MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t   51/1984

Multigrid Methods

8.12. bis 15.12.1984

The conference was held under the chairmanship of Prof. D. Braess (Bochum), Prof. W. Hackbusch (Kiel) and Prof. U. Trottenberg (Essen).

During the 5 days of the conference, 31 talks were given; there were speakers from 8 different countries.

The centre of interest in the more theoretical contributions was the exact convergence proof for multigrid methods.

In the last years some remarkable improvements were obtained in the theory of multigrid methods especially with respect to quantitative results: it was possible to close the gap between the theoretically provable rate of convergence and the efficiency which is practically observed at least for simple model problems. At the conference, it became evident that those multigrid experts which are more interested in theory now treat also more complicated problems. For example, they presented multigrid methods for the solution of finite-element approximations especially for the Stokes- and the biharmonic problem. Here, however, the gap between theory and practice mentioned above is not yet closed.

On the other hand in the talks with a more practical background, the main point of interest was the development of highly efficent and fast algorithms, for which, however, an exact convergence proof is not yet possible. In these talks multigrid solvers for the Stokes- and for the biharmonic equation were also discussed. In order to make a objective and reliable comparison possible between the different algorithms for this class of problems in regard to efficency and accuracy, it was

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suggested that the GAMM-Fachausschuss "Effiziente numerische Verfahren für partielle Differentialgleichungen" should organize a competition in which all these methods should be tested with some well defined model problems. The participants of the conference were asked to propose suitable model problems for this purpose.
In the more practical talks, questions of software development were also discussed. A number of contributions were further concerned with the multigrid solution of very difficult problems arising in physical applications; most of the examples treated here originate in fluid - or aerodynamics (Euler equations, Navier-Stokes equations). The importance of 3D-problems for practical applications and their special difficulties were mentioned; first multigrid results for a simple 3D-model problem (Poissons equation in a cube) as well as some results of a theoretical analysis were presented in two talks.

Another central point of the conference was the multigrid solution of eigenvalue problems. Five lectures were given on this subject reaching form theoretical analysis and development of new algorithms up to the application to a highly complicated physical problem.

On two evenings, those open questions and problems were taken up which were of special interest in the morning- and afternoon- lectures and which were partly discussed in a rather controversial way.
The first evening was dedicated to the field of "multigrid methods and finite - element approximation".In the second one, Prof. A. Brandt gave a detailed introduction to his theoretical view of elliptic differential- and difference operators and to the importance of ellipiticity in the multigrid context.

Abstracts

O. AXELSSON:

An efficient finite element method for nonlinear diffusion problems

A mixed variable f.e. method is used for the derivation of an efficient iterative method for diffusion problems. The formulation has two advantages as compared to classical finite element methods:
(i) Updating of the material coefficients is simplified.
(ii) The discrete approximation is much more accurate for problems with (almost) discontinuous coefficients, where the discontinuity occurs in the interior of the elements.
An iterative method based on preconditioning by the lower order (piecewise linear b.f.) is used for the solution of the higher order (piecewise quadratic b.f.) approximations. The solution of the linear b.f. equations on the form $BM(u)^{-1}B^T\alpha=F$ is done efficiently by use of the generalized inverses of B and B^T. This means that two Poisson solvers are applied at every nonlinear iteration. A similar application on the Stokes problem is also discussed.

R. BANK:

The use of accelerated smoothing procedures in multigrid iterations

We consider the use of acceleration procedures (e.g. Chebyshev and conjugate gradient) for enhancing the effectiveness of the basic smoother in the multigrid iteration. For a fixed smoothing procedure the use of acceleration can increase the convergence rate from $O(\frac{1}{m})$ to $O(\frac{1}{m^2})$, where m is the number of smoothing steps. It is shown that the minimum residual version of the conjugate gradient algorithm computes an optimal sequence of acceleration parameters.

K. BÖHMER:

Mesh Independence Principle for Newtons method and multi-level applications

Let $Fz=0$ be an operator equation discretized into $F^h z^h=0$. We compute z and z^h, resp., by Newtons method (NM) $F'(z_\nu)(z_{\nu+1}-z_\nu)=-Fz_\nu$ and $F^{h'}(z_\nu^h)(z_{\nu+1}^h-z_\nu^h)=F^h z_\nu^h$, $z_0^h:=I^h z_0$, where we assume the usual conditions for quadratic convergence to z for F. Under regularity assumptions for z and z_0, z_ν (available) and the usual stability and consistency conditions (order p) for F and F^h, and (unusually) for F' and $F^{h'}$ we have $z_\nu^h-I^h z_\nu=O(h^p)$, $F^h z^h{}_\nu-\hat{I}^h F z_\nu= O(h^p)$. Furthermore, the number of iterations to obtain a certain tolerance τ in (NM) is independent of the stepsize h for h sufficiently small. This result is used to formulate efficient strategies to solve nonlinear equations arising in discretization. This allows to obtain z^h, essentially independent of $z_0^{h_0}$ on a coarse grid, in the equivalence of 2-3 iterations on the final grid.

C. BOLLRATH:

Multigrid algorithms for the dam problem

We deal with the application of multigrid techniques to the two dimensional dam problem. Especially H.W.Alt's variational inequality formulation of the problem is considered, which works in a quite general situation. Two multilevel algorithms have been developed. The first one consists of relaxation steps and of solving auxiliary problems of obstacle type in the saturated part of the dam. These auxiliary problems only have the discrete pressure as unknown. They are solved approximatively by a two-grid method.
The second method is a pure multigrid algorithm of FAS type. The natural f.e. residual weighting operator has to be modified in boundary-nodes, in order to handle a condition of complementarity in the right way.
Solving several test-problems our first algorithm turned out to be faster than the relaxation method of H.W. Alt, however the pure multigrid algorithm is significantly the fastest. The speed of convergence is nearly independent of meshsize and not sensitive to changes in the permeability.

D. BRAESS:

On the numerical solution of the biharmonic equation

The formulation of the biharmonic equation $\Delta^2 u=f$ in $\Omega, u=\partial u/\partial n=0$ on $\partial\Omega$, in mixed form leads to an equation with the structure $\begin{pmatrix} M & B^T \\ B & 0 \end{pmatrix}\begin{pmatrix} w \\ u \end{pmatrix} = \begin{pmatrix} 0 \\ f \end{pmatrix}$. We discuss the preconditioned cg-method for the reduced system $BM^{-1}B^T u=f$. The first step for preconditioning is the modification of the boundary conditions to get two Poisson equations. This gives rise to a condition number $O(h^{-1})$. The second step consist in applying the incomplete Choleski decomposition. The standard way leads to a condition number $O(h^{-3})$, i.e. not to a squaring of the usual $O(h^{-1})$ result for the Poisson equation. It is shown for which norms multigrid procedures are assumed to have good convergence factors such that they are good preconditioners.

A. BRANDT:

Accuracy and fast solution of non-elliptic boundary value problems

For general linear algebraic systems and appropriate relaxation schemes, a general property of slow-to-converge errors is formulated. This property makes the error approximable on much lower dimensional spaces. In case of elliptic operators this property is equivalent to smoothness. Scale-dependent measures of ellipticity are defined for differential and discrete operators. Types of non-ellipticity and corresponding types of multigrid solvers. They typically solve to truncation level in one-cycle FMG, and their solutions can in fact be better approximation than the exact discrete solutions. Theoretical and practical tools used to predict and design the multigrid efficiency are modified for non-elliptic cases. Examples include non-staggered approximation to Stokes equations and convection-dominated boundary value problems.

W. HACKBUSCH:

Simultaneous eigenvalue calculation

To compute k eigenvalues and eigenvectors of a general unsymmetric matrix L, one can determine k vectors u_i collected in U and an upper triangular kxk matrix A such that $LU-UA=0$, $U^T U=I$. The Newton method applied to this problem leads to linear equations of the form $L\,\delta U - \delta U\,A - U\,\delta A = F$, $\delta U^T U + U^T \delta U = G$ (F, G given). It is shown that these equations form a staggered system. According to this structure, a multigrid algorithm is constructed.

P.W. HEMKER:

MG methods for the Euler equations

New results for multigrid methods for the Euler-equations will be discussed.
The emphasis will be on the solution of the discrete steady-state equations, obtained by Osher upwind discretizations - both 1st and 2nd order - for non-isentropic flow in 2 dimensions. Several relaxations in combination with multigrid will be considered.

G. HOFMANN:

Solving elliptic eigenvalue problems by multigrid methods

Analogously to the SOR-method for eigenvalue problems one can find a multigrid method. This method involves projections on the orthogonal space of the eigenspaces to solve the singular coarse grid equations. In this talk estimates are given, which show the effect of dropping these projections and of errors in coarse-grid-eigenvalues to the convergence of this multigrid method.

H. HOLSTEIN, G. PAPAMANOLIS:

A multigrid treatment of Stokesian boundary conditions

In order to treat the steady two-dimensional Stokes problem as coupled stream function and vorticity Dirichlet systems, the method of Thom is used to obtain a discrete boundary condition on vorticity. Such a boundary condition requires special considerations when a multigrid solver employing pointwise relaxation is used. We demonstrate some boundary treatments that exhibit ideal global smoothing rates, when lexicographic collective Gauss-Seidel interior relaxations with full weighting and W-cycles are used. For one particular treatment, ideal rates are also obtained with injection.

J. KARLISCH:

Convergence rates of the multigrid method for a three dimensional model problem

For the Poisson-equation on a subspace of $\mathbb{R}^3$ we study a finite element discretization by cubical elements and an alternating multilevel algorithm.
We compute exact convergence rates via a strengthened Chauchy inequality by the application of an algebraic method. A comparison of these rates with some rates we derive by the method of Fourier analysis for finite difference discretizations shows that they are too pessimistic for many cases. However we have to take into account that they also hold for non-convex domains.

R. KETTLER:

Some aspects of the application of multigrid in oil reservoir simulation

- An up-to-date reservoir simulator should be able to cope with a variety of oil/gas flow problems without loosing its reliability. Simulators therefore consist of huge program packages , which are usually very expensive in their set-up. A problem in reservoir simulation is the large amount of computing time per simulation. Frequently, much of this time is spent in the solution of sparse linear algebraic systems.
- In 1980, multigrid (MG) had proved to be a very efficient solution method for large linear systems arising from discretizations of simple cases of partial differential equations.
- Hence, it was decided to start a four-year project to make MG a fast and reliable sparse system solver in reservoir simulation, though it was understood that MG might have further possibilities as well. Investigation of the different types of discretizations taught us the necessity to extend MG to e.g. non-rectangular regions, strong inhomogenities, strong anisotropies, convection - dominated equations, systems of partial differential equations and 3D. This involved careful choices of the MG components. The result is a fast and reliable sparse system solver in reservoir simulation, but the implementation into the existing software turned out to be very involved.

C. LACOR:

Calculation of 3D, inviscid, rotational flows using a multigrid method

A method to calculate the three-dimensional, inviscid, rotational flow in ducts and turbomachines has been developed. The three-dimensional flow is split into a potential part, described by an elliptic equation, and a rotational part, described by two convective equations. An energy equation completes the set.
The equations are solved using a multigrid method, based on a Galerkin finite element approach. The program allows the use of arbitrary body fitted meshes. A procedure is

followed, assuring that the space of finite elements on a coarse grid is always an inclusion of the space of finite elements on the next finer grid. The construction of non-uniform interpolation and residual weighting operators, consistent with this approach, leads to simple expressions, without having to make any approximation due to the non cartesian mesh. A Gauss Seidel zebra alternating line relaxation is used as smoother. The program is based on the Full Approximation Scheme and uses V-cycles. Results will be presented for the potential flow as well as the complete rotational flow in ducts and turbomachines.

J. LINDEN:

A Navier-Stokes problem in stream function-vorticity formulation

In this talk a multigrid method for the computation of stationary flow fields in the gap between two coaxially rotating spheres is presented. The question of introducing artificial viscosity in order to keep the discrete equations h-elliptic is discussed as well as the question how to treat the no-slip boundary conditions of the stream function correctly in MG methods. This is studied in some detail for the model case of the biharmonic equation. A very efficient multigrid solver for this fourth order bvp. is presented.
Finally numerical results are shown for the flow in the gap where the outer sphere is rotating and the inner one is at rest.

J.F. MAITRE, F. MUSY:

Multigrid methods in a variational framework; formalization, basic concepts, estimation of convergence factors for some smoothers

To solve the symmetric variational problem: $u \in H$, $a(u,v) = L(v)$, $\forall v \in H$, multigrid methods are constructed using a sequence of nested subspaces of H. Bounds for the convergence rate in the "energy-norm" are obtained in two ways. The first gives a result which depends on the integer μ characterizing the cycle and does not prove the case of the V-cycle. The second gives a bound not depending on μ and proves the V-cycle case. The convergence factors are made more precise for different classes of smoothers: S.O.R., p steps of Richardson, p steps of two block G.S.,... All the factors are computed exactly for the monodimensional Laplacian on a regular mesh with piecewise linear approximation for the smoother of Richardson. The sharpness of the convergence bounds is very good for such a model problem.

J. MANDEL:

Algebraic analysis of multigrid methods

Multigrid methods in a variational setting (finite elements) and in the positive definite case converge with h-independent rate of convergence even with one smoothing step only. This is well known for H^2-regular problems. The case of less regular problems and W-cycles is new. Also, the convergence proof is extremely simple once the approximation assumption is verified.

I. MAREK:

On some aspects of homogenisation and aggregation in the multi-level context

A general two-level algorithm of solving linear equations in Banach space is shown to be locally convergent. As particular cases of this algorithm one can consider the aggregation method used in algebraic problems of economical sciences, the classical homogenisation method of nuclear reactor physics on the one hand, and some new techniques of reducing the dimensionality of some models of mathematical physics on the other hand. A multi-level version of the algorithm is also discussed.

S. McCORMICK:

The fast adaptive composite grid method (FAC) for $LU = \lambda FU$

Several authors have considered generally two categories of multigrid methods for solving the generalized eigenproblem $LU = \lambda FU$ for elliptic operators. One uses a linear multigrid solver for the inner loop of a linearized (e.g. by inverse iteration) problem; the other integrates multigrid into the nonlinear problem and may be called FAS-type. We, however, present what might be called a simpler but somewhat more attactive approach (in the self-adjoint case) that is based on minimization of the Rayleigh quotient (e.g., by coordinate relaxation) and a variationally formulated coarse grid correction.
We develop the algorithm for both global and local grid cases, establish a simple V-cycle theory, and exhibit numerical results for a single group neutron diffusion model.

M.L. MERRIAM:

Similarities between multigrid & cyclic reduction

A technique is shown whereby it is possible to relate a particular multigrid process to cyclic reduction using purely mathematical arguments. The necessary transfers and interpolations between grids are not neglected but are made the focus of the analysis. This technique suggests methods for solving Poisson's equation in 1-, 2-, or 3-dimensions with Dirichlet or Neumann boundary conditions. In one dimension the method is exact and, in fact reduces to cyclic reduction. This provides a valuable reference point for understanding multigrid techniques. The particular multigrid process analyzed is referred to here as Approximate Cyclic Reduction (ACR) and is one of a class known as MGR methods in the literature. It involves one approximation with a known error term. It is possible to relate the error term in this approximation with certain eigenvector components of the error. These are sharply reduced in amplitude by classical relaxation techniques. The approximation can thus be made a very good one.

H.D. MITTELMANN :

Multi-level continuation techniques

A technique is presented to solve parameter-dependent nonlinear elliptic boundary value problems. Several new ideas have been combined with known methods to yield a very efficient and reliable algorithm for continuation along solution branches. Some of the features of the program in which this algorithm is implemented are: continuation on the coarsest mesh, adaptive local mesh refinement in the multi-grid iteration, it allows to hit target points, runs in interactive mode, locates singular points and switches branches. Important further applications of the program are linear eigenvalue problems and homotopy continuation. Numerical results are presented for several problems including one with a tertiary symmetry-breaking bifurcation point. This paper represents joint work with R. Bank and T. Chan.

F. MUSY:

Multigrid methods in a variational framework: The case of the saddle point problem

We consider a class of multigrid methods for the numerical solution of saddle point problems. The methods are constructed from a sequence of nested subspaces of both Hilbert spaces on which the variational problem to solve is defined. The subspaces have to fit together such that an inf sup condition is satisfied at each level with a constant independent of the level. The Uzawa method is studied as smoothing process. For the two level convergence factor we establish a bound which proves the multigrid convergence for the W cycle scheme. The 2-dimensional Stokes equation with appropriate discretizations is considered as an application. We repeat some numerical experiments where the Uzawa smoother compares formally with the distributive relaxation.

P. PEISKER:

A multilevel algorithm for the biharmonic problem

We consider a finite element discretization of the mixed variable formulation of the biharmonic equation. For the numerical solution of the discrete equations a multilevel algorithm is applied. Convergence is proved under the assumption of H^3 - regularity and using piecewise quadratic finite elements. In particular the analysis with the assumption of H^3 - regularity includes domains, which are convex polygons.

J. PERIAUX:

Domain decomposition methods for the Stokes and the Navier-Stokes problems

The main goal of this paper is to present iterative and direct methods, using domain decomposition, for solving the Stokes problem. The idea is to decompose the global domain in subdomains with or without overlapping, thensolve local Stokes problems and recouple these local solutions in order to obtain a method for solving the global problem. Schwarz method can be used but also more sophisticated iterative or direct methods matching the velocity and the pressure on the interfaces of adjacents subdomains. The treatment of the incompressibility introduces extra difficulty, compared, for example, to the solution of a standard Poisson problem ; this is particularily time for the finite element approximation of the Stokes problems, specially if $\nabla.u=0$ is weakly satisfied. Then,this methodology is used to solve unsteady incompressible Navier Stokes equations for which the formulation is founded on a time discretization by operator splitting method. These methods providean efficient local way to decouple the two main difficulties of the problem, i.c. the incompressibility (local Quasi Stokes problems) and the non linearity (local non linear problems solved by least squares). These methods are well suited for a solution by multiprocessor machines, and numerical results obtained using such computer systems will be presented.

J. PITKÄRANTA:

On the multigrid solution of Stokes equations

The bilinear/constant finite element method for the Stokes problem on a rectangular domain yields a simple finite difference method which works well when combined with direct solvers, despite the known lack of stability of the method. We show by numerical experiments that the lack of stability causes severe difficulties in the multigrid solution of the system. However, the convergence of the multigrid method can be recovered by adding an appropriate pressure smoothing step after each relaxation step. The added step can also be interpreted as stabilization of the finite element method. A similar strategy is shown to work also in connection with some other simple but unstable methods for the Stokes problem in two or three dimensions.

J. RUGE:

Algebraic multigrid methods applied to systems of PDE's

Algebraic multigrid is a method of applying multigrid ideas to the solution of a matrix equation without explicit use of the geometry or origin of the original problem. All information needed for the choice of the coarse grid and for the definition of grid transfer and coarse grid operators is taken from the matrix itself. This method works well for a number of scalar problems (those for which all unknowns represent the same quantity) including finite difference and finite element discretizations of anisotropic problems, diffusion problems with discontinuous coefficients, and convection-diffusion problems.
This talk briefly explains the ideas involved in AMG and its relation to multigrid methods. Also, some of the problems which arise when attempting to apply AMG to discretizations of systems of PDE's. More information must be provided, in particular which unknowns correspond to the same quantity in the continuous problem. Using this information, it is shown how AMG can be extended to cover 2-d linear elasticity problems. In addition, further modifications to relaxation and interpolation allow more complicated systems, such as Stokes equations.
Also discussed are several alternate approaches to systems, each of which have some advantages in different cases.

B. STEFFEN:

Mehrgitterverfahren zur Berechnung der Eigenschwingungen von Hohlraumresonatoren

Für die Auslegung der gegenwärtig vielerorts geplanten Hochleistungs - Teilchenbeschleuniger müssen die beschleunigenden Felder mit sehr hoher Genauigkeit bekannt sein. Dazu werden dreidimensionale Verfahren zur Berechnung der Eigenlösungen der Maxwellgleichung benötigt. Eine sorgfältig optimierte Kombination der Treppeniteration nach Bauer - Rutishauser als Eigenwert - Algorithmus mit Mehrgitterverfahren für die Differentialgleichung und adaptiver Spektralverschiebung verspricht, die geforderte Genauigkeit ohne exzessiven Rechenaufwand erreichen zu können. Wegen der Komplexität des Gebiets und der Randbedingungen sind hier aber sowohl in der Lösung der Differentialgleichung als auch in der Steuerung des Eigenwertalgorithmus noch eine Reihe von Detailproblemen zu lösen.

U. TROTTENBERG:

Basic smoothing methods for anisotropic 3D operators; some remarks about the SUPRENUM project

In this talk the "3D activities" of the GMD-MG group are surveyed.
Some systematical investigations for the anisotropic operator ($au_{xx}+bu_{yy}+cu_{zz}$) by model problem analysis have been made. It is shown that in certain cases plane-relaxation is needed (if standard coarsening is used). Contrary to the expectations of many experts suitable plane-relaxation algorithms (very crude 2D-MG cycles) turn out to be very efficient, and easily implementable. In all cases FMG solutions (up to the level of truncation error) can be obtained within 4-11 work units (3D point relaxations).
The SUPRENUM project (Numerischer Superrechner) and its connection to non-adaptive and adaptive 3D-MG-methods is described.

R. VERFORTH:

Iterative solution of mixed finite element approximations of the Stokes problem

We present a preconditioned conjugate residual algorithm for mixed finite element approximations of the Stokes problem. The preconditioning consists in replacing the H^1-scalar product for the velocity by a mesh dependent scalar product which is spectrally equivalent up to a factor $O(|\log h|)$. Using hierachical basis functions this scalar product has a diagonal stiffness matrix except a very small diagonal block. Hence it can easily be inverted. The cost of the preconditioning roughly corresponds to the calculation of 3 vector products. The resulting algorithm has a quasi optimal convergence rate of $1-O(|\log h|)$. We give some numerical results for Stokes and Navier-Stokes problems and compare them with those obtained with other iterative methods.

P. WESSELING:

Applications of multigrid methods in computational fluid dynamics

The application of multigrid methods to transonic potential and Euler flows, and to the Navier-Stokes equations will be discussed.

K. WITSCH, U. TROTTENBERG:

Survey on several MG activities in the GMD

Four activities of the GMD-MG group are surveyed.
1) A MG code for the 2D full potential equation describing the flow around an airfoil has been developed. The approach is characterized by the following difficulties:
(1) use of cartesian coordinates
(2) Neumann b.c. on the airfoil
(3) the far field b.c. at infinity
(4) the Kutta-Joukowski condition.
By proper MG-treatment one obtains convergence factors of 0.1 in 3-4 WU. A special feature of the code is the adaptive local refinement of the grids (massive concentration of gridpoints) near the profil.
2) Advanced refinements techniques (with local relaxation and λ-FMG) have been systematically studied for the Poisson equation in 2D-regions with reentrant corners.
3) The "NUSIMOT" project is described. Here a MG code for the simulation of flow in the combustion chamber of otto engines is under development.
4) Guided waves in optical components can be described by an unsymmetric eigenvalue problem for two coupled 2D elliptic b.v.p., derived from Maxwells equations. A MG code for this problem is described and first results are shown.

H. YSERENTANT:

A nonstandard multi-level method not depending on regularity

In this talk the use of hierarchical bases in finite element computations has been discussed. It has been shown that for plane elliptic boundary value problems the use of hierarchical bases reduces the exponential growth of the condition numbers of the discretization matrices when using nodal bases to a quadratic growth in the number of refinement levels. In combination with simple but very fast algorithms for handling hierarchical bases this leads to iterative schemes which are of nearly optimal computational complexity and are very easy to program.

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