MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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(letzter Bericht 1984)

Functional Equations

16.12 to 22.12. 1984

The Twenty-second International Symposium on Functional Equations, which was dedicated especially to Professor J. Aczél on the occasion of his 60th birthday, was held from December 16 to December 22, 1984 in Oberwolfach, Germany. The organizational committee consisted of J. Aczél (Waterloo), W. Benz (Hamburg), and J. Rätz (Bern); Professor Aczél, however, took the liberty of not participating in some activities of the committee, given the special circumstances. B. Ebanks (Lubbock) acted as the secretary of the symposium. The meeting was opened by W. Benz, who also used the opportunity to express with all sincerity the thanks of the participants to J. Aczél as a mathematician and as a person.

The symposium was attended by 46 participants from 10 European and 2 American countries. We were pleased to see that some mathematicians from Czechoslovakia, Poland, and Yugoslavia were able to come.

Among the fields represented were iterative equations, and equations of classical and generalized types. Relations to functional analysis, approximation theory, differential equations, special functions, measure theory, theory of stability, information theory, stochastics, geometry, and economics were discussed extensively.

At the end of each of the 9 sessions, which included 42 lectures, there was time dedicated to remarks, new open problems, and solutions of both old and new problems. These were even more stimulating and successful than at previous meetings. Sometimes problems were solved surprisingly fast and gave rise immediately to new problems. All this contributed to the usual creative and supportive atmosphere.

The traditional Weinabend took place on Thursday. Tributes to the personality and to the scientific mastery and virtuosity of Professor János Aczél were paid by J. Rätz, L. Reich, B. Schweitzer, A. Sklar, W. Eichhorn, B. Choczewski, Z. Daróczy, and Pl. Kannappan.

Towards the end of the last session, J. Aczél thanked the speakers of the meeting and the authors of the special issue of Aequationes for their kind contributions on this occasion.

The meeting was closed by J. Rätz, who expressed the gratitude of the participants to the Institute for its warm hospitality. The Twenty-third International Symposium on Functional Equations will be held in Gargnano, Italy.

Abstracts of the Talks

J. ACZÉL:

Functional equations applied to Korovkin approximation and similar problems

In order to prove that the space spanned by $f_1(x)=x$ and $f_2(x)=x^2$ is the Korovkin closure of $\{f_1, f_2\}$ in $C(]0,1])$, H. Bauer suggested showing that the general continuous solution of

$$\text{(K)}\quad \frac{f(x)+f(y)}{x+y} = \frac{x+y}{x^2+y^2}\, f\left(\frac{x^2+y^2}{x+y}\right)\ (x>0,y>0) \text{ is } f(x)=ax+bx^2.$$

Both this and the general continuous solution $f(x)=ax+bx^{-1}$ of

$$\frac{f(x)+f(y)}{x+y} = \frac{1}{\sqrt{xy}}\, f(\sqrt{xy})\ (x>0,y>0),$$

also suggested by H. Bauer, follows from a uniqueness theorem in Aczél, Acta Math. Hungar. 15 (1964), 355-362.

But no answer to the question raised by H. Bauer about the functional equation

$$\text{(B)}\quad \frac{f(x)+f(y)}{x+y} = \frac{x+y}{2xy}\, f\left(\frac{2xy}{x+y}\right)\ (x>0,y>0)$$

follows from that theorem. It is proved both by reduction to a general theorem by Chung-Kannappan-Ng (Linear Algebra Appl., to appear) and directly by use of $([(\alpha+\frac{\alpha+\beta}{2})/2]+[(\frac{\alpha+\beta}{2}+\beta)/2])/2 = \frac{\alpha+\beta}{2}$ that the only solution (without any regularity condition) of (B) is $f(x)=cx$ (c, as well as a, b above, are arbitrary constants).

C. ALSINA:

The associative solutions of the functional equation $\tau(F,G)+\hat{\tau}(F,G)=F+G$

Let Δ^+ be the space of probability distributions vanishing at 0 and let τ be a continuous binary operation on Δ^+ which is commutative, nondecreasing in each place and with ε_0 as a unit element. Then we have

Theorem. The operations $\tau(F,G)$ and $F+G-\tau(F,G)$ are associative if and only if there exists a t-norm T on [0,1] such that

$$\tau(F,G)(x) = T(F(x),G(x))$$

and the operations $T(x,y)$ and $x+y-T(x,y)$ are associative. The problem of the simultaneous associativity of $T(x,y)$ and $x+y-T(x,y)$ was completely solved by M.J. Frank (Aequat. Math. 19, 194-226 (1979)).

K. BARON:

On the convergence of sequences of iterates of random-valued functions

It has been proved in [1] that for every $x\in[0,1]$ the sequence $(f^n(x,\cdot): n\in\mathbb{N})$ of iterates of a so-called random-valued function f converges to 0 almost surely provided f is continuous and $E(f(x,\cdot))<x$ for every $x\in(0,1]$.

The question arises, how fast is this convergence? A result in this direction is contained in the paper [3] by M. Kuczma. Ph. Diamond in [2] also deals with such a problem. The aim of this talk is to present another result in this direction.

References

1. K. Baron and M. Kuczma, Iteration of random-valued functions on the unit interval, Colloquium Mathematicum 37 (1977), 263-269.

2. Ph. Diamond, A stochastic functional equation, Aequationes Mathematicae 15 (1977), 225-233.

3. M. Kuczma, Normalizing factors for iterates of random valued functions, Prace naukowe Uniwersytetu Slaskiego nr 87. Prace matematyczne 6 (1975), 67-71.

N. BELLŰOT (BRILLOUËT):

On the functional equations $f[xf(y)+yf(x)]=\alpha f(x)f(y)$ and $f[xf(y)+yf(x)] = \alpha f(xy)$

Consider the two following functional equations:

$$\left.\begin{array}{ll}(1) & f[xf(y)+yf(x)] = \alpha f(x)f(y) \\ (2) & f[xf(y)+yf(x)] = \alpha f(xy)\end{array}\right\}$$ where α is a non-negative real number.

We have the following results:

Theorem 1. Let E be a real topological vector space. When $\alpha=0$, the unique continuous solution $f: E \to \mathbb{R}$ of (1) and (2) is: $f \equiv 0$.

Theorem 2. Let E be a real locally convex topological vector space. When α is a strictly positive real number, all continuous solutions $f: E \to \mathbb{R}$ of (1) are given by:

(i) $f \equiv 0$, (ii) $f \equiv \frac{1}{\alpha}$,

and, in the case $\alpha=2$ only:

(iii) $f(x) = \begin{cases} \langle x^*,x\rangle & \text{for } x \in \bar{K} \\ 0 & \text{for } x \notin \bar{K}, \end{cases}$ (iv) $f(x) = \begin{cases} \langle x^*,x\rangle & \text{for } x \in \overline{K \cup (-K)} \\ 0 & \text{for } x \notin \overline{K \cup (-K)} \end{cases}$

where $x^* \in E^* \setminus \{0\}$ and K is a non-empty open convex cone with vertex 0 contained in $\{x \in E \mid \langle x^*, x\rangle < 0\}$.

Theorem 3. When $\alpha=1$, all continuous solutions $f: \mathbb{R} \to \mathbb{R}$ of (2) are given by:

(i) $f \equiv a, a \in R$; (ii) $f(x) = \frac{1}{2}x$;

(iii) $f(x) = \operatorname{Inf}(x, 0)$; (iv) $f(x) = \operatorname{Inf}(-x, \frac{x}{2})$.

W. BENZ:

A contribution to a problem of I.S. Fenyö

Fenyö (1982) asks for the general solution of

$$(*) \quad f(xf(y)+f(x)y-xy) = f(x)f(y).$$

Volkmann, Weigel (1984) determine the general continuous solution $f: \mathbb{R} \to \mathbb{R}$ of (*), a class which contains $\aleph$ many functions. We show that there are $2^{\aleph}$ many discontinuous solutions $f: \mathbb{R} \to \mathbb{R}$ of (*). We present classes of solutions, which are nowhere continuous, which are continuous in only one point of $\mathbb{R}$, which are discontinuous in one, in two points of $\mathbb{R}$. The construction methods come via multiplicatively closed curves (Dhombres (1982)) and from semigroup theory.

B. CHOCZEWSKI:

Iterative roots of Laguerre polynomials (joint work with M. Kuczma)

The system of functional equations

$$(1) \quad \begin{aligned} &H^n(t) = \frac{t}{t-1}, \\ &\prod_{i=0}^{n-1} G(H^i(t)) = (1-t)^{-\alpha-1}, \quad \alpha > -1 \end{aligned}$$

occurs when looking for n-th roots, under umbral composition (cf. G.C. Rota, D. Kahaner and A. Odlyzko; On the foundation of combinatorial theory, VIII. Finite operator calculus. J. Math. Anal. Appl. 42 (1973), 684-760), of the sequence of Laguerre polynomials. Here H and G are generating formal power series of an n-th root, whereas the right-hand sides of (1) represent those of the Laguerre sequence. The general solution of system (1) will be presented.

The paper is related to one by J.M. Brown and M. Kuczma, Self-inverse Sheffer sequences, SIAM J. Math. Anal. 7 (1976), 723-728.

Z. DARÓCZY:

Intervallfüllende Folgen und volladditive Funktionen (gemeinsam mit A. Járai und I. Kátai)

Es sei $\lambda_n > \lambda_{n+1} > 0$ und $L := \sum_{n=1}^{\infty} \lambda_n < \infty$. Die Folge $\{\lambda_n\}$ wird intervallfüllend genannt, falls für beliebiges $x \in [0,L]$ es eine Folge $\varepsilon_n \in \{0,1\}$ gibt derart, dass $x = \sum_{n=1}^{\infty} \varepsilon_n \lambda_n$ ist. Wir nennen eine Funktion $F: [0,L] \to \mathbb{R}$ volladditiv (bezüglich der intervallfüllenden Folge $\{\lambda_n\}$), falls

$$F\left(\sum_{n=1}^{\infty} \varepsilon_n \lambda_n\right) = \sum_{n=1}^{\infty} \varepsilon_n F(\lambda_n)$$

für jede Folge $\varepsilon_n \in \{0,1\}$ gilt. Die Zahl $x \in [0,L]$ wird eindeutig genannt, falls es genau eine Folge $\varepsilon_n \in \{0,1\}$ gibt, für welche $x = \sum_{n=1}^{\infty} \varepsilon_n \lambda_n$ gilt. Wir nennen die intervallfüllende Folge $\{\lambda_n\}$ ergiebig, falls jede Zahl $x \in]0,L[$ nicht eindeutig ist. Es gelten die folgenden Ergebnisse: (1) Ist F volladditiv, so ist F in [0,L] stetig. (2) Ist $\{\lambda_n\}$ intervallfüllend und ergiebig, und ist F volladditiv bezüglich $\{\lambda_n\}$, so gibt es ein $c \in \mathbb{R}$ derart, dass $F(x)=cx$ fur alle $x \in [0,L]$.

Vermutung: Ist $\{\lambda_n\}$ intervallfüllend und ist F volladditiv, dann muss F linear sein. Ist diese Vermutung nicht richtig, dann existiert eine volladditive Funktion, die in $[0,L]$ stetig und nirgends differenzierbar ist.

J. DHOMBRES:

Functional equations and characterization of inner product spaces

Let ABC be a triangle in the euclidean space. It is well known that it is possible to compute the length of any median of the triangle from the length of its three sides. In a real normed space, this mere possibility (median property) implies that the norm derives from an inner product (Lorch 1948). Known proofs either rely on an earlier result of Ficken where all computations are not indicated, or on the proof of some smoothness of the unit shere, an approach which does not seem the most direct way.

Our purpose is first to provide a self-contained proof of Lorch's result, second to systematically use functional equations and more precisely conditional functional equations to perform the proof, and third to generalize Lorch's result by showing the role played by the field on which the normed space E is built. In this process, some interesting questions arise for functional equations, and all are not solved.

B. EBANKS:

Some recent results about inset entropies on open domains

Several problems regarding inset entropies on open domains (i.e. without empty sets and zero probabilities) have been solved recently. Among these are determinations of (i) all semisymmetric, β-recursive entropies (including those with weak regularity properties, say, measurability), (ii) all additive inset entropies with measurable sum property, and (iii) all semisymmetric entropies which are recursive of multiplicative type. Some of these results and other work in progress will be discussed.

W. EICHHORN:

Bellman's functional equation and the optimal investment ratio of an economy

Problem: Given (the poduction structure of) an economy, its initial capital stock [= amount of capital goods (buildings, machines) involved in the production process] at the beginning of year 1, and a time horizon T ($\geq$ 3, finite or infinite), find a vector $(u_1^*, u_2^*, \ldots, u_T^*)$ of investment ratios

$$u_t = \frac{\text{gross domestic investment during year } t}{\text{gross domestic product during year } t}$$

which maximizes the macroeconomic consumption during the years 1 to T.

The problem is solved within the framework of a model of an aggregated economy by means of Bellman's principle of backward dynamic programming applied to Bellman's functional equation(s). The functional equation(s) come from the model.

I. FENYÖ:

Some properties of the Jacobian snz function

Applying some theorems on Cauchy functional equations it is proved that the graph of $\int_0^x \mathrm{sn}^2 t\,dt$ remains in a strip: $\{\alpha x-1, \alpha x+1\}$ where α is a constant depending only on the modulus of the Jacobian snz function. Some new functional equations are discussed related with the snz functions.

B. FORTE:

A Cauchy type functional equation for stochastic processes (joint work with W. Hughes)

Consider a stochastic process $\Phi(t): t\epsilon\mathbf{R}^+$ with $\Phi(t) = X_t$ a real valued measurable function on some probability space (Ω,F,P). If we set

$$X_{s+t} = X_s + X_t \text{ prob } 1 \qquad \text{(i)}$$

where the functions add pointwise on the elements ω of the sample space, we can consider this to be a generalization of the Cauchy functional equation $f(x+y) = f(x)+f(y)$ to stochastic processes.

If we restrict ourselves to random variables with finite expectation and we assume Φ continuous in the mean, then the only solutions are

$$X_t = tX_1$$

with X_1 a measurable function on the probability space.

Equation (i) becomes more interesting if interpreted in distribution rather than with probability one. We consider the following equation

$$X_{s+t} - X_t \overset{\text{dist}}{=} X_s .$$

G.L. FORTI:

On a difference-functional equation

In 1980 C. Borelli Forti and I. Fenyö considered (see [1]) the difference equation

$$(\Delta^n f)(x;y_1,\ldots,y_n) = d(x;y_1,\ldots,y_n)$$

where $f:X\to E$, $d:X\times X^n\to E$ (X is an abelian group, E is a Banach space), d is a given bounded function, and gave the explicit expression of the general solution. The previous result is used for solving an equation of the form

$$(1)\quad (\Delta^n f)(x;y_1,\ldots,y_n) = \Phi\{x;y_1,\ldots,y_n;(\Delta^i f)(x;y_{k_1},\ldots,y_{k_i});f(x)\}$$

$i=1,\ldots,n$ (we have indicated only one of the positions of the variables depending on i and $(k_1,\ldots,k_i)$, actually there are 2^n-1), when Φ satisfies a Lipschitz condition and a condition of boundedness.

More precisely we prove that there exists at most one bounded solution of (1) and it is the uniform limit of a sequence of functions explicitly described.

REFERENCE

[1] C. Borelli Forti, I. Fenyo: Sulle equazioni alle differenze con incrementi variabili, Stochastica 4 (1980), 93-101.

M.J. FRANK:

Randomness and orbit structure (joint work with W. Darsow, T. Erber, and P. Johnson)

Sequences that are intended to imitate randomness are usually generated by iterating an appropriate function f on a finite set. The natural requirement that such sequences exhibit the distributional behavior of simple urn models imposes surprisingly severe constraints on the orbit structure of f. First, almost exactly 80% of all points lie in just one f-orbit. Second, in this dominant orbit roughly 75% of the points lie on branches that enter the terminal loop at the same point, and a large fraction of these on a single branch. The proofs involve, among other things, delicate combinatorial arguments. Computer experiments with various mixing transformations and standard pseudo-random number generators confirm this structure.

R. GER:

Positive multilinear functionals and n-unitary spaces

Given a real vector space X, fix an even positive integer n and consider a symmetric functional $A: X^n \to \mathbb{R}$ which is linear in each variable and such that its diagonalization $A(x,x,\dots,x)$ is positive provided $x \neq 0$. Having the case $n = 2$ in mind as a pattern model we propose axiomatics of a generalized unitary space (n-unitary space) and introduce a suitable norm with respect to which A is continuous. We deal with an analogue of the celebrated Jordan-von Neumann theorem characterizing the classical unitary spaces. The latter are just 2-unitary spaces in our terminology. Some nontrivial examples of n-unitary spaces are also presented.

H. HARUKI:

A new characterization of Euler's gamma function

The purpose of this talk is to prove the following theorem.

Theorem. Let D denote the complement of the set of all nonpositive integers. If

(A) f is a meromorphic function of a complex variable z in $\mathbb{C}$;

(B) f is analytic and never vanishes in D;

(C) f satisfies the functional equation

$$n^{z-\frac{1}{2}} \prod_{k=0}^{n-1} f\left(\frac{z+k}{n}\right) = (2\pi)^{\frac{n-1}{2}} f(z)$$

on D, where n is an arbitrarily fixed integer greater than 1 and $n^{z-\frac{1}{2}}$ denotes the principal value $\exp((z-\frac{1}{2})\operatorname{Log} n)$; then $f(z) = \exp(a(z-\frac{1}{2}) + \frac{2m\pi i}{n-1})\Gamma(z)$, where a is an arbitrary complex constant and m is an arbitrary integer.

A. JÁRAI:

On Steinhaus-type theorems

It is proved, that the mapping

$$t \mapsto \int_Y h(f_1(g_1(t,y)),\ldots,f_n(g_n(t,y)))\,d\nu(y)$$

is continuous for continuous h and g_i satisfying certain conditions and for functions $f_i \in \mathbb{L}^\infty$. From this a general "convolution is continuous"-type theorem is derived. Namely, it is proved, that the interior of the multi-dimensional set

$$F(A_1 \times A_2 \times \ldots \times A_n)$$

is nonvoid for measurable A_i's with positive Lebesgue measure.

H.-H. KAIRIES:

Funktionalgleichungen für stetige, nirgends differenzierbare Funktionen

Es sei p eine feste Primzahl, $[p]: = \{p^n | n \in \mathbb{N} \cup \{0\}\}$. Durch $S_p(x) = \sum_{n=0}^{\infty} \frac{1}{p^n} \sin 2\pi p^n x$ wird eine reelle Funktion erlkärt, die stetig, 1-periodisch und ungerade ist. S_p gehört zu einer von Weierstrass eingeführten Klasse nirgends differenzierbarer Funktionen. Weiter gilt $S_p(x) = \sum_{m=0}^{k-1} S_p(\frac{x+m}{k})$ für alle $k \in [p]$ und $\sum_{m=0}^{k-1} S_p(\frac{x+m}{k}) = 0$ für alle $k \in \mathbb{N} \setminus [p]$. Wir untersuchen, inwiefern S_p (und damit insbesondere die Eigenschaft "nirgends differenzierbar") durch die oben aufgeführten Funktionalgleichungen charakterisiert ist.

SATZ 1. $f:\mathbb{R} \to \mathbb{R}$ sei stetig, 1-periodisch und ungerade. Ferner gelte

$$\sum_{m=0}^{p-1} f\left(\frac{x+m}{p}\right) = f(x) \text{ sowie } \sum_{m=0}^{q-1} f\left(\frac{x+m}{q}\right) = 0 \text{ für alle von } p \text{ verschiedenen}$$

Primzahlen q. Ist dann $f \neq 0$, so ist f nirgends differenzierbar. Ist $f(1/2p) = \sin\pi/p$, so ist $f = S_p$.

SATZ 2. $f:\mathbb{R} \to \mathbb{R}$ sei stetig, 1-periodisch und ungerade. Ferner gelte

$$\sum_{m=0}^{p-1} f\left(\frac{x+m}{p}\right) = f(x) \text{ sowie } f(x) - \sum_{m=0}^{k-1} f\left(\frac{x+m}{k}\right) = \Phi_k(x) \text{ für alle } k \in \mathbb{N} \setminus [p],$$

$\Phi_k:\mathbb{R} \to \mathbb{R}$. Es folgt $f = S_p$ genau dann, wenn $2\int_0^1 \Phi_k(t)\sin 2\pi t\, dt = 1$ für alle $k \in \mathbb{N} \setminus [p]$.

PL. KANNAPPAN:

On a generalized fundamental equation of information (joint work with C.T. Ng)

The aim is to determine the general solution of (*) $f(x)+\alpha(1-x)g\left(\frac{y}{1-x}\right) = h(y)+\alpha(1-y)k\left(\frac{x}{1-y}\right)$, for $x,y \in [0,1[$ with $x+y \in [0,1]$. It is shown that, f,g,h,k are given by

$$\begin{aligned}
f(x) &= \Phi(1-x) + a_1\alpha(x) + b_1\alpha(1-x) + c \text{ on } [0,1[\\
g(x) &= \Phi(x) + a_2\alpha(x) + b_2\alpha(1-x) - b_1 \text{ on } [0,1]\\
h(x) &= \Phi(x) + a_2\alpha(x) + b_3\alpha(1-x) + c \text{ on } [0,1[\\
k(x) &= \Phi(1-x) + a_1\alpha(x) + b_2\alpha(1-x) - b_3 \text{ on } [0,1]
\end{aligned}$$

where

either $\alpha(x) = x^2$ and $\Phi(x) = D(x)$,

or $\alpha(x) = |\phi(x)|^2$ and $\Phi(x) = a\,\mathrm{Im}\,\phi(x)$,

or $\alpha(x) = x$ and $\Phi(x) = xL(x) + (1-x)L(1-x)$,

or α is an arbitrary multiplicative map and $\Phi = 0$.

L is logarithmic, D is a real derivation and $\phi:\mathbb{R} \to \mathbb{C}$ is an isomorphism, with constants a, a_i, b_i, c.

A. KRAPEŽ:

Configurations in webs (joint work with M.A. Taylor)

There are certain problems with Belousov's proof (Configurations in algebraic webs (nets)), Stiinca, Kisinev, 1979) of the following result:

A configuration in a web corresponds to a system of functional equations on quasigroups related to a coordinate system of a web.

We comment on Belousov's proof and give some further relevant results.

M. LACZKOVICH:

On a generalized difference equation

Theorem. Suppose that $f:\mathbb{R}\to\mathbb{R}^+$ is measurable and satisfies the equation

$$f(x) = \sum_{i=1}^{k} b_i f(x+a_i) \quad (x\in\mathbb{R}),$$

where $b_i > 0$ $(i=1,\dots,k)$ and $a_1,\dots,a_k$ are linearly independent over the rationals. Then $f(x) = \sum_{j=1}^{N} c_j e^{\gamma_j x}$ for a.e. x, where $\gamma_1,\dots,\gamma_N$ are the roots of the characteristic equation $\sum_{i=1}^{k} b_i e^{a_i x} = 1$.

Simple examples show that neither of the conditions imposed on f, b_i, and a_i can be deleted.

Corollary. If $f: \mathbb{R}^+ \to \mathbb{R}^+$ is measurable and satisfies $f(x) = f(\frac{x}{3}) + f(\frac{2x}{3})$ $(x \in \mathbb{R}^+)$, tben there is a constant $c \geq 0$ such that $f(x) = cx$ for a.e. $x \in \mathbb{R}^+$.

This gives a positive answer to a problem by G. Székely.

K. LAJKÓ:

Functional equations in the spectral theory of random fields

M.I. Yadrenko (Kiev) proposed the following problems:

The correlation function of a homogeneous, isotropic, Gaussian random field of Markovian type on the Hilbert space ℓ^2 or on the sphere S^n satisfies the functional equations

(1) $B(R\sqrt{2})B(R_2) = B(R)B(\sqrt{R^2+R_2^2})$ $(R > R_2 > 0)$

and

(2) $B(\cos\theta_1)B(\cos^2\theta_2) = B(\cos\theta_1\cos\theta_2)B(\cos\theta_2)$ $(\theta_1, \theta_2 \in [0,\pi], \theta_1 > \theta_2)$

respectively. Find the general solutions of (1) and (2) if B is continuous.

Functional equations (1) and (2) can be reduced to

(3) $f(2x)f(y) = f(x)f(x+y) \quad (y>x>0)$.

The general continuous solutions of (3) are determined.

GY. MAKSA:

On completely additive functions

A strictly decreasing sequence of positive real numbers (λ_n) is called an interval-filling sequence if $\sum_{n=1}^{\infty} \lambda_n = L \in \mathbb{R}$ and for any $x \in [0,L]$ there exists a sequence $(\varepsilon_n):\mathbb{N} \to \{0,1\}$ such that $x = \sum_{n=1}^{\infty} \varepsilon_n \lambda_n$. The function $F:[0,L] \to \mathbb{R}$ is called completely additive (with respect to the interval-filling sequence (λ_n)) if $F(\sum_{n=1}^{\infty} \varepsilon_n \lambda_n) = \sum_{n=1}^{\infty} \varepsilon_n F(\lambda_n)$ for all $(\varepsilon_n):\mathbb{N} \to \{0,1\}$. Professors Daróczy, Járai and Kátai, under various further assumptions on the interval-filling sequence, have determined all completely additive functions by showing that they are linear. In this talk we suppose nothing on the interval-filling sequence and we prove the same for those completely additive functions which are nonnegative or differentiable at a point.

J. MATKOWSKI:

Cauchy functional equations on restricted domains and applications to a characterization of the L^p norm and to iteration theory

Suppose that $\varphi:\mathbb{R} \to \mathbb{R}$ is continuous at least at one point and let $a,b \in \mathbb{R}$ be not commensurable. If φ satisfies the Cauchy functional equation $\varphi(x+y) = \varphi(x) + \varphi(y)$ for $(x,y) \in X$ where X is an union of two perpendicular straight lines

$$(x=a, y=t);\ (x=t), y=b),\ t\in R,$$

or two parallel straight lines

$$(x=a, y=t);\ (x=b, y=t),\ t\in R,$$

thenthere exists a $c\in R$ such that $\varphi(t)=ct$ for $t\in R$.

This result can be applied to a characterization of the L^p norm as well as to some problems in iteration theory.

Z. MOSZNER:

Sur les différentes définitions de la stabilité de l'équation fonctionnelle

On donne les relations entre les différentes définitions de la stabilité de l'équation d'homomorphisme

(1) $$f_1(x\cdot y) = f_1(x)\cdot f_1(y),$$

ou $f_1:E_1\to V_1$ et E_1, V_1 forment des groupoides. Soit

(2) $$f_2(x\cdot y) = f_2(x)\cdot f_2(y)$$

une autre équation d'homomorphisme, ou $f_2:E_2\to V_2$, et $\alpha_1:E_1\to E_2, \alpha_2:E_2\to E_1, \beta_1:V_1\to V_2, \beta_2:V_2\to V_1$ des homomorphismes tels que $\alpha_1(\alpha_2) = id_{E_2}$ et $\beta_1(\beta_2) = id_{V_2}$. On donne les conditions complémentaires au sujet de β_1 et β_2 sous lesquelles la stabilité de (1) entraîne (2).

REFERENCES

[1] Janusz Baster, Zenon Moszner, Jozef Tabor, On the stability of a class of functional equations, sous presse dans Rocznik Nauk.-Dydak. WSP w Krakowie.

[2] Zenon Moszner, Sur la stabilité de l'équation d'homomorphisme, sous presse dans Aeq. Math.

[3] Jozef Tabor, On mappings preserving the stability of Cauchy functional equation, en préparation.

F. NEUMAN:

Functional and differential equations

Several types of functional equations occur when dealing with global properties and transformations of differential equations. A vector functional equation

(1) $$y(x) = A.f(x).y(h(x))$$

describes global transformations of a linear differential equation of the n-th order into itself, $n \geq 2$. Here y denotes an n-tuple of linearly independent solutions of the equation defined on an interval $I \subset \mathbb{R}$, and A is an n by n regular constant matrix, f and h are functions such that $f: I \to \mathbb{R}$, $f \in C^n(I)$, $f(x) \neq 0$ on I, $h: I \to I$, $h \in C^n(I)$, $h(I) = I$.

For a given y all solutions $\langle A,f,h \rangle$ of (1) were described. Especially, it was shown that all h satisfying (1) form a group with at most three parameters regardless of the dimension n of the vector equation.

L. PAGANONI:

On an inhomogeneous Cauchy equation connected with Jacobi's elliptic functions (joint work with I. Fenyö)

Consider the following functional equation in the complex domain $\mathbb{C}$:

(1) $$g(z_1+z_2) - g(z_1) - g(z_2) = f(z_1)f(z_2)f(z_1+z_2).$$

The following theorem holds:

Theorem. Let (f,g) be a pair of analytic functions, defined in a neighborhood of the origin and solutions of (1).

(i) If $f(0) \neq 0$ then there exist $\alpha,\gamma \in \mathbb{C}$ such that

$$f(z) = \alpha \qquad g(z) = -\alpha^3 + \gamma z$$

(ii) If $f(0) = 0$ and $f'(z) = \alpha$, then there exists $\gamma \in \mathbb{C}$ such that

$$f(z) = \alpha z \qquad g(z) = \alpha^3 \frac{z^3}{3} + \alpha z$$

(iii) If $f(0) = 0$ and f' is not constant, then there exist $\alpha,\beta,\gamma,\kappa \in \mathbb{C}$ such that

$$f(z) = \alpha \operatorname{sn}(\beta z,\kappa) \qquad g(z) = \alpha^3 \int_0^{\beta z} \operatorname{sn}^2(t,\kappa)\,dt.$$

(Here $\operatorname{sn}(z,\kappa)$ denotes the classical Jacobi elliptic function).

Z. PÁLES:

On generalized homogeneous mean values (joint work with Z. Daróczy)

Let $M_n : \mathbb{R}_+^n \to \mathbb{R}_+, n \in \mathbb{N}$, be a given mean on $\mathbb{R}_+$, and $k \geq 1$ be fixed. The mean M_n is said to be k-homogeneous if

$$\text{(1)} \quad \begin{aligned} &M_k(t_1,\dots,t_k)M_n(x_1,\dots,x_n) = \\ &M_{nk}(t_1x_1,\dots,t_kx_1,\dots,t_1x_n,\dots,t_kx_n) \end{aligned}$$

for all values $t_1,\dots,t_k,x_1,\dots,x_n \in \mathbb{R}_+$, $n \in \mathbb{N}$. In the lecture we study functional equation (1) for deviation means and for $k \geq 2$ fixed. Under certain regularity assumptions we determine all solutions of (1) in

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the class of deviation means.

J. RÄTZ:

Some new results on orthogonally additive mappings

If in Def. 1 of [1], we replace the scalar field $\mathbb{R}$ by an arbitrary ordered field K, we obtain the notion of K-orthogonality space. Unless stated otherwise, $(X,\perp)$ will denote such a space and $(Y,+)$ an abelian group. The main results of the case $K = \mathbb{R}$ (cf. [1], Theorems 5 and 6) remain true.

LEMMA. If an even orthogonally additive $g:X\to Y$ is additive on a one-dimensional subspace of X, then $g = 0$.

Definition. $\perp$ is called left-unique (right-unique) if for all $x,z\in X$, $x \neq 0$, there is at most one $\alpha\in K$ such that $\alpha x+z \perp x$ (one $\beta\in K$ such that $x\perp\beta x+z$).

THEOREM. If $\perp$ is not left-unique or not right-unique or not symmetric, then every even orthogonally additive $g: X\to Y$ is 0.

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L. REICH:

Über Iteration ohne Regularitätsbedingungen in Ringen formaler Potenzreihen in einer Unbestimmten

Wir betrachten die Gruppe Γ der Automorphismen F der Form $F: x = {}^t(x_1,\ldots,x_n) \mapsto Ax+P(x)$ des Rings der formalen Potenzreihen, $\mathbb{C}[x_1,\ldots,x_n]$ und behandeln die Frage, ob die Bedingung der Einbettbarkeit eines gegebenen $F \in \Gamma$ in eine einparametrige analytische Untergruppe von Γ dasselbe ist wie die Bedingung der Einbettbarkeit von F in eine einparametrige Untergruppe $(F_t)_{t\in\mathbb{C}}$ ohne Regularitätsbedingung, d.h. die Lösung der Translationsgleichung $F_{t+s}(x) = F_t(F_s(x))$ unter der Randbedingung $F_1(x) = F(x)$, ohne Regularitätsbedingung an die Koeffizientenfunktionen von $F_t(x)$. Für n=1 läßt sich beweisen: $F(z) = \rho z+c_2z^2+\ldots(\rho \neq 0)$ ist iterierbar genau dann, wenn es analytisch iterierbar ist. Der einzige komplizierte Fall liegt vor, wenn $\rho \neq 1$, ρ eine Einheitswurzel. Durch Methoden von A. Kräuter und vom Verfasser, welche den Zusammenhang zwischen fraktioneller Iteration (Lösung der Gleichung von Babbage) und analytischer Iteration betreffen, läßt sich zeigen, daß jedes iterierbare F(z) linearisierbar und damit analytisch iterierbar ist.

D.C. RUSSELL:

Tauberian-type results for convolutions of sequences and functions

For semi-infinite sequences $a:=(a_n)_{n\geq 0}$, $p:=(p_n)_{n\geq 0}$, define $(a*p)_n := \sum_{j=0}^{n} a_{n-j}p_j$. Copson (1970) proved a theorem which can be expressed: If $1=p_0>p_1>\ldots>p_N=p_{N+1}=\ldots = 0$, if a is bounded, and if a*p is real and monotone, then a is a convergent sequence. Shortly afterwards Borwein (assuming $\sum|p_j|<\infty$ and $\sum p_j \neq 0$) and Russell (assuming p a finite sequence) published necessary and sufficient conditions for the conclusion. Here we generalize the result to a Tauberian form where the conclusion bears an obvious relationship to the hypothesis. Thus, given a suitable pre-condition on p, and a sequence space λ, we ask for a theorem of the form $a*p\in\lambda \Rightarrow a\in\lambda$, provided that the Tauberian condition $a\in\ell^{\infty}$ holds. If we take λ = bv (sequences of bounded variation) the result contains the theorems of Copson, Borwein and Russell. However, the result extends to bi-infinite sequences, and to any sequence space with the property $(u\in\lambda,\ v\in\ell^{1}) \Rightarrow u*v\in\lambda$. Moreover, for semi-infinite sequences we can, by strengthening the condition on p, eliminate the Tauberian condition and obtain the conclusion without an explicit boundedness assumption on the sequence a. There are analogues for functions.

W. SANDER:

Non-additive information measures

Let $I,G,L: [0,1] \to \mathbb{R}$ and let $\Delta_k=\{P=(p_1,\dots,p_k):\ p_i\geq 0,\ \sum_{i=1}^{k} p_i=1\}$, $k\in\mathbb{N}$.

The functional equation

$$(1)\quad \sum_{i=1}^{n}\sum_{j=1}^{m} I(p_iq_j)=\sum_{i=1}^{n}\sum_{j=1}^{m} G(p_i)I(q_j)+\sum_{i=1}^{n}\sum_{j=1}^{m} L(q_j)I(p_i),\ P\in\Delta_n, Q\in\Delta_m,$$

is of interest in information theory, since the special cases

$$G(p) = p + \lambda I(p),\ L(p) = p,\ \lambda\in\mathbb{R},$$

respectively

$$G(p) = p^{\alpha},\ L(p) = p^{\beta},\ \alpha,\beta,\in\mathbb{R},$$

play important roles in the characterization of the entropies of degree α and of the entropies of degree (α,β).

We determine all measurable triples (I,G,L) satisfying (1) when $G(0)=L(0)=0$ and holding for some fixed pair (n,m), $n\geq 3$, $m\geq 3$. Especially we get all measurable functions I satisfying (1) with

$$G(p) = p^{\alpha} + \lambda I(p), \quad L(p) = p^{\beta}, \quad \alpha,\beta,\lambda \in \mathbb{R},$$

and we get a new characterization of the entropies of degree (α,β). Our results are extensions of some recent results about this topic.

J. SCHWAIGER:

On the stability of a functional equation for homogeneous functions

Consider an abelian group G and a Banach space H, and consider furthermore for $\alpha = (\alpha_1,\ldots,\alpha_n)\in \mathbb{Z}^n$, $x=(x_1,\ldots,x_n)\in G$, $f\colon G\to H$ the expression

$$(1) \quad L_{n,\alpha}f(x) := f\Big(\sum_{i=1}^{n} \alpha_i x_i\Big) - \sum_{i=1}^{n} \alpha_i^n f(x_i) - \sum_{s\in S} \frac{\alpha^s}{s!} K_n f(x^s),$$

where

$$S=\{s=(s_1,\ldots,s_n)\in \mathbb{N}_0^n \mid s_i\geq 1, |s| := \sum_{i=1}^{n} s_i = n\}, \; s! := \Pi s_i!, \; \alpha^s := \Pi \alpha_i^{s_i},$$

$$x^s = (\underbrace{x_1,\ldots,x_1}_{s_1 \text{ times}},\ldots,\underbrace{x_n,\ldots,x_n}_{s_n \text{ times}}), \; K_n f(x_1,\ldots,x_n) = \Delta_{x_1,\ldots,x_n} f(0) + (-1)^{n+1} f(0)$$

$$(\Delta_{x_1,\ldots,x_n} = \Delta_{x_1}\circ\ldots\circ\Delta_{x_n}, \; \Delta_x f(y) := f(x+y) - f(y)).$$

In this context K.J. Heuvers in 1980 showed that $L_{n,\alpha}f=0$ for all α is necessary and sufficient for f to be a homogeneous polynomial of degree n. Now the following 'Hyers type' stability theorem, generalizing a result of P.W. Cholewa, holds.

Theorem. Let $f:G\to H$ be a mapping such that for all $\alpha \in \mathbb{Z}^n$ there is some constant $\delta(\alpha)$ with $|L_{n,\alpha}f(x)|\leq\delta(\alpha)$ for all $x\in G$. Then there exists a unique homogeneous polynomial g of degree n, such that $|f-g|\leq\delta_0(2^n-1)^{-1}$, where $\delta_0=\delta(\alpha^0)$ and $\alpha^0=(2,0,0,\ldots,0)$.

B. SCHWEIZER:

Conjugacy of hat functions

For $0<s<1$, $0<t<1$, the hat function $f_{st}:[0,1]\to[0,1]$ is defined by

$$f_{st}(x) = \begin{cases} \frac{t}{s}x, & 0\leq x\leq s, \\ \frac{t}{1-s}(1-x), & s\leq x\leq 1. \end{cases}$$

By explicitly determining the orbit structure, we show that the functions in the following sets are orbit-isomorphic, i.e., conjugate: (1) $\{f_{st}|t<s\}$; (2) $\{f_{st}|s=t\}$; (3) $\{f_{st}|s<t$ and $s+t<1\}$; (4) $\{f_{st}|s<t$ and $s+t=1\}$. We further show that f_{st} has a 3-cycle (and hence cycles of all orders) if and only if $s^2+st+t^2\geq s+t$ and $s<t$.

A. SKLAR:

The suspension construction for arbitrary functions

The well-known "suspension" construction is a method for producing a flow starting from a single function. The construction goes back at least to E. Hopf's "Ergodentheorie" of 1937, where it is applied to bijections of a set onto itself. Since then, it has been widely employed in differential topology and the theory of dynamical systems (cf., e.g., M.W. Hirsch, Bull. AMS 11 (1984), p.32). But its employment has been restricted to homeomorphisms or diffeomophisms, i.e., to classes of functions even narrower than the class of bijections. Upon analysis, however, the suspension construction turns out to be purely combinatorial in essence, and can actually be applied to <u>any function whatever</u>, provided that the resulting flow is restricted to non-negative reals instead of being defined over all reals. The reasons for the success of the construction can be related to the criteria for "mateability" ("Verbindbarkeit") of orbits established in G. Zimmermann's Inaugural-Dissertation "Uber die Existenz iterativer Wurzeln von Abbildungen" (Marburg, 1978).

K. STRAMBACH:

<u>Akivis-Identität</u>

Es wurden sog. "Akivis-Algebren" vorgestellt; das sind Vektorräume, auf denen eine bilineare antikommutative Multiplikation [,] und eine ternäre trilineare Multiplikation $<, \;,>$ so erklärt sind, daß die folgende verallgemeinerte Jacobi-Identität, die wir Akivis-Identität nennen, gilt

$$[[x_1,x_2],x_3] + [[x_2,x_3],x_1] + [[x_3,x_1],x_2] =$$

$$\sum_{g\in\Upsilon_3} \operatorname{sgn}(g) <x_{g(1)},x_{g(2)},x_{g(3)}>.$$

Die Tangentialalgebra jeder lokalen analytischen Loop ist eine Akivis-Algebra, und umgekehrt gibt es zu jeder Akivis-Algebra A so viele lokale analytische Loops, deren Tangentialalgebra A ist, wieviele symmetrische trilineare Abbildungen zwischen den Vektorräumen $A\times A\times A$ und A existieren.

L. SZÉKELYHIDI:

Exponential polynomials and functional equations

The Fourier transform of exponential polynomials is introduced and its properties are investigated. As a first step, we define the mean operator on the set of all exponential polynomials defined on an Abelian group. This mean operator is a polynomial-valued linear operator which commutes with all translations. The mean operator is an analogue of the integral mean value of trigonometric polynomials, and it can be used to define the Fourier transform of exponential polynomials. As an application, we show that some linear functional equations, differential equations and partial differential equations can be solved easily by any thin Fourier transform, if it is known that all solutions are exponential polynomials.

J. TABOR:

Ideal stability of the Cauchy and Pexider equations

Let $(X,+)$ be a group, I_1 a p.l.i. σ-ideal in X, I_2 an ideal in X^2 conjugated to I_1, S a subsemigroup of X such that $S-S=X$, $S\not\subset I_1$. Let $(Y,+)$ be a sequentially complete linear Q space and let $V\subset Y$ be a Q-convex, symmetric with respect to zero and bounded set containing zero. Assume additionally the following two conditions:

$$A\in I_1 \Rightarrow \{x\in X: \underset{n\in N\cup\{0\}}{\exists} 2^n x\in A\} \in I_1,$$

$$\underset{x,y\in S}{\forall}\ \underset{m\in N,}{\exists}\ 2^m(x+y) = 2^m x+2^m y.$$

Under the above assumptions, if

$$f: S\to Y,\ f(x+y)-f(x)-f(y) \in V \text{ for } (x,y) \in S^2\setminus M,$$

where $M \in I_2$, then there exist an additive mapping $\alpha: X\to Y$ and a set $T \in I_1$ such that

$$\alpha(x)-f(x) \in 3 \text{ seqcl } V \text{ for } x\in S\setminus T.$$

In the case when S=X a similar result is valid for the Pexider equation.

G. TARGONSKI:

A functional equation for phantom iterative square roots

The following functional equation is proposed for given $f \in C(0,1)$ and unknown $\alpha,\beta \in C(0,1)$.

(1) $f(x) = \alpha[\alpha(x)] + \beta[\alpha(x)] + \alpha[\beta(x)] + \beta[\beta(x)]$

with the conditions

(2) $\alpha(x)\beta(x) \not\equiv 0$

(3) (a) $\alpha[\alpha(x)]\beta[\alpha(x)] \equiv 0$ (b) $\alpha[\alpha(x)]\alpha[\beta(x)] \equiv 0$ (c) $\alpha[\alpha(x)]\beta[\beta(x)] \equiv 0$
(d) $\beta[\alpha(x)]\alpha[\beta(x)] \equiv 0$ (e) $\beta[\alpha(x)]\beta[\beta(x)] \equiv 0$ (f) $\alpha[\beta(x)]\beta[\beta(x)] \equiv 0$.

If a solution exists, then, defining the linear operators A,B,Ω on $C(0,1)$ by

(4) $A\varphi: = \varphi\circ\alpha, B\varphi: = \varphi\circ\beta, \Omega\varphi: = \varphi\circ f,$

we find that A+B is a phantom iterative square root of f in the sense that

(5) $\underset{\varphi \in C(0,1)}{\forall \varphi} \quad (A+B)^2 \varphi = \Omega\varphi = \varphi \circ f$

P. VOLKMANN:

Bedingungen, unter welchen das System $f(1+x)=1+f(x)$, $f(\varphi(x))=\varphi(f(x))$ nur $f(x)=x$ als Lösung besitzt (gemeinsame Arbeit mit L. Volkmann)

Sei $f: \mathbb{R} \to \mathbb{R}$ eine Lösung des im Titel angegebenen Funktionalgleichungssystems, wobei $\varphi: \mathbb{R} \to \mathbb{R}$ der Bedingung

$$\overline{\lim_{n\to\infty}} |\varphi^n(x) - \varphi^n(y)| = \infty \quad (x,y>a;\ x \neq y)$$

genügt. Satz 1: Ist f auf einem Intervall $[b,b+1]$ beschränkt, so ist $f(x) = x$. Satz 2: Ist φ eine gerade, stetige Funktion, so ist $f(x)=x$.

W. WALTER:

Functional-differential equations

As a model for parabolic functional differential equations, the problem

$$(*) \qquad \begin{aligned} u_{it} &= d_i \Delta u_i + f_i(t,x,u(\cdot)) \text{ in } S := (0,T] \times \mathbb{R}^n, \\ u(t,x) &= u_0(t,x) \text{ in } S_0 := [-r,0] \times \mathbb{R}^n \end{aligned}$$

is considered, where $d_i \geq 0$. The notation $u(\cdot)$ indicates that $f(t,x,u(\cdot))$ may depend on "past" values of u such as $u(t-h(t,x), x+k(t,x))$, $\int_0^t k(\tau,x)u(\tau,x)\,d\tau$, etc. (Such dependence occurs, e.g., in biological models.) Under the assumption

$$|f(t,x,\varphi) - f(t,x,\psi)| \leq L \sup_{\tau \leq t} |\varphi(\tau,x) - \psi(\tau,x)|,$$

problem (*) is transformed into an integral equation to which the contraction principle applies. In this way, existence, uniqueness, continuous dependence on u_0, and growth estimates are derived. The proper Banach space for this problem is the function space $C^0(S\cup S_0)$ with norm $||\varphi|| := \sup|\varphi(t,x)|e^{-\alpha t}$. Problems in which f depends also on u_x can be handled in the same way if the norm $||\varphi|| + ||\sqrt{t}\,\varphi_x||$ is used. The same approach works also for boundary value problems in a cylinder $S = [0,T] \times D$ with $D \subset \mathbb{R}^n$. This method is more flexible than semigroup theory; e.g., time delays h which depend on x can be handled.

M.C. ZDUN:

Embedding of continuous self-mappings of the circle in measurable iteration groups

A measurable iteration group (MIG) on S^1 of $T: S^1 \to S^1$ is an iteration group $\{T^t, t\in\mathbb{R}\}$ of continuous mappings S^1 into S^1 such that for every $x\in S^1$ $t \to T^t x$ is measurable and $T^1 = T$.

THEOREM. A continuous function $T: S^1 \to S^1$ has a MIG iff $T^2[S^1] = T[S^1]$, $T_{|T[S^1]}$ is a homeomorphism with positive orientation and one of the following cases occurs: (i) $\{x: Tx = x\} \neq S^1$ and $\emptyset$, (ii) $\forall_{m\in\mathbb{N}}\ \exists_{x\in S^1}\ T^m x = x$, (iii) $\exists_{x\in S^1}\ \overline{\{T^n x, n\in\mathbb{N}\}} = S^1$.

In the case (i) the construction of all MIG's of T can be reduced to the construction of all MIG's on an interval I of a real function defined in I. In the cases (ii) and (iii) there exists exactly one homeomorphism $f: \mathbb{R} \to \mathbb{R}$ such that $0 \leq f(0) < 1$ and $T\, e^{2\pi i x} = e^{2\pi i f(x)}$. In these cases the general form of all MIG's of T is the following

$$T^t e^{2\pi i x} = e^{2\pi i \alpha^{-1}[(a+k)t + \alpha(x)]}, \quad t,x \in \mathbb{R},$$

where a is the rotation number of T, $k \in \mathbb{Z}$ and in the case (iii) $\alpha^{-1}(x) = \inf\{f^k(0)+n\colon k+na>x/a\}$, while in the case (ii) α is an arbitrary homeomorphism satisfying the Abel equation

$$\alpha(f^p(x)-pa+1/m) = \alpha(x)+1/m, \quad x \in \mathbb{R},$$

where m is the minimal positive integer such that $T^m x \equiv x$ and p is an integer such that $0 \le p \le m-1$ and $pma = 1 (\mathrm{mod}\ m)$.

Reporter: B. Ebanks

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