MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 8/1985

Einhüllende Algebren von Lie-Algebren

3.2. bis 9.2.1985

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Dies war die fünfte Oberwolfacher Tagung über Einhüllende Algebren von Lie-Algebren nach den vorausgegangenen in den Jahren 1973, 1975, 1978 und 1982.

Das Bild wurde geprägt von den neuen Methoden im halbeinfachen Fall. Zu ihnen gehören die Verwendung der Theorie der $\mathcal{D}$-Moduln, die sich Anfang der achtziger Jahre umwälzend entwickelt hatte, sowie die Ausnutzung der Zusammenhänge von $\mathcal{D}$-Moduln, Einhüllenden Algebren und unitären Darstellungen. Exemplarisch dazu (Collingwood) die Behandlung von Fragen der $\mathfrak{n}$-Homologie eines Harish-Chandra-Moduls und die Konstruktion eines geometrischen Analogons des sogenannten Jacquet-Funktors unter Verwendung des Zerlegungtheorems (vgl. Vortrag Björk) von Deligne-Beilinson-Bernstern-Gabber in der $\mathcal{D}$-Modul-Theorie. Die sogenannte Bernstein-Beilinson Lokalisierung, die im Jahre 1981 den Beweis der Kazhdan-Lusztig Vermutung über die Multiplizitäten der Verma-Moduln durch Bernstein-Beilinson und Brylinski-Kashiwara ermöglichte, gehört nun zu den gebräuchlichen Techniken ebenso wie die Verwendung von Kazhdan-Lusztig Polynomen.

Die seit einigen Jahren offene, von Bernstein gestellte Frage, ob, wie im Fall $n = 1$, die irreduziblen Moduln über einer n - ten Weyl-Algebra (d.h. de Algebra der Differentialoperatoren mit polynomialen Koeffizienten auf $\mathbb{C}^n$), automatisch holonom sind (d.h. die Gelfand-Kirillov Dimension n haben) wurde negative beantwortet (Stafford).

Die erwähnte Bernstein-Beilinson Lokalisierung eröffnet die Möglichkeit, den primitiven Idealen (deren Studium eines der Hauptprobleme darstellt) der Einhüllenden Algebra $U(\mathfrak{g})$ der Lie-Algebra $\mathfrak{g}$ einer halbeinfachen (zusammenhängenden) linearen algebraischen Gruppe G (über $\mathbb{C}$) charakteristische Varietäten in dem Cotangentialbündel der Fahnenmannigfaltigkeit von G zuzuordnen. Versieht man diese Varietäten mit entsprechenden Multiplizitäten, so gelangt man zu charakteristischen Zyklen, die ihrerseits zu topologischen Invarianten in der Cohomologie und in der equivarianten K-Theorie führen und eine überraschend klare Interpretation von Josephs Goldie-Rang Polynomen ermöglichen weitere interessante Informationen werden von der neven Schnitthomologie geliefert (Arbeiten von Borho-Brylinski-Mac Pherson).

Unbekannt ist nach wie vor der skalare Proportionalitätsfaktor dieser Goldie-Rang Polynome. Unter ihrer entscheidenden Verwendung war es nach langen Anstrengungen ca. 1982 gelungen, die primitiven Ideale im halbeinfachen Fall zu klassifizieren (mit Hilfe des Translationsprinzips, von gewissen Weyl-Gruppen Darstellungen und von nunmehr kombinatorisch beschreibbaren Linkszellen in Weyl-Gruppen). Ungelöst blieb jedoch (im halbeinfachen Fall), von wenigen Fällen wie sl_3 (Dixmier 1974) abgesehen, die Bestimmung der vollprimen primitiven Ideale von $U(\mathfrak{g})$. Für sl_n wurde auf dieser Tagung die Lösung präsentiert (Moeglin); diese besagt insbesondere, dass im Fall von sl_n die vollprimen primitiven Ideale von eindimensionalen Darstellungen parabolischen Unteralgebren induziert sind (für andere einfache Lie-Algebren ist eine solche Eigenschaft schon für das Joseph-Ideal nicht gegeben), also Annulatoren von verallgemeinerten Verma - Moduln sind. Joseph gab einen Einblick in die allgemeine Probelmatik und in die Hürden dieses aufgeworfenen Klassifikationsproblems. Unter einer mässigen Nichtausartungsbedingung sind die Annulatoren von verallgemeinerten Verma-Moduln überraschenderweise durch ihren Durchschnitt mit der isotypischen Komponente vom adjungierten Typ (und ihrem Durchschnitt mit dem Zentrum) erzeugt (Gupta). Erwähnt sei, dass es gelungen ist, die Krull - Dimension von $U(sl_3)$ zu bestimmen.

Eine wichtige Rolle spielt die Untersuchung der Gelfandschen Kategorie $\mathcal{O}$ (z.B. Fragen von Multiplizitäten, von Ext-Gruppen und von gewissen natürlichen Filtrierungen), deren Objekte alles $U(\mathfrak{g})$-Moduln endlicher Länge sing. Zu $\mathcal{O}$ gehören insbesondere die verallgemeinerten Verma-Moduln und ihre Subquotienten.

Tadic stellte seine Klassifizierung der irreduziblen unitären Darstellungen von allen $GL(m, \mathbb{R})$'s und $GL(m, \mathbb{C})$'s vor.

Fragen der Geometrie, herrührend von bestimmten S-Tripeln in $\mathfrak{g}$ ($\mathfrak{g}$ halbeinfach), wurden von Kostant, geometrische Fragen von sogenannten vollständigen symmetrischen Räumen wurden von Springer behandelt.

Weitere Vorträge betrafen neuere Fragen (injektive Hüllen) im auflösbaren Fall (Theorie der Links), Eigenschaften des Semizentrums von $U(\mathfrak{g})$ ($\mathfrak{g}$ allgemein und mit beliebigem Grundkörper der Char 0), sowie die Möglichkeiten algebraischer Methoden zur Herleitung der Fourier-Plancherel Transformation im nilpotenten Fall.

Es sei noch angemerkt, dass man seit 1983 die Bestimmung der primitiven Ideale von $U(\mathfrak{g})$ für allgemeines $\mathfrak{g}$ auf den halbeinfachen Fall zurückführen kann. Im allgemeinen Fall fehlen jedoch bislang gänzlich die geometrischen und topologischen Methoden, die nunmehr im halbeinfachen Fall so eindrucksvoll zur Anwendung kommen.

Vortragsauszüge

J.E. BJÖRK : THE DECOMPOSITION THEOREM BY DELIGNE, BEILINSON, BERSTEIN AND GABBER

Let X and Y be two complex analytic manifolds and let $\pi : Y \longrightarrow X$ be a holomorphic map which can be factored through a projective map , i.e. a commutative diagram

$$\begin{array}{ccc} Y & \overset{j}{\hookrightarrow} & \mathbb{P}_N(\mathbb{C}) \times X \\ & \searrow^{\pi} & \downarrow \\ & & X \end{array}$$

exists where j is a closed embedding .

We consider $\mathcal{O}_Y$ as a sheaf of left $\mathcal{D}_Y$ - modules where $\mathcal{D}_Y$ is the sheaf of differential operators with holomorphic coefficients.

Consider the (derived !) direct image $\int_\pi \mathcal{O}_Y$. The Decomposition Theorem shows that for each integer ν the single direct image $\int^\nu \mathcal{O}_Y$ is isomorphic to a direct sum of simple regular holonomic $\mathcal{D}_X$ - modules. This was proved by Deligne, Beilison, Bernstein and Gabber (see Asterisque volume 100). A rather different proof is available (which also extends the cited work since we can use complex analytic maps).

Also, the new proof uses distributions on the underlying real analytic manifolds and residue Calculus from the work by Herrera-Lieberman (Math. Annalen 194 (1971)) and D. Barlet's notion of Trace class functions (Ann. Inst. Fourier, 33, 2 (1983)) and Kashiwara's conjugation Functor which is defined on the abelian categories of regular holonomic $\mathcal{D}$ - modules (see Kashiwara : "$\mathcal{D}$ - modules and distributions on complex manifolds", to appear (1985) in Advances of Mathematics). For the proof of the Decomposition Theorem I refer to my article "Kashiwara's Conjugation Functor and the Decomposition Theorem" to appear in Seminaire Bony-Sjöstrand-Meyer (Ecole Polytechnique, Palaiseau), January 1985.

W. BORHO : PRIMITIVE IDEALS AND CHARACTERISTIC CLASSES

This is a report on joint work J.-L. Brylinski and R. Mac Pherson. Let G be a complex semisimple connected linear algebraic group, $B \subset G$ a Borel subgroup, $T \subset B$ a maximal torus, and $\mathfrak{t} \subset \mathfrak{b} \subset \mathfrak{g}$ the corresponding Lie algebras. For a primitive ideal J with trivial central character in the enveloping algebra $\mathcal{U}(\mathfrak{g})$, we define a "characteristic class" $p_J \in H^*(X)$ in the cohomology of the flag variety $X=G/B$ as follows : the Beilinson-Bernstein localization of the left $\mathfrak{g}$-module $M=\mathcal{U}(\mathfrak{g})/J$ as a coherent $\mathcal{D}_X$-module on X has a characteristic variety $Ch(M)$ of pure codimension d in the contangent bundle T^*X, it even determines a ("characteristic") algebraic cycle and hence a certain cohomology class in $H^{2d}(T^*X)$, which is finally intersected homologically with the zero-section $\sigma: X \hookrightarrow T^*X$ to define the characteristic class p_J. Considering p_J as a homogeneous harmonic polynomial on $\mathfrak{t}$ by Borel's description of $H^*(X)$, we can state the following

Theorem : Up to a non-zero constant factor, p_J is Joseph's Goldie rank polynomial of p_J. It is possible to use this geometric interpretation of Joseph's Goldie rank polynomial for a new, more geometric development of Joseph's classification theory of primitive ideals. Parallel results are formulated and proved for the study of nilpotent orbits in $\mathfrak{g}$ and their corresponding "orbital cone bundles". The methods used include equivariant K-theory, in addition to some $\mathcal{D}$- module and intersection homology theory.

K. BROWN : MODULES OVER SOLVABLE ENVELOPING ALGEBRAS

We describe the structure of the injective hull of a finitely generated uniform module over $\mathcal{U}(\mathfrak{g})$, where $\mathfrak{g}$ is a finite dimensional solvable Lie algebra, in terms of the graph of ideal links of the enveloping algebra.

K. CARLIN : EXTENSIONS OF VERMA MODULES

The problem order consideration is the determination of the extensions (in $\mathcal{O}$) of one Verma module by another. In the case that the highest weights are integral and regular, Gabber and Joseph (1981) conjectured that the dimensions of the extensions are given by the coefficients of a Kazhdan-Lusztig polynomial.

A spectral sequence is introduced which computes extensions in $\mathcal{O}$ in terms of derived functors. When this is applied to the problem described above, most of the known results can be proved in this setting. However the main result of the Gabber-Joseph paper has not been recovered by this approach.

This method is used to prove that the last non-zero extension group is one-dimensional.

D.H. COLLINGWOOD : COMPUTING THE $\mathfrak{n}$-HOMOGOLY OF A HARISH-CHANDRA MODULE

Let $\mathfrak{g} = \mathfrak{k} \oplus \mathfrak{a} \oplus \mathfrak{n}$ be the complexified Iwasawa decomposition of the Lie algebra of a connected semisimple real matrix group G. Given a Harish-Chandra module V for G, it is an unsolved problem to compute $H_*(\mathfrak{n}, V)$. We discuss two aspects of this problem, which have recently been solved. First, when G has real rank one and V is a discrete series representation, the $\mathfrak{n}$-homology is now known. Secondly, we determine precisely when $H_*(\mathfrak{n}, V)$ is given by an analogue of Kostant's formula, in the Hermitian symmetric case. In both settings, we must analyze the structure of certain generalized Verma modules and use the Kazhdan-Lusztig conjectures for Verma modules and Harish-Chandra modules. We close by describing joint work with Luis Casian (IAS, Princeton). We state a theorem which asserts there exists a geometric analogue of the usual Jacquet functor, call it J_{geo}. We have J_{geo} inducing a Hecke algebra map of Hecke modules, whose image is computable via a successful algorithm. J_{geo} has all the usual properties of J, same character, etc... Also, the weights in the "weighted character" $J_{geo}(\ldots)$ correspond to levels in a $\mathfrak{g}$-filtration of $J_{geo}(\ldots)$, with semisimple subquotients. We conjecture this corresponds to a filtration of J(...), with semisimple subquotients. This is verified on $\mathbb{R}$-rank one and other law $\mathbb{R}$-ranks. Our techniques rely on the "Decomposition theorem" (Björk's talk) and the philosophy of "going to positive characteristic".

T. ENRIGHT : MULTIPLICITIES AND KLV-POLYNIMIALS FOR SU(p,q) (joint work with BRAD SHELTON)

Let $\mathfrak{g} = \mathfrak{sl}(n, \mathbb{C})$, $\mathfrak{p} = \mathfrak{m} + \mathfrak{n}$ a maximal parabolic subalgebra and by a C S A. Set $P_{\mathfrak{m}} = \{ \lambda \in \mathfrak{h}^* \mid \lambda$ is integral and $\Delta^+(\mathfrak{m})$ - dominant and regular $\}$. For $\lambda \in P_{\mathfrak{m}}$, let N_λ , L_λ and P_λ be the generalized Verma module, its simple quotient and projective cover associated to λ in the category $\mathcal{O}(\mathfrak{g}, \mathfrak{p})$.

For fixed $\lambda \in P_{\mathfrak{m}}$ we define two sets of orthogonal roots which determine explicitly the composition factors of N_λ , the socle of N_λ , the self-duality of P_λ and $\mathrm{Ext}^1(N_\lambda , L_\xi)$.

For $y , w \in P_{\mathfrak{m}}$ define :

$$Q_{y,w}(q) = \sum_i q^i \dim \mathrm{Ext}^{l(y)-l(w)-2i}(N_y, L_w) .$$

The same techniques which yield the previous results, give the recursion formula :

$$Q_{y,w}(q) = Q_{ys_\alpha,w}(q) + q^r R(q)$$

where $R(q)$ is an explicitly determined similar polynomial for $\mathfrak{sl}(n-2 , \mathbb{C})$. From this recursion formula we recover the results of Lascoux and Schutzenberger for Kazhdan-Lusztig polynomials.

D. GARFINKLE : ANNIHILATORS OF HARISH-CHANDRA MODULES

Let G be a real reductive Lie group, X the Harish-Chandra module of an irreducible representation of G . Then (under certain not very restrictive conditions on G) the annihilator of X in $\mathcal{U}(\mathfrak{g})$ (the universal enveloping algebra of the complexification on the Lie algebra of G) is a primitive ideal. In this talk we present, for $G = U(p,q)$ and $G = GL(n,\mathbb{R})$ an algorithm for assigning to each such X its annihilator (given in terms of the current classification of primitive ideals in $\mathcal{U}(\mathfrak{g})$. We also discuss some relationships between this problem and the duality (due to Vogan) between (a block of) irreducible representations of G and those of a dual group $\check{G}$.

R. GUPTA : OCCURENCES OF THE ADJOINT REPRESENTATION IN ANN $M_{\mathfrak{p}}(\lambda)$

Let g be a complex semi-simple Lie algebra of rank ℓ with universal enveloping algebra U. U carries a representation of g coming from the adjoint action of g on itself. The simplest $\mathfrak{g}$-compenents of U are the invariants : these make up the center Z of U and are very well understood. For instance, primitive ideals J of U are partially classified by their intersection with Z .

The next simplest $\mathfrak{g}$-componets of U are the copies of $\mathfrak{g}$ itself, c.f., the space A spanned by $f(\mathfrak{g})$ for all $f \in \mathrm{Hom}_{\mathfrak{g}}(\mathfrak{g}, U)$. A is well-understood as a Z-module (Kostant), but other things are not understood, for instance $A \cap J$ is not known in general. [For other $\mathfrak{g}$-compenents, even the Z-module structure is not known].

In the talk we considered the case where J is the annihilator of a generalized Verma module $M = M_{\mathfrak{p}}(\lambda)$ induced from a character of a parabolic. We explicitly deseribed $A \cap J$ as a subspace of A (as the vanishing locus of certain linear functionals on A) for λ "good" in a technical regularity sort of sense. This also led to somewhat surprising generation results for J and for to the annihilator of the highest weight vector of M .

H.P. JAKOBSEN : HOMOMORPHISMS BETWEEN GENERALIZED VERMA MODULES

The set of homomorphisms originating or terminating in scalar generalized Verma modules for hermitian symmetric spaces, is described. Equivalently, the space of covariant differential operators D between holomorphically induced representations, of the form D = an $n \times m$ matrix (with constant coefficient differential operators as entries) and either $n = 1$ or $m = 1$, is determined. By viewing more general objects as being built up out of hermitian symmetric spaces it is then exemplified how the above results can be used attack a conjecture of Lepowskys concerning conical vectors.

A. JOSEPH : COMPLETELY PRIME PRIMITIVE IDEALS AND THE BERLINSON-BERNSTEIN THEOREM

Let $\mathfrak{g}$ be a complex semisimple Lie algebra, $U(\mathfrak{g})$ its enveloping algebra and Prim $U(\mathfrak{g})$ the set of primitive ideals of $U(\mathfrak{g})$. We study the subset $\text{Prim}_c\, U(\mathfrak{g}) \subset \text{Prim}\, U(\mathfrak{g})$ of completely prime primitive ideals. For each $J \in \text{Prim}\, U(\mathfrak{g})$ it is known that its associated variety admits a unique dense orbit $\mathcal{O}_J$. Since the notion of induction applies to both orbits and ideals it is natural to conjecture (an idea going back to Borho) that $J \in \text{Prim}_c\, U(\mathfrak{g})$ is induced exactly in as much as $\mathcal{O}_J$ is induced. Nevertheless this statement is totally false for $J \in \text{Prim}\, U(\mathfrak{g})$ so we have to understand how J completely prime imposes a restriction that can be used. Our main result shows how this can be done and up to a technical restriction the conjecture is proved for the case when $\mathcal{O}_J$ is induced from the zero orbit. The later restriction is probably not essential; but the technical restriction involves difficulties which are not easy to overcome. This restriction is not necessary in type A_n so in particular we recover Moeglin's theorem reported in this conference. It involves a difficulty which has many essentially equivalent forms including a negative answer to the Kostant problem concerning all $\mathfrak{g}$-finite vectors in End M : M a simple $U(\mathfrak{g})$ module and the non-birationality of the moment map of $T^*(G/P) \longrightarrow G\mathfrak{m}$. Our proof involves the use of bimodules with a locally ad $\mathfrak{g}$ finite action and a translation principle inherent in the Hodges-Smith interpretation of the Berlinson-Bernstein theorem and which can be read off from the work of Gelfand and Kirillov as simplified by Berline and Duflo.

B. KOSTANT : GENERALIZED EXPONENTS AND THE CENTER OF $U(\mathfrak{n})$

Each nilpotent element e in a semi-simple Lie algebra $\mathfrak{g}$ and a corresponding S-triple (h, e, f) defines a homogeneous polynomial p on $\mathfrak{g}_2 = \{y \in \mathfrak{g} \mid [h, y] = 2y\}$ such that if $G = \text{Ad}\,\mathfrak{g}$ than $\mathfrak{g}_2 \setminus ((Ge) \cap \mathfrak{g}_2)$ is defined by the equation $p = 0$. Using the polynomial p one may describe the ideal $I \subset S(\mathfrak{g})$ which defines the boundary $\overline{Ge} - Ge$ of the G-orbit of e. The statement is that if $w \in S(\mathfrak{g})$ is a lowest weight vector (and it supplies to known such vector) and $\tilde{\mathfrak{g}}_2 = \mathfrak{g}_2 + \mathfrak{g}_3 + \dots$ then w vanishes on $\overline{Ge} - Ge$ if and only if $w \mid \mathfrak{g}_2$ is divisible by p.

Th. LEVASSEUR : THE KRULL DIMENSION OF $U(\mathfrak{sl}(3))$

The Krull dimension of $U(\mathfrak{g})$, $\mathfrak{g}$ a finite dimensional Lie algebra, is easily computed in case $\mathfrak{g}$ solvable : it is $\dim_{\mathbb{C}} \mathfrak{g}$. In the semi-simple case J.E. Roos conjectured that it is $\dim_{\mathbb{C}} \mathfrak{h}$, $\mathfrak{h}$ a Borel subalgebra of $\mathfrak{g}$. We offer a proof in case $\mathfrak{g} = \mathfrak{sl}(2) \times \ldots \times \mathfrak{sl}(2)$ or $\mathfrak{g} = \mathfrak{sl}(3)$. The last case needs the analysis of differential operators on some quadratic cone in $\mathbb{C}^6$. So we study the differential operators ring on a cone in $\mathbb{C}^n$, and realize an embedding of the Lie algebra $\mathfrak{so}(n+2)$ in this ring (Goncharov's construction). Some $U(\mathfrak{so}(n+2))$-modules of minimal ($\neq 0$) GK dimension are also constructed.

C. MOEGLIN : IDEAUX PRIMITIFS COMPLETEMENT PREMIERS DE L'ALGEBRE ENVELOPPANTE DE $\mathfrak{gl}(n , \mathbb{C})$

Suivant des idées de Dixmier, Borho a défini une application du dual de l'algèbre de Lie de $\mathfrak{gl}(n , \mathbb{C})$ dans l'ensemble des idéaux primitifs complètement premier de l'algèbre enveloppante de $\mathfrak{gl}(n , \mathbb{C})$; il a prouvé que cette application est injective et le but de mon exposé était d'esquisser la preuve de la surjectivité. Pour cela, on note $\mathfrak{p}$ la sous-algèbre de $\mathfrak{gl}(n , \mathbb{C})$ formée des matrices dont la première colonne est nulle et on étudie en détail l'application suivante

$$\mathrm{Prim}_{cp} U(\mathfrak{g}) \in I \longrightarrow (I \cap U(\mathfrak{p}) , I \cap Z(\mathfrak{g}))$$

où $\mathrm{Prim}_{cp} U(\mathfrak{g})$ est l'ensemble des idéaux primitifs complètement premiers et $Z(\mathfrak{g})$ est le centre de $U(\mathfrak{gl}(n , \mathbb{C}))$. On montre que cette application est injective et on détermine son image; cela permet de conclure grâce à une hypothèse de récurrence.

X.H. NGHIEM : ALGEBRAIC DETERMINATION OF THE FOURIER INTEGRAL

Using a well situated Weyl algebra of the enveloping algebra, one defines a representation of the corresponding simply connected nilpotent Lie Group by Fourier Integral symbols. Algebraic computation of the exponentation of this symbol gives the Fourier integral amplitude which etablishes the Fourier-Plancherel transformation on the group. In the case of homogeneous spaces the same procedure works. In that way one gets a purely algebraic computation of the Fourier Transform.

A. OOMS : PRIMITIVE CENTRAL LOCALIZATION OF ENVELOPING ALGEBRAS

Let L be a finite dimensional Lie algebra over a field k of characteristic zero, $\mathcal{U}(L)$ its enveloping algebra with quotient division ring D(L). We show that the weights of the semi-invariants of D(L) form a finitely generated, free abelian group G . This implies, among other things, that the semi-center of D(L) is isomorphic to the group algebra of G over the center Z(D(L)) . We also report on joint work with E. Nauwelaerts . In particular, let S be a subalgebra of the center $Z(\mathcal{U}(L))$ and put $S_o = S \setminus \{0\}$. Suppose that $ab \in S_o$ always implies that $a, b \in S_o$ whenever $a, b \in \mathcal{U}(L)$. Then the following conditions are equivalent :

1. Z(D(L)) is the quotient field of S .
2. $\mathcal{U}(L)$ has only a finite number (up to scalars) of irreducible semi-invariants not contained in S .
3. $\mathcal{U}(L)$ has a nonzero semi-invariant which is contained in each nonzero prime ideal P of $\mathcal{U}(L)$ with $P \cap S = \{0\}$.
4. The localization of $\mathcal{U}(L)$ at S_o is primitive.

Furthermore, under these circumstances we have that $S = Z(\mathcal{U}(L))$ and the semi-center of $\mathcal{U}(L)$ is a polynomial ring over $Z(\mathcal{U}(L))$.

T.A. SPRINGER : BETTI NUMBERS OF COMPLETE SYMMETRIC VARIETIES

Let G be an adjoint complex semi-simple linear algebraic group, with an involutorial automorphism θ . Let H be the fixed point group of θ . De Concini and Procesi [Proc. Montecatini Conf. on Invariant Theory, Lect. Notes in Math. no 996] constructed a "compactification" X of the affine variety G/H, with very nice properties. Such an X is called a <u>complete symmetric variety</u>. The talk reported on results of C. de Concini and the speaker, describing the Poincaré polynomial P_X of a complete symmetric variety X . Two methods were described to obtain a formula for P_X . The first one follows an approach already introduced in the paper quoted above. The second one involves counting the number of rational points of a variety like X over a finite field. The two methods produce different formulas for P_X .

J.T. STAFFORD : NON-HOLONOMIC MODULES

If $R = A_n(\mathbb{C})$ is the n^{th} Weyl algebra or $R = U(\mathfrak{g})$ is the enveloping algebra of a finite dimensional Lie algebra, then a simple R-module M is called holonomic if GKdim M = 1/2 GKdim R/ann M . In both cases, holonomic modules have particularly pleasant properties, and the standard simple modules are holonomic. This raises the question of whether all simple modules are holonomic. We answer this in the negative as follows. Let x_i and $y_j = \frac{\partial}{\partial x_j}$ be the canonical generators of A_n and put

$$\alpha = x_1 + y_1 \textstyle\sum_{i\geq 2} \lambda_i x_i y_i + \sum_{i\geq 2} (x_i + y_i)$$

for some $\lambda_i \in \mathbb{C}$ that are linearly independent over $\mathbb{Q}$. Then :

<u>Theorem</u> $A_n/\alpha A_n$ is a simple A_n-module with GKdim $2n-1$ (of course, GKdim $A_n = 2n$).

This module seems to have none of the pleasant properties of hotonomic A_n-modules. For example, $\dim_{\mathbb{C}} \mathrm{Ext}^2(A_n/\alpha A_n , A_n/\alpha A_n) = \infty$. Also $\mathbb{C}(\alpha)$ is a maximal commutative subfield of $\mathrm{Fract}(A_n)$.

Similarly we provide a simple, non-holonomic module S over $U = U(\mathfrak{sl}_2 \times \mathfrak{sl}_2)$. Again this has some nasty properties. For example, there exists a finite dimensional $U(\mathfrak{sl}_2 \times \mathfrak{sl}_2)$ module E such that $S \otimes E$ has infinite length, as a U-module under the diagonal action (thereby answering a question of Kostant). Further, there exists an ideal P of U such that $S \otimes_U P$ has infinite length (thereby showing that U is not weakly ideal invariant).

TADIC : UNITARY REPRESENTATIONS OF GL(n)

We fix the field $F = \mathbb{R}$ or $\mathbb{C}$. The irreducible representations $u(\delta , n)$ indexed by square integrable representations of $GL(m , F)$'s and natural numbers are introduced. The complementary series determinded by them is denoted by $\Pi(u(\delta , n), \alpha)$, $0 < \alpha < 1/2$. Now $u(\delta , n)$ and $\Pi(u(\delta , n) , \alpha)$ are irreducible unitary. We explaine how irreducible unitary representations $GL(m , F)$'s are described as induced representations by representations $u(\delta , n)$, $\Pi(u(\delta , n) , \alpha)$.

T. TANISAKI : CHARACTERISTIC CYCLES OF THE $\mathcal{D}$-MODULES CORRESPONDING TO THE HIGHEST WEIGHT MODULES

We give the answer to the following problem for semisimple Lie algebras of rank ≤ 3 .

Problem Determine the characteristic cycles of the regular holonomic system on a flag manifolds, corresponding to the simple highest weight modules with trivial central character.

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