Math. Forschungsinstitut Oberwolfach
E 20 /01949

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 10/1985

Funktionalanalysis

17.2 bis 23.2.1985

Auch die diesjährige Tagung über Funktionalanalysis stand wieder unter der Leitung der Herren Professoren K.-D. Bierstedt (Paderborn), Heinz König (Saarbrücken) und H.H. Schaefer (Tübingen). 40 Mathematiker aus 13 Ländern nahmen an der Tagung teil und trugen in vielen Diskussionen zum Erfolg (und zur harmonischen Atmosphäre) der Konferenz bei.

Bei der Größe der Tagung konnten wieder mehrere Interessenten aus In- und Ausland nicht mehr eingeladen werden. Auch mußten leider einige der Eingeladenen wegen Krankheit kurzfristig absagen.

In 31 Vorträgen wurde über die verschiedensten Teilgebiete der Funktionalanalysis berichtet, z.B. Approximationstheorie, Fréchet-räume und lokalkonvexe Räume, Folgen- und Funktionenräume, Netzwerktheorie, Banachraumtheorie und Banachalgebren, schwache Kompaktheit, topologische Tensorprodukte, Distributionentheorie, partielle Differentialgleichungen, Operatorentheorie, Spektraltheorie und nichtarchimedische Funktionalanalysis. Außerdem fanden sich die Teilnehmer der Tagung zu vielen fruchtbaren Einzelgesprächen zusammen.

Gefördert durch DFG Deutsche Forschungsgemeinschaft

Vortragsauszüge

E. ALBRECHT:

Maximal abelian quasinilpotent subalgebras of B(H)

If M is a maximal commutative subset of the set Q(H) of all quasinilpotent operators in B(H) (H a Hilbert space) then M is a closed abelian subalgebra of B(H) contained in Q(H) and will be called a MAQS (maximal abelian quasinilpotent subalgebra) of B(H). We investigate the structure (up to similarity) of the MAQS in B(H). If $M \subset B(H)$ is a MAQS then we write $\ker M := \bigcap_{A\in M} \ker A$ and $MH := LH\{Ax: A\in M, x\in X\}$. If $1 \leq \dim H < \infty$ then it is well known that $\ker M \neq \{0\}$ and $\overline{MH} \neq H$. However for infinite dimensional H the cases $\overline{MH}=H$ and/or $\ker M = \{0\}$ are possible. This causes new phenomena. A typical result is

Theorem: Let $\dim H \geq 2$, M a MAQS in B(H).

(a) If M is nilpotent then $\ker M \subset \overline{MH}$ (Kravchuk).

(b) If $\dim H$ is infinite then either $\ker M \subset \overline{MH}$ or

(*) $\ker M \cap \overline{MH} = \{0\}$, $H = \ker M \oplus \overline{MH}$, and $\dim \ker M = 1$.

Examples show that (*) is possible.

The results have been obtained in joint work with H. ZASSENHAUS (Columbo, Ohio).

F. ALTOMARE:

Convergence of nets of linear operators (Korovkin type theorems) in the context of Banach algebras

I shall present a short survey and some new results about the problem to determine the Korovkin closure and the universal Korovkin closure of a Banach algebra with respect to nets of linear contractions (or other classes of linear operators). The particular cases concerning function algebras and C*-algebras will be considered and some open questions will be discussed.

E. BEHRENDS:

Approximation of bounded operators by compact operators

A closed subspace I of a Banach space X is called an M-ideal if the polar I^{π} of I in X' has an L-complement. A well-known example (due to Dixmier) is the case $I=K(H)$ (=the compact operators) in $X=L(H)$

Gefördert durch DFG Deutsche Forschungsgemeinschaft
©

(=the bounded operators) for Hilbert spaces H which has a number of interesting consequences for the problem to approximate bounded operators by compact operators. In the talk one systematically investigates the problem to find more general Banach spaces X and Y such that K(X,Y) is an M-ideal in L(X,Y). For example, it is shown that this is the case if X and Y are subspaces of l^p and l^q, respectively, with the compact bounded approximation property.

J. BONET:

On weighted inductive limits of spaces of continuous functions

The problem of projective description of weighted inductive limits of spaces of continuous functions was stated and considered by Bierstedt, Meise and Summers in the following way:
If V is a decreasing sequence of strictly positive weights on a completely regular Hausdorff space X and E is a locally convex space, determine when (a) $V_0C(X,E)=C\overline{V}_0(X,E)$ and (b) $VC(X,E)=C\overline{V}(X,E)$ hold algebraically and topologically.
The following results are presented:
1. If V is a regularly decreasing sequence of continuous weights on a locally convex space X and E is a complete (gDF)-space, then (a) and (b) hold algebraically and topologically.
2. A reflexive Fréchet space E is a quojection (in the sense of Bellenot and Dubinsky) if and only if $V_0C(X,E)$ is a topological subspace of $C\overline{V}_0(X,E)$ for every sequence V of continuous weights on any locally compact space X.
3. For every normal space X and every normed space E, $C\overline{V}(X,E)$ is a (DF)-space.
4. Let $\lambda(A)$ a Köthe echelon space of order 1. $\lambda(A)\hat{\otimes}_\pi F$ is distinguished for every distinguished Fréchet space F if and only if $\lambda(A)$ is Montel (This is part of a joint article with A. DEFANT).

H.G. DALES:

Embedding algebras in Banach algebras

Automatic continuity theory for Banach algebras considers the question whether or not each homomorphism $\theta: A \rightarrow B$ is necessary continuous. Counter-examples exist in a number of cases - in particular when $A=C(K)$ for K an infinite space. All such examples require the construction of embeddings from certain integral domains into Banach

Gefördert durch DFG Deutsche Forschungsgemeinschaft ©

algebras.
I present a general theorem of this type. We construct a certain algebra C, and we show that C is universal in the class of "algebraically closed, β_1-, η_1-valuation algebras". It is claimed firstly that it is considerably easier to prove that there are embeddings from C than to follow the original proofs, secondly, that it is sufficient to work with C, thirdly, that the subtle rôle of the Continuum Hypothesis in this area is clarified.
The construction of C was discussed in the lecture: it involves the theory of ordered sets and of generalized formal power series algebras.

A. DEFANT:

Tensor norms and a product of operator ideals

Let α and β be two tensornorms in the sense of Grothendieck and let $(\mathcal{A},A)$ and $(\mathcal{B},B)$ be the according maximal normed operator ideals in the sense of Pietsch. The aim of the talk is to give a reasonable description (in terms of a certain product of $\mathcal{A}$ and $\mathcal{B}$) of the operator ideal of all $T:E\to F$ such that

$$id \otimes T: G \otimes_\alpha E \to G \otimes_\beta F$$

is continuous for all Banach spaces G (resp. for a fixed Banach space G). Applications to absolutely (r,p,q)-summing and (r,p,q)-integral operators are given.

B. GRAMSCH:

Lifting of idempotent elements

Let B be a Banach algebra with a unit over $\mathbb{C}$ and I a closed two-sided ideal, $\upsilon: B\to B/I$, $P(B)=\{p=p^2 \in B\}$. Under which conditions exist $p \in P(B)$ for a given $p' \in P(B/I)$ with $\upsilon(p)=p'$? When is $\upsilon_0: P(B)\to P(B/I)$ onto? Interesting cases have been considered by Calkin 1941, Rickart 1960 (see the survey article of de la Harpe, Springer Lect. Notes 925(1979), L. Brown 1982, Choi 1983 and other authors).In this talk results are discussed for the lifting of continuous (or holomorphic) functions $f:\Omega\to P(B/I)$.
Let $M_p := \{q' \in P(B/I) : \exists\, (e' - (q' - p)^2)^{-1}\}$ (T. Kato, 1955).

Theorem 1: Assume Ω to be a holomorphy region in $\mathbb{C}^n$, $p' \in P(B/I)$, $p \in P(B)$, $\upsilon(p)=p'$ and $f:\Omega\to M_{p'}$ continuous (resp. holomorphic). Then

Gefördert durch
DFG Deutsche Forschungsgemeinschaft

there exists $\tilde{f}:\Omega\to P(B)$ continuous (or holomorphic) such that $\upsilon\circ\tilde{f}=f$. The indicated proof shows also that $M_{p'}$ is analytically contractible over $P(B/I)$.

Theorem 2: Let M' be a connected component of $P(B/I)$ and assume that $M:=\upsilon^{-1}(M')\cap P(B)\neq\emptyset$. Then $\upsilon_0:M\to M'$ is onto and defines a fibre bundle $(M,\upsilon_0,F_{p'},G',M')$.

Theorem 1 and Theorem 2 are refinements and generalizations of methods in the paper of B. Gramsch, Math.Ann.269,27-71(1984).

Theorem 3 : Let $h:\Omega\to P(B/I)$ be a holomorphic map, $\Omega\subset\mathbb{C}^n$ a holomorphy region, and assume that there exists a continuous map $f:\Omega\to P(B)$ such that $\upsilon\circ f=h$. Then there exists a holomorphic map $\tilde{h}:\Omega\to P(B)$ with $\upsilon\circ\tilde{h}=h$.

The proof depends on a Mittag-Leffler-Weierstraß-product method for projective limits of Banach analytic homogeneous spaces.

R. HOLLSTEIN :

Extension and lifting of bounded linear mappings in locally convex spaces

A locally convex space (l.c.s.) E is said to have the left-extension property (L-EP) resp. the left-weak extension property (L-WEP) if for each l.c.s. F and each l.c.s. G containing E as a topological subspace each bounded linear mapping $A\in LB(E,F)$ has an extension $\bar{A}\in LB(G,F)$ resp. $\bar{A}\in LB(G,F''_n)$ where F''_n is the bidual of F equipped with the natural topology. We say that a l.c.s. F has the R-EP resp. the R-WEP if for each l.c.s. E and $G\supset E$ each $A\in LB(E,F)$ has an extension $\bar{A}\in LB(G,F)$ resp. $\bar{A}\in LB(G,F''_n)$.

A locally convex quotient space G/H is said to have the BL-property if each bounded subset $B\subset G/H$ can be lifted to a bounded set in G. We say that E has the L-LP resp. L-WLP if for each quotient space G/H having the BL-property each $A\in LB(E,G/H)$ has a lifting $\hat{A}\in LB(E,G)$ resp. $\hat{A}\in LB(E,G''_n)$. A l.c.s. F is said to have the R-LP resp. R-WLP if for each l.c.s. E and each quotient space $G/H\cong F$ with the BL-property each $A\in LB(E,G/H)$ has a lifting $\hat{A}\in LB(E,G''_n)$.

Let Γ_p resp. F_p, $1\le p\le\infty$, be the operator ideal of all p-factorable resp. discretely p-factorable operators. For $\mathfrak{A}=\Gamma_p$ resp. F_p, the $\mathfrak{A}$- and w-$\mathfrak{A}$-spaces in the sense of A. Pietsch and H. Junek have the following extension and lifting properties, respectively.

Gefördert durch DFG Deutsche Forschungsgemeinschaft ©

Theorem: Let E be a l.c.s. The following assertions hold:

(1) E is an F_∞-space $\Leftrightarrow$ E has the L-EP

(1') E is an F_1-space $\Leftrightarrow$ E has the L-LP

(2) E is a co-F_∞-space $\Leftrightarrow$ E has the R-EP

(2') E is a co-F_1-space $\Rightarrow$ E has the R-LP

(3) E is a Γ_∞-space $\Leftrightarrow$ E has the L-WEP

(3') E is a Γ_1-space $\Leftrightarrow$ E has the L-WLP

(4) E is a co-Γ_∞-space $\Rightarrow$ E has the R-WEP

(4') E is a co-Γ_1-space $\Rightarrow$ E has the R-WLP

The converse in (2') holds if E is ultrabornological and the converse in (4) resp. (4') is true if E is normed or semi-reflexive.

Examples of F_1-, F_∞-, Γ_1- and Γ_∞-spaces were given. Furthermore, the above extension and lifting theorem was applied to compact linear mappings, to holomorphic mappings between locally convex spaces, and to vector-valued functions lying in function spaces $\mathcal{F}(\Omega,F)$ which are isomorphic to the Schwartz ε-product $\mathcal{F}(\Omega)\varepsilon F$.

H. JARCHOW:

On Hahn-Banach Extension for Operators in Certain Ideals

Let A be an operator ideal. Say that a Banach space X has the A-EP ("extension property") if, given any Banach spaces Y,Z such that $Y\subset Z$, every $S\in A(Y,X)$ admits an extension $\tilde{S}\in A(Z,X)$. The general problem is to characterize all B-spaces having A-EP.

It is well-known that in case $A=K$ (compact operators), one obtains exactly all L_∞-spaces with Schur property.

It is shown that for a quite number of closed ideals, only the finite-dimensional spaces appear as extension spaces. If one starts with any closed ideal A which is injective and surjective, then a whole class of closed ideals is constructed leading to the same extension spaces as A. This class is shown to be particularly big if $A=W$, but it doesn't comprise e.g. the ideal of Banach-Saks operators. The latter is seen by considering (the dual of) Tsirelson's space.

HERMANN KÖNIG:

Spaces with large projection constants

Banach spaces with large projection constants are related to the existence of sets of equiangular lines in $\mathbb{R}^n$ or $\mathbb{C}^n$. We show that

Gefördert durch DFG Deutsche Forschungsgemeinschaft ©

there are complex k-dimensional spaces with projection constant equal to $\sqrt{k}\,(1-1/k)+1/k$, for prime numbers k. This is (almost) worst possible. In the real case, slightly weaker estimates hold; the equiangular lines are constructed using finite projective geometry.

G. KÖTHE:

Convergence-free spaces

σ_α, σ^*_α, $\sigma_\alpha \oplus \sigma^*_\alpha$ are the usual forms of spaces of countable degree α, $1<\alpha<\Omega$. The composition of two of these spaces is again a space of countable degree, e.g. $\sigma_\alpha(\sigma_\beta)=\sigma_{\beta+\alpha}$ and so on. The problem to determine the completed tensor product of two spaces of countable degree is much more involved. It is possible to prove that $\sigma_\alpha \hat{\otimes} \sigma_\beta$ is again a space of countable degree $\sigma_{\nu_1(\alpha,\beta)}$, but the function $\nu_1(\alpha,\beta)$ cannot be explicitely determined. I gave a lot of examples where $\nu_1(\alpha,\beta)$ can be explicitely determined.

N. KUHN:

Countable convex combinations and applications

Let X be a set, $X^\infty := X^{\mathbb{N}}$ and $Q:=\{(t_l)_l:\ t_l \geq 0\ \forall l \in \mathbb{N},\ \sum_{l=1}^{\infty} t_l = 1\}$. A superconvex structure on X is defined to be a map

$$I : Q \times X^\infty \to X,\ (t,x) \to I(t,x) = \mathop{I}_{l=1}^{\infty} (t_l, x_l)$$

with the properties

i) $\mathop{I}_{l=1}^{\infty} (\delta^p_l, x_l) = x_p\ \forall p \in \mathbb{N}$ (δ^p_l = Kronecker symbol),

ii) $\mathop{I}_{p=1}^{\infty} (t_p, \mathop{I}_{l=1}^{\infty} (t^p_l, x_l)) = \mathop{I}_{l=1}^{\infty} (\sum_{p=1}^{\infty} t_p t^p_l, x_l)\ \forall t, t^p \in Q\ (p=1,2,\cdots)$.

Examples: 1) On a σ-convex subset X of a Hausd.TVS we have the superconvex structure $\mathop{I}_{l=1}^{\infty} (t_l, x_l) = \sum_{l=1}^{\infty} t_l x_l$.

2) Let S be a topological space and let $X:=\{f: S \to [-\infty, 0]:\ f \text{ is USC}\}$. Then $\mathop{I}_{l=1}^{\infty} (t_l, f_l) = \sum_{l=1}^{\infty} t_l f_l$ is a superconvex structure.

Let $P:=\{(t_l)_l \in Q:\ t_l = 0$ for almost all $l \in \mathbb{N}\}$. $A \subset X$ is called superconvex iff $I(t,x) \in A\ \forall t \in Q,\ x \in A^\infty$.

Gefördert durch DFG Deutsche Forschungsgemeinschaft ©

Theorem 1 Let A be a subset of X such that for every countable set $B \subset A$ there exists some $t \in Q \setminus P$ with $I(t,x) \in A \ \forall x \in B^{\infty}$. Then A is superconvex.

Cor. Let A be a bounded subset of a Hausd.TVS and suppose that for every convergent sequence $(a_l)_l$ in A with $\lim a_l \in A$ the series $\sum_{l=1}^{\infty} 2^{-l} a_l$ has a limit in A. Then A is σ-convex.

A function $f: X \to [-\infty, \infty[$ is called convex if $f(I(t,x)) \leq \sum_{l=1}^{\infty} t_l f(x_l)$ for all $t \in P$, $x \in X^{\infty}$. f is called superconvex if the same inequality holds for all $t \in Q$ and $x \in X^{\infty}$ with $\sup f(x_l) < \infty$.
For a set $A \subset X$ let sco A denote the smallest superconvex subset of X which contains A.

Theorem 2 Let $f: X \to \mathbb{R}$ be convex and bounded below. Let $a \in X^{\infty}$, $\varepsilon > 0$ and $t \in Q$ with $t_l > 0 \ \forall l \in \mathbb{N}$. Then there exists a sequence $x \in X^{\infty}$ such that for all $n \in \mathbb{N}$ we have $x_n \in \operatorname{sco}\{a_l : l \geq n\}$ and

$$f\Big(\mathop{I}_{l=1}^{n} (t_l/T_n, x_l)\Big) \leq f(I(t,x)) + \varepsilon \sum_{l=n+1}^{\infty} t_l ,$$

where $T_n := \sum_{l=1}^{n} t_l$.

Cor. Given superconvex functions $f_1 \leq f_2 \leq \cdots \leq f: X \to [-\infty, \infty[$ and a sequence $1 \leq t_n \uparrow \infty$ we have

$$\inf_{a \in X} \Big(f(a) + \sup_{n \in \mathbb{N}} t_n (f_n(a) - f(a)) \Big) \leq \lim_{n \to \infty} \inf_{x \in X} f_n(x) .$$

A another consequence of Theorem 2 is the following theorem which implies the famous James theorem about weakly compact subsets of Banach spaces.

Theorem 3 (Ródé). Let E be a normed space and $T \subset \operatorname{Ball} E'$ such that for every $x \in E$ there exists some $\varphi \in T$ with $\varphi(x) = \|x\|$. Then for a convex set $A \subset E$ the following are equivalent:

i) A is relatively weakly compact.

ii) For every sequence $(a_l)_l$ in A there exists some $a \in E$ such that

$$\liminf_{l \to \infty} \varphi(a_l) \leq \varphi(a) \leq \limsup_{l \to \infty} \varphi(a_l) \quad \forall \varphi \in T.$$

Gefördert durch DFG Deutsche Forschungsgemeinschaft ©

M. LANGENBRUCH:

Sequence space representations for weighted solution spaces of partial differential operators

Let P(D) be a hypoelliptic system of pdo with constant coefficients and let M be a weight function satisfying certain technical conditions. For $0<p<\infty$ let $\mathcal{W}_p:=\{f\in C^\infty(\mathbb{R}^N)^s : P(D)f=0, |f(\alpha)|\leq Ce^{M(r\zeta)}\ \forall r>p\}$ and $\mathcal{W}^p:=\{f\in C^\infty(\mathbb{R}^N)^s : P(D)f=0, |f(\alpha)|\leq Ce^{M(r\zeta)}\ \exists r<p\}$ with their natural projective (resp. inductive) topology. The conditions (DN) and (Ω) (by D. Vogt) are generalized to triples $(d,X,\tilde{X})$, where $X,\tilde{X}$ are (F)-spaces and $d\in L(X,X)$. This enables us to determine the diametral dimension $\Delta(\mathcal{W}_p)$ to obtain the following theorem: There is a sequence $\alpha=(\alpha_n)$ such that for any $0<p,r<\infty$: $\mathcal{W}_p=\Lambda_0(\alpha)$ and $\mathcal{W}^r=\Lambda_0(\alpha)'_b$. Especially, $(\mathcal{W}_p)'_b=\mathcal{W}^r$. The sequence α may be calculated for special systems. Sequence space representations for solution spaces defined by more general weight systems can be proved by constructing suitable projections and using a variant of Pelczynski's trick.

K.B. LAURSEN:

Automatic continuity in C*-algebras

We examine homomorphisms of a C*-algebra A into commutative Banach algebras. Thus we need the commutator ideal C_A:= ideal generated by all commutators ab-ba of A.

Fact: $\overline{C_A} = \cap\{M: M \text{ primitive ideal of codim } 1\}$

Cor.1: $C_{L(H)}=L(H)$, if $\dim H>1$.

Cor.2: If A is a separable AW*-algebra with type I_1 part A_1, then $A=A_1\oplus C_A$.

Question: If I is a closed ideal and C_A is closed, is C_I closed? If A is AW*, then the answer is yes (E. Albrecht, K.B. Laursen). If $Z_I I=I$ (Z_I is the center of I), then the answer is yes. This applies to any C*-subalgebra of L(H), the compact operators on H.

W. LUSKY:

On Banach spaces with the bounded approximation property

We study the interrelation between Banach spaces with the bounded approximation property (BAP), Banach spaces with finite dimensional decompositions (FDD) and those with bases.

Gefördert durch DFG Deutsche Forschungsgemeinschaft ©

Theorem A: If X is separable and has the BAP then $X\oplus C_\infty$, $X\oplus C_1$ have bases.

Here, $C_\infty=(\Sigma\oplus E_n)_{(0)}$, $C_1=(\Sigma\oplus E_n)_{(1)}$, where the E_n satisfy the following condition:

For each $\varepsilon>0$ and every finite dimensional Banach space F there is E_n such that $d(E_n,F)\leq 1+\varepsilon$ (Banach-Mazur-distance).

Theorem B: If X has a sequence of finite rank projections R_n, and $R_n\to id$ pointwise, then $X\oplus(\Sigma\oplus l_\infty^n)_{(1)}$ has an FDD.

Theorem C: For any separable Banach space X there is a subspace Y with basis such that $X/Y\sim c_0/C_\infty$. Moreover c_0/C_∞ has a basis.

R. MEISE:

On a problem of P. Lelong

The proof of the following results (joint work with S. DINEEN, R. MEISE and D. VOGT) were presented, which answer a question of P. Lelong.

Theorem A: For a (DFN)-space E the following are equivalent:

(i) E contains a bounded set which is not pluripolar.

(ii) E contains a bounded set which is not uniformly pluripolar.

(iii) E_b' has property (DN).

Theorem B: For an (FN)-space E the following are equivalent:

(i) E contains a bounded set which is not uniformly pluripolar.

(ii) E has property $(\tilde{\Omega})$.

If every pseudoconvex domain in E is a domain of existence then (i) and (ii) are equivalent to

(iii) E contains a bounded set which is not pluripolar.

Recall: Let F be a Frêchet space with a fundamental system $(\|\cdot\|_n)_n$ of semi-norms, and define $\|\cdot\|_n^*:F'\to[0,\infty]$ by $\|y\|_n^*:=\sup\{|y(x)|:\|x\|_n\leq 1\}$. Then F has property (DN) (resp. $\tilde{\Omega}$) introduced by D. Vogt, if the following holds:

(DN) $\exists$ cont. norm $\|\cdot\|$ on F: $\forall k\in\mathbb{N}\ \exists m\in\mathbb{N}, d>0, c>0$: $\|\cdot\|_n^{1+d}\leq c\|\cdot\|^d\ \|\cdot\|_m$

$(\tilde{\Omega})$ $\forall p\in\mathbb{N}\ \exists q\in\mathbb{N}, d>0\ \forall k\in\mathbb{N}\ \exists c>0$: $\|\cdot\|_q^{*\,1+d}\leq c\|\cdot\|_n^*\ \|\cdot\|_p^{*\,d}$

R. NAGEL:

Positive solutions of abstract Cauchy problems with delay

By means of the theory of positive semigroups and in particular by

Gefördert durch DFG Deutsche Forschungsgemeinschaft ©

a result of W. Kerscher and the author (Acta.Appl.Math.2, 297-309 (1984)) we show that the stability of the solutions of the transport equation with delayed neutrons

$$\frac{d}{dt}f(x,v,t) = -v\, \mathrm{grad}_x\, f(x,v,t) - a(x,v)\cdot f(x,v,t) + \int_{-1}^{0}\int_V k(x,v,v',\tau)\, f(x,v',t-\tau)\, dv'd\tau$$

is independent of the delay.

M. NEUMANN:

Flows in infinite networks

The lecture deals with generalizations of the theorems of Ford-Fulkerson and Gale to the case of arbitrary networks. In our general context, flows are certain biadditive set functions $\nu: \Sigma \times \Sigma \to X$, where Σ is some algebra of sets and X is a Dedekind complete ordered vector space. The present approach to flows in infinite networks is completely different from the various classical proofs in the finite case. Here, the emphasis lies on sublinear generators and the interpolation theorem due to Mazur-Orlicz; these techniques are close in spirit to those of Fuchssteiner and König-Neumann in somewhat related situations. In the last part of this talk, we indicate some applications concerning the existence of measures with given marginals and certain supply and demand problems from mathematical economics.

A. PELCZYNSKI:

Classification of Banach spaces of smooth functions and a Sobolev embedding type theorem

Let $L^1_{[k,n]}(T^2)$ denote the Sobolev space of functions which are 2π-periodic in each variable together with their k pure partial derivations in the first variable and n pure partial derivations in the second variable, and these partial derivations belong to $L^1(T^2)$. The following conjecture is discussed:

If $u \in L^1_{[k,n]}(T^2)$ then

$$\sum_{-\infty<p,q<\infty} |p|^{k-1}\, |q|^{n-1}\, |\hat{u}(p,q)|^2 < \infty .$$

Gefördert durch DFG Deutsche Forschungsgemeinschaft ©

<u>Theorem:</u> (K. Senator and A.P.)
The conjecture is valid in the following cases: 1) k,n odd numbers, 2) k=n, 3) min(k,n)=1.

Remark: If the conjecture is valid then there is a bounded non absolutely summing operator from $L^1_{[k,n]}$ into a Hilbert space.

H.J. PETZSCHE:

<u>On Spectral Synthesis</u>

The first part of the talk dealt with the solutions of $P(-D)f=0$. Hereby P is a polynomial in N variables and f a C^∞ or ultradifferentiable function or a distribution or an ultra-distribution on an open convex set U. The aim was to find a space of distributions $\mathcal{G}_P(U)$ which are defined on $\mathbb{C}^n=\mathbb{R}^{2n}$ and have support in $Z(P)=\{z\in\mathbb{C}^n: P(z)=0\}$ such that the map $\mathcal{G}_P(U)\to\mathcal{N}_P(U)=\{f: P(-D)f=0\}$, $u\to\langle u(z),e^{ixz}\rangle$ is defined and surjective.
To that end a space of finitely differentiable functions F, a closed subspace F_0 of F, and an embedding $k: \hat{\mathcal{E}}'(U)/P\hat{\mathcal{E}}'(U)\to F/F_0$ were constructed. Hereby $\hat{\mathcal{E}}'(U)$ contains exactly the Fouriertransforms of distributions with compact support in U and P (U) denotes the closed ideal generated by P. $\hat{\mathcal{E}}'(U)$ is the space to be used in the C^∞-case and it must be replaced by suitable spaces in all other cases. Because k has closed image its adjoint $k^t:(F/F_0)'\to(\hat{\mathcal{E}}'(U)/P\hat{\mathcal{E}}'(U))'$ is surjective. Using the isomorphisms

$$(\hat{\mathcal{E}}'(U)/P\hat{\mathcal{E}}'(U))'\cong\mathcal{N}''_P(U)=\mathcal{N}_P(U) \quad\text{and}\quad (F/F_0)'\cong F_0^\perp\subset F'$$

we get a surjective map $\mathfrak{F}: F_0^\perp\to\mathcal{N}_P(U)$. $F_0^\perp$ can be identified with a space of distributions $\mathcal{G}_P(U)$ as required above and $\mathfrak{F}$ is the map wanted. In the second part we discussed extensions of this approach to convolution equations and asked some open problems.

F. RÄBIGER:

<u>On some structure theoretical characterizations of Grothendieck spaces</u>

Let E be a Banach space. E is called a <u>Grothendieck space</u> if in the dual E' weak* convergent sequences are weakly convergent. And E has <u>property (V)</u> -resp. <u>property (V_0)</u>- if every non-weakly compact operator from E into a Banach space F -resp. into c_0- fixes a copy of c_0.

Gefördert durch DFG Deutsche Forschungsgemeinschaft ©

Theorem 1 Let E be a Banach space. TFAE:

(i) E is a Grothendieck space.

(ii) E' is weakly sequentially complete and no quotient of E is isomorphic to c_0.

(iii) E has property (V_0) and no complemented subspace of E is isomorphic to c_0.

Theorem 2 Let E be a complemented subspace of a Banach lattice. TFAE:

(i) E is a Grothendieck space.

(ii) No quotient of E is isomorphic to c_0.

(iii) E has property (V) and no complemented subspace of E is isomorphic to c_0.

W. RUESS:

Asymptotic behaviour of motions of dynamical systems

The study of the asymptotic behaviour of solutions to the abstract Cauchy problem $\dot{x}(t)=Ax(t)$, $x(0)=x_0 \in D(A)$ X, X Banach, leads to the following general problems:

Problem 1: Let $f \in C_b(\mathbb{R}^+,X)$, and $H^+(f)=\{f_w : w \in \mathbb{R}^+\}$, where $f_w(t)=f(t+w)$. Characterize those f, for which

a) $H^+(f)$ is relatively compact in $(C_b(\mathbb{R}^+,X), \|\cdot\|_\infty)$;

b) $H^+(f)$ is weakly relatively compact in $(C_b(\mathbb{R}^+,X), \|\cdot\|_\infty)$.

Problem 2: Given $f \in C_b(\mathbb{R}^+,X)$, let $F(t)=\int_0^t f(u)du$. Under which conditions on f,X and F is $H^+(F)$ (weakly) relatively compact in $C_b(\mathbb{R}^+,X)$?

The following are two of the main results to be presented. Let $f \in C_b(\mathbb{R}^+,X)$.

Definition: F is symptotically almost periodic: for every $\varepsilon>0$, there exist $M=M(\varepsilon)>0$ and P_ε relatively dense in $[M,\infty)$ such that $\|f(t+\tau)-f(t)\|<\varepsilon$ for all $t \geq M$ and all $\tau \in P_\varepsilon$.

Theorem 1: The following are equivalent:

a) f is asymptotically almost periodic.

b) $H^+(f)$ is relatively compact in $(C_b(\mathbb{R}^+,X), \|\cdot\|_\infty)$.

c) There exists a unique decomposition $f=g|_{\mathbb{R}^+}+\varphi$ with $g:\mathbb{R}\to X$ almost periodic and $\varphi \in C_0(\mathbb{R}^+,X)$.

Theorem 2: Let $f=g|_{\mathbb{R}^+}+\varphi$ be asymptotically almost periodic and $F(t)=\int_0^t f(u)du$. If a) $F(\mathbb{R}^+)$ is bounded in X and $c_0 \not\subset X$, or

b) $F(\mathbb{R}^+)$ is weakly relatively compact in X,

then the following are equivalent:

Gefördert durch DFG Deutsche Forschungsgemeinschaft ©

(1) $H^+(F)$ is relatively compact in $(C_b(\mathbb{R}^+,X), \|\cdot\|_\infty)$.
(2) $H^+(F)$ is weakly relatively compact in $(C_b(\mathbb{R}^+,X), \|\cdot\|_\infty)$.
(3) φ is improperly Riemann-integrable on $\mathbb{R}^+$.

References: R. Ruess, W.H. Summers: 1) Compactness in spaces of vector valued continous functions and asymptotic almost periodicity. 2) Integration of asymptotically almost periodic functions and weak almost periodicity.

J. SCHMETS:

Spaces of continuous functions vanishing on a fixed subset

Let F be a closed subset of a Hausdorff completely regular space X and E be a Hausdorff locally convex space. Then C(X,F;E) denotes the space of the continuous functions on X with values in E which vanish identically on F. As soon as P is a family of bounded subsets of νX which union contains F and is dense in νX, C(X,F;E) is a closed subspace of $C_P(X;E)$, i.e. the space of the continuous functions on X with values in E, endowed with the topology of uniform convergence on the elements of P. However in general C(X,F;E) is not complemented in $C_P(X;E)$. We present a way to consider $C_P X,F;E)$ as a copy of a complemented subspace of $C_{P'}(Y;E)$ where Y is a Hausdorff completely regular space obtained by use of X and F and where P' is a suitable family of bounding subsets of νY. We use this construction to get the characterization of the ultrabornological, barrelled, quasi-barrelled, ... $C_P(X,F;E)$ spaces as well as the associated spaces.

C. STEGALL:

Some results about weak compactness

The following was discussed:

Theorem: Let X be a Banach space. Denote by $B_1(X) \subset X^{**}$ the space of sequential pointwise limits of X. For $E \subset X$, E bounded, are equivalent:
(i) Every sequence in E has a pointwise (or X*) Cauchy subsequence.
(ii) For each sequence $(x_n)_n$ in E there exists $x^{**} \in B_1(X)$ such that $\liminf x^*(x_n) \leq x^{**}(x_n) \leq \limsup x^*(x_n)$ $\forall x^* \in \text{ext Ball } X^*$.
If the x^{**} may be chosen in X, then E is relatively weakly compact.

Gefördert durch DFG Deutsche Forschungsgemeinschaft ©

T. TERZIOGLU:

Unbounded operators between F-spaces

A linear operator is called bounded if it maps a neighbourhood onto a bounded subset. Let E and F be Fréchet spaces and suppose F admits a continuous norm and has a basis. If a continuous linear operator $T:E \to F$ is unbounded, then there is a infinite-dimensional, closed, nuclear subspace E_0 of E such that the restriction of T to E_0 is an isomorphism onto $T(E_0)$.

T.V. TONEV:

Embedding of big discs in the maximal ideal space

Let $S=\{f^p\}_p$ be a multiplicative subsemigroup in a commutative Banach algebra A, algebraically isomorphic to an additive subsemigroup of $Q_+=\mathrm{Rat}[0,\infty)$, and let G be the dual group of the group generated by the set $S\cup(-S)$. A function on the big disc $\overline{\Delta}_G=\{[0,1)\times G/\{0\}\times G\}^-$ we call generalized-analytic if it can be approximated on $\overline{\Delta}_G$ by linear combinations over $\mathbb{C}$ of functions $\chi^p(\lambda,g)=\lambda^p\chi^p(g)$, $\chi^p(g)=g(p)$, $p\in S$. The algebra A_G of gen.- analytic functions on $\overline{\Delta}_G$ is an interesting object in commutative Banach algebra theory. For instance it is the minimal uniform algebra extension of disc algebra, in which all functions $z\to z^n$ have arbitrary powers from S; A_G does not admit corona, just as in $\mathbb{C}^1$.

Theorem: Let A and $S\cong Q_+$ are as above, A a uniform algebra. If $|f^p|_{\delta A}\equiv\mathrm{const}\neq 0$ and if for some $\varphi\in \mathrm{sp}\,A$ we have $\mathrm{Ker}\,\varphi=\{\bigcup_{p\in S} f^pA\}^-$, then the set $\mathrm{sp}\,A\setminus\delta A$ is homeomorphic to a big disc and the functions of A are gen.-analytic there, i.e. $A|_{\mathrm{sp}\,A\setminus\delta A}\subset A(\Delta_G)$.

R. URBANSKI:

On modular and Orlics spaces over a field with valuation

Let X be a vector space over a field K with valuation $|\cdot|$. Let $s>0$, $s\geq t\geq 0$, $k,l>0$. For $i=0,1$ and $a,b\geq 0$ we define $a \textcircled{i} b=(1-i)\max(a,b)+i(a+b)$. A functional $\rho:X\to[0,\infty]$ is called $(s,it)_i$-modular if it satisfies:

(m1) $\rho(0)=0$, and $\rho(\alpha x)=0$ for all $\alpha\neq 0$ implies $x=0$.

(m2) $\rho(\alpha x+\beta y)\geq l^i|\alpha|^{it}\rho(x) \textcircled{i} l^i|\beta|^{it}\rho(y)$ for $k|\alpha|^s+k|\beta|^s\leq 1$.

(m3) $\rho(x)=\rho(-x)$

The vector space $X_\rho=\{x\in X:\ \lim\rho(\alpha x)=0 \text{ as } \alpha\to 0\}$ is called the modular space generated by ρ. If ρ is $(s,it)_i$-modular, then the functional

Gefördert durch DFG Deutsche Forschungsgemeinschaft ©

$\|x\|_{s,t}=\inf\{|\alpha|^s>0:\ \rho(x/\alpha)\le|\alpha|^{i(s-t)}\}$ is an F-quasinorm on X_ρ with the constant $c=\inf\{|\alpha|>1:\ \alpha\in K\}\cdot\max(k,1^i)$ in the triangle inequality. The $(s,t)_1$-modular is called (s,t)-modular. If s=1 the $(1,0)_0$-modular is called k-quasiconvex. The spaces L^p $(0<p<\infty)$ are examples of modular spaces generated by k-quasiconvex modulars. If the valuation is non-archimedean, μ a finite non-atomic measure and Φ satisfies the condition Δ_2, then there exists a non-trivial continuous linear functional on the Orlics space L^Φ.

M. VALDIVIA:

On (LF)-spaces

Let E_n be a Fréchet-Montel space, dim E_n = (n=1,2,...). $E:=\bigoplus_{n=1}^{\infty} E_n$.
The following conditions are equivalent:

1) Every separated quotient of $E'[\mu(E',E)]$ is complete.
2) Every separated quotient of E is complete.
3) E is B_r-complete.
4) $E'[\mu(E',E)]$ is B_r-complete.
5) E_n is isomorphic to ω (n=1,2,...).

Let $M_0,M_1,\cdots,M_n$ be positive numbers such that:
a) $M_0=1$ b) $M_n^2\ M_{n-1}M_{n+1}$ (n=1,2,...) c) $\sum_{n=1}^{\infty} M_{n-1}/M_N<\infty$
Let Ω be a non-empty open subset of $\mathbb{R}^m$.
Then we have:

1) $\mathcal{D}^{(M_n)}(\Omega)$ is not B_r-complete.
2) $\mathcal{D}^{(M_n)'}(\Omega)$ is not B_r-complete.
3) $\mathcal{D}^{(M_n)'}(\Omega)$ has a separated quotient which has a closed subspace isomorphic to a dense proper subspace of ω.
4) $\mathcal{D}^{(M_n)}(\Omega)$ has a quotient isomorphic to a dense proper subspace of ω.

D. VOGT:

Solution operators for convolution equations

The existence of right inverses for convolution operators $T_\mu=\mu*\cdot$, $\mu\in\mathcal{E}'(\mathbb{R}^n)$, acting on $C^\infty(\mathbb{R}^n)$ was investigated. It was shown: If $n\ge2$ and T_μ is hypoelliptic and has nontrivial zero solutions, then it does not have a right inverse. This holds in particular for hypoelliptic partial differential operators. It extends results of A. Grothendieck, D.K. Cohoon. In the case of semielliptic PDE the ex-

Gefördert durch DFG Deutsche Forschungsgemeinschaft ©

istence of right inverses in Gevrey classes was investigated (c.f. results of D.K. Cohoon), an ultradistributional elementary solution for the one-dimensional heat equation (n=2) with support on a half-ray was presented.

In a second part, which is based on joint work with R. MEISE, the case n=1 was treated and completely solved. There exist right inverses iff T_μ is hyperbolic in the sense of Ehrenpreis. It is the case iff the expansions of the local zero solutions into series of exponential polynomials always converge globally. This corresponds to the Ehrenpreis characterization of hyperbolicity by means of the zeros of the Fouriertransform.

J. VOIGT:

Interpolation for (positive) C_0-semigroups on L_p-spaces

For the generator T of a C_0-semigroup $(U(f);t\geq 0)$ on a Banach space we define the spectral bound $s(T):=\sup\{\mathrm{Re}\, z : z\in\sigma(T)\}$ and the uniform spectral bound $s_u(T):=\inf\{\alpha>s(T): \sup\{\|(z-T)^{-1}\| : \mathrm{Re}\, z\geq\alpha\}<\infty\}$.
If the semigroup consists of positive operators on a Banach lattice then $s_u(T)=s(T)$ holds.

It is shown that the uniform spectral bound, considered as a function of p, has a convexity property if $(U(t);t\geq 0)$ acts as a C_0-semigroup on different L_p-spaces. An application of this result to linear transport theory is sketched.

P. WOJTASZCZYK:

On homogenous polynomials

Let S_d be a unit sphere in C^d - the d-dimensional complex space - and let σ be a normalized rotation invariant measure on S_d. P_N denotes the space of all N-homogenous polynomials on C^d restricted to S_d. The following theorem was proved:

For every c>0 there is a constant C=C(c,d) such that for every subspace $E\subset P_N$ with $\dim E\geq \dim P_N$ there exists $\varphi\in E$ such that $\sup\{|\varphi(z)|: z\in S_d\}=1$ and $\left(\int_{S_d} |\varphi(z)|^2 d\sigma(z)\right)^{1/2}\geq C$.

This improves slightly an earlier result of Kashin and Ryll.

Berichterstatter: N.Kuhn

Gefördert durch DFG Deutsche Forschungsgemeinschaft ©

Liste der Tagungsteilnehmer

E. Albrecht

Fachbereich Mathematik
Universität des Saarlandes

D-6600 Saarbrücken, BRD

F. Altomare

Istituto di Analisi Matematici
Universitá di Bari

I-70121 Bari, Italien

E. Behrends

Institut für Mathematik I
Freie Universität Berlin
Arnimallee 3

1000 Berlin 33, BRD

K.-D. Bierstedt

Universität-GH-Paderborn
Fachbereich 17
Warburger Str. 100

4790 Paderborn, BRD

J. Bonet

c/ Angel Guimerá 47, 12 a/

Valencia (8), Spanien

R. B. Burckel

Dept. of Mathematics
Kansas State University

Manhattan, Kansas 66506, USA

P. P. Carreras

Departamento de Matemâticas
-Escuela Superior de
Ingenieros Industriales
Camino de Vera s/n

Valencia (10), Spanien

H. G. Dales

School of Mathematics
University of Leeds

Leeds LS 2 9JT, England

A. Defant

FB 6 der Universität
Postfach 25 03

2900 Oldenburg, BRD

B. Fuchssteiner

Universität-GH-Paderborn
Fachbereich 17
Warburger Str. 100

4790 Paderborn, BRD

B. Gramsch

Fachbereich Mathematik
Universität Mainz
Saarstr. 21

D-6500 Mainz, BRD

R. Hollstein

Universität-GH-Paderborn
Fachbereich 17
Warburger Str. 100

4790 Paderborn, BRD

H. Jarchow

Institut für Angewandte
Mathematik
Universität Zürich
Rämistr. 74

CH-8001 Zürich, Schweiz

K. John

Mathematical Institute
Czechoslovak Academy of Sciences
Zitna 25

CS-11567 Praha 1, CSSR

Gefördert durch
DFG Deutsche Forschungsgemeinschaft

©

Heinz König
Fachbereich Mathematik
Universität des Saarlandes
D-6600 Saarbrücken, BRD

Hermann König
Math. Seminar
Universität Kiel
Olshausenstr. 40-60
2300 Kiel, BRD

G. Köthe
Universität Frankfurt
Fachbereich Mathematik
Robert-Meyer-Str. 10
6000 Frankfurt, BRD

N. Kuhn
Fachbereich Mathematik
Universität des Saarlandes
D-6600 Saarbrücken, BRD

M. Langenbruch
Universität Münster
Mathematisches Institut
Einstein-Str. 64
4400 Münster, BRD

K. B. Laursen
Mathematics Institute
University of Copenhagen
DK-2100 Copenhagen, Denmark

W. Lusky
Universität-GH-Paderborn
Fachbereich 17
Warburger Str. 100
4790 Paderborn, BRD

R. Meise
Math. Institut
Universität Düsseldorf
Universitätsstr. 1
D-4000 Düsseldorf 1, BRD

R. Nagel
Universität Tübingen
Mathematische Fakultät
Auf der Morgenstelle 10
7400 Tübingen, BRD

M. Neumann
Gesamthochschule Essen
Fachbereich 6 Mathematik
Universitätsstr. 2
4300 Essen, BRD

A. Pelczynski
Mathematical Institute
Polish Academy of Sciences
PL 00-950 Warszawa
Sniadeckich 8, Poland

H. J. Petzsche
University of Maryland
Dep. of Math., College Park
MD 20742, USA

F. Räbiger
Universität Tübingen
Mathematische Fakultät
Auf der Morgenstelle 10
7400 Tübingen, BRD

W. Ruess
Gesamthochschule Essen
Fachbereich 6 Mathematik
Universitätsstr. 2
4300 Essen, BRD

Gefördert durch
DFG Deutsche Forschungsgemeinschaft
©

H. H. Schaefer

Math.Inst.
Universität Tübingen
Auf der Morgenstelle 10

D-7400 Tübingen 1, BRD

J. Schmets

Institut de Mathématique
Avenue des Tilleuls, 15

B-4000 Liège, Belgien

Ch. Stegall

Math. Institut
Universität

A-4040 Linz, Österreich

T. Terzioglu

Matematik Bölümii Orta Doğu
Tekuik Universitesi

Ankara, Türkei

H. G. Tillmann

Mathematisches Institut
Einsteinstr. 64

D-4400 Münster, BRD

T. V. Tonev

Mathematical Institute
Bulgarian Academy of Sciences
P.O. Box 376

1090 Sophia, Bulgarien

R. Urbanski

Universität München
Mathematisches Institut

D-8000 München 2, BRD

M. Valdivia

Departamento de Matematicas
Facultad de Matematicas

Burjasot (Valencia), Spanien

D. Vogt

Universität-GH-Wuppertal
Fachbereich 7
Gaußstr. 20

D-5600 Wuppertal 1, BRD

J. Voigt

Universität München
Mathematisches Institut
Theresienstr. 39

D-8000 München 2, BRD

G. Wittstock

Fachbereich Mathematik
Universität des Saarlandes

D-6600 Saarbrücken, BRD

P. Wojtaszczyk

Institute of Mathematics
Polish Academy of Sciences
00950 Warsaw
Sniadeckich 8, Poland

Gefördert durch
DFG Deutsche Forschungsgemeinschaft
©