MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 21/1985

Mathematical Methods in the Kinetic Theory

5.5. bis 11.5.1985

Die Tagung wurde geleitet von H. Neunzert (Kaiserslautern) und D.C. Pack (Glasgow). Die meisten Vorträge beschäftigten sich mit einem der drei folgenden Problemkreise: Theorie der Boltzmanngleichung, Theorie der Vlasovgleichung und allgemeine (lineare und nichtlineare) Transporttheorie. Daneben gab es einzelne Beiträge zur irreversiblen Thermodynamik und Strömungsdynamik. Im Brennpunkt des Interesses standen Fragen der globalen Existenz von Lösungen der Boltzmann- und Vlasovgleichung; obwohl für beide Gleichungen immer noch keine allgemeingültigen Sätze vorliegen, sind doch deutliche Fortschritte während der letzten 10 Jahre unverkennbar. Dasselbe gilt für die Theorie der diskreten Geschwindigkeitsmodelle und die Gültigkeitsfrage der Boltzmanngleichung.

In der abstrakten Transporttheorie zeigt sich ein Trend zum Zusammenfassen der umfangreichen vorhandenen Theorie und zur Erweiterung auf allgemeine Randbedingungen.

In der abschließenden Diskussion wurde der Nutzen einer solch interdisziplinären Tagung erörtert; alle Teilnehmer votierten in überzeugender Weise für eine Fortsetzung der bisherigen Tagungspraxis. Gerade der übergreifende Charakter dieser Konferenz wurde als besonders lehrreich und anregend gelobt. Die Tagung war in ungewöhnlichem Maße international: Es waren 12 Nationen vertreten, und die Zahl der deutschen Teilnehmer war nur unwesentlich größer als die anderer Nationen.

Vortragsauszüge

K. ARKERYD:

Loeb Solutions of the Boltzmann Equation

As an introduction, some relevant facts about non-standard analysis and Loeb integration are sketched. That is followed by a survey of existence results of Loeb type for the non-linear, space-dependent Boltzmann equation far from equilibrium. This includes existence in the periodic case, for C^1-boundary and exterior forces, and for unbounded space together with an Euler limit. The method of proof is also indicated.

H. BABOVSKY:

Boundary Conditions in the Diffusion Limit

For a model transport equation, Maxwellian boundary conditions with accommodation coefficient are analyzed in the case of small mean free path. It is shown that in the limit of zero mean free path these boundary conditions lead to total absorption of the impulse components tangential to the wall.
This result is obtained without using any regularity conditions of solutions but taking into account the recurrence property of an associated stochastic process.

K. BÄRWINKEL:

The Boltzmann Equation Corrected with Regard to Binary Interaction Effects

A review is given of some of the main implications of the kinetic equation which has been derived for moderately dense monatomic gases via evaluation of Kadanoff's and Baym's theory in T-matrix approximation up to second order in the density and under the assumption of long-wave-length behaviour. This generalized Boltzmann equation yields the correct state functions of thermodynamic equilibrium, if considered up to

second order in the particle density n and in the classical limit. Moreover, the pressure $p = nkT(1+nB(T))$ is also quantum-mechanically correct. By modifying the collision integral, thus producing density corrections to the equilibrium Maxwellian, it is not possible to achieve quantum-mechanical correctness for both the internal energy and the pressure. In comparison with the Kadanoff-Baym theory this is a drawback which is, however, unavoidable if one eliminates the energy as an independent variable in order to obtain a closed equation for the Wigner distribution function $f(\bar{p},\bar{r};t)$.
In the case of sufficiently small particle density a weak version of an H-theorem holds.

J. BATT:

A Virial Theorem for the Vlasov-Poisson System

We consider the Vlasov-Poisson system in the configuration of spherical symmetry and study the asymptotic behavior of the solutions as $t\to\infty$. In particular, if the initial condition ϕ_o is continuously differentiable, nonnegative and with compact support, we prove that the four limits

$$\lim_{t\to\infty} \frac{1}{h} \int_0^t E_{kin}(s)\,ds =: \hat{E}_{kin} ,$$

$$\lim_{t\to\infty} \frac{1}{h} \int_0^t E_{pot}(s)\,ds =: \hat{E}_{pot} ,$$

$$\lim_{t\to\infty} \frac{\dot{I}(t)}{t} =: A , \qquad \lim_{t\to\infty} \frac{I(t)}{t^2} =: B$$

exist and satisfy $A = 2\hat{E}_{kin} + \hat{E}_{pot} = 2B$ (I is the moment of inertia). We have

$$A = \int_{S_e} u^2(\infty,P_o)\,\phi_o(P_o)\,dP_o$$

where S_e is the set of escaping particles and $u(\infty,P_o)$ their limit velocity. The case $A = 0$ can be characterized by a growth condition of the system; this yields a virial theorem for self-gravitating mass systems.

C. BARDOS:

Half Space Problem for Transport Equation and Related Problems

We consider the following equations:

(1) $$\frac{\partial \mu}{\partial x} + \mu - \frac{1}{2} \int_{-1}^{1} \mu(x,\mu')d\mu' = 0$$

(2) $$\xi_1 \frac{\partial f}{\partial x} + Lf = 0$$ (L is the linearized (near a Maxwellian) collision operator)

(3) $$\mu\frac{\partial u}{\partial x} - \frac{\partial}{\partial \mu}(1-\mu^2)\frac{\partial u}{\partial \mu} = 0$$ (Fokker Planck)

(4) $$\zeta\frac{\partial I}{\partial \nu}(\mu,\nu,\lambda) + \sigma(T)(I-B(T)) = 0$$

$$\int_0^{\infty} d\mu \int_{-1}^{1} d\mu\sigma(T)(I-B(T)) = 0$$ (Radiative Transfer)

in the half space $x \geqq 0$; these equations appear when one computes a boundary layer in the diffusion approximation.
For these problems we prove existence, uniqueness and asymptotic behaviour $(\lambda \to \infty)$ of the solution. The method rest uniquely on energy-type estimates. It has been devised by the author and two coworkers: Santos and Sentis for the transport equation. It has been adapted to the Boltzmann equation (by R. Caflish, B. Nicolaenko and C. Bardos), to the Fokker Planck by P. Degond and S. Gallic and for the radiative transfer equation by B. Perthame, F. Golse and C. Bardos.

N. BELLOMO:

Global Existence and Asymptotic Stability of the Nonlinear Boltzmann Equation

The full nonlinear Boltzmann Equation with very general gas-particles interaction potential with cut-off is considered and a proof of global existence and asymptotic behaviour of the solution of the initial value problem in unbounded domains is supplied for initial data which decay at infinity with several behaviours. The results can be compared and discussed on the ground of their consistency.

V.C. BOFFI, G. SPIGA:

Some Recent Results in Kinetic Theory of Multicomponent Gas Mixtures

A dynamical analysis of a spatially homogeneous N-component gas mixture, including both removal and self-generation effects, is carried out. First, the system of N first-order nonlinear ordinary differential equations governing the N number densities $\rho_j(t) = \int_{R_3} f_j(\bar{v},t)d\bar{v}$ $(j=1,2,\ldots,N)$ is derived by integrating over the velocity domain the corresponding nonlinear integro-differential Boltzmann system for the relevant distribution functions $f_j(\bar{v},t)$ in the limiting case of constant microscopic binary collisions and with initial conditions of the form $f_j(\bar{v},0) = Q_j s(v)$ $(\int_{R_3} s(\bar{v})d\bar{v} = 1)$. The derived system, of the type $\dot{\rho}_j = \phi_j(\rho_1,\rho_2,\ldots,\rho_N)$, where ϕ_j is a quadratic form with constant coefficients plus a linear term in ρ_j, is then studied from the point of view of the theory of the dynamical systems. The divergence, which in the present situation can be even positive as due to the presence of self-generation effects, is calculated, and several explicit analytical solutions are constructed for the simple case $N=2$. Numerical preliminary results for $N=2$ and $N=3$ are also illustrated, showing the existence of periodic orbits and bifurcation points in the phase trajectory space. Curves plotting the single ρ_j's versus time are given. A systematic numerical campaign for $N \geq 3$ is in frogressin collaboration with V. Franceschini, University of Modena.

H. CABANNES:

Global Existence of Solutions, Close to Equilibrium, for Discrete Velocity Models of the Boltzmann Equation

The general form of a discrete velocity model of the Boltzmann equation is given by the following quasi-linear hyperbolic system of equations:

$$\frac{\partial F_i}{\partial t} + v_i \cdot \nabla_x F_i = Q_i(F,F) \qquad (i = 1,\dots,m)$$

where $F_i(t,x)$ represents the mass density of particles with the velocity $v_i = (v_i^1,\dots,v_i^n)$ (constant vector). Let M be an absolute Maxwellian state and the initial value $F(0,x) = F_0(x)$ close to M, i.e. $\|F_0 - M\|$ is small. Shizuta and Kawashima have given a criterion for the global existence of the solution of the initial values problem. The criterion is that the summational invariants are not eigen-vectors of the matrix $V(\omega) = \mathrm{diag}(v_i \cdot \omega)$ where ω is an arbitrary unit vector.

The verification of this condition is very complicated. Recently, Cercignani has proposed criteria (sufficient conditions) which are simpler. A first criterion consists of computation of determinants of order $\frac{1}{2}(n^2+5n+2)$; if one is not zero, the global existence is proved. This criterion is valid when the number of summational invariants is n+2.

As a consequence it is easy to prove the global existence for the two regular models with 14 velocities related to the cube.

C. CERCIGNANI:

Global Existence Theorems for Space Inhomogeneous Problems in Kinetic Theory

As is well known, the theory of global existence for the Boltzmann equation in the space inhomogeneous case is far from being satisfactory with the exceptions of small deviations from either equilibrium or vacuum and the nonstandard approach of Arkeryd.

In this paper a class of global solutions corresponding to special initial data is shown to exist. The situation is non-homogeneous and the initial density can be chosen arbitrarily large.

In the second part of the talk existence theorems for the Enskog equation will be discussed.

P. DEGOND:

Global Existence for the Vlasov-Fokker-Planck Equation by a Deterministic method

We present a deterministic proof of global existence of smooth solutions for the Vlasov-Fokker-Planck equation in 1 and 2 dimensions (results using stochastic ideas have been obtained by H. Neunzert and al.). The key idea is to get estimates of the decay at infinity in v of the distribution function. One can obtain an equation for the quantity Y(u,v,t) defined by

$$Y(u,v,t) = (1+|v|^2)^{\gamma/2} f(u,v,t).$$

Using the maximumprinciple and interpolation estimates one proves that $\|Y(\cdot,\cdot,t)\|_\infty$ satisfies a Gronwall inequality which happens to be linear in dimensions 1 and 2, and nonlinear in dimension 3. This yields global existence in dimensions 1 and 2.

M. H. ERNST:

Nonlinear Kinetic Equations in the Theory of Clustering

Smoluchowski's coagulation equation,

$$\dot{c}_k = \frac{1}{2} \sum_{i+j=k} K_{ij} c_i c_j - c_k \sum_{j=1}^{\infty} K_{kj} c_j ,$$

describes irreversible aggregation of units (monomers) into clusters of size k $(k = 1,2,\ldots)$. It constitutes an infinite set of coupled nonlinear rate equations for the concentration $c_k(t)$ of clusters of size k, and one is interested in global solutions to the initial value problem for a given $c_k(0)$. The equation resembles the nonlinear Boltzmann equation for the spatially uniform case.

I will review: the global solutions for the exactly solved cases with rate coefficients: $K_{ij} = 1$, $K_{ij} = i+j$ and $K_{ij} = ij$; what (little) is known about existence and uniqueness of such solutions for more general K_{ij}; limiting solutions for large k-values; for large or small t-values; similarity solutions and "gelling" solutions, and I will indicate some of the existing mathematical difficulties.

W. GREENBERG:

Stationary One-Dimensional Kinetic Equations in Multiplying Media

An abstract model for stationary transport in multiplying media in slab geometry is presented. Existence and uniqueness for a large class of (non-dissipative) collision operators below a critical diameter is obtained. For collision operators which are compact perturbations of the identity, additional information on the smoothness of solutions and behavior at the critical diameter may be obtained. The method depends upon approximation of the slab albedo operator by an albedo operator for dissipative media, and application of Pontrjagin's Theorem and the spectral theorem for operators self-adjoint in a π_k-space.

J. HEJTMANEK:

The Spectral Mapping Theory for the Exponential Function, Examples and Applications to Linear Transport Theory

The asymptotic behavior of the Fermi problem in neutron transport theory is connected with the spectral mapping theorem for the exponential function of the generator of a semigroup. There exist only two types of spectral mapping theorems: (i) one for eventually uniformly continuous semigroups and (ii) Gearhart's Theorem in a separable Hilbert space. While the first type of SMT has been applied to linear transport theory by Jörgens, Vidav, Voigt, Greiner, Weis, Gearhart's Theorem is applied for the first time. Finally an example of a semigroup is presented where we can give the spectrum of the generator and the spectrum of the semigroup for all times, see Hejtmanek, Kaper [1985]; Proceedings AMS, and generalizations of Gearhart's Theorem are discussed.

E. HORST:

What Would a Counterexample to Global Existence for the Vlasov-Poisson System Look Like?

We consider the initial value problem for the nonlinear Vlasov-Poisson system:

$$(\partial_t + v\partial_x + E(t,x)\partial_v)\phi(t,x,v) = 0$$

$$E(t,x) = -\partial_x U(t,x)$$

$$U(t,x) = \pm \int_{\mathbb{R}^3} \rho(t,y)/|x-y|\,dy$$

$$\rho(t,x) = \int_{\mathbb{R}^3} \phi(t,x,v)\,dv$$

$$\phi(0) = \varphi, \varphi \in C_c^1(\mathbb{R}^6), \ \varphi \geq 0, \ x,v \in \mathbb{R}^3 .$$

If there exists a strong solution on a finite interval $[0,T[$, which cannot be continued to a larger interval, then the following functions are unbounded on $[0,T[$:

$$\int_{\mathbb{R}^6} |x|^{\alpha} \phi(t,x,v)\,d(x,v), \ 3 < \alpha < \infty$$

$$\int_{\mathbb{R}^6} |v|^{\beta} \phi(t,x,v)\,d(x,v), \ 3 \leq \alpha < \infty$$

$$\|\rho(t)\|_p, \ 2 \leq p \leq \infty \ \ (\|\cdot\|_p \text{ means } L^p\text{-norm})$$

$$\|E(t)\|_q, \ 6 \leq q \leq \infty$$

$$\|\partial_t U(t)\|_r, \ r = \infty.$$

In contrast the same functions are bounded, if $0 \leq \alpha \leq 2$, $0 \leq \beta \leq 2$, $1 \leq p \leq 5/3$, $3/2 \leq q \leq 15/4$, $3/2 \leq r \leq 15/7$.

R. ILLNER:

<u>Some Recent Progress in (Discrete) Kinetic Theory: New Identities, Some Examples</u>

For a system of N hard spheres interacting through k binary elastic collisions at times $0 \leq t_1 \leq \ldots \leq t_k \leq t$ with incoming velocities u_j, w_j and collision parameter $n_j \in S^2$, $j = 1,\ldots,k$, the identity

$$\sum_{i=1}^{N} x_i(t)^2 = \sum_{i=1}^{N} (x_i(0)+v_i(0)t)^2 + 2\cdot d \sum_{j=1}^{k} (t_{j+1}-t_j) n_j (u_j - w_j)$$

is derived (here $t_{k+1} := t$, and $x_i(t)$, $v_i(t)$ denote position and momentum of the i-th particle at time $t > 0$). For the Boltzmann equation, the corresponding identity is

$$\iint x^2 f(t,x,v)\,dx\,dv = \iint x^2 f_0(x-tv,v)\,dx\,dv,$$

which says that the moment of inertia of a Boltzmann gas is equal to the moment of inertia of a Knudsen gas with the same initial data.
In the second part of the lecture, an example from discrete kinetic theory is presented: For the standard 4-velocity model in 2 dimensions, integrable and bounded initial data are given for which the $\|\cdot\|_\infty$-norm of the solution is unbounded as $t \to \infty$. Generalizations are discussed.

E. A. JOHNSON:

Rarefied Plane Poisseuille Flow

In the classical plane Poisseuille flow problem, the flow rate of the gas diverges as the mean free path becomes infinite. But if one instead approximates the problem in a simple way one can obtain an exact solution for the distribution function, for a collisionless gas which gives a finite, closed-form solution for the flow. For z the distance parallel to the plates and x that normal to the plates (in dimensionless units), one can replace the usually-assumed pressure $p = p_o(1-\beta z)$ by $p = p_o[1-\beta z(\exp-(\beta z)^2)]$. This new problem allows an exact solution for the distribution function which gives

$$U(x) = \frac{\beta}{2\sqrt{\pi}} \int_0^1 c^2 e^{-c^2} dc \left\{ \frac{(1+x)}{[c^2+\beta^2(1+x)^2]^{3/2}} + \frac{(1-x)}{[c^2+\beta^2(1-x)^2]^{3/2}} \right\}$$

for the dimensionless flow velocity, finite for all $|x| \leq 1$ and all β. For $\beta \to 0$, $U \to [-\beta \ln \beta/\sqrt{\pi}]$, independent on x; the dimensionless flow rate $Q \to [-2\beta \ln \beta/\sqrt{\pi}]$. The flow divergence of the original plane Poisseuille flow problem is mirrored here in the fact that (U/β) and (Q/β) diverge as $\beta \to 0$.

I. KUSCER:

Irreversible Thermodynamics of Rarefied Gases

For transport phenomena in gases the Onsager scheme can be extended so as to include the quantities occurring in a complete set of linearized Burnett equations. To show this, the linearized Boltzmann equation is solved in the Chapman-Enskog manner, allowing for $\nabla\nabla T$ and $\overline{\nabla\nabla \underset{\sim}{v}}$ terms among the thermodynamic forces. Microscopic and macroscopic Burnett fluxes are introduced, and the phenomenological equations and formal expressions for the transport coefficients derived. The matrix of these coefficients is symmetric in the Onsager way, in agreement with an expression one can derive for the density of entropy source. The scheme simplifies for the bulk part of plane flows between parallel plates, with their separation assumed to be greater than a few mean free paths. Boundary layers must be treated separately. The corresponding expression for the entropy source suggests another set of phenomenological equations, involving slip coefficients. The symmetry relations among the coefficients, that were earlier obtained indirectly, are thereby explained in Onsager's sense.

M. LACHOWICZ:

<u>Initial Layer and Existence of Solutions of the Boltzmann Equation</u>

I use the nonlinear Boltzmann equation with a small parameter ε and study the raising singular perturbation problem in a rectangular domain with the specular reflection boundary condition. I educe the existence theorem of this problem from the existence of solution to the compressible Euler equations. I look for a solution in the form of the sum of a truncated Hilbert series, a truncated initial layer series and a remainder term. This enables me to replace the singularly perturbed Boltzmann equation by the following set of equations: the Hilbert procedure equations, the initial layer equations and the equation for the remainder. The equation for the remainder is nonlinear and nonhomogeneous, but the nonlinear and the nonhomogeneous terms are multiplied by a small parameter ε^a (a-positive number). Hence, this is a weakly nonlinear equation and can be solved for any time interval. To sum up, I obtain the existence theorem to the nonlinear Boltzmann equation for initial data sufficiently close (in the sense $O(1)$ as $\varepsilon \downarrow 0$) to a local Maxwellian.

S. K. LOYALKA:

Kinetic Theory of Transport of Mass, Momentum and Heat to Single Particles

Kinetic Theory of Transport of Mass, Momentum and Heat to Single Particles in rarefied gases is discussed. Recent advances in analytical and numerical methods of solutions are described. Specific problems of specific interest include those of condensational growth of particles and motion of particles under concentration, temperature and other gradients.

C. VAN DER MEE, M. KLAUS, V. PROTOPOPESCU:

Generalized kinetic equations and applications to Sturm-Liouville type diffusion equations

In the talk we discuss boundary value problems of the form

$$\begin{cases} w(\mu)\dfrac{\partial\psi}{\partial x} = \dfrac{\partial}{\partial\mu}\left(p(\partial)\dfrac{\partial\psi}{\partial\mu}\right) - q(\mu)\psi(x,\mu) & \mu \in (a,b),\ x \in [0,\infty) \\ \psi(0,\mu) = \varphi_{+}(\mu) \text{ if } w(\mu) > 0 & \\ \int_a^b |w(\mu)|,|\psi(x,\mu)|^2 d\mu = O(1) \text{ or } o(1) & (x \to \infty), \end{cases}$$

where $p(\mu) > 0$ is absolutely continuous, $q(\mu)$ is continuous, $w(\mu)$ is indefinite and does not vanish on a set of positive measure, and $h \mapsto -(ph')'+qh$ + selfadjoint boundary conditions is positive selfadjoint. Assuming the zero eigenvalue (simple when present) to be isolated when present, the problem was proved to be uniquely solvable by R. Beals (J. Diff. Eqs. 56, 391 (1985)). Posing the problem in an abstract way on $H_T = L_2((a,b);|w(\mu)|d\mu)$, one may write the solution at $x = 0$ as $\psi(0,\mu) = (E_{+}\varphi_{+})(\mu)$, where E_{+} is the albedo operator. Hitherto no explicit solutions were known for the above problem. Two methods are presented to compute the solution. The first method uses the eigenfunctions of the Sturm-Liouville operators to derive an integral equation for $g(\mu) = \psi(0,\mu)$ where $w(\mu) < 0$. The second method consists of converting the problem into a Wiener-Hopf type integral equation, which is solved by Wiener-Hopf factorization. The restrictions of both methods are discussed.

J. MIKA, D. C. PACK:

Approximation to inverses of normal operators

In the calculation of certain fluxes in the kinetic theory of gases (and for other purposes), involving the estimation of upper and lower bounds to bilinear forms $<g_o,f>$, where f is the solution to the operator equation $Af = f_o$ and g_o and f_o are given, it is useful to obtain a close approximation to the inverse A^{-1} of the operator A in some well-understood sense.
For the class of bounded normal operators such approximations can be expressed in terms of polynomials in A. Using the fundamental properties of normal operators we reduce the problem to finding the polynomial of best approximation in the sense of maximum norm to the function $\frac{1}{\lambda}$ over the spectrum $\sigma(A)$ of the operator A.
The application of the procedure to Fredholm equations of the second kind with a self-adjoint positive semi-definite operator shows that the error in the first-order approximation is at worst one-eighth of that connected with the Neumann expansion of the same order.

H. NARNHOFER:

Vlasov Hydrodynamics of a Quantum Mechanical Model

Vlasov hydrodynamics can be derived from a quantum mechanical model in an appropriate scaling that is relevant for a system with gravitation. Quantum effects only appear in the initial conditions.

P. NELSON:

On the Nonlinear Integrodifferential Initial-Value Problem of the Reflection Kernel for Transport in a Plane-Parallel Layer with Isotropic Scattering

A local existence theorem is proved for nonnegative solutions of the problem of the title, under suitable conditions. It is shown that any such solution can be indefinitely continued in the spatial variable, so long as certain norms (the L^1-operator norm as a flux-to-flux transformation, and the L^{∞}-operator norm as a current-to-current transformation) remain bounded, and further that such solution is continuous in the spatial variable, relative to these norms. Finally, by suitably estimating the composite flux-to-flux and current-to-current L^{1}-operator norm, a global existence theorem is obtained, for secondary scattering ratio bounded by unity.

F.-J. PFREUNDT:

Stationary Solutions of the 1-Dimensional Vlasov Equation

For the 1-dimensional Vlasov equation with ion background and periodic boundary conditions, results concerning existence and uniqueness of inhomogeneous stationary solutions were presented. Two methods were described, how to calculate stationary solutions:

I. Prescribe the potential u(x), and the energy distribution for the untrapped particles ⇒ integral equations of Abel type.

II. Prescribe the trapped and untrapped distributions ⇒ periodic boundary value problems for the potential u(x).

Then results about stability properties for a system of ordinary differential equations, which has been derivated from the Vlasov equation, were presented.

A. PIGNEDOLI:

On the Differential Equations of the Brownian Motion

One considers the differential equations of the Brownian Motion and makes a discussion about the operators of the probabilistic theory in general (in opposition with the deterministic points of view). The equations of the Fokker-Planck type are particularly studied. Some cases concerning the diffusion of particles in certain regims of energy are investigated.

T. PLATKOWSKI:

Boltzmann-Enskog Equation and Discrete Velocity Models

A global existence theorem for the Boltzmann-Enskog equation of hard spheres is proved for small (in an appropriate norm) initial data, for the Enskog correlation functions vanishing at infinity. A simple discrete velocity model of the Boltzmann-Enskog equation is then proposed. Global existence theorems for such a model are proven for adequate sets of initial data.

M. PULVIRENTI:

Boltzmann-Grad Limit for Hard Discs

Consider N identical hard discs of diameter d in the plane. Suppose the joint distribution densities of the system, namely $f_j^{N,d}(x_1 \dots x_j, v_1 \dots v_j)$, decay at time zero sufficiently fast at infinity in both sets of variables. In the limit $N \to \infty$, $d \to 0$, $Nd \to \lambda^{-1}$ (i.e. the Boltzmann-Grad limit), if

$$f_j^{N,d}(x_1 \dots x_j, v_1 \dots v_j) \to \prod_{k=1}^{j} f_o(x_k, v_k)$$

uniformly on compact sets, and λ is sufficiently large, then for

$$t > 0,\ f_{j,t}^{N,d}\ (x_1 \dots x_j, v_1 \dots v_j) \to \prod_{k=1}^{j} f_t(x_k, v_k) \quad \text{a.e.}$$

where f_t solves the Boltzmann equation with initial datum f_o. This extends, in this context, a previous local result due to Lanford and gives a rigorous deduction of the Boltzmann equation.

S. RADEV:

Applications of the Direct Simulation Monte Carlo Method to Some Problems of the Molecular Gas Dynamics

The report discusses problems that arise during the application of the Belotserkovskiy-Yanitskiy scheme to the solution of problems of the molecular gas dynamics, when employing the direct simulation method. Special attention is paid to the satisfaction of the condition of molecular chaos in systems involving small number of particles. However, this condition is necessary for the existence of an approximation of the Boltzmann equation. What if further shown is that reasonable satisfaction of the condition of molecular chaos, for systems with an average number of particles $\langle N_o \rangle \sim 1$ in each cell of the grid, might be attained by an appropriate choice of the parameters of the computation scheme. Results from the numerical simulation of vapour-gas flow between two liquid (solid) phases are given.

M. SHINBROT:

The H-Theorem and Reversibility

A transparent example is given of a deterministic, reversible dynamical system with an H-theorem. This is followed by a discussion of some of the possible meanings of "reversibility".

H. SPOHN:

An Example for the Failure of the Navier-Stokes Correction

We consider the simple, asymmetric exclusion process on $\mathbb{Z}$: At each site in $\mathbb{Z}$ there is at most one particle. The dynamics of the particles is given by the stochastic process where each particle, independently of all others, jumps after an exponential holding time with probability p to the right and with probability 1-p to the left, provided the final site is empty. By a theorem of H. Rost, Z. Wahrscheinlichkeitstheorie 58, 41 (1981), improved by T. Liggett, Interacting Stochastic Particle Systems, Springer, Grundlehren (1985), in the hydrodynamic limit the density profile is governed by $\partial_t \rho = (1-2p)\partial_x(\rho(1-\rho))$. The naive Navier-Stokes correction would be $\partial_t \rho = (1-2p)\rho_x(\rho(1-\rho) + \partial_x(D(\rho)\partial_x)\rho$. We show that this correction is invalid because density fluctuations spread as $t^{4/3}$ rather than t. For spatial dimensions $d \geq 3$ the Navier-Stokes correction presumably holds.

L. TRIOLO:

On the Vlasov-Poisson/Fokker-Planck Equation

An existence theorem for the Cauchy problem to which the title is referred, is given.
The existence is global, for dimensions ≥ 3, but as it's based on an expansion around the free-field case, is valid for

suitably small initial data.

M. TROCHERIS:

On the Derivation of the One Dimensional Vlasov Equation

A solution of the one dimensional Vlasov equation is compared to the motion of a system of points interacting through the one dimensional equivalent of the Coulomb potential, namely $\phi(x) = -|x|$. For each value of the time t, both the distribution function f(x,v,t) and the system of points P(t) are considered as elements of the dual space of a suitable vector space of test functions. In the absence of any smoothing of the potential, the most natural test functions and in particular the very convenient C_b^1 functions cannot be used. The chosen class of test functions is a little more complicated and is related to one of the possible definitions of functions of two variables of bounded variation. Then it can be shown that for a given and fixed solution f(x,v,t) of the Vlasov equation, if P(O) tends to f(x,v,O) in the chosen dual space, then P(t) tends to f(x,v,t) for all $t > 0$. For easier comparison to problems of numerical analysis, periodic boundary conditions on the space variable x have been used.

F. VERHULST:

Dynamical System Theory Applied to Vlasov Models

We consider stationary solutions of the Liouville-Poisson system of equations and the corresponding characteristic equations. The solutions of the characteristic equations for all initial values determine the distribution function. We consider two classes of models. First the axisymmetric case. The equations of motion represent a two degrees of freedom Hamiltonian system with resonances 1:1, 1:2, 1:3 and higher order. The analysis is based on averaging and Birkhoff normalization and produces two sets of invariant manifolds: one set parametrized by the energy, the second set corresponding with the invariant KAM tori around the shortperiodic solutions. In the case of the 1:1 resonance both sets of invariant manifolds are prominent in phase space, outside this resonance the invariant manifolds correspond

Gefördert durch DFG Deutsche Forschungsgemeinschaft

with the energies in the respective degrees of freedom. We discuss some implications for the construction of distribution functions. In the second case we consider models for triaxial elliptical galaxies which involve three degrees of freedom. We describe as an illustration the work by P.T. de Zeeuw.

J. WICK, M. PULVIRENTI:

Convergence of Galerkin-Approximation for Two-Dimensional Vlasov-Poisson Equation

We consider the two-dimensional Vlasov-Poisson equation with periodical boundary condition in space. This problem has for a nonnegative C^{∞}- initial function with compact support in V a unique solution. This solution will be expanded in a serie w.r.t. an orthonormal L_2-basis and an infinite system of ODE's can be considered as an equivalent formulation of the Vlasov-Poisson equation. For the truncated system we show that for each fixed time-interval the coefficients approximate the original one in an arbitrary order of the truncation number.

P. F. ZWEIFEL, C. BURNAP:

Kinetic Equations with Non-Self-Adjoint Collision Kernels

The spectral theorem is a powerful tool for dealing with operation equations. For example, the functional calculus yields useful representations of A^{-1} and e^{-At} if A is an operator in a Hilbert space. An operator with a spectral representation $A = \int \lambda dE_A(\lambda)$ is called a "spectral operator of scalar type" by Dunford and Schwartz. It is necessary and sufficient (Kermer) that such an operator be related to a normal by $A = SNS^{-1}$, S, S^{-1} bounded. If S^{-1} is unbounded, we say A is quasi-similar to N. Clearly, such an A has a generalized spectral representation $A = \int \lambda dSE_N(\lambda)S^{-1}$. Such a spectral family, $SE_N(\lambda)S^{-1}$ is unbounded and is a generalization of the usual spectral representation (we call it a U-spectral representation). We give sufficient conditions that an operator be U-spectral which includes, as a subset, the operators which are quasi-similar to normals.

We conjecture that all U-spectral operators are of this form. We also conjecture that an operator with non empty residual spectrum is not U-spectral. Examples are given.

Berichterstatter: T. Platkowski

Liste der Tagungsteilnehmer

Dr. L Arkeryd	Dept. of Math., Chalmers Univ. of Techn. 40220 GÖTEBORG, Schweden
Dr. H. Babovsky	Fachbereich Math., Univ. Kaiserslautern, Postfach 3049, 6750 KAISERSLAUTERN
Prof. Dr. K. Bärwinkel	Phys. Inst., Univ. Osnabrück, 4500 OSNABRÜCK
Prof. Dr. C. Bardos	École Normale Supérieure, 45, Rue d'Ulm, 75230 PARIS Cedex 05, Frankreich
Prof. Dr. J. Batt	Math. Inst., Univ. München, Theresienstr. 39, 8000 MÜNCHEN 2
Prof. Dr. N. Bellomo	Dipt. di Matematica, Univ. di Torino, Corso Duce degli Abruzzi, 24 10129 TORINO, Italien
Prof. Dr. V. Boffi	Laborat. Ingegn. Nucleare, Univ. di Bologna, Via dei Colli, 16, 40136 BOLOGNA, Italien
Prof. Dr. G. Busoni	Ist. Matematico "U.Dini", Univ. di Firenze, Viale Morgagni 67/A, 50134 FIRENZE, Italien
Prof. Dr. H. Cabannes	Mécanique Théorique, Univ. Paris VI, Tour 66, 4, Place Jussieu, 75230 PARIS Cedex 05
Prof. Dr. C. Cercignani	Ist. di Matematica, Politecn. di Milano, Piazza Leonardo da Vinci, 20133 MILANO, Italien
Dr. P. Degond	Centre de Math. Appl., École Polytechnique, 91128 PALAISEAU Cedex
Prof. Dr. M.H. Ernst	Inst. vor Theor. Fysica, Rijksuniversiteit Utrecht, UTRECHT, Niederlande
Prof. Dr. W. Greenberg	Dept. of Math., Virginia Polytechnic Inst. and State Univ., BLACKSBURG, Virg. 24061, U.S.A.
Prof. Dr. J. Hejtmanek	Inst. für Math., Univ. Wien, Strudlhofgasse 4, 1090 WIEN, Österreich
Dr. E. Horst	Math. Inst., Univ. München, Theresienstr. 39, 8000 MÜNCHEN 2
Prof. Dr. R. Illner	Fachbereich Mathematik, Univ. Kaiserslautern Postfach 3049, 6750 KAISERSLAUTERN
Dr. E.A. Johnson	Dept. of Physics, Univ. of Surrey, GUILDFORD GU25xH, England

Prof. Dr. H.G. Kaper	Argonne Nat. Laboratory, 9700 South Cass Avenue, ARGONNE, Ill. 60439, U.S.A.
Prof. Dr. I. Kuščer	VTO Oddelek za Fiziko, Univ. Ljubljani Jadranska c. 19, LJUBLJANA 61001, Jugoslav.
Dr. M. Lachowicz	Dept. of Math. and Mechanics, Univ. of Warsaw, WARSAW, PKiN, r. 936, Polen
Prof. Dr. S.K. Loyalka	Nuclear Eng. Dept., Univ. of Missouri, COLUMBIA, MO 65202, U.S.A.
Prof. Dr. C.V.M. van der Mee	Dept. of Math., Virginia Polytech. Inst., State Univ., BLACKSBURG, Virg. 24061, U.S.A.
Prof. Dr. J. Mika	Dept. of Math., Univ. of Strathclyde, Livingston Tower, Richmond Street, GLASGOW 1xH, Schottland
Dr. H. Narnhofer	Inst. f. Theor. Physik, Univ. Wien, Boltzmanngasse 5, WIEN 1090, Österreich
Prof. Dr. P. Nelson	Dept. of Math., Texas Tech Univ., LUBBOCK, Texas 79409, U.S.A.
Prof. Dr. H. Neunzert	Fachbereich Math., Univ. Kaiserslautern, Postfach 3049, 6750 Kaiserslautern
Prof. Dr. D.C. Pack	Dept. of Math., Univ. of Strathclyde, Livingston Tower, Richmond Street, GLASGOW 1xH, Schottland
Dipl.-Math. F.J. Pfreundt	Fachbereich Math., Univ. Kaiserslautern, Postfach 3049, 6750 Kaiserslautern
Prof. Dr. A. Pignedoli	Dipt. di Matematica, Univ. di Bologna, 40127 BOLOGNA, Italien
Dr. T. Platkowski	Fachbereich Math., Univ. Kaiserslautern, Postfach 3049, 6750 Kaiserslautern
Prof. Dr. M. Pulvirenti	Istit. Matematico, Univ. di Roma I, Piazzale A. Moro, 5, 00185 ROMA, Italien
Prof. Dr. S. Radev	Inst. of Mech. and Biomech., Univ. of Sofia, SOFIA 1090, Bulgarien
Prof. Dr. M. Shinbrot	Dept. of Math., Univ. of Victoria, P.O.Box 1700, VICTORIA B.C., V8W, Canada
Dr. H. Spohn	Math. Inst., Univ. München, Theresienstr. 39 8000 MÜNCHEN 2
Prof. Dr. L. Triolo	Istit. di Matematica Appl., Univ. di Roma, Via A. Scarpa, 10, 00161 ROMA, Italien

Prof. Dr. M. Trocheris	Dept. de Recherches sur la Fusion Controlée, Ass. Eurotom-Cea sur la Fusion, Boîte Postale no. 6, 92260 FONTENEY-AUX-ROSES, Frankreich
Prof. Dr. F. Verhulst	Math. Inst., State Univ. of Utrecht, P.O.Box 80.010, 3508 TA UTRECHT, Niederlande
Prof. Dr. J. Wick	Fachbereich Math., Univ. Kaiserslautern Postfach 3049, 6750 KAISERSLAUTERN
Prof. Dr. T. Ytrehus	Univ. I Trondheim, Norges Tekniske Høgskole, Inst. for Mekanikk, 7034 TRONDHEIM, Norwegen
Prof. Dr. P. Zweifel	Virg. Polytechnic Inst., State Univ., BLACKSBURG, Virg. 24061, U.S.A.