MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 47/1984

Stochastische Analysis

4.11. bis 10.11.1984

Die Tagung fand unter der Leitung von J.M. Bismut (Orsay) und H. Föllmer (Zürich) statt. In 33 Vorträgen ergab sich ein breites Spektrum von aktuellen Themen der stochastischen Analysis, unter anderem mit den folgenden Schwerpunkten:

- Malliavin-Kalkül über dem Wiener- und Poisson-Raum,
- Lyapunov-Exponenten von stochastischen Flüssen,
- Vlasov-Limes für interagierende stochastische Differentialgleichungen,
- wahrscheinlichkeitstheoretischer Zugang zum Index-Satz von Atiyah-Singer.

Eine besondere Rolle spielten die Querverbindungen zwischen stochastischer Analysis, stochastischer Mechanik und stochastischen Feldern der Quantenfeldtheorie, die sich in den letzten Jahren immer stärker entwickeln. Hierzu gab es mehrere Uebersichtsvorträge, sowohl von Wahrscheinlichkeitstheoretikern wie von mathematischen Physikern. Eine Reihe von neuen mathematischen Resultaten, zum Beispiel zur Hyperkontraktivität von Diffusionalshalbgruppen und zu den Ueberschneidungen Brownscher Pfade, war direkt oder indirekt von diesen Querverbindungen her motiviert.

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Vortragsauszüge

S. ALBEVERIO

Some Recent Work on Dirichlet Forms and Quantum Theory

We review some recent work on Dirichlet forms over $\mathbb{R}^n$ and over infinite dimensional spaces, with particular attention to problems which also have a counterpart in quantum mechanics and quantum field theory. In both cases we consider in particular the existence problem (closability; new sufficient conditions are mentioned) and the uniqueness problem. The condition of N. Willems' in finite dimensions and the ones of Takeda and Kusuoka in infinite dimensions are also discussed. The significance of uniqueness is stressed, especially in connection with problems of quantum field theory. In finite dimensions the problems of unattainability of zeros of the wave function, the ergodicity of the process and the quantum mechanical tunneling are also discussed.

L. ARNOLD

Asymptotic Results for the Density of States and Lyapunov Exponent for the One-Dimensional Schrödinger Operator

Given the Schrödinger operator

$$Hy = -\ddot{y} + \sigma\,\xi(t)y$$

in $L^2(\mathbb{R},dt)$, where $\xi(t)$ is a nice diffusion on an interval $(\alpha,\beta)\subset\mathbb{R}$ with invariant distribution γ and $\langle\xi(t)\rangle = 0$.
Let $\lambda_\sigma(E)$ be its Lyapunov number and $N_\sigma(E)$ its integrated density of states.

Theorem: Let $\sigma\to\infty$, $E = E_1\sigma$, $E_1 \in \mathbb{R}$.

Then
$$\lambda_\sigma(E_1\sigma) = \sqrt{\sigma}\left\langle\sqrt{(\xi - E_1)^+}\right\rangle + 0(1) \quad \text{for} \quad E_1 < \beta$$
$$= \frac{1}{32}\left\langle\frac{b^2(\xi)}{(\xi - E_1)^2}\right\rangle + 0\left(\frac{1}{\sqrt{\sigma}}\right) \quad \text{for} \quad E_1 > \beta$$

(b = diffusion coefficient of ξ)

$$\pi N(E_1\sigma) \equiv 0 \qquad \text{for} \quad E_1 < \alpha$$
$$= \sqrt{\sigma}\left\langle\sqrt{(E_1-\xi)^+}\right\rangle + o\left(\frac{1}{\sqrt{\sigma}}\right), \quad E_1 > \alpha$$

D. BAKRY

Riesz Transforms for some symmetric Semigroups

X_r being a Markov process, symmetric with respect to μ, with semigroup P_r and generator L, such that there is an algebra of functions on the state space D, stable by L and by P_r, define on D Γ and Γ_2 like in Emery's abstract.

Put $C = -(-L)^{1/2}$, and suppose $\Gamma_2 \geqslant 0$; then you can compare the norms of $\Gamma(f, f)^{1/2}$ and Cf in $L^p(\mu)$ in the following way:

$$p \geqslant 2 : \quad \| \Gamma(f, f)^{1/2} \|_p \leqslant c_p \| Cf \|_p \quad ; 1 \leqslant p \leqslant 2 : \quad \| Cf \|_p \leqslant c_p \| \Gamma(f,f)^{1/2} \|_p$$

Furthermore, if L is a diffusion, then

$$\| Cf \|_p \leqslant c_p \| \Gamma(f,f)^{1/2} \|_p \ , \quad \forall \ 1 < p < +\infty$$

K. BICHTELER

Joint Regularity of Solutions to SDE driven by Poisson Measures

This continues Jacod's exposition (p. 8). Under suitable conditions on the coefficients a,b,c, the transition kernel $P_t(x, dy)$ of the solution X_t has a density $p_t(x,y)$ regular in (t,x,y).

J.M. BISMUT

The Atiyah-Singer Theorem in a Probabilistic Approach

The proof of the index theorem given in JFA 57, 56-99,(1984) has been presented. The case of the Dirac operator on the spin complex has been completely treated.

H. DOSS

Probability Proof of Some Quasi - Classical Expansions

For $\varepsilon \neq 0$ consider the solution of:

$$\begin{cases} \frac{\partial\psi}{\partial t}(t,x) = L_\varepsilon \psi(t,x) = \varepsilon^2/2 \sum_{j=1}^{n} A_j^2 \psi(t,x) + A_0\psi(t,x) + \frac{1}{\varepsilon^2} V(x)\psi(t,x) \\ \psi(o,x) = f(x) \exp\left(-\frac{s(x)}{\varepsilon^2}\right), \quad t \in [o,T], \ x \in M \end{cases}$$

where M is a smooth manifold, $A_0, A_1, \ldots, A_n$ are vector fields on M, V, f, s regular functions from M to $\mathbb{C}$.

We give, when the second order differential operator L_ε is smooth but eventually completely degenerated, an asymptotic expansion with respect to ε ($\varepsilon \to 0$) for the solution of (1). The estimates obtained in the real case allow one to study, under some conditions, the Schrödinger equation with complex analytic coefficients, and also some situations where L_ε is hyperbolic.

D. DUERR

The Smoluchowski Limit for a Simple Mechanical Model

The motion of a Brownian particle in a potential $U^A(x) = U(\frac{1}{\sqrt{A}} X)$, which varies on a macroscopic scale ($\sqrt{A}$) may be described on a macroscopic scale by a diffusion process for the position $X(t)$ of the molecule at time t. This diffusion is given by the Smoluchowski equation

$$(1) \quad dX(t) = -\frac{\nabla U}{\gamma} dt + \frac{\sigma}{\gamma} dW_t$$

For a stationary system the stationary solution for the position is the Gibbs state $\sim e^{-U/kT}$, k Boltzmann konstant, T absolute Temperature. Therefore, in (1) $\frac{2\gamma}{\sigma^2} = 1/kT$, which is the famous Einstein relation. We wish to derive (1) from "first principles", starting from a deterministic mechanical system, where the stochasticity comes only from the random initial configurations. A system, which may be treated, is a one dimensional ideal gas with an identical tagged particle serving as the molecule, on which a force is acting. The motion of the molecule is then determined by elastic collisions with the gas particles and by the Newtonian motion in the force field. The ideal gas is described by a Poisson point process in position-velocity $(q - v)$ space, where $f(v)$

is the density of the velocity distribution. $X(t)$ and $V(t)$ are position and velocity of the molecule and $V(0)$ is distributed also by $f(v)$. We considered the simplest case $f(v) = 1/2\,\delta_1(v) + 1/2\,\delta_{-1}(v)$.
The force field is $\frac{1}{\sqrt{A}} F \ (\equiv \nabla U^A(x))$ and we consider $X_A(t) = \frac{1}{\sqrt{A}} X(A\cdot t)$.
We obtain the surprising result that $X_A(t) \to W(t)$, i.e. there is no drift in the limit. This fact is due to recollisions and a typical one-dimensional phenomenon.In a two-dimensional model, where the molecule is a stick moving along the x-axis without rotation and $f(v)$ is supported equally by velocities on the diagonals we obtain an equation like (1) in the scaling limit.

The reason that the Einstein relation is violated comes from the fact, that our velocity distribution is not the stationary - Maxwellian - one. In fact, we conjecture that if $f(v)$ has a density at zero, equation (1) comes out and γ and z are related by $\frac{2\gamma}{z^2} = \frac{1}{\langle v^2\rangle}$. It should be noted that the derivation of (1) in more than one dimension with the Maxwellian distribution is out of reach at the moment, even the free case, i.e. $U = 0$ has not been treated until now.

E. DYNKIN

<u>Markov Processes and Field Theory</u>

With every symmetric Markov process X a Gaussian random field φ is associated with the Hamiltonian $H(\varphi) = \frac{1}{2} E(\varphi, \varphi)$ where E is the Dirichlet form of X. It turns out that the path of X can be used for investigating not only properties of φ but also for investigating non-Gaussian fields with the Hamiltonians $H(\varphi) = \frac{1}{2} E(\varphi,\varphi) + V(\xi)$ where V is a functional of the field $\xi_x = \frac{1}{2}\varphi_x^{\,2}$. In particular, the specification corresponding to H can be expressed in terms of the occupation field or of the hitting field for the Poisson "gas" in the space of paths and loops.

The program outlined in the talk can be viewed as an interpretation, from a point of view of a probabilist, of Symanzik's ideas in quantum field theory.

Th. EISELE

Equilibrium and Non-Equilibrium Theory of a Magnetic Model

(Summary of joint works with R. Ellis, F. Comets, M. Schatzmann)

A magnetic model with a long-range interaction is discussed. It has paramagnetic, ferromagnetic and antiferromagnetic phases. In both its equilibrium and non-equilibrium theory, we prove the thermodynamic limit, resp. the law of large numbers, and we show the fluctuation theorems. The equilibrium fluctuation fields represent stationary distributions of the non-equilibrium fluctuation processes. This holds true also in the critical borderline cases of second-order phase transitions, where the usual Gaussian fields break down. Here, we get degenerate non-Gaussian fields with densities $\exp(-t^4c)$.

K.D. ELWORTHY

Lyapunov Exponents for Stochastic Flows

Let $(F_t)_{t\geq 0}$ be the flow of the S.D.E. $dx_y = \sum_i X^i(x_t) \cdot dB_t^i + A(x_t)\, dt$ on a compact Riemannian manifold M, and let A be its differential generator. Results of Carverhill have extended Ruelle's ergodic theory of diffeomorphisms to show that Lyapunov exponent can be defined for F_t which describe how sample solutions from different points of M behave in relationship to one another in the long term. Simple examples for $M = S^1$ with $A = \frac{1}{2}\Delta$ show that these exponents are not determined by A. However, computations with M. Chappell show that the sum of the exponents, given by

$$\lambda_\varepsilon = \lim_{t\to\infty} \frac{1}{t} \log \det T_{x_0} F_t$$

is always non positive for $A = \frac{1}{2}\Delta$ and is dominated by (the principal eigenvalue of $\frac{1}{2}\Delta$) $\times$ dim M if $A = \frac{1}{2}\Delta$ and each X^i is a gradient.
Estimates for the canonical flow on the frame bundle of M have been obtained by Carverhill and Elworthy.

M. EMERY

Hypercontractivité de Semigroups de Diffusion

(Travail commun avec Bakry)

Si (X_r) est une diffusion markovienne continue, stationnaire, réversible de loi μ sur un espace d'états E, une condition suffisante pour que X soit hypercontractif et que soit satisfaite l'inégalité de Sobolev logarithmique de Gross est

$$\forall x \forall f \qquad \Gamma_2(f,f)(x) \geq c\, \Gamma(f,f)(x),$$

où, désignant par L le générateur de X,

on a posé

$$\Gamma(f,g) = \frac{1}{2}\left[L(fg) - fLg - gLf \right]$$

$$\Gamma_2(f,g) = \frac{1}{2}\left[L\Gamma(f,g) - \Gamma(f,Lg) - \Gamma(Lf,g)\right]$$

L. GROSS

Where do Random Fields come from in Quantum Field Theory?

The notion of ground state representation can be used to explain easily how random fields enter into the construction of quantum fields. This is an entirely expository lecture, which takes advantage of the familiarity of the audience with Markov processes, infinite dimensional integration theory and the Ornstein-Uhlenbeck process to motivate, starting with quantum mechanics, the definition of the free Euclidean Markov field and the study of the additive functional $\int_\Lambda :\varphi:^4\, dx \quad , \Lambda \subset R^2$

U. HAUSSMANN

Non-Linear Filtering - the Degenerate Case

Control of diffusions with partial information leads to the problem: find the conditional density of X_t given $\{Y_s : s \leq t\}$ where

$$dX_t = b(t,X_t,Y)dt + \sigma(t,X_t,Y)dw$$

$$dY_t = h(t,X_t)dt + d\tilde{w}$$

where one may not assume any regularity of b, σ with respect to (t,Y), e.g. no continuity in t. It is shown, under suitable hypotheses, that even when $\sigma\sigma^T$ is degenerate a conditional density exists and its unnormalized version satisfies the pathwise (non-stochastic) p. d.e. usually associated with the Zakai equation.

R. HOEGH KROHN

Gauge Fields as Generalized Markov Processes

To every Markov process on a compact Lie group G, which is stationary and left as well as right invariant, there exists a Markov stochastic gauge field in two dimensions such that the curvature is a homogeneous chaos with values in the Lie-algebra of g. This homogeneous chaos is the same as the one which governs the increments of the Markov process. These continuous Gauge theories are exactly the theories which can be described as limits of lattice Gauge theories as the lattice converges to zero. The Gauge Field of Physics is obtained by starting with the solution of the Markov process which is the solution of the heat equation. This gives the white noise Gaussian homogeneous chaos $\xi(x)$, and the corresponding Gauge field is constructed by solving the non linear stochastic differential equation $Dw = \xi$ for the connection form w, where Dw is the covariant exterior derivative given by the connection form w.

J. JACOD

Malliavin Operator for Poisson Space

Let $(\Omega,\mathcal{F},P)$ be the canonical space of Poisson measure (μ) with $\nu(ds, dy) = ds \otimes dy$, on $[0, T) \times E$ (E = open subset of $\mathbb{R}^n$). We construct a Malliavin operator (L, R) on this space, starting with "regular functionals" and such that the coordinates of the solution of the equation

$$dX_t = a(X_t)dt + b(X_t)dW_t + c(X_{t-})\,(d\mu - dv)$$

be in the domain. We can add Wiener process as above, and take the "product" Malliavin operator.

This allows to study the smoothness (and existence) of a density for the random variables X_t.

G. JONA-LASINIO

Stochastic Quatization: Construction of Renormalized Diffusion Processes

(Joint work wirh P.K. Mitler)

Using techniques from constructive field theory we prove the existence of weak solutions for the stochastic differential equation

$$d\varphi_t = -\frac{1}{2}(\varphi_t + \lambda c_* :\varphi_t^3:)\, dt + dw_t$$

where $\varphi_t(x)$ is a random field on $\mathbb{R}^2$, w_t is a brownian motion with covariance

$$E\, w_t + w_{t'} = \min(t, t^1)\, C(x, x^1)$$

and $C(x,x') = (-\Delta + 1)^{-1}$. $-\Delta$ is the laplacian in two dimensions. This process is ergodic for x restricted to a finite domain and it has as stationary measure the Euclidean Φ_2^4 measure in finite volume.

Y. KIFER

Characteristic Exponents and Invariant Subbundles for Random Diffeomorphisms

Let $f_1, f_2, \ldots$ be a sequence of i.i.d. random diffeomorphisms of a compact Riemannian manifold M. Then under natural conditions for any x outside of some exceptional set and each ξ from the tangent space T_xM at x with probability one the limit $\lim_{n\to\infty} \frac{1}{n} \log\| Df_n \circ \ldots \circ Df_1 \| = \beta(\xi)$ exists and it is non-random.

Moreover, for these x there exists a filtration of (non-random) subspaces

$$L_x^{r(x)} \subset \ldots \subset L_x^1 \subset L_x^0 = T_x M \text{ and (non-random) numbers } \beta_x^{r(x)} < \ldots < \beta_x^0$$

such that $\beta(\xi) = \beta_x^i$ for any $\xi \in L_x^i \setminus L_x^{i-1}$.

P. KOTELENZ

Linear Parabolic Differential Equations as Limits of Space-Time Jump Markov Processes

A class of linear parabolic differential equations on a bounded domain in $\mathbb{R}^n$ is obtained as the class of deterministic limits of space-time jump Markov processes X^N. These X^N describe particle systems which are spatially inhomogeneous due to diffusion and random change in the number of particles. The deviation of X^N from its deterministic limit is a distribution valued generalized Ornstein-Uhlenbeck process and can be represented as the mild solution of a stochastic partial differential equation, whose driving term is the sum of two independent Gaussian martingales arising from diffusion and change in the number of particles, respectively.

R. LEANDRE

Regularity of Jump Processes in the Degenerate Case

We give examples of self-interactions in a jump process, which allow the process to have a density. We determine a type of jump process which is always on a submanifold of $\mathbb{R}^d$, and we determine the regularity of a jump process whose Lévy measure is fixed and supported by a smooth curve . We use techniques of the calculus of variation of Bismut.

Y. LEJAN

Lyapunov Exponents for Isotropic Flows

We investigate the qualitative behaviour of flows defined by S.D.E. associated with isotropic homogeneous velocity fields. Our results include the computation of Lyapunov exponents and the proof of existence in the unstable case of a non trivial statistical equilibruim. The proof involves a detailed study of isotropic B.M.on matrices.

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T. LYONS

Quasi-Isometry of Riemmannian Manifolds and Discrete Approximates

If a property of a manifold can be shown to be dependent on a network having a similar property then that property is likely to be fairly robust and preserved under quasi-isometry. Many such properties have been so treated (particularly transcience). However, an example was outlined showing how the existence of non-constant positive or bounded harmonic functions (or a non-trivial shift invariant tail σ-field) is not preserved under quasi-isometry. Moser's Harnack theorem shows an important subclass of manifolds where these properties are preserved.

P.A. MEYER

Nelson's Stochastic Mechanics

A survey is given of some recent results of E. Carlen (Comm. Math. Physics, 1984) and W.A. Zeng (Ann. Inst. H. Poincaré, 1985) showing how to construct the diffusions with singular drifts needed to develop stochastic mechanics.

P.K. MITTER

Gauge Fields

A survey of the global aspects of non-abelian gauge fields was given.

M. NAGASAWA

Propagation of Chaos for Diffusion Processes with Mean-field Singular Interaction

We consider a system consisting of n white diffusing particles distributed left to n red diffusing particles. Reds and whites interact in a repulsive way so that the group of reds are kept segregated from the group of whites. (Take e.g. $f(x) = 1/x^4$ as an interaction). It is assumed that the absolute value of the interaction is a decreasing function of the distance of a pair of red and white particles. We prove first of all the existence of a unique solution for a system of equations of 2n particles and also for the corresponding one of non-linear diffusion processes in the mean-field limit. Then we prove the propagation of chaos, i.e. the convergence of the 2n particles, as $n \to \infty$, to the infinite independent copies of the mean-field diffusions. This is a report of what has been done by Tanaka and myself. This gives a mathematical justification of "statistical quantization", which is a possible interpretation of "Stochastic Quantization".

J. NORRIS

Non-Degeneracy of the Malliavin Covariance Matrix

Let φ_t denote the flow associated to the S.D.E.

$$dx_t = X_o(x_t)dt + X_i(x_t)dW_t^i \qquad \text{(with coefficients}$$

$X_o, X_1, \ldots, X_m \in C_o^\infty(\mathbb{R}^d, \mathbb{R}^d)$ and W_t a Brownian motion on $(\mathbb{R}^m)$).
The Malliavin covariance matrix for the solution $x_t \equiv \varphi_t(x)$ starting from x is given by

$$C_t = \int_0^t (\varphi_s^{*-1} X_i)(x) \otimes (\varphi_s^{*-1} X_i)(x)ds$$

Malliavin, Bismut, Stroock etc. have shown that provided $C_t^{-1} \in L^p(\mathbb{P}) \quad \forall p < \infty$ x_t will have a smooth density. It was further shown by Stroock that if

$$H_1: \; X_1, \ldots, X_m \; ; \; [X_i, X_j]_{i,\, j=o}^{m} \; ; \; [X_i, [X_j, X_k]]_{i,j,k=o}^{m} ; \; \ldots \text{ etc.}$$

evaluated at x span $\mathbb{R}^d$,

then indeed $C_t^{-1} \in L^p(\mathbb{P}) \quad \forall p < \infty$ (see Springer LNM 776). The following lemma was presented which affords a simplification of Stroocks proof:

Let T be a bounded stopping time; Let $Y_t = y + \int_0^t a_s ds + \int_0^t u_s^i dW_s^i$
where $y \in \mathbb{R}$, and $a_t, u_t^1, \ldots u_t^m$ all have the form

$$\alpha + \int_0^t \beta_s ds + \int_0^t \gamma_s^i dw_s^i \qquad \text{with } \alpha \in \mathbb{R}, \text{ and } \beta_s, \gamma_s^1, \ldots, \gamma_s^m \text{ previsible with}$$

$$\sup_{s \leq T} |\beta_s| , \sup_{s \leq T} |\gamma_s| \in L^p(\mathbb{P}) \qquad \forall p < \infty$$

Then $\mathbb{P}\left\{\int_0^T Y_t^2\, dt < \varepsilon^q \quad \text{and} \quad \int_0^T (|a_t|^2 + |u_t|^2) dt \geq \varepsilon\right\} = O(\varepsilon^p)$ for all $p < \infty$,

for each $q > 36$.

E. PARDOUX

Two-Sided Stochastic Integral and Stochastic Calculus

(joint work with Ph. Protter)

Consider a forward diffusion X_t, solution of

$$X_t = x_0 + \int_0^t b(s,X_s)ds + \int_0^t \sigma(s, X_s)dW_s, \qquad 0 \leq t \leq 1$$

and a backward diffusion Y^t, solution of

$$Y^t + \int_t^1 c(s,Y^s)ds + \int_t^1 \gamma(s, Y^s)dW_s = y, \qquad 0 \leq t \leq 1$$

A stochastic integral of the type $\int_0^1 \Phi(X_t, Y^t)\, d W_t$ is defined.

This permits to write down the differential of a process of the type $\psi(X_t,Y^t)$.

Both Ito and Stratonovitch calculus are considered.

J. POTHOFF

Stochastic Quantization: A Remark on Unstable Actions

(Joint work in progress with Ph. Blanchard and R. Sénéor)

It was explained how the procedure of stochastic quantization can possibly be used to give a meaning to the formal perturbation expansion of Euclidean quantum field theories with unstable action functionals.

I. SHIGEKAWA

Diffusions on the Wiener Space

We consider a diffusion on the abstract Wiener space (B,H,μ) generated by the operator of the form $A = \frac{1}{2} L + b$, L being the Ornstein-Uhlenbeck operator and b being an H-valued function on B, which we regard as a vector field on B. We discuss about invariant measures of this diffusion and obtain that there exists an invariant measure which is absolutely continuous with respect to μ. Moreover, the uniqueness holds if we restrict ourselves to measures which are of finite total variation and are absolutely continuous with respect to μ.

We prove the existence by two steps. First we show it in the finite dimensional case and secondly in the infinite dimensional case by the limiting procedure. In this procedure, the Gross logarithmic Sobolev inequality is crucial.

D. STROOCK

Wiener Chaos Revisited

Let $(H, \mathcal{W}, W)$ be an abstract Wiener space and define D^m to be the Sobolev extention of m^{th}-order differentiation in directions of H. Let δ^m be the adjoint of D^m. Then, Wiener's decomposition of a function into states of homogeneous chaos is:

$$\Phi = \sum_{0}^{\infty} \frac{1}{m!} \delta^m E^W \left[D^m \Phi \right]$$

A. WAKOLBINGER

Time Reversal of Infinite Dimensional Diffusions

(Joint work with H. Föllmer)

We consider the problem under which conditions the "time reversal" $\hat{X}_t = X_{1-t}$ of an infinite dimensional diffusion $X = (X_t^i)$, $i \in I$ countable, $dX_t^i = b^i(X_t,t)dt + dW^i$ (W^i independent Wiener processes) is again of the same type, with forward drift (b^i) and backward drift $(\hat{b}^i)$ being related by an analog of Kolmogorov's classical relation, namely

$$b^i((x^i\xi),t) = \hat{b}^i((x^i,\xi),\ 1-t) + \frac{\delta}{\delta x^i} \log \rho_t^i(x^i|\xi)$$

where $\rho_t^i(\cdot|\xi)$ is the conditional density of X_t^i, given $(X_t^j)_{j\neq i} = \xi$

Theorem: This holds true if, for all $i \in I$, the relative entropy of the law P of X with respect to P^i is finite (where P^i is the law of the process which arises from X by replacing X^i by a Wiener process independent of $(X^j)_{j\neq i}$).

Moreover, conditions on the drift (b^i) are given which guarantee this finite entropy condition, and which are not far from the usual conditions ensuring existence and uniqueness of the strong solution of an infinite dimensional stochastic differential equation.

M. YOR

Double Points of Brownian Motion in $\mathbb{R}^d$ (d=2,3) and Related Stochastic Calculus

Simpler proofs of Tanaka-Rosen formulae for the local time of intersection of complex or 3-dimensional Brownian motion, are given, taking advantage of Hardy's L^2 inequality, which is closely related to the second order equation:

$$\frac{1}{r} g'(r) + g''(r) = h(r).$$

In dimension 2, Varadhan' s renormalization appears a simple consequence of the new Tanaka-Rosen formula thus obtained.
In dimension 3, a new convergence in distribution for the renormalised local time of intersection is obtained However, the relationship which may exist between this limit in distribution and Westwater's renormalization is not understood .

Berichterstatter: H. Föllmer

Tagungsteilnehmer

S. Albeverio

Institut für Mathematik
Universitätsstrasse 150

D-463 Bochum

H. Bauer

Mathematisches Institut
Universität Erlangen-Nürnberg
Bismarckstrasse 1 1/2

D-852 Erlangen

L. Arnold

FB Mathematik, Forschungsschwerpunkt
Dynamische Systeme
Universität Bremen
Postfach 330440

D-2800 Bremen 33

G. Benarous

Département de Math. Appl.
Ecole Normale Supérieure
rue d'Ulm

F- 75 Paris

R. Azencott

Département de Mathématique
Université Paris VII, Tour 45-55
2, Place Jussieu

F-75221 Paris Cedex 05

K. Bichteler

Department of Mathematics
University of Texas
Austin, TX 78712
U.S.A.

C. Bachelet
Université Paris VI, Tour 46-56
UER Math. 47
4, place Jussieu

F-75007 Paris

J.M. Bismut

Mathématique, Bât. 425
Université de Paris-Sud

F-91405 Orsay

M. Barlow

Trinity College
Cambridge CB2 1TQ
England

E. Bolthausen

Fachbereich Mathematik der TU
Strasse des 17. Juni 135

D-1000 Berlin 12

D. Bakry

Département de Mathématique
Université de Strasbourg
7, rue René Descartes

F-67084 Strasbourg

T. Brox
Institut für Angew. Mathematik
Universität Zürich
Rämistrasse 74

CH-8001 Zürich

H. Doss

Laboratoire de Probabilités
Université de Paris VI, Tour 46-56
4, place Jussieu

F-75230 Paris

H. Föllmer

Abteilung für Mathematik
ETH-Zürich

CH-8092 Zürich

D. Dürr

Zentrum für interdisziplinäre
Forschung
Wellenberg 1

D-4800 Bielefeld 1

L. Gross

Department of Mathematics
Cornell University
Ithaca, N.Y. 14853
U.S.A.

E.B. Dynkin

Department of Mathematics
Cornell University
Ithaca, N.Y. 14853
U.S.A.

U. Haussmann

Department of Mathematics
University of British Columbia
121-1984 Mathematics Rd
Vancouver, B.C.
Canada, V6T 144

T. Eisele
Institut für Angew. Mathematik
Im Neuenheimer Feld 294

D-6900 Heidelberg

R. Høegh-Krohn
Mathematical Institute
University of Oslo
Blindern, Oslo 3
Norway

K.D. Elworthy

Mathematics Institute
University of Warwick
Coventry, CV4 7AL
England

J. Jacod

Laboratoire de Probabilités
Université de Paris VI, Tour 46-56
4, place Jussieu

F-85230 Paris

M. Emery

Université de Strasbourg
Département de Mathématique
7, rue René Descartes

F-67084 Strasbourg

G. Jona-Lasinio
Dipartimento di Fisica
Università "La Sapienza"
Piazza Aldo Moro 2

I-00185 Roma

Y. Kifer

Department of Mathematics
Hebrew University,
Givat Ram
Jerusalem
Israel

P. Kotelenez

Fachbereich Mathematik
Universität Bremen

D-2800 Bremen 33

R. Lang

Institut für Ang. Mathematik
Im Neuenheimer Feld 294

D-6900 Heidelberg

R. Léandre

Département de Mathématiques
Faculté des Sciences

F-25030 Besançon

Y. Le Jan
Laboratoire de Probabilités
Université de paris VI, Tour 46-56
4, place Jussieu

F-75230 Paris

T. Lyons

Mathematics Department
University of Edinburgh
King's Building
Edinburgh
Scotland

P.A. Meyer

Département de Mathématiques
Université de Strasbourg
7, rue René Descartes

F-67084 Strasbourg

P.K. Mitter

Laboratoire de Physique Théorique
et Hautes Energies
Université Paris VI, Tour 46-56
4, place Jussieu

F-75230 Paris

M. Nagasawa

Institut für Ang. Mathematik
Universität Zürich
Rämistrasse 74

CH-8001 Zürich

J.R. Norris

Department of Mathematics
University College Swansea
Singleton Park
Swansea, SA2 8PP
England

E. Pardoux

Département de Mathématiques
Université de Provence
3, place Victor Hugo

F-13331 Marseille Cedex 3

J. Potthoff

Fachbereich 3-Inst. für Mathematik
Technische-Universität Berlin
Strasse des 17. Juni 136

D-1000 Berlin 12

U. Rösler

Institut für Math. Stochastik
Universität Göttingen
Lotzestrasse 13

D-3400 Göttingen

A.S. Sznitman

Courant Institute
251, Mercer Street
New York, N.Y. 10012
U.S.A.

M . Scheutzow

Fachbereich Mathematik
Universität Kaiserslautern
Erwin-Schrödinger-Strasse

D-6750 Kaiserslautern

A. Wakolbinger

Mathematisches Institut
Johannes-Kepler-Universität

A-4040 Linz-Auhof

I. Shigekawa

Dept. of Mathematics
College of General Education
Osaka University
Toyonaka 560
Japan

H. von Weizsäcker

Fachbereich Mathematik
Universität Kaiserslautern
Erwin-Schröder-Strasse

D-6750 Kaiserslautern

D. Stroock

Dept. of Mathematics
M.I.T.
Cambridge, MA 02139
U.S.A.

M. Yor

Laboratoire de Probabilités
Université de Paris VI, Tour 46-56
4, place Jussieu

F-75230 Paris

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