# MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH 

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\text { Tagungsbericht } 9 / 1985
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p-adic Function Theory and Analysis
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10.2. bis 16.2.1985

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\text { Tagungsleiter: } & \text { Y. Amice (Paris) } \\
& \text { L. Gerritzen (Bochum) }
\end{aligned}
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Participants from 10 countries used the opportunity to meet other people interested in p-adic theory. There have been 25 lectures at the conference. A wide range of different topics was discussed. The following list gives an idea of the subjects that were mainly treated:
p-adic differential equations and the Boyarski principles, p-adic L-series, $\mathrm{SL}_{2}\left(\mathbb{Q}_{\mathrm{p}}\right)$-representations, Mumford curves, rigid and crystalline cohomology of varieties, uniformization and stable reduction of abelian varieties, theory of Drinfeld modular forms and Zeta-functions, modular theory of Mumford curves, transcendence theory, locally convex spaces, analytic extension and T-filter.

## B. DWORK: Boyarsky Principle II

We indicate how the Laplacetransform may be used to study how the Frobenius matrix of generalized hypergeometric functions vary with exponents.

Let for example $f(\lambda, x) \in \mathbb{Z}\left[\lambda, x_{1}, \ldots x_{n}\right]$, $a \in U=\mathbb{Q} \cap \mathbb{Z}_{p} \mathbb{Z}$.
Consider $\lambda$ as variable element of $\mathbb{C}_{p}, R_{\lambda}=\mathbb{C}_{p}\left[x, \frac{1}{f(\lambda, x)}\right]$,
$\mathrm{R}_{\lambda, \mathrm{a}}=\mathrm{R}_{\lambda} \cdot \frac{1}{\mathrm{f}^{\mathrm{a}}}, \mathrm{W}_{\mathrm{a}, \lambda}=\frac{\mathrm{R}_{\lambda, \mathrm{a}}}{\sum \mathrm{x}_{\mathrm{i}} \frac{\partial}{\partial \mathrm{x}_{\mathrm{i}}} \mathrm{R}_{\lambda, a}}$.
Put $\mathscr{L}_{\lambda}=\mathbb{a}_{p}[x, t],(t=$ new variable $), \mathscr{L}_{\lambda, a}=t^{a} \mathscr{L}_{\lambda}, \quad$ by the formal Laplacetransform $T_{\lambda}: \xi(x, t) \rightarrow \int_{0}^{\infty} \xi(x, t) \exp (-t \pi f(x, \lambda)) \frac{d t}{t} \cdot \frac{\left(-\pi^{a}\right)}{\Gamma(a)}$ we map $V_{\mathrm{a}, \lambda}=\mathscr{L}_{\lambda, \mathrm{a}} / \Sigma \mathrm{D}_{\mathrm{i}, \lambda} \mathcal{L}_{\lambda, \mathrm{a}}$ onto $\mathrm{W}_{\mathrm{a}, \lambda}$ where $D_{i, \lambda}=\exp (\pi t f) \bullet x_{i} \frac{\partial}{\partial x_{i}} \bullet \exp (-\pi t f), \pi^{p-1}=-p$.
Replacing $R_{\lambda}$ by a corresponding Reich space (completion) and $\mathscr{L}_{\lambda}$ by the space of powerseries in $x, t$ converging in a poly disk of radius $1+\varepsilon$, we deduce (subject to some hyperthesis on f) the commutative diagram

$$
F=\exp \left(-\pi t f(x, \lambda)+\pi t^{p_{f}}\left(x^{p}, \lambda^{p}\right)\right)
$$


where $\mathrm{pb}-\mathrm{a} \in \mathbb{Z}$.
This remains valid for $a, b \in \mathbb{Z}$ provided we take $a=b=1$.
A. ROBERT: p-adic Representations of open subgroup of $\mathrm{SL}_{2}\left(\mathbb{Q}_{\mathrm{p}}\right)$

I: Iwahori subgroup ( $c \equiv 0$ ) in $K=S L_{2}\left(\mathbb{Z}{ }_{p}\right) \subset G=S L_{2}\left(Q_{p}\right)$ $\pi_{k}(s) \phi(x)=\left(c x_{x}+d\right)^{k} \phi\left(\frac{a x+b}{c x+d}\right)$ if $s^{-1}=\left(\begin{array}{ll}a & b \\ c & d\end{array}\right) \in I$ defines a unitary (continuous) representation of $I$ in the Tate algebra of the unit disc $|x| \leq 1, \phi(x)=\sum_{i \geq 0} a_{i} x^{i}, a_{i} \in \mathbb{Q}_{p}$, $\left|a_{i}\right| \rightarrow 0 \quad H=\{\phi$ anal. on $|x| \leq 1\}$
$k \in X=\mathbb{Z} /(p-1) \mathbb{Z}^{\times} \mathbb{Z}_{p} \quad$ (if $\left.p \neq 2\right)$.
Theorem: $\pi_{k}$ is top. irreducible when $k_{j} \notin \mathbb{N}$ uncountable family of inquivalent irreducible such restric. of $\pi_{k}$ to any open subgroup of $I$ remains top. irred. $\pi_{k}$ cannot be extended to $K$.
These $\pi_{k}$ are analytic: $\pi_{k}^{\prime}: \operatorname{SL}_{2}\left(Q_{p}\right) \rightarrow$ End (H)
$h=\left(\begin{array}{cc}1 & 0 \\ 0 & -1\end{array}\right) \rightarrow k_{1}-2 x \frac{d}{d x}, c=\left(\begin{array}{ll}0 & 1 \\ 0 & 0\end{array}\right) \rightarrow-\frac{d}{d x}, f=\left(\begin{array}{ll}0 & 0 \\ 1 & 0\end{array}\right) \rightarrow x^{2} \frac{d}{d x}-k_{1} x$.
Projection operator on maximal weight space (constr. functions)

$$
\prod_{j \geq 0}\left[1-\left(\sum_{p}^{D} v(j)\right) /\left(v_{v(j)}^{i}\right)\right] \quad \text { strongly converges }
$$

For subgroup $b \equiv c \equiv 0$ of $I$ can define
$\pi_{1, k}(s) \phi(x)=(c x+d)^{k}\left(a+\frac{b}{x}\right)^{1} \phi\left(\frac{a x+b}{c x+d}\right) \quad(k, 1 \in X)$.
Connection with Morita-Murase: $\operatorname{Ind}_{\mathrm{I}}^{\mathrm{K}} \pi_{k}=\tilde{\pi}_{k}$ acts in $\tilde{H}=\underset{i=0}{\underset{\oplus}{\oplus}} \mathrm{H}_{\mathrm{i}}$
contains a dense subspace (Fréchet) in which repro. of $K$ extends To $G=S L_{2}\left(\mathbb{Q}_{p}\right)$. This Fréchet space consists of analytic functions over $\mathbb{C}_{p}-\mathbb{Q}_{\mathrm{p}}$.
Ref.: C.R. Acad. Sc. Paris t. 298 (1984) pp. 237-340.
M. van der PUT: Vectorbundle on a Mumfordkurve

Main Result: E vectorbundle on a Mumfordkurve $X$, defined over a field $K$. If $E$ is semi-stable and deg $E=0$, then exists a $\Phi$-bounded representation (unique up to isomorphism $\rho$ such that $E$ is isomorphic to the vectorbundle $E(\rho)$ derived from the representation $\rho$
of the fundamental group $\Gamma$ of $X$.
This result has been proved by G. Faltings for the case of a field $K$ with a discrete valuation. The proof that we gave (or indicated) works for a general field $K$. A paper on the subject is in presentation and will be written by M. Reversat (Bordeaux) and M. van der Put (Groningen). In the lecture, the relatively simple case of a Tate-curve $X=K^{*} /\{q\rangle^{\text {is }}$ presented. Let $\pi: K^{*} \rightarrow K^{*} /(q)=X$ denote the uniformisation of the Tate-curve. The vectorbundle $\pi^{*} E$ on $K^{*}$ is trivial, and $\Gamma$-equivariant. The vectorspace $V \subset \pi^{*} E\left(K^{*}\right)$ consisting of the sections $s \in \pi^{*} E\left(K^{*}\right)$ with the following conditions:
(i) $\sup _{\mathrm{n}<0}\left(|q|^{|n|}\|s\|\right.$ on $\left.|q|^{n+1} \leq|z| \leq|q|^{n}\right)<\infty$.
(ii) $\lim _{\mathrm{n} \rightarrow+\infty}\left(\| \mathrm{si}|q|^{\mathrm{n}+1} \leq|z| \leq|q|^{\mathrm{n}}\right)=0$
turns out to be $\Gamma$-invariant, $r$-dimensionaland $V$ generates $\pi * E$.
So the representation $\rho$ corresponding to $E$ is the $\Gamma$-action on the r-dimensional vectorspace $V$ (N.B. $r=r a n k E)$.
In the proof one needs to consider certain completion $K(\langle\Gamma\rangle)$ and $k(\langle\Gamma\rangle) \hat{\oplus}_{k} K$ of the group-algebra $K[\Gamma]$.
D. GOSS: Zeta-Functions for Function Fields

In this talk we described various developments in the theory of such functions. These developments-concern: The converse to the Herbrand criterion; the interpretation due to $S$. Okada and the author of the Bernoulli-Carlitz numbers; and two criteria for cyclicity of components. In particular, we are able to show that these criteria work for some components but for all primes. The components that arise seem to be related to numerial evidence for a functional equation of such functions. We also presented a
possible analogue of the above structure in the theory of classical p-adic L-series.

## P.ROBBA: Symmetric powers of the p-adic Bessel equation

Let $\omega_{p}$ be a nontrivial additive character on $\mathbb{F}_{p}(p \neq 2)$ and $\omega_{q}:=\omega_{p} \circ \operatorname{Tr}_{\mathbb{F}_{q}}: \mathbb{F}_{p}$ its extension to $\mathbb{F}_{q}\left(q=p^{s}\right)$. For $\lambda \in \mathbb{F}_{q}$, one defines the Kloosterman sum $K_{q}(\lambda):=\sum_{x \in \mathbb{F}_{q}^{*}} \omega_{q}\left(x+\frac{\lambda}{x}\right)$. For $\lambda \in \mathbb{F}_{\infty}=\mathbb{F}_{p}^{\text {alg }}$ define $\operatorname{deg} \lambda:=\left[\mathbb{F}_{p}(\lambda): \mathbb{F}_{p}\right]$. It is well known that the L-function associated to the Kloosterman sums is a polynomial of degree 2.
$L(\lambda, T):=\exp \left(\sum_{n \geq 1} K_{p} \mathrm{sm}^{(\lambda)} \mathrm{T}^{\mathrm{m}} / \mathrm{m}\right)=\left(1-\pi_{1}(\lambda) \mathrm{T}\right)\left(1-\pi_{2}(\lambda) T\right)$ (where $s=\operatorname{deg} \lambda$ ).
In this talk we give a p-adic theory for the infinite product..
$M_{k}(t)=\sum_{\lambda \in \mathbb{F}_{*}^{*}} \prod_{\nu=0}^{k}\left(1-\pi_{1}\left(-\lambda^{\nu}\right)^{k-\nu} \pi_{2}\left(-\lambda^{\nu}\right)^{\nu} t^{\operatorname{deg} \lambda}\right)^{-1} / \operatorname{deg} \cdot \lambda$
( $k$ positive integer).
We show that $M_{k}$ is a polynomial of degree
$\operatorname{deg} M_{k}=\left\{k-2\left[\frac{k}{2 p}\right], k\right.$ even

$$
\mathrm{k}+1-2\left[\frac{\mathrm{k}}{2 \mathrm{p}}+\frac{1}{2}\right], \mathrm{k} \text { odd. }
$$

This is done by interpreting $M_{k}$ using Dwork's cohomology and using own results on index of differential operatiors. The operators to consider are the symmetric powers of order $k$ of the Bessel differential equation.

Using Dwork's dual theory, one shows also that one has $M_{k}(t)=P_{k}(t) \tilde{M}_{k}(t)$ where $\tilde{M}_{k}$ satisfies the functional equation $\tilde{M}_{k}(t)=$ const $\cdot t^{\delta} \tilde{M}_{k}(1 / p t)\left(\delta=\operatorname{deg} \tilde{M}_{k}\right)$ and the degree of $P_{k}$ is given by $\operatorname{deg} P_{k}=\left\{\begin{array}{cc}2+2\left[\frac{k}{2 p}\right] & \text { if } k \text { even } \\ 1 & \text { if } k \text { odd. }\end{array}\right.$

## C. SCHMIDT: p-adic L-functions attached to Rankin convolutions of modular forms

For an arbitrary newform $f \in S_{k}(N, \nu)$ with rational Fourier coefficients we consider the associated "symmetric square function"

$$
D_{\infty}(f, s)=\frac{L_{N}\left(2 s+2-2 k, v^{2}\right)}{L_{N}(s+1-k, v)} \quad \sum_{n \geq 1} a_{n}^{2} n^{-s} \text { where } f(z)=\sum_{n=1}^{\infty} a_{n} q^{n}
$$

and its twists $D_{\infty}(f, x, s)$ by Dirichlet characters $x$. For the primitive symmetric square $D_{\infty}(f, x, s)$ which is a certain modification of $D_{\infty}(f, x, s)$ at finitely many Euler factors, we prove

1. holomorphic continuation and a functional equation for $s \rightarrow 2 k-1-s$
2. algebraicity of the special values $D_{\infty}(f, x, m) \cdot \pi$ - power $/\langle f, f\rangle$

$$
\text { for } m=1, \ldots, 2 k-1
$$

3. p-adic interpolation of the special values by a p-adic L-function $D_{p}(m, s)$ for any prime $p \nmid 2 N a_{p}$
4. functionai equation of the p-adic L-functions:

$$
D_{p}(m, s)=D_{p}(2 k-1-m, 2-s)
$$

Using and generalising work of Shimma, W.Li., Sturm and Arnaud. The results above were initiated by joint work with Coates, where we have been working in the special case $k=2, v=$ trivial character. These, for the corresponding modular elliptic curve $E / \mathbb{Q}$ one looks at the "symmetric square function"

$$
L\left(S y m^{2}, s\right):=\prod_{\text {primes } q} \operatorname{det}\left(1-\operatorname{Frob}_{q}^{-1} q^{-s} ;\left(\operatorname{Sym}^{2} H_{1}^{1}(E)\right)^{I}\right)^{-1}
$$

and shows holomorphic continuation and functional equation for $s \rightarrow 3-\mathrm{s}$ by showing
5. $L\left(S y m^{2}, x, s\right)=D_{\infty}(f, x, s)$.

These is a conjecture which relates the p-adic L-function $D_{p}(E, s)$, that one gets in this way, with an arithmetically defined p-adic

L-function from Iwasawa theory on $E$ or rather on $E \times E$. For CM-curves this conjecture follows from the "2-variable Main-Conjecture".
H.B.SIEBURG: Exponential functions and algebraic independence over non-local-fields

Let $R$ denote any commutative ring with 1 . For any real multiplicative valuation $\phi$ on $R$ define

$$
\delta(\phi):=\inf \{\phi(a): 0 \neq a \in R\} .
$$

It is easily seen that $\delta(\phi) \in\{0,1\}$. Let

$$
D:=\{R: \exists \phi \text { non-trivial: } \delta(\phi)=1\}
$$

and let $\bar{D}$ be the class of all fields being quotient field of rings in $D$.

Let $(k, \phi) \in \bar{D}$ be fixed and let $K$ denote a complete, algebraically closed extension of $k$. Assuming char ( $k$ ) = 0 we present general criteria from which one can deduce transcendence and algebraic independence results for values of the exponential function associated to $K$ (analogons results hoid for char ( $k$ ) >0).

An interesting application is the following: Let exp be the exponential function defined over a complete, algebraically closed extension of $k=\mathbb{Q}\left(\frac{1}{e}\right)$. Then $\exp \left(e^{N}\right) \notin \overline{\mathbb{Q}}$ for any positive integer $N$. Here $e^{2}$ denotes the "real" exponential.

## E.U.GEKELER: Geometry of Drinfeld modular curves

Let $K$ be a function field in one variable over the finite field $F_{q}$, " ${ }^{\infty}$ "a fixed place of degree $\delta$ and $A$ the ring of functions integer outside $\infty$. Let further $C$ be the completion of the algebraic closure of $K_{\infty}$, and $\Omega=\mathrm{C} \backslash \mathrm{K}_{\infty}$.

The discrete group $\Gamma=G L(2, A)$ acts by fractional linear transformaLion on the , upper half plane" $\Omega$, and $\Gamma \Omega$ is, by a theorem of Drinfell, the set of $C$-points of an affine algebraic curve $M_{\Gamma}$, defined over a certain finite abelian extension of K . It follows by results of Deligne, that the genus $g\left(M_{\Gamma}\right)$ is the dimension $h(\Gamma)$ of $H^{1}(\Gamma, \mathbb{Q})$; a similar result holds for congruence subgroups $\Gamma^{\prime}$ of $\Gamma$.
Now, by the analytic theory of modular forms for $\Gamma$ on $\Omega$, it is possible to compute $g\left(M_{\Gamma}\right)$, if one is able to describe the sets of cusps resp. elliptic points for $\Gamma$, and if one knows the divisor of just one modular form, which is, unlike the classical case, the most difficult part.
In the talk, I formulated answers to this questions. As a corollary, one obtains the following formula for $h(\Gamma)=g\left(M_{\Gamma}\right)$, improving a result of Serve for sufficiently small subgroups $\Gamma^{\prime}$ of $\Gamma$ :

$$
h(\Gamma)=1+\frac{\left(\frac{q^{\delta}-1}{q-1}\right) P(q)-q / 2^{\left[\delta(q+1) P(1)+\left\{\left[\begin{array}{c}
0 \\
(q-1) P(-1) \text { oxen } \delta\}
\end{array}\right]\right.\right.}}{q^{2}-1},
$$

where $P(X)$ is the polynomial of degree $2 \times$ genus ( $K$ ) in the nominator of the $\zeta$-function of $K$.

By the complete description of possible points and types of tamifictions, it is easy to compute $h\left(\Gamma^{\prime}\right)$ for any given congruence subgroups $\Gamma^{\prime}$ of $\Gamma$.

Further, some connections with Stark's conjectures and questions of diophantine geometry over $K$ were indicated.
W. RADTKE: Diskontinuous arithmetic groups in the function field case

Let $F / \mathbb{F}{ }_{q}, q=p^{n}$ a function field in one variable and $\infty$ a fixed place of $F$. The ring $A:=\{f \in F / f$ regular outside $\infty\}$ is a Dedekind domain of finite class number $h$. Let $k$ be the completition of $F$
with respect to $\infty$ and $K$ the smallest complete alg. closed field containing $k$. The group. $\mathrm{PGL}_{2}(\mathrm{~A})$ acts on the tree of the local field $k$ and also acts on $F, k, \Omega=K i k$. Serre proved a structure theorem for this group

$$
\mathrm{PGL}_{2}(\mathrm{~A}) \cong\left(\left(\Gamma_{o}^{*} \mathrm{G}_{1} \Gamma_{1}\right){ }^{*} \mathrm{G}_{2} \Gamma_{2} * \ldots\right) * \mathrm{G}_{\mathrm{h}} \Gamma_{\mathrm{h}},
$$

where $\Gamma_{o}$ is finitely generated, the $\Gamma_{i}$ are stabilizers of cusps and the $G_{i}$ are the intersections $\Gamma_{o} \cap \Gamma_{i}$. Each $\Gamma_{i}$ is a torsiongroup and each $G_{i}$ is finite.

The quotient $\Omega / \Gamma$ can be compactified byadding $h$ points $s_{1}, \ldots, s_{n}$. The compactified quotient turns out to be the set $X(K)$ of $K$-points of a projective algebraic curve $X$. This is known by work of Drinfeld. By considering products of the form

$$
\theta(\omega, \eta ; z)=\prod_{\gamma \in \Gamma} \frac{z-\gamma \omega}{z-\gamma \eta}, \quad \omega, \eta, z \in \Omega, \quad \Gamma:=P^{2} L_{2}(A),
$$

one can construct a nonconstant meromorphic function on the curve $X$.
Such a function $f$ can be obtained by a quotient of two such products $\theta_{1}, \theta_{2}$, which have the same factor of automorphy. It is a theorem of Deligne, that the genus $g(X)$ of $X$ equals the rank of $\Gamma^{\mathrm{ab}}=\Gamma /[\Gamma, \Gamma]$. (This is finite because of Serre's theorem). The construction of this function $g$ shows that $g(X) \leq r k r^{a b}$. For $\alpha \in \Gamma$ define $u_{\alpha}:=\theta(\omega, \alpha \omega ;$, $)$, which is independent of the choice of $\omega$. $\frac{d u_{\alpha}}{u_{\alpha}}$ are regular 1 -forms on $X$. For a proof of Deligne's theorem in terms of nonarchimedean function-theory it remains to show, that the forms $\frac{d u_{\alpha_{i}}}{u_{\alpha_{i}}}, i=1, \ldots, r k \Gamma^{a b}, \alpha_{i}$ a basis for the free part of $\Gamma^{a b}$, are linearly independent.

## F.BALDASSARI: p-adic GAGA

Let $X_{o}$ be non-singular algebraic variety defined over $K_{o}=\bar{Q}^{\mathrm{alg}}$; we assume to be given a locally free $\boldsymbol{O}_{X_{0}}$-module of finite type $\mathcal{V}_{0}$, endowed with an integrable connection $\nabla_{0}$. To the triple ( $x_{0}, V_{0}, \nabla_{0}$ ) one classically associates a ©-analytic object ( $x_{c 1}, V_{c 1}, \nabla_{c 1}$ ) made of a smooth $\mathbb{C}$-analytic space $X_{c l}$ with a connection $\left(V_{c 1}, \nabla_{c 1}\right)$ on it. Analogously, in p-adic rigid analytic geometry, one has a natural function:

$$
\left(x_{0}, V_{0}, \nabla_{0}\right) \rightarrow\left(x_{r i g}, V_{r i g}, \nabla_{r i g}\right)
$$

where $X_{\text {rig }}$ is the regular rigid variety over $K=a l g$. closed, complete, p-adic field, associated to $X_{o}$. The Grothendieck-Deligne comparison theorem asserts that if $\left(V_{0}, \nabla_{0}\right)$ has regular singularities at infinity there are natural isomorphisms:

$$
\mathbb{C} \otimes \mathrm{H}_{\mathrm{DR}}^{\mathrm{q}}\left(\mathrm{x}_{\mathrm{o}} ;\left(\mathcal{V}_{\mathrm{O}}, \nabla_{\mathrm{o}}\right)\right) \stackrel{\mathrm{H}_{\mathrm{DR}}^{\mathrm{q}}\left(\mathrm{x}_{\mathrm{c} 1} ;\left(\vartheta_{\mathrm{c} 1}, \nabla_{\mathrm{c} 1}\right)\right)}{ }
$$

where $H_{D R}^{q}$ stands for the de Rham cohomology (i.e. hypercohomology of the de Rham complex) of a connection. In the rigid analytic case we conjecture that

$$
K \otimes_{K_{0}} H_{D R}^{q}\left(x_{o} ;\left(V_{0}, \nabla_{o}\right)\right) \stackrel{\sim}{\rightarrow} H_{D R}^{q}\left(x_{r i g} ;\left(\mathcal{V}_{r i g}, \nabla_{r i g}\right)\right)
$$

holds in füll generality, and in particular when $\left(\mathcal{V}_{0}, \nabla_{0}\right)$ is irregular at infinity. We can prove the conjecture in the following to particular cases:
a) when $X_{o}$ is a curve
b) when $\left(X_{0}, \nabla_{0}\right)$ is regular.

## M. MATIGNON: Topological genus of valued fields

All absolute values are non archimedean.
Def.: Let $K$ be a field. A topological function field L of one variable over $K$ is the completion of a valued function field of one variable over K. Its topological genus is: $g_{t}(L)=:$ min $g(M)$ for $M$ function field of one variable over $K$ dense in $L \quad(G(M)$ is the genus of $M)$. We prove the following.
Th.: Let $K$ be a complete valued field, $\left(K^{a l g}\right) \wedge$ be the completion of the algebraic closure of $K$. Let $L \supset K, L \notin\left(K^{a l g}\right)^{\wedge}$ be a topological function field of one variable over $K$, then $\exists$ K'כ $K$ finite such that for all $K^{\prime \prime} ; K^{\prime} \subset K^{\prime \prime} \subset\left(K^{\text {alg }}\right)^{\wedge}$, the field (LK' $)^{\wedge}$ is a topological function field of one variable over $K^{\prime \prime}$ and
i) if $\overline{\mathrm{L}} \mid \overline{\mathrm{K}}^{\mathrm{alg}} \Rightarrow \mathrm{g}_{\text {top }}\left(\left(L K^{\prime \prime}\right)^{\wedge}\right)=\mathrm{g}\left(L K^{\prime \prime}\right)$
$\overline{\mathrm{M}}$ is the residual field, $\mathrm{M}^{\sim}$ the completion)
ii) if $\overline{\mathrm{L}} \subset \overline{\mathrm{K}}^{\mathrm{alg}} \Rightarrow \mathrm{g}_{\text {top }}\left(\left(\mathrm{LK}^{\prime \prime}\right)^{\wedge}\right)=0$
for i): The inequality $g_{\text {top }}\left(L^{\prime \prime \wedge}\right) \geq g\left(\overline{L K^{\prime \prime}}\right)$ comes from a generalisation of a previous result of H. Mathịeu, Arch. Math. 1969. Our proof uses reduction of algebraic curves, the other inequality comes from the lifting porperty of algebraic plane curves with only nods as sịngularities (Popp, Arch. Math. 1965).
for ii): It was obtained by M.v.d. Put (Stable reduction of algebraic curves Indagationes 1984) for maximally complete algebraically closed field K.
Y. MORITA: Irreducibility of analytic representations of $\mathrm{SL}_{2}$ over a p-adic number field

Let $L$ be a finite extension of $Q_{p}$, and let $k$ be a maximally complete field containing $L$. Let $X: L^{\#} \rightarrow k^{*}$ be a locally analytic character. Put

$$
G=S L_{2}(L) \nexists P=\left\{\left(\begin{array}{ll}
a & 0 \\
c & d
\end{array}\right) \in G\right\}
$$

We define a representation of the parabolic subgroup $P$ of $G$ by

$$
p \ni p=\left(\begin{array}{ll}
a & 0 \\
c & d
\end{array}\right) \rightarrow x(a) \in k
$$

Let Ind ( $\mathrm{P}, \mathrm{G}, \mathrm{X}$ ) be the space consisting of all functions $\mathrm{F}: \mathrm{G} \rightarrow \mathrm{k}$ such that $F$ is locally analytic, and

$$
F(p g)=x(p) F(g)
$$

holds for any $p \in P$. Then this space $I(P, G, X)$ has a natural topology, and $G$ acts on this space by

$$
T_{\chi}\left(g_{1}\right) F(g)=F\left(g_{1}\right) \quad\left(g, g_{1} \in G, F \in \operatorname{Ind}(P, G, \chi)\right)
$$

We can determine all closed G-invariant subspaces of Ind ( $\mathrm{P}, \mathrm{G}, \mathrm{X}$ ), and can determine the equivalences between all ineducibily representations obtained from the $T_{X}^{\prime}$ s.

## E. ROBINSON: p-adic Spectra

Valued fields first-order equivalent to the p-adic numbers are called "p-adically closed". This theory has been extensively investigated by logicions (c.f. - Roquette, SLN 1050) interested in such properties as quantifier elimination. Logical techniques lead to the following result:
Any subset of $K^{n}$, $K$ a p-adically closed field, which is first-order definable (with parameters from $K$ ) and closed in the valuation topology is expressible as a finite union of finite intersections of sets of the form

$$
\left\{\vec{x} \in K^{n} \mid p(\vec{x}) \text { has an nth root }\right\}
$$

where $p(\vec{x})$ is a polynomial over $K$.
For example $(p \neq 2)$, the ring of integers is $\left\{x \mid 1+p x^{2}\right.$ is a perfect square\} . ~ T h e r e ~ i s ~ a n ~ a n a l o g o u s ~ c h a r a c t e r i s a t i o n ~ o f ~ definable open sets.

We investigate the Grothendieck-topology given by finite etale coverings surjective on p-adically closed points of the corresponding varieties, and show that this leads to the definition of a spectrum functorial on rings which has a structural sheaf of p-adically closed local rings (ie. henselian local rings with p-adically closed residue field). We show further that this spectram can be represented as a topological space, and use the above characterisation to prove that, for finitely-generated $\mathbb{Q}_{\mathrm{p}}$-algebras, the subspace of the closed points of the spectrum is isomorphic to the points over $\mathbb{Q}_{\mathrm{p}}$ of the corresponding variety equipped with the valuation topology.

## F. HERRLICH: Non-archimedean Teichmü1ler spaces

Let $\Gamma$ be a finitely generated group, $K$ a complete non-archimedean field. We call $T_{K}(\Gamma):=\left\{\tau \in \operatorname{Hom}\left(\Gamma, P G L_{2}(K)\right): \tau\right.$ infective, $\tau(\Gamma)$ discontinuous and without parabolic elements\} the Teichmüller space of $\Gamma$ over $K$, and we put $\bar{T}_{K}(\Gamma):=\operatorname{PGL}_{2}(K) T_{K}(\Gamma)$ and $M_{K}(\Gamma):=\bar{T}_{K}(\Gamma) /$ Aus $\Gamma$.

We define a $\Gamma$-family over an analytic space $S$ to be a group homomorphism $\psi: \Gamma \rightarrow \operatorname{Aut}_{S}\left(S \times \mathbb{P}^{1}(K)\right)$ such that the induced homomorphisms $\psi_{S}: \Gamma \rightarrow \operatorname{PGL}_{2}(K), \psi_{S}(\gamma)(z):=p_{2}(\psi(\gamma)(s, z))$ are elements of $T_{K}(\Gamma)$ for all $s \in S$ ( $p_{2}$ is the projection onto the second component). A \{ $\Gamma$ \}-family (resp. [ $\Gamma$-family) is then an equivalence class for the identification of $\psi$ and $\beta \circ \psi$ (resp. $\beta \circ \psi \bullet \alpha$ ) for inner automorphisms $B$ of Auth $\left(S \times \mathbb{P}^{1}(K)\right)$ and $\alpha \in$ fut $\Gamma$.

If $S=T_{K}(\Gamma)$, then by $\psi_{0}(\gamma)(\tau, z):=(\tau, \tau(\gamma)(z))$ a $\Gamma$-family is defined. As there exists a section $\sigma: \bar{T}_{K}(\Gamma) \rightarrow T_{K}(\Gamma)$, we can define a $\{\Gamma\}$-family over $\bar{T}_{K}(\Gamma)$ by $\psi_{1}(\gamma)(\bar{\tau}, z):=(\bar{\tau}, \sigma(\bar{\tau})(\gamma)(z))$.

We can show that $\quad \psi_{0} \quad\left(\psi_{1}\right)$ is a universal $\Gamma$-family (\{Г\}-family), whereas $M_{K}(\Gamma)$ is only a coarse moduli space for [ $\Gamma$ ]-families. If $T_{K}(\Gamma)$ is non empty, then $\Gamma$ contains a free normal subgroup $\Gamma$ of finite index. Every $\Gamma$-family then induces a family of Mumford curves of genus $g=r a n k\left(\Gamma_{0}\right)$, each of which has an automorphism group contraining a subgroup isomorphic with $\Gamma / \Gamma_{o}$. Then .
$M\left(\Gamma, \Gamma_{0}\right):=\bar{T}_{K}(\Gamma) /\left\{\alpha \in \operatorname{Aut} \Gamma: \alpha\left(\Gamma_{0}\right)=\Gamma_{o}\right\}$ is the closure of a stratum of the moduli space of Mumford curves of genus $g$.

We mention the following properties of the Teichmüller spaces.
a) The definitions are functorial in $K$ and $\Gamma$ (for infective group homomorphisms)
b) $T_{K}(\Gamma)$ is contained in the affine algebraic K-variety $S_{K}(\Gamma):=\operatorname{Hom}\left(\Gamma, \mathrm{PGL}_{2}(\mathrm{~K})\right)$
c) If $\Gamma=\Gamma_{1}^{*}{ }_{c}, \Gamma_{2}$, then $S(\Gamma)=S\left(\Gamma_{1}\right) x_{S(C)} S\left(\Gamma_{2}\right)$
d) $S_{K}(\Gamma)$ and therefore $T_{K}(\Gamma)$ are nonsingular
e) $\operatorname{dim} T_{K}(\Gamma)=3 g+3(D-d)+2(C-c)$ if $T_{K}(\Gamma)$ is nonempty. Here $g$ is the cyclomatic number of a graph of groups with fundamental group $\Gamma$. $D(d)$ is number of noncyclic vertex (edge) groups. C (c) its number of nontrivial cyclic vertex (edge) groups.
f) The analytic structure on $\bar{T}_{K}(\Gamma)$ can be defined by a covering consisting of analytic polyedra. This makes it possible to define the analytic structure on $M_{k}(\Gamma)$.
S. BOSCH: Neron models from the rigid analytic viewpoint

Let $K$ be a field with a discrete non-Archimedean valuation and consider a scheme f flat and locally of finite type over the valuation ring $K$. Then $\underset{X}{ }$ is uniquely characterized by its geometric fibre $\forall_{n}$ and by the formal analytic variety $\bar{*}$ associated to the formal completion $\hat{X}$ of $\boldsymbol{X}$. Using this fact and the uniformization
of abelian varieties (see the lecture of $W$. Lütkebohmert), the following results were shown:

Theorem: (Neron, Raynaud). Let $A$ be an abelian variety over $K$. Then the Neron model of $A$ exists and is of finite type over $K$.

Theorem: ( Grbthendieck). There exists a finite separable extension L of $K$ with a unique extension of the valuation from $K$ to $L$ such that the Neron model of $A \otimes L$ has semi-abelian reduction.

Theorem: (Raynaud). Let $n \geq 3$ be an integer prime to char $\tilde{K}$, where $\tilde{K}$ is the residue field of $K$. Assume that all $n$-torsion points of $A$ are rational over $K$. Then Grothendieck's result on the semi-abelian reduction holds for $L=K$. (joint work with W. Lütkebohmert).
W. LUTKEBOHMERT: Uniformization of abelian varieties

Let $k$ be a field with a discrete non-Archimedean valuation assumed to be complete and algebraically closed. Let $A$ be an abelian variety over $k$. The following results were shown:

1. There exists a unique open analytic subgroup $\bar{A}$ of $A$ which is a connected, quasi-compact, formal analytic group having.: semiabelian reduction $\tilde{A}$.
2. $\bar{A}$ has the universal mapping property:

If $X$ is a formal analytic variety, smooth over $k$ and connected, and if $\phi: X \rightarrow A$ is a rigid morphism such that $\operatorname{im} \phi \cap \AA \neq \phi$, then im $\dot{\phi} \subset \overline{\mathrm{A}}$ and $\phi: \mathrm{X} \rightarrow \overline{\mathrm{A}}$ is förmal.
3. Let $A^{\prime}$ be the dual of $A$. Then there are canonical isomorphisms

$$
H^{1}\left(A^{\prime}, \mathbb{Z}\right) \stackrel{\sim}{\rightarrow} \operatorname{Hom}_{a n}\left(\mathbb{C}_{m}, A\right) \xrightarrow{\sim} \operatorname{Hom}_{a n}\left(\mathbb{C}_{m} ; A\right) .
$$

4. Let $\overline{\mathrm{T}} \subset \overline{\mathrm{A}}$ be the maximal affinoid torus (given by $\mathrm{H}^{1}(\mathrm{~A}, \mathbb{Z})$ ). Let $B:=\bar{A} / \bar{T}$ be the quotient. Then $B$ is an abelian variety with good reduction. The exact sequence $1 \rightarrow \bar{T} \rightarrow \overline{\mathrm{~A}} \rightarrow \mathrm{~B} \rightarrow 1$. is a lifting of the chevalley decomosition

$$
1 \rightarrow \tilde{\mathrm{~T}} \rightarrow \tilde{\mathrm{~A}} \rightarrow \tilde{\mathrm{~B}} \rightarrow 1
$$

5. The uniformization of $A$ can be expressed in the diagramm:

where
$\mathrm{T} \supset \overline{\mathrm{T}}$ is the affine torus containing $\overline{\mathrm{T}}$ as torus of units
$\left.\hat{\mathrm{A}}:=\overline{\mathrm{A}} \times \mathrm{T} /\left\{\mathrm{cs}^{-1}, \mathrm{~s}\right) \mathrm{s} \in \overline{\mathrm{T}}\right\}$
$p$ is a surjective covering map
$\Gamma=\operatorname{ker} p \subset \hat{A}$ is a lattice of rank $t=\operatorname{dim} T$
Moreover, $H^{1}(\hat{A}, \mathbb{Z})=H^{1}(\bar{A}, \mathbb{Z})=0$ and $H^{1}(A, \mathbb{Z}) \cong \mathbb{Z}^{t}$.
Now let $k$ be discretely valued, $k$ the completion of the algebraic closure of $k$.
6. There exists a unique open analytic subgroup $\bar{A}$ of $A$ such that $A \otimes k=\overline{A_{\bar{k}}}$, where $\overline{A_{\bar{k}}}$ is the group mentioned in 1 . which is associated to $A \otimes \bar{k}$. And $\bar{A}$ is a formal group over $\stackrel{\circ}{k}$ (not necessarily smooth over k ).
7. If $\bar{A}$ is smooth over $\dot{k}$. there exists an open analytic subgroup $\bar{A}^{e t}$ of $A$ which is formal, quasi-compact and smooth over $\stackrel{\circ}{k}$ with the following mapping property:
If $X$ is a formal analytic variety, smooth over $\stackrel{\circ}{k}$, and if $\phi: X \rightarrow A$ is a rigid morphism, then $i m \phi \subset \bar{A}^{-t}$ and $\phi: X \rightarrow \bar{A}^{\text {et }}$ is formal.
8. Let $n \geq 3$, prime to char $\tilde{k}$. If $\bar{A}[n]$ consists of k-rational points, then $\bar{A}$ is smooth over $\stackrel{\circ}{k}$.
(joint work with S. Bosch).

## p. ULLRICH: Analytic lifting of algebraic tori

Let $G$ be a formal analytic group that is smooth over the valuation ring of a completely non-archimedean valued field k. Suppose that there is a closed immersion $\tilde{\sigma}$ of group schemes over the residue field of $k$ defining the split d-dimensional affine-algebraic torus $\tilde{T}$ as a normal subgroup of the reduction $\tilde{G}$ of $G$.
Then by means of rigid analysis one can lift $\tilde{\sigma}$, i.e. there exists a closed immersion $\sigma$ of formal analytic groups defining the split d-dimensional affinoid torus $T$ over $k$ as a subgroup of $G$ in a way that the closed immersions and the reduction maps commute. This lifting is unique and $T$ is a normal subgroup of $G$.
W. SCHIKHOFF: Duality theory for locally convex spaces over nonspherically complete fields

Let E be a locally convex space over a complete non archimedean valued field $K$ with a dense valuation. A continuous seminorm $p$ on $E$ is a polar seminorm if $p=\sup \left\{|f|: f \in E^{\prime},|f| \leq p\right\}$. E is a polar space if there exists a family polar seminorms defining the topology.
$E$ is strongly polar if each continuous seminorm is polar. If $K$. is spherically (= maximally ) complete each $E$ is strongly polar. If $K$ is not, $1^{\infty}$ is polar, but not strongly polar., $1^{\infty} / c_{o}$ is not polar as $\left(1^{\infty} / c_{0}\right)^{1}=(0)$.
The class of polar spaces has nice stability properties. Also we have Theorem: On compactoids of a polar space, the initial topology coincides with the week topology."Bounded" = "weekly bounded'".

Theorem: E strongly polar $\Leftrightarrow(1+\varepsilon)$-Hahn-Banach theorem holds $E$ is of countabletype is an extension $\tilde{f}$ with $|\tilde{f}| \leq(1+\varepsilon) p$ )

Further theorems can be proved, for example:
Thm.: Each Frechet space of countable type over $\mathbb{C}_{\mathrm{p}}$ is reflexive.

## N. DE GRANDE-De KIMPE: Projective locally K-convex spaces

Let $K$ be a field with a non-trivial, non-archimedean valuation. Let $E$ be a locally convex space over $K, F$ any closed subspace of $E$ and $\pi: E \rightarrow E / F$ the quotient map. A locally convex space $X$ over $K$ is said to have the "lifting property" with respect to $E$ if for every linear continuous map $f: X \rightarrow E / F$ there exists a linear continuous map $g: X \rightarrow E$ such that $\pi \circ g=f$.

If $(\mathcal{C})$ is a class of locally convex spaces over $K$ then a locally convex space $X$ is called "projective with respect to ( $\mathcal{C}$ ) " if $X$ has the lifting property with respect to every element of ( $\boldsymbol{C}$ ). Let (NJ) denote the class of all nuclear Frechet spaces over K . Main result: If $X$ is a locally convex space such that its strong dual space is metrizable then $X$ is projective with respect to the class ( N F).

Corollaries: (i) Every Banachspace is projective with respect to (N $\mathcal{F}^{\text {) }}$. (ii) If the valuation on $K$ is discrete then every Banach space is projective with respect to (B) $\cup\left(N^{\Im}\right)((B)=$ the class of all Banach spaces over $K$ ).

## E. KANI: "Real-analytic" functions on non-archimedean curves

Let $K$ be an algebraically closed field, complete with respect to an absolute value $|\cdot|$, and let $C$ be a smooth projective curve defined over $K$. Nontrivial examples of "real-analytic" functions $f: U \rightarrow \mathbb{R}$ on open sets $U C$ C are furnished by (i) Néron's pairing on $C \times C$-diagonal and (ii) Green's functions on subdomains of $C$. In the archimedean case, both (i) and (ii) may be "constructed" by integration of suitable positive (1,1)-forms defined on $C$ (or on subsets thereof). The purpose of this talk was to show that the same can be done also in the non-archimedean case and in particular, that there exists a notion of "integration of (1,1)--forms" on C.

This was done as follows. Let $X$ be a rigid analytic variety with a reduction map $p: X \rightarrow \tilde{X}$ (s. th. $\tilde{X}$ is of finite Type /K). For each irreducible component $\tilde{X}_{i}$ of $\tilde{X}$ (with $\operatorname{dim} \tilde{X}_{i}=\operatorname{dim} \tilde{X}$ ) define a (real-valued) additive set function $\mu_{i}$ on the boolean algebra Con ( $X_{i}$ ) of (Zariski) constructive subsets of $\tilde{X}_{i}$ by

$$
\tilde{\mu}_{i}(A)= \begin{cases}1 & \text { if Ais(Zariski-) dense in } \tilde{X}_{i} \\ 0 & \text { else }\end{cases}
$$

where $A \in \operatorname{Con}\left(\tilde{X}_{i}\right)$. Let $\operatorname{Con}_{r i g}(X)$ denote the algebra of rigid constructible sets of X , ie. the boolean algebra generated by the allowable subsets of $X$, and define $\mu_{p}: \operatorname{Con}_{r i g}(X) \rightarrow \mathbb{R}$ by

$$
\mu_{p}(A)=\sum_{i} \tilde{\mu}_{i}\left(p(A) \cap \tilde{x}_{i}\right), \quad A \subset \operatorname{Con}_{r i g}(X) .
$$

Then $\mu_{p}$ is an additive set function on Con $_{\text {rig }}(X)$ which may be used to "construct" (i) Néron pairings (ar la Arakelov) and (ii) Green's functions (a 1 a Rumbly) on affined subdomains $X$ of $C$.
P. BERTELOT: On the duality theorem for rigid cohomology

Let $K$ be a field of characteristic 0 , complete under a non-archimedea absolute value, $\mathcal{O}_{\text {its }}$ ring of integers, $k$ its residue field; we assume $k$ to be perfect, of characteristic $p>0$.

If $X_{o}$ is a smooth, quasi-projective variety over $k$, one can attach to $X_{o}$ two types of p-adic cohomology groups which are $K$-vector spaces: - it's rigid cohomology groups $H^{i}{ }_{r i g}\left(X_{o / K}\right)$ : they coincide with crystalline cohomology (tensored with $K$ ) if $X_{o}$ is projective, with Monsky- Washnitzer cohomology if $X_{o}$ is affine, and allow to recover Dork's analytic cohomology.

- it's rigid cohomology groups with compact supports $H_{c}^{i}\left(X_{o / K}\right)$ : they coincide with crystalline cohomology (censored with $K$ ) if $X_{o}$ projective, and allow to recover Dork's dual analytic cohomology. The purpose of the talk was to present results about the duality between these groups, with a special emphasis on the affine case. The main points are the following:

1) $H_{c}^{i}\left(X_{o / K}\right)=0$ if i $>2 \mathrm{dim} X_{o}$
2) There exists a trace map: $H_{C}^{2 n}\left(X_{o / K}\right) \rightarrow K$
3) There exist canonical pairings

$$
H_{r i g}^{i}\left(X_{o / K}\right) \times H_{c}^{j}\left(X_{o / K}\right) \rightarrow H_{c}^{i+j}\left(X_{o / K}\right)
$$

4) The spaces $H_{c}^{i}\left(X_{o / K}\right)$ have a canonical topology of quotient of Freshet spaces, and the above pairings define a map

$$
H_{r i g}^{i}\left(X_{o / K}\right) \rightarrow \text { How cont } K_{K}\left(H_{c}^{2 n-i}\left(X_{/ K}\right), K\right) .
$$

5) If the spaces $H_{c}^{j}(0 / K)$ are Hausdorff; the above map is an isomorphism. In the general case there exists a duality theorem at the level of cochin complexes, from which this statement on
cohomology can be derived.
As an application, one can prove the following
Theorem: Suppose $X_{0}$ is smooth and affine, and is the complement of a divisor with normal crossings in a projective smooth variety. Then the spaces $H_{c}^{i}\left(X_{o / K}\right)$ are finite-dimensional and Hausdorff; hence the spaces $H_{\text {rig }}^{i}\left(X_{o / K}\right)$ (equal to Monsky-Washnitzer cohomology) are finite dimensional dual to $H_{c}^{i}\left(X_{o / K}\right)$.

## M. Piwek: Families of Schottky groups

Let $k$ be an algebraically closed field, maximally complete w.r.t. a non-archimedean valuation, let $S$ bea reduced $k$-analytic space and $\boldsymbol{O}=\mathcal{O}_{\mathrm{S}}$ its structure sheaf.
Then $\operatorname{PGL}_{2} \mathcal{O}_{(U)}:=\operatorname{Aut}_{U}\left(U \times \mathbb{P}^{1}\right)$ for each admissible $U \subset S$.
Def.: A subgroup $\quad \Gamma<\operatorname{PGL}_{2} \mathcal{O}(S)$ is called Schottky-group on $\mathbb{P}_{S}^{1}$ iff for each $s \in S$, the group $\Gamma_{s}:=\{z \rightarrow \gamma(s, z) \mid \gamma \in \Gamma\} \subset \operatorname{PGL}_{2}(k)$ is a Schottky-group and the canonical map $\Gamma \rightarrow \Gamma_{s}$ is an isomorphism. With the help of a result on coverings given by units, one can find locally for the Grothendieck-topology Schottky-bases for $\Gamma$ and good fundamental domains:
Theorem: (i) The set $Z:=\left\{(s, z) \in S \mathbb{P}^{1} \mid z\right.$ is not a limit point for $\left.\Gamma_{s}\right\}$ is an admissible open subset of $S \times \mathbb{P}^{1}$,
(ii) There exists an admissible covering of $S$ with affinoid domains $U_{i}$ and for each $i$ an admissible $F_{i} \subset p^{-1}\left(U_{i}\right) \cap Z \operatorname{s}$. th.
(a) $\underset{\gamma \in \Gamma}{\cup} \gamma F_{i}=p^{-1}\left(U_{i}\right) \cap z$
(B) $\gamma \mathrm{F}_{\mathrm{i}} \cap \mathrm{F}_{\mathbf{i}}=\varnothing$ for almost all $\gamma \in \Gamma$
( $\gamma$ ) $F_{i} \cap\left(\{s\} \times \mathbb{P}^{1}\right)$ is a good fundamental domain for $\Gamma_{s}$.

One can now easily construct the quotients $\ell_{i}=\left(p^{-1}\left(U_{i}\right) \cap Z\right) / \Gamma \rightarrow U_{i}$ and glue them to a quotient $\boldsymbol{\ell}=Z / \Gamma_{\Gamma} \rightarrow S$, which is a family of Mumford-curves.

As an example consider the group $[\Gamma, \Gamma]$, with $\Gamma=\langle\alpha\rangle *\langle\beta\rangle, \alpha, \beta$ parabolic transformations with fixed points $0, \infty, 1, s, 0<|s-1|<1$ as a family over $\{s|0<|s-1|<1\}$. This leads to the family
$\left\{\left(\lambda, y_{1}, y_{2}\right)\left|0<|\lambda-1|<1, y_{1}=\frac{\mathrm{r}_{2}{ }_{2}^{r_{2}-\lambda}}{y_{2}{ }_{2}{ }_{2}-1}\right\}\right.$ over $\{0<|\lambda-1|<1\}$.
L. GERRITZEN: Theta functions and horizontal elements

On a curve of genus $g$ over a p-adic field which admits a p-adic Schottky uniformization there is a canonical $\mathbb{Z}$-module of rank $g$ of differentials of the first kind.

These differentials are obtained by non-vanishing analytic automorphic forms with constant factors of automorphy which are sometimes called multiplicative periods and give the multiplication period form or period matrix.

These period forms allow the construction of the Jacobian variety of the curve. If you take the Riemann theta function in these automorphic forms whose coefficients are given by the multiplicative period form you can obtain the Riemann vanishing theorem which implies that the $\theta$-divisor is the ( $\mathrm{g}-1$ )-fold product of the curve canonically embedded into the Jacobian variety of the curve. Moreover you can take certain derivatives of $\theta$ which give non-regular rational differentials of the second kind which can be integrated and which have additive periods in the classical sense of the word. These differentials can be chosen in such a way that
these additive periods are integers for any family of curves which admits a split p-adic horizontal element. for the Gauss-Manin-connection.

For a Legendre-type family of abelian coverings of the projective line the diffentials with multiplicative periods and the differenttials with integral additive periods are computed and expressed in algebraic terms.

## A. ESCASSUT: Analytic extension through a T-filter

Let $K$ be a complete ultrametric algebraically closed field. For every closed bounded subset $D \subset K$, let $H(D)$ be the Banach algebra. of the analytic elements on $D$ provided with the norm of uniform convergence on $D, \mathbb{\|}_{\mathrm{D}}$. We recall a family of holes ( $\mathrm{T}_{\mathrm{m}, \mathrm{i}}$ ) ${ }_{1 \leq i \leq k_{m}}$ with $T_{m, i} \subset C\left(a, d_{m}\right)$ circle of center $a$, of diameter $d_{m}, \quad m \in \mathbb{N}$ with $d_{m}<d_{m+1}, \lim _{m \rightarrow \infty} d_{m}=R$, provided with a family of positive integers $\left(q_{m, i}\right)$ is called a $T$-sequence when
$0=\lim _{m \rightarrow \infty}\left[\max _{1 \leq i \leq k_{m}} \underset{\left(\frac{d_{m}}{\rho_{m, i}}\right)}{q_{m, i}} \underset{\substack{j \neq i \\ 1 \leq i \leq k_{m}}}{\pi}\left(\frac{d_{m}}{\alpha_{m, i}}{ }_{m, j}^{\alpha_{m} \mid}\right) \quad{ }^{q_{m, j}}\right]\left[\prod_{1=1}^{m}\left(_{d_{1}}^{d_{1}}\right)^{q_{1}}\right]$
with $\alpha_{m, i} \in T_{m, i}$ and $q_{m}=\sum_{i=1}^{k_{m}} q_{m, i}$.
Similarly we define decreasing T-sequences. T-sequences characterize the $T$-filters on $D$ : here the filter $\mathcal{F}$ of base the annuli $r<|k-a|<R$ is called the $T$-filter of center 0 , of diameter $R$. We denote by $B(F)$ the set $\{\kappa \in D||\kappa-a| \geq R\}$. Fealso defines a continuous multiplidative semi-norm $\psi_{\mathcal{F}}$ on $H(D): \psi_{\mathcal{F}}(f)=\lim |f(k)|$ for $f \in H(D)$.
$\mathcal{F}_{\text {is }}$ said to be well pierced if $\underset{(m, i)}{\inf } \rho_{m, i}>0$.

Theorem 1: Let $D$ be a bounded closed infraconnected set with a well pierced $T$-filter $\mathcal{F}$ with $B(F)=\phi$. Then $H(D) / K e r \psi_{\oiint}$ is a field $\Gamma$; the quotient of the norm $\left\|\|_{D}\right.$ and the absolute value $\bar{\psi}_{\mathcal{F}^{\prime}}$ quotient of $\psi_{F}$ on $\Gamma$ are equivalent.
Theorem 2: Let $f(k)$ be a taylor series that converges for $|\kappa| \leq R$ and let $D$ be an infraconnected closed set with a well pierced decreasing $T$-filter Foo center 0 , of diameter $R$ that contains $d(0, R)$. There does exist a bounded element $g \in H(D)$ such that $g(k)=f(\kappa)$ whenever $\kappa \in \cdot d(0, R)$. All the $1 \in H(D)$ such that $l(k)=f(k)$ for $|k| \leq R$ are the $g+h$ with $h \in \operatorname{ker} \psi_{\mathcal{F f}}$.
Theorem 3: Let $D$ be a bounded closed infraconnected set with a well pierced $T$-filter Fe and let $\mathcal{F}(F)$ the ideal of the $f \in H(D)$ such that $f(\kappa)=0$ whenever $k \in B(F)$. Then $H(B(F))$ is algebrically and toplogically isomorphic to $H(D) / f(F)$.
(Remark: if $B(\mathbb{F}$ ) has no T-filter complementary to Fethen
$\mathcal{f}(\boldsymbol{F})=$ er $\psi_{F}$, hence $\mathrm{H}\left(\mathrm{B}\left(F_{F}\right)\right)=\mathrm{H}(\mathrm{D}) /$ ger $\left.\Psi_{F}\right)$.
M. SARMANT: Construction pratique d'un prolongement d'une série entière de rayon de convergence $\geq 1^{+}$
(Les notations et les hypotheses son les mêmes que dan l'article precedent).

$$
f(\kappa)=\sum_{n=0}^{+\infty} a_{n} \kappa^{n} \quad \lim _{n \rightarrow+\infty} a_{n}=0
$$

On cherche:

1) $g(k)=\sum_{i \in \mathbb{N}} \frac{\varepsilon_{i}}{1-\frac{k}{b_{i}}}$
tel que: $\left|b_{i}\right|>\left|b_{i+1}\right|>1$ et $g(k)=f(k)$
$\forall k \in C(0,1)$.

En posant $\beta_{i}=\frac{1}{b_{i}}$, on voit que
$f(k)=g(k) \Leftrightarrow \sum_{i \in \mathbb{N}} a_{n} k^{n}=\sum_{i \in \mathbb{N}} \frac{\varepsilon_{i}}{1-\beta_{i} k} \quad \forall \kappa \in C(0,1)$
Ce système de $+\infty$ d'équations est équivalent à:

$$
\begin{aligned}
& \mathrm{A}=\mathrm{BE} \\
& \text { ou } \\
& \mathrm{A}=\left(\begin{array}{c}
\mathrm{a}_{\mathrm{o}} \\
\mathrm{a}_{1} \\
\vdots \\
\overline{\tilde{a}}^{\mathrm{n}} \\
\vdots
\end{array}\right) \\
& E=\left(\begin{array}{c}
\varepsilon_{0} \\
\varepsilon_{1} \\
\vdots \\
\vdots \\
\cdot
\end{array}\right) \\
& , B=\left(\begin{array}{llll}
1 & \cdots & \cdots & 1 \\
\beta_{0} & \cdots \cdots & \beta_{i} \\
\vdots & & & \cdots \\
\beta_{0}^{n} & & \beta_{i}^{n} \\
\vdots & & & \vdots
\end{array}\right)
\end{aligned}
$$

soient des matrices infinis.
On démontre alors:

1) la matrice $B$ est invertible si la suite $\left(D\left(B_{i}, \phi\right), 1\right) \underset{i \in N}{ }$ est une T-suite idempotente et alors:
2) A étant donnéla suite ( $\beta_{i}$ ) étant bien choisic, il existe une matrice $\infty$ B telle que:
$B\left(B^{\prime} A\right)=\left(B B^{\prime}\right) A=A$.
Ce qui entraine que $E=B^{\prime} A$ est solution de $A=B E$ (la verification de $B\left(B^{\prime} A\right)=\left(B B^{\prime}\right) A=A$ étant indispensable car le produit de matrices infinis n'est ni toujours possible, ni associatif), d'oú la solution du problême.

- Berichterstatter: Meinolf Piwek


## Tagungsteilnehmer

Yvette Amice
UER Mathematiques
Univ. Paris 7, Aile 45-55
2 place Jussieu
75251 Paris cedex 05
Frankreich

Prancesco Baldassarri
Seminario Mathematico
Via Belzoni 7
35100 Padova
Italien

Daniel Barsky
UER Mathematiques
Univ. Paris 7, Aile 45-55
2 place Jussieu
75251 Cedex 05
Frankreich

Pierre Berthelot
IRMAR
Univ. de Rennes 1
Campus de Beaulieu 35042 Rennes Cedex Frankreich
G. Borm

Math. Institut, K.U. Toernooiveld
6325 ED Nijmegen
Niederlande

Siegfried Bosch Math. Institut
Univ. Münster
Einsteinstr.. 62
4400 Münster

Selim Bouras
Dept. of Math.
U.S.T.A.
B.P.nO 9

Alger - Dar el Beida, Algerie

Abdelbaki Boutabaâ Dept. of Math.
U.S.T.A.
B.P. $\mathrm{n}^{\circ}$ 9,

Alger,- Dar el Beida Algerie
P. Cassou Nogues

Math. et Informatique
Univ. Bordeaux 1
351 cours de la Libération 33405 Talence Cedex Frankreich
M.F. Coste-Roy I RMAR
Univ. de Rennes 1
Campus de Beaulieu 35042 Rennes Cedex Frankreich
B. Deshommes

15 rue de l'ancienne Comédie 7500 Paris
Frankreich
B. Dwork

Dept. of Math. Fine Hall
Princeton University
Princeton NJ 08540
USA
A. Esscassut

UER Math. et Informatique Univ. de Bordeaux I
351 de la Liberation 35405 Talence Cedex Frankreich
J.Y. Etesse IRMAR
Univ. de Rennes 1 Campus de Beaulieu 35042 Rennes Cedex Frankreich

## J. Fresnel

Math. et Informatique
Univ. de Bordeaux 351 cours de la Libération 33405 Talence Cedex Frankreich

Gerhard Frey
Fachbereich 9 Mathematik Univ. des Saarlandes Bau 27
6600 Saarbrücken
E.U. Gekeler

Math. Institut
Univ. Bonn
Beringstr. 4
5300 Bonn 1

Lothar Gerritzen
Math. Institut
Univ. Bochum Postfach 102148 4630 Bochum 1

David Goss
Dept. of Math.
Univ of California
Berkely, California 94720 USA
N. de Grande-de Kimpe

Vrije Univ Brussel
Faculteit Wetenschappen
Pleinlaan 2, F7
1050 Brussel
Belgien
L. van Hamme

Fac. of Appl. Sciences
Vrije Univ. Brussel
Pleinlaan 2, F7
1050 Brussel
Belgien

Frank Herrlich
Math. Institut
Univ. Bochum
Postfach 102148
4630 Bochum 1
P. Jarraud

5 Avenue de la Porte de Villiers 75017 Paris Frankreich
E. Kani

Math. Institut Univ. Heidelberg
Im Neuenheimer Feld 288
6900 Heidelberg
W. Lütkebohmert

Math. Institut
Univ. Münster
Einsteinstr. 62
4400 Münster
M. Mathieu

58 Quai Pompadour
94600 Choisy-Le-Roy
Frankreich
M. Matignon

Math. et Informatique
Univ. de Bordeaux 1
351 cours de la Libération
33405 Talence Cedex
Frankreich

E1hanan Motzkin
196 rue du chateau des rentiers
750 Paris
Frankreich
M. van der Put

Math. Instituut
Rijksuniveriseit Groningen Postbus 800
9700 AV Groningen
Niederlande

Liu Quing
Math. et Informatiques
Univ. de Bordeaux 1
351 cours de la Libération 33405 Talence Cedex Frankreich

Wolfgang Radtke
Fachbereich Mathematik
Fernuniversität Hagen
Postfach 940
5800 Hagen

Reinhold Remmert Math. Institut Univ. Münster Roxeler Str. 64 4400 Münster

Marc Reversat
Univ. Bordeaux et Ec. Polytechnique
12 rue du Cos des Obiers
33170 Gradignan
Frankreich

Philippe Robba
Dept. of Math.
I.U.T. Univ. Paris-Sud

Boite postale 25
91406 Orsay Cedex
Frankreich

Alain Robert
Institut de Math.
Univ. Neuchâtel
Chantemerle 20
Schweiz

Edmund Robinson
Dept of Computer Sciences The King's building
Mayfield Road
Endinburgh EH 93 IZ
Großbritanien

Marie-Claude Sarmant
16 Boulevard Jourdan
75014 Paris
Frankreich
W.H. Schikhof

Math. Institut
Katholike Universiteit
Toernooiveld
Nijmegen
Niederlande

Claus Schmidt
Math. Institut
Univ. Saarbrücken
Bau 27
6600 Saarbrücken
H.B. Sieburg Dept. of Math. Stanford University
Stanford
Ca1 94305
USA

Guido van Steen
Dept. of Math.
Rijksuniv. Centrum Antwerpen
171 Groenenborgerlaan
2020 Antwerpen
Belgien

Peter Ullrich
Math. Institut
Univ. Münster
Roxeler Str. 64
4400 Münster

