indiana 47907

USA

Dr. T. Miyakawa
Department of Mathematics
Hiroshima University
Higashisenda-machi

Hiroshima City

Japan

Dr. E. Sinestrari
Istituto Guido Castelnuovo
Universitâ degli studi Roma

I-00100 Roma

Prof. H. Pecher
Fachbereich Mathematik
der Universität
Gaußstr. 20

5600 Wuppertal

Prof. Dr. H. Sohr
Fachbereich Mathematik
der Universität
Warburger Str. 100

4790 Paderborn

Dr. J. Prüss
Fachbereich Mathematik
der Universität
Warburger Str. 100

4790 Paderborn

Prof. H. Tanabe
Department of Mathematics
Faculty of Science
Osaka University

Toyonaka 560

Japan

Prof. Dr. R. Rautmann
Fachbereich Mathematik
der Universität
Warburger Str. 100

4790 Paderborn

Dr. M. Tsutsumi
Department of Applied Physics
Waseda University
341 Okubo, Shinjuku-ku

Tokyo

Japan

Prof. G. Velo
Istituto di Fisica "A. Righi"
Via Irnerio no. 46

40126 Bologna

Italy

Prof. C. Weinacht
Department of Mathematics
University of Delaware

Newark, Delaware 19711

USA

Dr. J. Voigt
Mathematisches Institut
der Universität
Theresienstr. 39

8000 München

Prof. F. Weissler
Department of Mathematics
University of Minnesota
514 Vincent Hall

Minneapolis, Minnesota 55455

USA

Prof. Dr. W. von Wahl
Mathematisches Institut
der Universität
Postfach 3008

8580 Bayreuth

Dr. J. Weyer
Mathematisches Institut
der Universität
Weyertal 86-90

5000 Köln

Dr. P. Weidemaier
Mathematisches Institut
der Universität
Postfach 3008

8580 Bayreuth