MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 16/1985

Dependence in Probability and Statistics

31.3. bis 6.4.1985

Die Tagung fand unter der Leitung von E. Eberlein (Freiburg), M. Rosenblatt (La Jolla, USA) und M. Taqqu (Ithaca, USA) statt. Im Mittelpunkt des Interesses standen stochastische Prozesse und Folgen von Zufallsvariablen, die eine gewisse Abhängigkeitsstruktur besitzen. Insbesondere wurde die entsprechende Grenzwerttheorie diskutiert.

Bei der Formulierung der klassischen Grenzwertaussagen in Wahrscheinlichkeitstheorie und Statistik geht man in der Regel davon aus, daß die zugrundeliegenden Variablen bzw. Beobachtungen unabhängig voneinander sind. In realistischen Modellen ist diese Annahme jedoch vielfach nicht zu rechtfertigen. Insbesondere ist etwa in Interaktionsprozessen die Abhängigkeit zwischen den Variablen gerade der wesentliche Aspekt. In jüngerer Zeit wurden große Fortschritte im Verständnis diverser Abhängigkeitsstrukturen und des entsprechenden Grenzverhaltens erzielt. Eine Tagung über dieses Thema stieß konsequenterweise auf starkes internationales Interesse. Nicht alle Interessenten konnten eingeladen werden. Die 45 Teilnehmer aus 13 Ländern bildeten einen sehr homogenen und fruchtbaren Diskussionskreis. In insgesamt 36 Vorträgen wurden die Zuhörer über eine Fülle neuer Ergebnisse unterrichtet.

Vortragsauszüge

J. BERAN:

A modified t-test for the location parameter in the presence of long-range serial correlations

Long-range correlations have disastrous effects on confidence intervals for the arithmetic mean. Although some investigations have been done how to tackle this problem (Mandelbrot 69, Mohr 81) no satisfactory method for reasonably short series and unknown self-similarity parameter H has been proposed yet. Here a test is proposed (for the case of fractional Gaussian noise) which also works well for short series. H is estimated by Graf's method (Graf 83) using the periodogram or equivalently by approximate maximum likelihood (Fox & Taqqu 84, Beran 84). Mandelbrot's t-statistic is used replacing H by its estimate $\hat{H}$. An approximation to its distribution using asymptotical results together with a finite sample correction turns out to be surprisingly good. A generalized version of the test for more general Gaussian processes with long-range dependence is also given together with an approximation to the distribution of the test statistic.

N. H. BINGHAM:

On the laws of large numbers of Lai and Chow

The law of large numbers of Lai extends the Kolmogorov strong law by linking existence of means with Abel, as well as Cesàro, convergence. The law of large numbers of Chow links existence of variances with Borel and Euler convergence. One may link the two by showing the equivalence of t^{th}

moments (for any $t \geqq 1$) with certain Riesz and Valiron means. This result (Bingham & G. Tenenbaum, to appear) is closely linked to rate-of-convergence complements to the strong law by Baum and Katz and others. The Baum-Katz result has recently been extended from the i.i.d. case to the ϕ-mixing case by Peligrad (to appear). Similar extensions of the other results above to the ϕ-mixing case are given. Connections with related results are discussed.

R. C. BRADLEY:

On the central limit question under strong mixing conditions

I. A. Ibragimov (1975) proved two central limit theorems for strictly stationary random sequences satisfying the ρ-mixing condition. In one of these theorems, finite $(2+\delta)$-th moments were assumed for some $\delta > 0$, and in the other, only finite second moments along with a mixing rate. The main result here is that both of these theorems are essentially sharp.

W. BRYC:

Some remarks on mixing measures of dependence

A large class of measures of dependence is shown to reduce to well known ones, i. e. strong mixing of Rosenblatt, absolute regularity, ϕ-mixing of Ibragimov and ρ-mixing. The talk summarizes papers by R. Bradley, W. Bryc and R. Bradley, W. Bryc, S. Janson and many others.

R. BURTON:

Infinitely divisible random measures, association and a central limit theorem

This is joint work with E. Waymire. Let M be the Polish space of locally finite measures on a locally compact Polish space S and X be a random measure, $X: \Omega \to M$ measurable, Ω probability space. M is ordered: $\mu \geq \nu$ if $\mu - \nu \in M$. X is *associated* if $F, G: M \to \mathbb{R}$ increasing, measurable implies $\mathrm{Cov}(F(X), G(X)) \geq 0$ when defined. It is shown that infinitely divisible X are associated. If $S = \mathbb{R}^d$ and X is translation invariant (stationary) then we say X satisfies the Central Limit Theorem if $\lim_{\lambda \to \infty} X_\lambda$ = Gaussian white noise (as generalized random fields) where $X_\lambda(B) = (X(\lambda B) - E(X(\lambda B)))\, \lambda^{-d/2}$ is the renormalized random field. The property of association is used to characterize those stationary infinitely divisible random measures that satisfy the central limit theorem via some ideas of C. M. Newman.

A. R. DABROWSKI:

Functional laws of the iterated logarithm for multiplicative and associated sequences

Berkes (73, ZW) proved a functional law of the iterated logarithm for general dependent sequences. Such a result is particularly useful for sequences whose dependence structure is neither ϕ-mixing, nor one of the many similar conditions. Berkes (73, Stud. Sci. Math. Hung.) applied his theorem to multiplicative sequences. Recently associated sequences were shown to satisfy the functional central limit theorem (Newman and Wright (AP 1982)). By using the techniques of these authors, and the methods of Berkes, we prove a functional law of the iterated logarithm for associated sequences satisfying a rate requirement.

R. A. DAVIS:

<u>On limit distributions for the sample correlation function of moving averages</u>

Let $X_t = \Sigma_{j=-\infty}^{\infty} c_j Z_{t-j}$ be a moving average process where $\{Z_t\}$ is i.i.d. with common distribution in the domain of attraction of a stable law with index α, $0 < \alpha \leq 2$. If $0 < \alpha < 2$, $E|Z_1|^{\alpha} < \infty$, and the distributions of $|Z_1|$ and $|Z_1 Z_2|$ are tail equivalent, then the sample correlation function of $\{X_t\}$ suitably normalized converges in distribution to the ratio of two dependent stable random variables. On the other hand if $E|Z_1|^{\alpha} = \infty$, the limit distribution is the ratio of two independent stable variables. The derivations of these results rely heavily on point process techniques. In the $\alpha = 2$ case the sample correlations are shown to be asymptotically normal which extends the classical result.

H. DEHLING:

<u>The functional law of the iterated logarithm and the almost sure invariance principle for degenerate U-statistics</u>

First we state a bounded LIL for Hilbert space valued martingales under a Feller-type condition. This is applied to prove the following theorem about degenerate U-statistics.

<u>Theorem:</u> Let $X_1, X_2, \ldots$ be i.i.d., uniformly distributed on $[0,1]$, let $h: [0,1]^m \to \mathbb{R}$ be symmetric, degenerate and satisfy $E|h(X_1,\ldots,X_m)|^{2+\delta} < \infty$, $\delta > 0$. Put

$U_n(h) = m! \, \Sigma_{1 \leq i_1 < \ldots < i_m \leq n} h(X_{i_1},\ldots,X_{i_m})$. Then

(i) $\{(n \log\log n)^{-m/2} U_{[nt]}(h), \ 0 \leq t \leq 1\}, \ n \geq 1$ is almost surely relatively compact and has the following set of limit points: $K_h = \{x \in C[0,1]: x(t) =$

$$\int_0^1 \ldots \int_0^1 h(x_1,\ldots x_m) \int_0^t f(x_1,s)ds \ldots \int_0^t f(x_m,s)ds \, dx_1 \ldots dx_m, \text{ where}$$

$$\int_0^1 \int_0^1 f^2(x,s)dxds \leq 1\}.$$

(ii) There exists a Kiefer process $\{K(s,t): 0 \leq s \leq 1, \ t \geq 0\}$ such that

$$U_n(h) - \int_0^1 \ldots \int_0^1 h(x_1,\ldots,x_m) \, K(dx_1,n)\ldots K(dx_m,n) = o((n\log\log n)^{m/2}).$$

As a corollary we obtain the functional LIL for multiple stochastic integrals with respect to the Kiefer process.

This is partially joint work with M. Denker and W. Philipp.

M. DENKER:

<u>Limit theorems for some statistics</u>

This is joint work with G. Keller. In the beginning some motivations for studying certain statistics under weak dependence are given: First we discuss the symmetry problem for certain analytic perturbations of the map $x \to 1 - |2x - 1|$. Secondly a way is discussed how to estimate the fractal dimension for the map

$(x,y) \to (2x \bmod 1, \lambda y + \cos 4\pi x)$. Studying these questions one needs to prove limit theorems for Hölder continuous functionals (or functionals of bounded oscillation) of functions of absolutely regular processes when applied to U-statistics or rank statistics.

E. EBERLEIN:

Strong invariance principles for martingale generalizations

Strong invariance principles with order of approximation $O(t^{1/2 - \kappa})$ are obtained for sequences of dependent random variables. The basic dependence assumptions include various generalizations of martingales such as amarts, semiamarts and mixingales as well as processes characterized by a condition on the Doléans measure. Provided the partial sum process is uniformly integrable also martingales in the limit and games fairer with time are included. Sufficient conditions for linear growth of the covariance function of the partial sums are given.

R. FOX:

Parameter estimates for strongly dependent random variables

An approximative ML-Estimator as introduced by Whittle is investigated for strongly dependent stationary Gaussian sequences. As in the weakly dependent case (Walker (1963), Hannan (1973)) asymptotic normality is obtained.

J. FRITZ:

Hydrodynamical rescaling of a stochastic model: The Euler equation

We consider the following system of stochastic differential equations when $\varepsilon > 0$ goes to 0.

$$d\delta_k = \varepsilon^{-2}(U'(\delta_{k+1}) + U'(\delta_{k-1}) - 2U'(\delta_k))dt + \frac{\sqrt{2}}{\varepsilon}(dW_{k+1} - dW_k),$$

where U is convex, $\delta_k \in \mathbb{R}$, $k \in \mathbb{Z}$.

Given a smooth $\lambda:\mathbb{R} \to \mathbb{R}$ we define the initial distribution μ_λ as

$$d\mu_\lambda = \Pi\, Z_k^{-1} \exp(-U(\delta_k) + \lambda(\varepsilon k)\delta_k)\, d\delta_k,$$

and denote by $\lambda_\varepsilon(t,x)$ the expectation of $U'(\delta_{[x/\varepsilon]}(t))$. Then $\lambda_\varepsilon(t,x) \to \lambda(t,x)$, where $\lambda(0,x) = \lambda(x)$ and

$$\frac{\partial\lambda}{\partial t} = \rho(\lambda)\frac{\partial^2\lambda}{\partial x^2},$$

where $\rho(\lambda) = Z(\lambda)^{-1}\int U''(\delta)\exp(-U(\delta) + \lambda\delta)\, d\delta$.

P. GÄNSSLER:

Central limit theory for martingales. Part I

(continued with part II by E. Häusler)

Part I is expository in nature emphasizing on a direct and — as we think — rather effective way of proving central limit theorems (CLT's) for martingales starting with a martingale version of Lindeberg's proof of the classical CLT and going on up to functional CLT's for time-continuous local martingales known through the work of Rebolledo, Liptser and Shiryayev, and Helland.

Ch. M. GOLDIE:

Mixing in $\mathbb{Z}^d$ and set-indexed limit theory

This talk reports on joint work with P. E. Greenwood, on central limit theory for set-indexed processes under mixing conditions. In $\mathbb{R}^d$ we consider a regular lattice of cubes of side $1/n$, each having a random mass, which is uniformly spread through it. The random masses satisfy mixing conditions relative to the distances of the cubes from each other. Let $Z_n(A)$ denote the resultant mass of an arbitrary measurable set A in $\mathbb{R}^d$. Then Z_n is a set-indexed partial-sum process. The problems we address are I: convergence of finite-dimensional laws to Brownian motion, and characterisation of the latter; II: uniform bounds on $\operatorname{var} Z_n(A) / \operatorname{measure}(A)$ under a logarithmic mixing rate; III: tightness under a metric entropy condition, implying a functional central limit theorem.

E. HÄUSLER:

Central limit theory for martingales. Part II: Continuous time local martingales and rates of convergence

The first half of part II of the present paper completes the exposition of martingale central limit theory presented in part I by a discussion of results for continuous time local martingales obtained by Rebolledo, Liptser and Shiryayev, and Helland. We compare several sets of sufficient conditions for asymptotic normality and show that they are all asymptotically equivalent. Applying then Helland's discretization method to the most appropriate set of conditions leads to a short and elementary derivation of the continuous time theory from the discrete time one. In the second half of the paper it is shown that discretization arguments may also be used to obtain estimates of the rate of convergence in the continuous time theory from corresponding results of the discrete time theory.

J. HÜSLER:

Extreme values and rare events in non-stationary sequences

We consider the limit behaviour of the extreme values and related random variables of a generally non-stationary random sequence. The given approach extends the method of Leadbetter used for stationary sequences to the non-stationary case. By considering the exceedances and more generally the rare events related to extreme values, our results imply e. g. the joint limiting behaviour of the maxima and the minima, the k-th largest values, the k-th smallest values. As particular example, we discuss Gaussian sequences which satisfy Berman's condition on the correlation function.

W. KOHNE:

On the rate at which the sample extremes become independent

The order statistics $X_{1:n},\dots,X_{n:n}$ belonging to a sample $X_1,\dots,X_n$ of n independent and identically distributed random variables possess well known dependence structures, e. g. they are positively quadrant dependent and for a continuous parent distribution they form a Markov chain. For an increasing sample size n the lower and the upper extreme order statistics become less dependent. In fact, they are asymptotically independent. We derive rates for this asymptotic independence which hold uniformly over all Borel sets and for arbitrary parent distributions.

NORIO KÔNO:

Hausdorff dimension of sample paths for self-similar processes

We consider the Hausdorff dimension of the graph $G = \{(t,X(t,\omega)) : 0 \leq t \leq 1\}$ and the range $R = \{X(t,\omega) : 0 \leq t \leq 1\}$ of sample paths for a stationary increment self-similar process under certain conditions.

J. KUELBS:

A Gaussian central limit theorem for stochastically compact sums

If $\{X_j\}$ is a strictly stationary ϕ-mixing sequence with values in a Hilbert space H, $\phi(1) < 1$, and stochastically compact sums $\{S_n\}$, then by deleting an appropriate number of maximal terms it is possible to produce non-degenerate Gaussian limits. For example, if $\{S_n/d(n)\} \xrightarrow{w} Z$ where Z is a stable law of index $p \in (0,2]$, the Gaussian limits are obtainable. These results are joint with M. Ledoux.

MAKOTO MAEJIMA:

Sojourns of multidimensional Gaussian processes

This talk surveys some recent results on sojourns of multidimensional stationary Gaussian processes with strongly dependent structures, given by S. M. Berman, M. S. Taqqu and the speaker.

Let $\{X(t),\ t \geqq 0\}$ be a measurable separable p-dimensional stationary Gaussian process with some long-range dependence and with some special correlation structure. In this talk, the limiting behaviour of sojourn functionals $M(t) = \int_0^t 1[X(s) \in D]ds$ is discussed, where

$$D = D(a_1,\ldots,a_p;\ u(t)) = \{(x_1,\ldots,x_p)' \mid \Sigma_{j=1}^p\ a_j^2\ x_j^2 \leqq u(t)\}.$$

Especially, the following types of D's are considered:

I. $p = 2,\quad D = D(a,b;1)$,

II. $p \geqq 2,\quad D = D(1,\ldots,1;1)$,

III. $p \geqq 2,\quad D = D(1,\ldots,1;u(t))^c,\quad u(t)\uparrow\infty$, or

$D = D(1,\ldots,1;v(t)),\ v(t)\downarrow 0$,

IV. $p \geqq 2,\quad D = D(1/b,\ldots,1/b;1) - D(1/a,\ldots,1/a;1),\quad 0 < a < b.$

P. MAJOR:

Statistical physical models with continuous group of symmetry - On Dyson's hierarchical model

We are interested in the behaviour of vector valued statistical physical models with continuous symmetry at low temperatures. It is believed that in such models in the direction orthogonal to the spontaneous magnetization one has to normalize in an unusual way in order to get a large scale limit. Moreover, in 3-dimensional models an unusual normalization is expected in the direction of the spontaneous magnetization, too. These conjectures are unsolved, and they seem to be very hard. On the other hand in a special case, for Dyson's hierarchical model, we can solve the analogous problems. They are in accordance with the above conjectures, and can help to understand the situation in the general case.

TOSHIO MORI:

The functional iterated logarithm law for stochastic processes represented by multiple Wiener integrals

The functional law of the iterated logarithm is obtained for a certain class of self-similar processes represented by multiple Wiener integrals. This class includes processes which arise as limit processes in the so-called non-central limit theorem. The main tool in the proof is an integration by parts formula for multiple Wiener integrals. Applying this formula we firstly establish a functional log log law for certain self-similar processes which are continuous mapping images of Brownian motion, and then obtain the desired result using approximation by such processes. The functional log log law is also obtained for certain processes defined by nonlinear functionals of some stationary Gaussian sequences or processes with long range dependence.

G. J. MORROW:

Contraction-type estimate for dyadic martingales

A very useful inequality in both the limit theory of probability and of analysis is the following maximal inequality. Let $f = (f_j)_{j\geq 0}$ be a martingale and put $f^* = \sup_{j\geq 0} |f_j|$. Then assuming that f is a one-parameter martingale, there is a universal constant C so that for every $p \geq 2$, $\|f^*\|_p \leq C\cdot p\cdot\|(\Sigma_{j\geq 0} (f_{j+1} - f_j)^2)^{1/2}\|_p$. Here $\|\cdot\|_p$ denotes the $L^P(dP)$ norm. The particular case $p = 2$ is generalized in a different direction for d-parameter dyadic martingales $f = (f_j)_{j\in\mathbb{Z}_+^d}$. Let $(\Delta_j f)$ denote the martingale differences, so that, when $d = 2$, $\Delta_j f = \Delta_{m,n} f =$

$= f_{m+1,n+1} - f_{m,n+1} - f_{m+1,n} + f_{m,n}$. It is proved that $\|f^*\|_p \leq q^d (p-1)^{d/2} (\Sigma_{j\geq 0} \|\Delta_j f\|_p^2)^{1/2}$ for all $p \geq 2$ with $1/q = 1 - 1/p$. This inequality is sharp. This inequality gives rise to a new class of random Fourier series $X(t) = \Sigma a_j \Delta_j f \exp(ij\cdot t)$ so that $X(.)$ satisfies the CLT in $C(T^d)$. Motivated by this connection, a general theorem on the CLT in $C(K)$ is also proved.

G. L. O'BRIEN:

Extreme values for stationary and Markov sequences

Let (X_n) be a strictly stationary sequence of random variables with marginal distribution function F. Let $M_{i,j} = \max(X_{i+1}, X_{i+2}, \ldots, X_j)$ and let $M_n = M_{0,n}$. Let (c_n) be a sequence of real numbers. If (X_n) satisfies a certain asymptotic independence condition, weaker than strong mixing, and if $(p) = (p_n)$ is a sequence of positive integers satisfying a certain growth rate condition, then $P(M_n \leq c_n) - (F(c_n))^{nP_n(M_{1,p} \leq c_n | X_1 > c_n)} \to 0$ as $n \to \infty$, provided $\liminf((F(c_n))^n + P(M_{1,p} \leq c_n | X_1 > c_n)) > 0$. Most earlier results about the asymptotic nature of $P(M_n \leq c_n)$ can be deduced from this result, often in a strengthened form. Some theorems about the simultaneous limiting behaviour of $P(M_n \leq c_n(x))$ for x in some index set T are also obtained. The results are applied to functions of positive Harris Markov sequences.

M. PELIGRAD:

The invariance principle for ϕ-mixing sequences under the Lindeberg-Feller condition

It is proved that the CLT for ϕ-mixing sequences having finite second moments, being second order stationary, centered and having the variances of the partial sums converging to ∞, is equivalent with the Lindeberg-Feller condition. This result replaces the Ibragimov-Iosifescu conjectures about the CLT or invariance principle for ϕ-mixing sequences by the study of the behaviour of the variances of the partial sums. So if the variance of the partial sum of n random variables goes to infinity faster than n the conjectures are true. However, if it will be possible to find a ϕ-mixing sequence with the variance of the S_n's satisfying $\liminf ES_n^2/n = 0$, then it will be possible to give a negative answer to Iosifescu's conjecture.

W. PHILIPP:

I: Central limit theorems for mixing sequences of random variables

Let $\{X_j, j \geqq 1\}$ be a strictly stationary sequence of random variables with mean zero, finite variance and satisfying a strong mixing condition. Let $S_n = \Sigma_{j\leq n} X_j$ and suppose that $\mathrm{Var}\, S_n$ is regularly varying of order 1. This talk reports on joint work with H. Dehling and M. Denker where we proved that if (*) $S_n(\mathrm{Var}\, S_n)^{-1/2}$ does not converge to zero in L^1 then $\{X_j, j \geqq 1\}$ is in the domain of partial attraction of a Gaussian law. If, however, no subsequence of (*) tends to zero in L^1 and if $E|S_n|$ is regularly varying of order 1/2 then $\{X_j, j \geqq 1\}$ is in the domain of attraction to a Gaussian law. In each case the norming constants can be chosen as $E|S_n|$.

II: Strong approximation of martingales

This talk reports on joint work with W. Stout. We obtain the almost sure approximation by a suitable Brownian motion of a martingale satisfying the martingale analogue of Kolmogorov's classical condition for the law of the iterated logarithm.

P. RÉVÉSZ:

Random walk in random environment

Let $X = \{X_i, i = 0,\pm1,\pm2,\dots\}$ be a sequence of i.i.d. r.v.'s with $0 < X_0 < 1$. The sequence X is called random environment. For any fixed sample sequence X of this environment define a random walk $\{S_n, n = 0,1,2,\dots\}$ by $S_0 = 0$, $P_X(S_{n+1} = i+1 \mid S_n = i) = X_i$, $P_X(S_{n+1} = i-1 \mid S_n = i) = 1 - X_i$ $(n = 0,1,2,\dots; i = 0,\pm1,\pm2,\dots)$. Define $M(n) = \max_{0\leq j\leq n} |S_j|$. The main result says:
$(\log n)^2 (\log\log n)^{-2-\varepsilon} \leq M(n) \leq (\log n)^2 (\log\log n)^{2+\varepsilon}$ for any $\varepsilon > 0$, a.s. except finitely many n. The main unsolved problem is to characterize the behaviour of the local time $\xi(x,n) = \#\{k: 0 \leq k \leq n, S_k = x\}$. The above formulated inequality is a joint result of myself and P. Deheuvels.

H. ROOTZÉN:

Maxima and exceedances of stationary Markov chains

Regenerative properties of stationary Markov chains (established by Athreya & Ney and by Nummelin for general state spaces) are used to study extremal behaviour. The results center on the "clustering" of extremes of adjacent values. In addition a criterion for convergence of extremes of general stationary sequences is found. The results are applied to waiting times in the GI/G/1 queue and to autoregressive processes.

M. ROSENBLATT:

The central limit theorem for spectral density estimates

Conditions sufficient for the validity of the central limit theorem for spectral density estimates of a stationary process are given in terms of a strong mixing assumption. This allows one to require only a limited number of moment assumptions.

R. J. SERFLING:

Coupling (and other topics)

Brief comments were given on the topics

(i) results (with W. Stout) on upper half LIL for strongly dependent Gaussian sequences,

(ii) open questions on the SLLN for multi-dimensionally indexed arrays of independent r.v.'s or U-statistic terms,

(iii) (with A. Karr) comparison of approaches to coupling constructions and applications to Poisson approximation of sums of r.v.'s or of point processes.

W. STRITTMATTER:

Invariance principles for set-indexed partial sum processes of weakly dependent random fields

Let $(X_j,\ j \in \mathbb{N}^q)$ be a weakly dependent stationary field of Banach space valued random elements, assumed to be strongly mixing in case the state space is finite-dimensional, and absolutely regular in the infinite-dimensional case. We obtain almost sure approximations of the partial sum process $(\Sigma_{j \in nA} X_j,\ A \in \mathcal{A},\ n \in \mathbb{N})$ by a partial sum process $(\Sigma_{j \in nA} Y_j, \ A \in \mathcal{A}, \ n \in \mathbb{N})$ uniformly over all sets A in a certain class $\mathcal{A}$ of subsets of the q-dimensional unit cube. Here $(Y_j, \ j \in \mathbb{N}^q)$ are i.i.d. Gaussian random vectors.

T. C. SUN:

Central and non-central limit theorems for non-linear functions of a Gaussian process

Suppose $\{X_n, \ n = 0,\pm 1,\ldots\}$ is a stationary Gaussian process with $EX_n = 0$, $EX_n^2 = 1$ and suppose $r(n) \approx n^{-\alpha}L(n)$. Consider $Y_n = H(X_n)$ with $EY_n = 0$, $EY_n^2 < \infty$. Suppose in the series expansion of Y_n in terms of multiple Wiener-Ito integrals, the leading term has rank k. Under the condition $0 < \alpha k < 1$, $Z_n^N(H) = A_N^{-1} \Sigma_{t=nN}^{(n+1)N-1} Y_t$, $n = 1,2,\ldots$ will have non-central limits if the integrand of the leading term in the expansion does not vanish at zero. If it does vanish at zero, both central and non-central limits can happen depending on the rate at which the integrand approaches zero at the origin and the properties of the remaining terms. We discuss some interesting cases in which central or non-central limit theorems are obtained.

D. SURGAILIS:

Limit theorems for functionals of i.i.d. random variables

CLT and non-CLT for non-linear functionals (the so-called subordinated processes) of i.i.d. random variables are discussed. Any such second order process can be expanded in orthogonal series of multilinear forms ("discrete stochastic integrals"), which coincide with the

Ito-Wiener expansion in the case of a Gaussian i.i.d. sequence. The CLT is formulated in terms of the behaviour of the Fourier transform of the weights of the expansion near "diagonals" $x_1 + \dots + x_k = 0$. The case of a non-linear function of a moving average process is considered also, where the Appell rank of the function plays the crucial role. Finally, a limit theorem for a triangular array of symmetric statistics is proved, with the limiting distribution being that of a multiple integral with respect to a Poisson process. The results were obtained jointly with L. Giraitis.

M. S. TAQQU:

Non-central limit theorems and generalized powers

Let $\{X_i,\ i \geq 1\}$ be a stationary moving average with long-range dependence. The Hermite polynomials play a fundamental role in the study of non-central limit theorems for functions of X_i, when the X_i are Gaussian. When the X_i are not Gaussian, we show that the relevant polynomials are "generalized powers". They satisfy a multinomial type expansion that can be used to establish non-central limit theorems.

W. VERVAAT:

Stationary self-similar extremal processes and random semicontinuous functions

In this talk all limiting processes are characterized of

$$M_n(t) := a_n \cdot \sup_{k \leq nt} \xi_k + b_n \qquad (a_n, b_n \in \mathbb{R},\ a_n > 0),$$

where $(\xi_k)_{k=1}^{\infty}$ may be any stationary sequence of random variables.

The limits are identified as the stationary self-similar extremal processes. Their properties are investigated. For this it is necessary to find an intrinsic definition of "extremal process". The right way to do this involves random upper semicontinuous functions, and opens unexpected connections with the theory of random closed sets. Results reviewed in this talk are due to G. L. O'Brien, P. J. J. F. Torfs, G. J. Gerritse and the speaker.

E. WAYMIRE:

Infinitely divisible distributions: Gibbs states and correlations

Consider ± 1 values randomly distributed over a countable set S according to a probability measure P. Two well-known, but not generally known to be related cases are considered; namely when P is a Gibbs distribution and when P is infinitely divisible in a sense which exploits the group structure of the configuration space. Results establishing connections between the two classes are presented, in particular, applications to formulae and inequalities for block correlations are given.

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