MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 5/1985

MEHRDIMENSIONALE KONSTRUKTIVE FUNKTIONENTHEORIE

20.1. bis 26.1.1985

Die vierte internationale Tagung über mehrdimensionale konstruktive Funktionentheorie fand unter der Leitung von Herrn Walter Schempp (Siegen) und Herrn Karl Zeller (Tübingen) statt. Im Mittelpunkt des Interesses standen wieder Fragen der Darstellung, Approximation und numerischen Behandlung von Funktionen mehrerer Variablen.

Als Schwerpunkte der Vorträge sind zu nennen:

- Polynomapproximation über Simplices,
- Multivariate Splines (Box-Splines, Dimension von Spline-Räumen),
- Blending-Methoden,
- Mehrdimensionale Hermite-Interpolation,
- Glättung von Daten und Flächendarstellung,
- Mehrdimensionale Summierungsmethoden -
- Eliminationsverfahren für Polynome in mehreren Variablen,
- Funktionen mit Singularitäten bei der numerischen Lösung von partiellen Differentialgleichungen.

Die erzielten Ergebnisse werden in einem von den Tagungsleitern herausgegebenen und vom Birkhäuser Verlag, Basel - Boston - Stuttgart, in der ISNM-Reihe zu veröffentlichenden Tagungsband

"Multivariate Approximation Theory, Vol.3"

einer breiteren wissenschaftlichen Öffentlichkeit zugänglich gemacht. Mit diesem Band wird die Reihe der zu den Oberwolfach-Tagungen gleichen Themas der Jahre 1976, 1979 und 1982 erschienenen Tagungsbände fortgesetzt:

"Constructive Theory of Functions of Several Variables". Lecture Notes in Mathematics 571 (Springer 1977) ,

"Multivariate Approximation Theory". ISNM 51 (Birkhäuser 1979),

"Multivariate Approximation Theory II". ISNM 61 (Birkhäuser 1982).

Mit 54 Teilnehmern aus der VR China, der DDR, Großbritannien, Israel, Italien, den Niederlanden, Norwegen, Polen, Schweden, Ungarn und der Bundesrepublik Deutschland war die Kapazität des Instituts so ausgeschöpft, daß eine Reihe weiterer Interessenten leider nicht mehr berücksichtigt werden konnte. Dem Direktor des Forschungsinstituts, Herrn Prof. Dr. Barner, und seinen Mitarbeitern sei für die Gastfreundschaft sehr herzlich gedankt.

Vortragsauszüge

L. Bamberger

Interpolation in bivariate spline-spaces

In one dimension interpolation played a crucial role in the development of spline theory. In two dimensions many problems concerning splines have turned out to be rather difficult (e. g. bases, approximation properties) so that the general answers cannot be given. This paper describes a practical method for interpolation in spline-spaces on uniform type-1-triangulations. A specific scheme is given for C^1-cubics but it can be seen that similar constructions can be found for more general degrees and smoothness. Numerical examples show the relationship to approximation properties of spline-spaces.

G. Baszenski

n-th Order Polynomial Spline Blending

The objective is to construct an n-th order discrete Blending scheme based on univariate spline projectors.

- Discrete Blending schemes have the general advantage of preserving an asymptotic interpolation error rate but with a reduced number of data (as compared to the corresponding tensor product scheme).
- The interpolation problem corresponding to this scheme is solved by an efficient procedure where only the univariate collocation matrices involved in the construction have to be inverted.
- A basis of tensor product B-splines for the Blending function space is constructed and the interpolant is expressed as a linear combination of these basis functions.

J. Boman

Reconstruction of a function from its weighted line integrals

Let $\mathcal{L}$ be the set of all straight lines in the plane and $\mathcal{M}$ the set of pairs (L,x), where $L \in \mathcal{L}$ and $x \in L$. If μ is a continuous function on $\mathcal{M}$ one defines the generalized Radon transform $R\mu$ by

$$(R\mu f)(L) = \int_L f\mu(L,.)ds \quad , \quad L \in \mathcal{L} ;$$

here $f \in C_0(R^2)$, the space of continuous functions with compact support, and ds denotes the arc length on L. The question is wether $R\mu$ is injective on $C_0(R^2)$. We prove that $R\mu$ is injective, if $\mu(L,x)$ is real analytic and positive. ($\mathcal{M}$ has a natural struc= ture of real analytic manifold, hence the assumption on μ makes sense.) On the other hand we construct an example showing that the assumption that μ is real analytic cannot be replaced by C^∞.

G.Z. Chang

Convexity and diminishing properties of Bernstein polynomials over triangles

1. We give another proof for the convexity theorem of Bernstein polynomials over triangles.
2. Define
$$V_1(g) = \iint_T |\Delta g| dxdy$$
for the functions g which have continuous partial derivatives up to second order, and define $V_1(\hat{f}_n;T)$ for the Bézier net $\hat{f}_n$ in a proper way. It has been shown that
$$V_1(B^n(f);T) \leq V_1(\hat{f}_n;T) ,$$
the equality holds if and only if $\hat{f}_n$ is convex over T, in which $B^n(f)$ is the n-th Bernstein triangular patch associated with $\hat{f}_n$.

C.K. Chui

Bivariate Vertex splines

This represents a joint work with M.J. Lai. We introduce the notion of Vertex splines (or V-splines), which are bivariate splines with one interior grid-point inside each support, and obtain existence and non-existence results. An algorithm for constructing V-splines is given and the order of approximation by their linear span is obtained. Applications to Hermite inter= polation and L^2 approximation on arbitrary triangular grid partitions will also be discussed.

Z. Ciesielski

Approximation by algebraic polynomials on a simplex

A new linear method of approximation by algebraic polynomials on a simplex in the L_p norm is established.
In the one-dimensional case it is a multiplier method for the Legendre polynomials and it is related to Bernstein polynomials and the finite interval L_p Hausdorff moment problem.

L. Collatz

Approximation von Funktionen mehrerer Veränderlicher mit gewissen Singularitäten

Ein wichtiges Anwendungsgebiet der Approximationstheorie ist die näherungsweise Lösung von Randwertaufgaben mit Hilfe von einseitiger Tschebyscheff Approximation. Falls für die Rand= wertaufgabe Monotonieprinzipien bestehen, kann man für die (als existierend vorausgesetzte) Lösung u einmal von oben her und einmal von unten her approximierende Funktionen w aus einer Klasse W berechnen und damit für die auf einem Computer berechnete Näherung w^* eine Garantie geben, wieviele der berechneten und ausgedruckten Dezimalen richtig sind.
Die Methode ist für einfache Modelle bei zahlreichen linearen und nichtlinearen, gewöhnlichen und partiellen Differential= gleichungen erprobt worden. Im Vortrag werden verschiedene Typen von Randwertaufgaben genannt, die in neuerer Zeit (meist im letzten Jahr) behandelt wurden, insbesondere elliptische Differentialgleichungen mit Singularitäten, die teils am Rande, teils aber an nicht a priori bestimmten Stellen liegen. Für die

Berechnung von w^* verwendet der Computer eine Optimierungs= aufgabe mit einer Lösung $\tilde{w}$. Wie nahe w^* bei der Lösung $\tilde{w}$ der Optimierungsaufgabe liegt, kann in einfachen Fällen mit Hilfe eines Algorithmus der H-Mengen-Theorie abgeschätzt werden. Eine Anzahl numerischer Beispiele wird bis zur Einschließung der Lösung u vorgeführt.

W. Dahmen

On the number of solutions to systems of linear diophantine equations and multivariate splines

Given some integer $s \times n$ matrix X such that the convex hull of its columns does not contain zero let for $\alpha \in \mathbb{Z}^s$

$$t(\alpha|X) = |\{\beta \in \mathbb{Z}_+^n | X\beta=\alpha\}| .$$

Applying various results on the algebraic properties of linear combinations of box splines leads to a complete characterization of $t(.|X)$ as a function of $\alpha \in \mathbb{Z}^s$. E.T. Bell's characterization for s=1 as well as some results on counting magic squares which were obtained by R. Stanley with the aid of totally different methods are covered as special cases.

F.J. Delvos

Intermediate blending

N-variate blending interpolation involves functions of (N-1) independent variables as data functions in contrast to N-variate product interpolation which depends on scalars only. We construct new Boolean interpolation schemes which have functions of N-r independent variables as interpolation parameters for r=1,...N.

W. Freeden

Multidimensional Euler and Poisson summation formulas

Multidimensional Euler summation formulas are developed based on the concept of Green's functions to elliptic differential operators and "boundary conditions" of periodicity. The appli= cability to problems of approximation theory and numerical analysis is discussed. Sufficient conditions for the validity of Poisson summation formula in $\mathbb{R}^q$ are given. The efficiency of the criteria for multidimensional alternating sums is illustrated for representative examples of Analytic Theory of Numbers.

M. v.Golitschek

Degree of best approximation by blending functions

Jackson-Favard type approximation theorems are derived for the tensor-product subspaces $U \otimes C(T) + C(S) \otimes V$ of continuous functions of two real variables, where S and T are intervals, U and V are finite-dimensional subspaces of C(S) and C(T), respectively.

T.N.T. Goodman

Shape preserving approximation by polyhedral splines

Approximation operators using polyhedral splines on a three-direction mesh are considered. It is shown they preserve monotonicity and convexity of the control net and diminish the total variation of this net and also the total variation of its gradient.

W. Haußmann

Best Harmonic L^1- Approximants to Subharmonic Functions

Let $n \in \mathbb{N}$, $n \leq 2$, $B := \{x \in \mathbb{R}^n \mid |x| < 1\}$, $B_o := \{x \in \mathbb{R}^n \mid |x| < 2^{-1/n}\}$ and $H(B) := \{u : B \to \mathbb{R} \mid u \in C^2(B) \cap C(\bar{B}), \Delta u = 0 \text{ in } B\}$.

Theorem

Let $s \in C^2(B) \cap C(\bar{B})$, $\Delta s < 0$ a.e. in B.
$h^* \in H(B)$ is an L^1- proximum to s on $\bar{B}$, i.e. with the $L^1(\bar{B})$-norm we have $\|s-h^*\|_1 \leq \|s-h\|_1$ for all $h \in H(B)$ iff

(i) $h^*|_{\partial B_o} = s|_{\partial B_o}$, and

(ii) $s-h^* > 0$ a.e. in $\bar{B} \setminus \bar{B}_o$.

In addition, if (i) and (ii) are satisfied, then h^* is the unique best harmonic L^1- approximant to s.

The sufficiency of (i) and (ii) is based on the mean value property and the maximum principle of harmonic functions. The necessity relies on an inverse mean value theorem as well as on an approximation theorem due to J.C. Polking (1972) and L.I. Hedberg (1973) combined with an approximation theorem of Runge type.

G. Heindl

Construction and applications of quadratic simplicial spline functions in two and three dimensions

It is shown how to construct spaces of quadratic simplicial C^1 spline functions with prescribed dual bases corresponding to Hermite interpolation data. These constructions are used to derive some new finite macro elements in two and three dimensions, to develop a plotting procedure for drawing contour lines, and to solve a Hermite interpolation problem on the sphere.

K. Höllig

Multivariate Cardinal Splines

Denote by $M(\cdot|T): \mathbb{R}^d \to \mathbb{R}$ the box-spline corresponding to a set T of d-vectors with integer coefficients. We characterize the limits of multivariate cardinal splines $S(T) := \operatorname{span}\{M(\cdot - j|T) : j \in \mathbb{Z}^d\}$ as their degree tends to infinity. Let $nT := \bigcup_1^n T$. Under the assumption that $f \in L^2(\mathbb{R}^d)$ and $\{f(j)\}_{j\in\mathbb{Z}^d} \in l_2(\mathbb{Z}^d)$ we prove that

$$\operatorname{dist}(f, S(n,T)) \to 0 \quad \text{as} \quad n \to \infty$$

if and only if $\operatorname{supp} \hat{f} \subset \bar{\Omega}$ where

$$\Omega := \{x \in \mathbb{R}^d : |\hat{M}(x+2\pi j|T)| < |\hat{M}(x)|, \quad j \in \mathbb{Z}^d \setminus 0\}.$$

H.B. Knoop

Hermite-Fejér und höhere Hermite-Fejér Interpolation mit Randbedingungen

Zunächst wird die Folge der Hermite-Fejér Interpolationspoly= nome bzgl. Jacobi-Knoten betrachtet, bei denen zusätzlich zu den Interpolationsbedingungen für die im Inneren des Intervalls [-1,1] liegenden Knoten noch an den Randpunkten ±1 Funktions= werte interpoliert und höhere Ableitungen Null gesetzt werden. Es werden Positivitätsbereiche, Konvergenz- und Divergenzbe= reiche bzgl. der gleichmäßigen Konvergenz und Konvergenzbe= reiche bzgl. der punktweisen Konvergenz angegeben. Anschließend wird auf entsprechende Fragen bei den Hermite-Fejér Operatoren eingegangen, bei denen zusätzlich zur ersten Ableitung noch die zweite und dritte Ableitung in den inneren Knoten Null gesetzt wird.

A. Kroó

Unicity of best L_1 - approximation

Let C_w denote the space of continuous functions endowed with the L_1 - norm with weight w. We shall give a characterization of those spaces, which are Chebyshev in C_w for all weights w. Some applications will be also given.

L. Lenarduzzi

Approximation methods for experimental data and applications

We give a survey of the developments of our experience in dealing with experimental data by numerical- statistical methods.
We used techniques to evaluate $f(\underline{x}^*)$ on the basis of a set of noised data $(\underline{x}_i, \tilde{f}_i)$, $i=1,\dots,N$, from points $\underline{x}_i$ scattered in $D \subset \mathbb{R}^k$, k large, $x^* \in D$.
The methods to solve the problem are different on the basis of the number of data. For $N \ll 2^k$ local techniques can be used, with a determination of the local neighbourhood which is adaptable to the information from the data.
A problem of heart potential mapping can be also handled by techniques of local type.
Last, we shall say about the problem of choosing a subset from a large set of data, so that the subset is significant for a good functional reconstruction.

D. Levin

Multidimensional Reconstruction by Set-valued Approximation

Some generalizations of the notation of univariate data interpolation are presented, including the concept of set-valued interpolation in a general metric space. Consequently methods for univariate interpolation or smoothing of multi=

dimensional geometrical data are suggested. In particular the application of these methods to 3-D body recognition from cross-sectional data is discussed. Preliminary analysis of the interpolation process is presented and the capability of reconstructing bodies of complex topologies is examplified.

W.A. Light

Projections on Bivariate function spaces

Some recent advances in the theory of minimal projections will be described, particularly in the spaces $C(S\times T)$ and $L_1(S\times T)$. Some results for tensor product spaces will also be given. The subspaces considered will always have a special form. For example a typical subspace of $C(S\times T)$ would be $C(S)+C(T)$.

F. Locher

Convergenz of Hermite-Fejér interpolation via Korivkin's theorem

We show with the aid of Korovkin's theorem that the sequence of the Hermite-Fejér polynomials to Jacobi nodes converges for every continuous f pointwise for $|x|<1$ and $\alpha,\beta>-1$ and uniform for $|x|\leq 1-\delta$, $\delta>0$ fixed. Moreover one has uniform convergence for $|x|\leq 1$ if $\max(\alpha,\beta)<0$. In order to prove this results of Szegö we show that the Hermite-Fejér functionals are asymptotically positive and the application of a functional of that type to the test function $g_x : t \to (x-t)^2$ yields a constant multiple of $\{P_m^{(\alpha,\beta)}\}^2$. These results may be applied to Hermite-Fejér interpolation in several variables if the operators are constructed by tensor product methods.

E. Luik

Cubature error bounds using degrees of approximation

Error estimates for cubature rules are usually given in terms of partial derivatives (Peano-Sard) or in terms of analytic properties (Davis-Hämmerlin). The approximation method has found little attention. This method can be refined by using biorthogonal systems (BOGS). We present these BOGS estimates both for cubature rules which are exact for P_m (the space of bivariate polynomials ot total degree notgreater than m) and for cubature rules which are exact for $P_{k,l}$ (the space of bivariate polynomials with degree in x notgreater than k and degree in y notgreater than l). As BOGS there will be used bivariate Chebyshev polynomials and the Fourier coefficient functionals. Furthermore, for product rules we get another error bound by reducing the cubature error to the quadrature errors (Nikol'skii) and taking the BOGS estimates for quadrature rules. As application we treat Clenshaw-Curtis product rules.

T. Lyche

Knot insertion and discrete Box-splines

Inserting extra knots in a B-spline series is a useful tool in theory and practice. Here we consider spline spaces formed by linear combinations of a scaled and translated Box-spline. We show that the transformation from Box-splines on a coarse mesh to Box-splines on a refined mesh involves certain discrete Box-splines. These discrete Box-splines satisfy a discrete convolution identity which makes efficient and stable algorithms possible. We also show that the spline coefficients converge to the Box-spline surface uniformly as the mesh is refined.

C. Micchelli

Local linear independence of translates of box splines

We prove that the box spline $B(x,X)$, $X \subset \mathbb{Z}^p \setminus \{0\}$, $\langle X \rangle = \mathbb{R}^p$ has translates which are locally linearly independent if and only if

$$|Y| = p,\ \langle Y \rangle = \mathbb{R}^p,\ Y \subset X \quad \Rightarrow \quad |\det Y| = 1.$$

H.M. Möller

Solutions of nonlinear equations by elimination

We consider the problem of finding all $x \in K_1^n$ satisfying $f_i(x)=0$, $i=1,\dots,r$, where $f_i \in K_2[x_1,\dots,x_n]$, K_1, K_2 fields. This problem can be solved by the algebraic method of elimi= nation. In 1970, Buchberger presented an algorithm, which performs, as pointed out by Trinks (1978), this elimination method. The algorithm is now implemented in some computer algebra systems allowing automatic and exact performance of the elimination, when K_2 is a field like $\mathbb{Z}_p$, $\mathbb{Q}$ or $\mathbb{Q}(\alpha)$, where α is algebraic over $\mathbb{Q}$, etc. Buchberger gave in 1979 some criteria for avoiding unnecessary eliminations in his algorithm. Here we show the framework for finding such criteria and present a modification of Buchberger's algorithm which derives more benefit from these criteria.

F. Móricz

Cesaro summability of double orthogonal series

We study the a.e. behaviour of the (C,α,β) - means of the double orthogonal series $\sum_{i=0}^{\infty} \sum_{k=0}^{\infty} a_{ik} \phi_{ik}(x)$ for $\alpha \geq 0$ and $\beta \geq 0$, where $\phi_{ik}(x)$ are orthonormal on a positive measure space and the a_{ik} are real numbers. Sufficient conditions are given for the a.e. (C,α,β) - summability, which are best possible in general. The so-called square functions introduced by Kolmogorov, Kaczmarz, and Zygmund are extended from single orthogonal series to double ones.

H. Nienhaus

Generalized Melkes Interpolation

An important class of rectangular finite elements are those of reduced Hermite interpolation type. In comparison with the corresponding tensor product interpolation the number of nodes are reduced, only the values of the function f and its derivatives up to the mixed maximal degree M in the vertices of the given rectangle are used by these schemes.
For every $M \in \mathbb{N}_0$ Melkes proved the existence and uniqueness of two interpolants in appropriate polynomial spaces. The elements differ in degree of conformity. Melkes type I inter= polation is $C^{[M/2]}$-conform, Melkes type II interpolation C^0-conform.
The objective of this paper is to establish a systematic method of reduced Hermite interpolation which yields besides the known C^0 and $C^{[M/2]}$-conforming Melkes elements new C^n-conforming schemes for every n with $0<n<[M/2]$.

G. Opfer

On certain minimal polynomials in complex domains

The minimal polynomials under consideration are defined by
$\min \|p\|_\infty$, $p \in \Pi_n$, $p(0)=1$ on a compact $S \subset \mathbb{C}$.
These polynomials play an important role in the determination of optimal iteration parameters for solving linear systems iteratively by Richardson's (or Chebyshev's) iteration method. For details consult LAA 58(1984), p.343-361. Results are given for the case where S is an annular sector. Cooperation with Glenn Schober, Indiana University is acknowledged.

P. Pfluger

The dimension of $S_2^1(\Delta)$ in special cases

It is known that the dimension of the spline space $S_2^1(\Delta)$ cannot be expressed in a formula depending on the number of vertices, edges and triangles of the triangulation Δ, even not if one takes into account the number of vertices with only two edges of different slopes (J. Morgan, R. Scott 1977). Using a representation of G. Heindl for quadratic C^1 splines we found more triangulations Δ for which the dimension of $S_2^1(\Delta)$ depends on the location of the vertices. In some of these cases the lower and upper bound on the dimension given by L. Schumaker are attained.

A. Quarteroni

Polynomial Approximation Theory and Analysis of Spectral Methods

Spectral methods are largely used in the numerical approximation of Partial Differential Equations. The discrete spectral solution is a finite expansion of some orthogonal functions, which are very often either the trigonometric or the Chebyshev polynomials. The convergence analysis relies upon the estimate of both the best approximation and the interpolation errors, in the Sobolev norms. In this framework, some Jackson-type theorems are given for both trigonometric and Chebyshev polynomials, and for their blending.

H.J. Rack

On Multivariate Polynomial L^1 - Approximation to Zero and Related Coefficient Inequalities

The well known L^1-extremal property of the univariate normalized (i.e., leading coefficient 1) Chebyshev polynomials $\bar{U}_n$ resp. $\bar{U}_{n-1}$

of the second kind is generalized to polynomials in several real variables of total degree $\leq m$.
In particular, sharp inequalities both for the single leading coefficients of degree m resp. m-1 and for the sum of those coefficients are established.

M. Reimer

Abschätzung von Lagrange-Quadratsummen für die Sphäre mit Hilfe gewisser Eigenwerte

Die direkte Berechnung der Norm eines Interpolationsopera= tors ist bei höher- oder hochdimensionalen Problemen eine vom Aufwand her nicht zu bewältigende Aufgabe. Man kann sie in manchen Fällen durch Abschätzung der Lagrange-Quadratsum= men mit Hilfe des kleinsten Eigenwerts einer gewissen positiv-definiten Systemmatrix umgehen. Dies gilt z.B. für die Räume der homogenen und/oder harmonischen Polynome bezüglich der Sphäre.

W. Schempp

CAWD (=Computer Aided Waveform Design)

The notions of radar auto- and cross-ambiguity function are on the borderline with mathematics, physics, and electrical engineering. This paper presents a solution of the radar synthesis problem (posed in 1953) and the invariance problem for radar ambiguity surfaces over the symplectic time-frequency plane via harmonic analysis on the real Heisenberg nilpotent group. In the same vein, the paper is concerned with the phase anomaly of Fourier and microwave optics. As a mathematical by-product of this research, an identity for Laguerre functions of different orders pops up. Some of its special cases, to wit, some holomorphic theta identities are explicitely calculated.

H.S. Shapiro

The Gram matrix of non-negative functions

Let $\phi_1,\ldots,\phi_n$ be real valued non-negative functions in the Hilbert space $L^2(X,\mu)$ where μ is a positive measure; then the Gramian of $\{\phi_1,\ldots,\phi_n\}$ is the matrix $\|a_{ij}\|$, $i,j=1,2,\ldots,n$, where $a_{ij} = \int\phi_i\phi_j \, d\mu$. Clearly this Gramian is positive semi-definit, and also $a_{ij}\geq 0$. It is shown that these conditions fully characterize such a Gramian, if and only if $n\leq 4$. The problem is closely related to known results on matrix factorization.

X.C. Shen

The Basis and Moment Problems of some systems of Analytic Functions

The talk is divided into three sections.

1. The characteristic properties of some incomplete systems
2. The Moment problem
3. The efficient solution for the multiple interpolation in H_p (Imz>0), $0<p\leq\infty$.

B. Sündermann

Normen von Projectionen in mehreren Veränderlichen

Wir betrachten Projektionen auf endlichdimensinale Polynomräume in mehreren Veränderlichen auf der Einheitssphäre S^{r-1} und der Einheitskugel B^r im $\mathbb{R}^r$. Die Normen derartiger Projektionen (in diesem Zusammenhang auch Lebesgue-Konstanten genannt) sind bekanntlich ein Maß für die Approximationsgüte dieser Operatoren. Für den Fall der Einheitssphäre kennt man Projektionen mit minimaler Norm, diese Ergebnisse lassen sich für $r\geq 3$ jedoch nicht auf die Vollkugel B^r übertragen. Wir benutzen einen bereits von Faber (der sich allerdings nur mit

Interpolationsoperatoren beschäftigt hat) eingeschlagenen Weg, um konstruktiv sowohl die oben genannten Schranken für die Einheitssphäre zu erhalten, als auch die Ergebnisse auf die Einheitskugel B^r zu übertragen. Die Abschätzungen, die man auf diese Art und Weise erhält, sind, bis auf eine genaue Bestimmung der Konstanten, asymptotisch scharf.

M. Tasche

A collocation method for some elliptic boundary value problems

This is a review of recent results of K. Gürlebeck, W. Sprößig (TH Karl-Marx-Stadt, Section Mathematik) and myself. It will be described an application of the theory of right-invertible operators to some elliptic boundary value problems.
Let $G \subset \mathbb{R}^3$ be a bounded domain with sufficient smooth boundary Γ. Further let Q be the quaternion algebra with the basic elements $e_o=1$, e_1, e_2, e_3. Let $u: G \to Q$ be a function

$$u(x) = u_o(x)\, e_o + \dots + u_3(x)\, e_3 = (u_o(x), \bar{u}(x)),$$
$$\bar{u}(x) = (u_1(x), u_2(x), u_3(x)), \qquad x = (x_1, x_2, x_3) \in G$$

and

$$\nabla = e_1 \frac{\partial}{\partial x_1} + e_2 \frac{\partial}{\partial x_2} + e_3 \frac{\partial}{\partial x_3}$$

the right-invertible Nabla-operator, which was considered by R. Fueter, A.V. Bizadse, I.N. Vekua, R.Delanghe and W. Sprößig. Especially we consider the Dirichlet problem

$$\Delta u = 0 \text{ in } G, \quad \gamma_o u = f \text{ on } \Gamma,$$

where $f \in H_o^s(\Gamma)$, $s>3/2$ is given and γ_o denotes the trace operator. We sketch a collocation method, investigated by K. Gürlebeck (by ideas of C. Müller). Numerical experiments will be shown.

R.H. Wang

The dimensions of spaces of bivariate splines

Let D be the rectangle $D=[a,b]\times[c,d]$. We use the lines $x=x_i$ and $y=y_j$, $i=1,\dots,m-1$, $j=1,\dots,n-1$ to partition D into mn rectangular cells $D_{ij}=[x_i,x_{i+1}]\times[y_j,y_{j+1}]$. By drawing the diagonal with positive slope in each D_{ij}, we obtain a uni-diagonal triangulation $\Delta^{(1)}_{m,n}$; and by drawing in both diagonal of each D_{ij}, we obtain a crisscross triangulation $\Delta^{(2)}_{m,n}$. Denote by $\mathbb{P}_k$ the collection of all polynomials with real coefficients and total degree k,

$$S^{\mu}_{k}(\Delta^{(i)}_{mn}) = \{s\in C^{\mu}(D):\ s\in\mathbb{P}_k \text{ in each cell}\}\ ,\quad i=1,2.$$

In this paper, we have calculated $\dim S^{\mu}_{k}(\Delta^{(i)}_{mn})$, $i=1,2$.

G.A. Watson

The solution of generalized least squares problems

The conventional approach to discrete data fitting using the least squares criterion is to assume that all the problem variables are exact except one. In many situations, however, this is an oversimplification, and use of the usual least squares method can lead to bias in the estimated parameter and variance values. It is then necessary to take proper account of errors in all variable values, and this is the motivation behind the idea of generalized least squares. In this paper some methods for solving generalized least squares problems are presented. Two general classes of problems are treated, depending on whether or not the underlying model can be interpreted as explicit or implicit. The problem is also adressed of fitting an n-dimensinal linear manifold in $\mathbb{R}^k$, $n<k$, to inexact data.

K. Zeller

Basic bivariate approximations

The approximations relate to Fourier-Chebyshev expansions and to binary decompositions. In the first realm the problem is reduced to univariate approximations (e.g. column-wise; allowing several procedures and bounds). In particular the Zolotarev case is treated by a simple procedure of CF-type (including rather sharp bounds for the error and for the degree of approximation). In the second realm the proximum to f(x) + g(y) is investigated using an H-set of simple structure. Further results concern product and composition of two functions.

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