Math. Forschungsinstitut Oberwolfach
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MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 28/1985

# Differentialgeometrie im Großen

23.6. bis 29.6.1985

Die Tagung fand unter der Leitung von Herrn Prof. Klingenberg statt. Die Teilnehmer kamen aus verschiedenen Ländern und vertraten einen breiten Themenkreis der Differentialgeometrie. Die Vorträge beschäftigten sich vor allem mit Riemannscher Geometrie, Geometrie eingebetteter und immersierter Untermannigfaltigkeiten, Geschlossenen Geodätischen, Affiner Geometrie und Anwendungen der Wärmegleichung. Aber hiermit sind bei weitem nicht alle Themen aufgezählt, und es wurden auch viele Querverbindungen aufgezeigt.

Die Vorträge präsentierten eine entsprechende Themenvielfalt. Vormittags gab es jeweils 4 Vorträge und am Nachmittag fanden verschiedene Workshops statt, zum Beispiel, Workshops über Affine Geometrie, Pinching Sätze und Anwendungen der Wärmegleichungsmethode.

Die Ergebnisse wurden in interessanter und verständlicher Weise vorgetragen. Sicherlich gab es auf der Tagung viele Anregungen.

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## VORTRAGSAUSZÜGE

U.Pinkall

### Total absolute curvature of immersed surfaces.

In 1960 N.Kuiper posed the following problems:

a) Within each regular homotopy class of immersed surfaces in $R^n$ determine the infimum of the total absolute curvature $\int |K| dA$.

b) Determine those regular homotopy classes where this infimum is attained.

In this talk we develop a geometric theory for the regular homotopy classes of immersed surfaces in $R^n$ and give the following partial answer to Kuiper´s problems.

Theorem: a) Within each regular homotopy class of immersions $f: M^2 \longrightarrow R^3$ one has

$$\inf \int_{M^2} |K| dA = 2\pi\beta(M^2),$$

where $\beta$ denotes the sum of the $Z_2$-Betti numbers.

b) Any tight torus in $R^3$ is regularly homotopic to the standard torus.

c) If the Euler characteristic $\chi$ of $M^2$ satisfy $\chi \leqslant -10$, then every immersion $f: M^2 \longrightarrow R^3$ is regularly homotopic to a tight immersion.

L.Simon

### Existence of Willmore surfaces.

We consider the "Willmore functional" $\mathcal{F}(\Sigma) = \int_\Sigma H^2$, where $\Sigma$ is a compact surface in $\mathbb{R}^3$ and H is the mean curvature of $\Sigma$. For $g \geqslant 0$, let

$$\beta_g = \inf \{ \mathcal{F}(\Sigma) : \Sigma \text{ has genus } g\}$$

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Theorem: For each integer $g \geq 0$ there is a genus g embedded real analytic surface $\Sigma$ with $\mathcal{F}(\Sigma) = \beta_g$

The proof uses the "direct method" of the calcules of variatoins: we take a sequence $\{\Sigma_k\}$ of genus g surfaces with $\mathcal{F}(\Sigma_k) \longrightarrow \beta_g$

After suitable normalizations ( involving rescaling and inversions in sphere, which leaves the value of $\mathcal{F}(\Sigma_k)$ unchanged ) we prove that $\Sigma_k$ converges ( in a measure theoretic sense ) to a minimum.

E.Ruh

The local structure of Riemannian manifolds.

The following assertion was discussed in the talk:

Every point in a Riemannian manifold has a neighborhood of a-priori size which carries a "geometric structure". To define this structure let U be a Neighborhood in the principal bundle of orthonormal frames and $\omega : TU \longrightarrow \mathfrak{g}$ a Maurer-Cartan form,i.e.a one-form which satisfies

(i) $\omega$ is non-degenerate,

(ii) $d\omega + [\omega , \omega] = 0$ (Maurer-Cartan equation)

Definition: A Riemannian manifold carries a r-micro-local geometric structure if every geodesic ball of radius r in the principal bundle of orthonormal frames admits a Maurer-Cartan form.

Theorem: There exists a universal $r = r(n)$ such that every Riemannian manifold with $|K| \leq 1$ carries an r-micro-local geometric structure.

Application: Estimate of the degree of nilpotency of the local fundamental group with small holonomy. This is an essential step in proving that almost flat manifolds are diffeomorphic to quotients of nil-manifolds.

Proof: Determination of Cartan connections.

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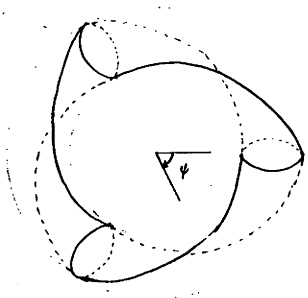
N.Kuiper

A real algebraic knot in $R^3$ of degree six.

A non-trivial knot in $R^3$ meets some plane transversally in at least six points ( Milnor ). Hence an algebraic knot has degree (defined over $\mathbb{C}$) at least six. Here is one of degree 2q=6:

| Torus, $\|\varepsilon\|$ small | perturbed torus |
| --- | --- |
| $x = \cos\phi(1+\varepsilon\cos\psi)$ | $x = \cos\phi/(1-\varepsilon\cos\psi)$ |
| $y = \sin\phi(1+\varepsilon\cos\psi)$ | $y = \sin\phi/(1-\varepsilon\cos\psi)$ |
| $z = \varepsilon\sin\psi$ | $z = \varepsilon\sin\psi/(1-\varepsilon\cos\psi)$ |

The p-q-torus knot $\gamma_{p,q}$ depends on the perturbed torus by $\phi$=pt, $\psi$=qt, 1<p<q, p,q coprime, has degree 2q (=6 for (p,q)=2,3).
Use $e^{ipt}$= cospt + isinpt = ( cost + isint $)^p$, sint $=(1-\omega^2)/(1+\omega^2)$ cost = $(2\omega)/(1+\omega^2)$ to determine the degree.




J.Eschenburg

Minimal surfaces in certain 4-sphere.

Let ( $M^2$,$ds^2$ ) always be a simply connected surface with Gaussian curvature K.

A function a: $M \longrightarrow [0,\infty)$ is called of absolute value type ( AVT ) if locally $a = a_o|h|$ where $a_o$ is a smooth positive function and h is holomorphic function.

1. Let $S_c^4$ be a 4-space form of constant sectional curvature c.

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Let $K_N \varepsilon C^\infty(M)$.

Theorem. There exists an isometric minimal immersion $j: M^2 \longrightarrow S^4_c$ with normal curvature $K_N$ if and only if $a_\pm := \sqrt{C-K\pm K_N}$ is AVT with $\Delta \log a_\pm = 2K\mp K_N$.

2. Let $P^4_\sigma$ be a Kähler surface with constant holomorphic sectional curvature $4\sigma$ and Kähler-form $\Phi$. If $M^2$ is oriented with volume element $dV$ and $j: M^2 \longrightarrow P^4_\sigma$ is an isometric immersion, we call $c := j^*\Phi/dV$ the Kähler function of this immersion.

Theorem. There exists an isometric minimal immersion $j: M^2 \longrightarrow P^4_\sigma$ with normal curvature $K_N$, Kähler-function c if and only if $a_\pm := \sqrt{1\pm c}$ and $a_o := \sqrt{2\sigma - K\pm K_N}$ are AVT with $\Delta \log a_\pm = \frac{1}{2}(K+K_N)\mp 3c$ and $\Delta \log a_o = 2K-K_N$.
We discussed these theorems which are in spirit of the classical theorem of Ricci about minimal surface in 3-space, and we derived global applications.

D.De Turck

Isospectral deformations.

If G is a simply connected nilpotent Lie group with Lie algebra ,
then an almost inner automorphism of G is the exponential of an almost inner derivation of $\mathfrak{g}$. The latter is defined as a derivation $\phi$ such that $\phi(X) = [Y,X]$ , but where Y may depend an X. Let $\Gamma$ be a uniform discrete subgroup of G.

Theorem. Let $\widetilde{M}$ be a manifold an which G acts in the left by diffeomorphism , such that $\Gamma\backslash\widetilde{M} = M$ is compact, Let g is "partially" invariant metric,

a) If G is the two-step nilpotent, then $\phi_t^* g$ is an isospectral deformation of g, where $\phi_t^* = \exp t\phi$.

b) If $\phi$ has rational coefficients with respect to a basis of $\Gamma$,

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then $\Phi_t^*$ is an isospectral deformation of g.
( Isospectral means that the eigenvalue of every natural differential operator on M are fixed. ) The deformations are nontrivial ( not isometries in M ), if $\Phi$ is almost-inner but not inner.

Hans-Bert Rademacher

On the equivariant Morse complex.

The concept of a relative ( $Z_m, S^1$ )-CW-complex (X,A) is defined. This is a certain CW-complex with a $S^1$-action on X such that the order of the isotropy group of each element in X-A divides the positive integer m. The corresponding cellular chain complex has a $Z_m$-action and it determines the singular homology of (X/G,A/G) where G is a subgroup of $Z_m$ or $G=S^1$.
The Hilbertmanifold $\Lambda M$ of closed curves on a compact Riemannian manifold M has a canonical $S^1$-action. If the energy functional $E:\Lambda M \longrightarrow \mathbb{R}$ is a Morse function , then there is a $S^1$-CW-complex (X,A) $S^1$-homotopy equivalent to $(\Lambda M, \Lambda^o M)$ with a filtration $(X_i, A)$ of $(Z_{m_i}, S^1)$-CW-complexes with cellular chain comlexes $C_i$. Their direct limit will be called equivariant Morse complex. It reflects the singular homology of $(\Lambda M/G, \Lambda^o M)$ for any G of $S^1$ and can be used to describe some homology classes in $H_*(\Lambda S^n/S^1, \Lambda^o S^n; Z)$.

F. Tricerri

Homogeneous Riemannian structures.

An homogeneous Riemannian structure on a Riemannian manifold (M,g) is a tensor field T of type (1,2) such that $\tilde{\nabla}g = \tilde{\nabla}T = \tilde{\nabla}\tilde{R} = 0$, where $\overline{\nabla} = \nabla - T$, $\nabla$ is the Levi-civita connection and $\tilde{R}$ is the curvature

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tensor of the linear connection $\tilde{\nabla}$.

Ambrose and Singer proved that a connected, simply connected, complete Riemannian manifold is homogeneous iff it carries an homogeneous structure. They suggested also the possibility of classifying the homogeneous Riemannian manifolds by the properties of tensor fields T.

Following this suggestion, we give a classification of the homogeneous structures (and therefore, of the simply connected homogeneous Riemannian manifolds) in eight type, based on the decomposition into O(n)-irreducible components of the space of tensors which have the some symmetries as the tensor field T.

(Joint work with L.Vanhecke)

CL.Epstein

<u>The Gauss map in $H^3$</u>.

Let $\Sigma \longrightarrow H^3$ be an immersed surface with unit normal field N. If $\psi^t(p,X)$, p $H^3$, X $T_pH^3$ denotes the directed geodesics with initial point p and direction X, then we define the Gauss maps $G_\pm(p) = \lim_{t\to\pm\infty} \psi^t(p,N_p)$, $H^3$ being modeled as $B^3$ with $ds^2 = 4(dx^2+dy^2+dz^2)/(1-r^2)^2$. On the other hand we define the support function as $\rho_\pm(\theta)=\langle p,G_\pm(p)\rangle$, $p = G_\pm(\theta)$, $\langle\cdot,\cdot\rangle$ is horospheric radius. We have a generalization of Gauss´formule for the curvature form KdA of $\Sigma$. $G_\pm(K_{\pm\infty}dA_{\pm\infty})=KdA$, $K_{\pm\infty}$ and $dA_{\pm\infty}$ refer to the curvature faced of the metries $ds^2_{\pm\infty} = e^{2\delta t}d\delta^2$ defined as Gauss images of $\Sigma$.

If $(k_1,k_2)$ are the principal curvature of $\Sigma$, then we can find about each $\theta$ in $G_\pm(\Sigma)$ a conformal coordinate so that

$$ds^2|_p = e^{2\rho\pm}\left(\frac{dx^2}{(1-k_1)^2}+\frac{dy^2}{(1-k_2)^2}\right) \text{ and } ds^2_\infty|_{G\pm(p)} = e^{2\rho\pm}(dx^2+dy^2).$$

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From these we prove:

Theorem. If $\Sigma \longrightarrow H^3$ is an embedded, comlete $\mathbb{R}^2$ with $K>-1$ and $\int_\Sigma |K|dA<\infty$, then $\partial_\infty\Sigma = \{1 \text{ point}\}$. ($\partial_\infty\Sigma = \bar{\Sigma}\subset S^2$, closure taken in euclidean topology of $\bar{B}^3$.)

One can also compute the dilatation of the map $\Lambda=G_-\cdot G_+^{-1}:S^2\longrightarrow S^2$ in terms of $(k_1,k_2)$ by :

$$K(\Lambda,\theta) = \max\left\{\left|\frac{1-k_1}{1+k_1}\cdot\frac{1+k_2}{1-k_2}\right|^{1/2} \quad \left|\frac{1+k_1}{1-k_1}\quad\frac{1-k_2}{1+k_2}\right|^{1/2}\right\}$$

We have the following geometric characterization of surface $\Sigma$ with $|k_i|< 1$, $i=1,2$.

Theorem. If $\Sigma \longrightarrow H^3$ is a complete surface with $|k_i|<1$, $i=1,2$. Then:

1) $\Sigma$ is properly embedded,
2) $\partial_\infty\Sigma$ is a Jordan curve,
3) $G_\pm$ are diffeomorphisms into the complement of $\partial_\infty\Sigma$ ,
4) If $\Sigma$ is normalized so that $(0,0,0) = \theta\in\Sigma$ , then $\exp_\Sigma:D_1\longrightarrow H^3$ given by $\exp(\log(\frac{1+r}{1-r})V_\theta)$, ( $V_\theta$ are identification of the unit tangent space at  with the unit circle) are uniformly equicontinuous for all surfaces described by the hypotheses of the theorem.
5) If $\sup_{\theta\in G_\pm(\Sigma)} K(\Lambda,\theta)\leq K < \infty$, then $\partial_\infty\Sigma$ is a quasicircle, the fixed point set of the quasiconformal reflection $\Lambda$.

U. Simon

Equiaffine hypersurfaces with constant curvature functions.

The following conjecture is due to W. Blaschke.

Conjecture: An ovaloid in real affine 3-space A, with constant equiaffine scalar curvature $\kappa$ is an ellipsoid.

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Blaschke proved instead the following result: Any infinitesimal normal deformatiom of an ellipsoid in $A_3$ with $\delta\kappa = 0$ is rigid.

In the first part we show that Blaschke's deformation result mainly is of local nature: Any infinitesimal deformation of a hypersurface of second order consists of a family of hypersurface of second order.

In the second part we give a proof of the original Blaschke conjecture, which follows from the following:

Theorem. (M.Kozlowski; U.Simon) A hyperovaloid with equiaffine Einstein metric and constant scalar curvature is an ellipsoid.

In the third part we give a survey on known results on hypersurface with constant curvature functions. Furthermore we prove results on compact hypersurfaces with boundary,e.g.

Theorem. Let M be compact with boundary $x:M \longrightarrow A$ a hypersurface which bounds a convex set in A. If the mean Curvature is constant and if on $\partial M$:

i) the Pick invariant vanishes;

ii) $x(\partial M)$ is a shadow line with respect to parallel light.

Then x(M) lies on a quadric. (A.Schwenk, U.Simon)

## Affine workshop

For a hypersurface $x:M \longrightarrow A$ (as above) the immersion x induces a linear connection ${}^1\Gamma$ on M, while the conormal-immersion $X:M \longrightarrow V^*$ ( V vector space corresponding to A, $V^*$ dual vector space ) induces another connection ${}^2\Gamma$, we prove:

Theorem. If for two immersions $x, x^*:M \longrightarrow A$ with relative normals $y, y^*$ the connections ${}^1\Gamma = {}^1\Gamma^*$, ${}^2\Gamma = {}^2\Gamma^*$ coincide and $\operatorname{rank}({}^2\mathrm{Ricci})=n$ x(M) and $x^*(M)$ differential by a regular affine mapping.

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There is a corresponding existence result.

Globally we have:

Theorem. A hyperovaloid $x:M \longrightarrow A$ is uniquely determined by ${}^2\Gamma$.

(U.Simon)

K.Grove

Closed Dopin hypersurfaces.

There are various geometric situations which in a natural way gives size to a decomposition of a manifold $M = DB_o \cup DB_1$ into the union of disc bundles $DB_i$ with common sphere bundle $E = \partial DB_o = \partial DB_1$, e.g. 1) manifolds of cohomogeneity 1 (Mostert); 2) Isoparametrc hypersurfaces in spheres (Münzner); 3) Closed Dupin hypersurfaces (in spheres) (Thorbergsson). In joint work with S.Halperin a complete classification theorem for the $\mathbb{Q}$-homology type of the homology fibre F of $E \longrightarrow M$ has been obtained. This can be used in the above situations. In particular one obtaines:

Theorem. (Grove-Halperin) For a closed Dupin hypersurface $E^n \subset S^{n+1}$ $(\mathbb{R}^{n+1}, H^{n+1})$ the number g of different principal curvature is $g = 1,2,3,4$ or 4 (proved first by Thorbergsson). More over there are (except for $g = 4$) the same restrictions on the possible multiplicities as for isoparametric hypersurfaces.

W.Ballmann

Manifolds of nonpositive curvature.

A survey of the work of M.Brin, K.Burns, P.Eberlein, R.Spatzier and the speaker on the structure of manifolds of nonpositive sectional curvature was given. The rank of such a manifold M is the maximal number k such that any geodesic in M is contained in an

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immersed k-flat. There are the following results:

Theorem 1. Suppose M is compact. Then the geodesic flow of M is ergodic if and only if rank(M) = 1.

Theorem 2. Suppose volume(M) $<\infty$ and that the sectional curvature of M is also bounded from below. If $\tilde{M}$ is irreducible, the M is either a space of rank one or a symmetric space of non-compact type.

An application of the latter result was discussed, namely a characterization of irreducible locally symmetric spaces of non-compact type of rank at least two and of finite volume in terms of the algebraic structure of its fundamental group. Here a Riemannian manifold is called irreducible if none of its finite covering spaces is a Riemannian product.

G.Thorbergsson

Tight immersion of simply connected four-manifolds.

It was proved that infinitely many simply connected four-manifolds amony them the Kummer surface, do not admit any tight immersion into a euclidean space. This was proved by showing that a simply connected four-manifold $M^4$ which admits a tight immersion with substantial codimension two splits differentiably as

$$M^4 = S^2 \times S^2 \# N^4.$$

and as

$$M^4 = P_2\mathbb{C} \# N^4$$

if the substantial codimension is greater or equal to three.

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U.Hamenstädt

Geometric aspects of Carnot metrics.

Given a Lie group G with left invariant metric g, a linear subspace $\mathfrak{g}$ of the Lie algebra $\mathfrak{g}$ generating $\mathfrak{g}$, the associated Carnot metric d has the following property:

For all $p \in G$, $X \in \mathfrak{g}$ there is a family of geodesic with respect to d, parametrized by $\mathbb{R}^{\dim G - \dim \mathfrak{g}}$ with starting point p, initial tangent X. Any $q \in G$ can be joined to p by a minimizing geodesic. All geodesic are smooth. Two situations, where Carnot metrics occur naturally, were discussed:

1) The moving Frenet frame of a nondegenerate closed curve on $S^{n-1}$ define a correspondence of the space of these curves to the space of closed curves on SO(n) tangent to an open cone of a left invariant distribution $\mathfrak{g}$ as above. Now the nondegenerate homotopy classes of curves on $S^n$ ( resp. $\mathbb{R}^n$ ) are classified for $n \geq 3$ by the element of $\pi_1 O(n+1)$ ( resp. $\pi_1 O(n)$ ).

2) The boundary of homogeneous manifold of strictly negative curvature is foliated by "submanifolds" carrying a Carnot metric in a natural way. Any quasiisometry between such spaces extend to a homeomorphism of the boundaries preserving the leafs This admits a classification of these spaces by quasiisometry.

E.Heintze

Submanifolds with parallel second fundamental form.

If the second fundamental form $\alpha$ of a submanifold $M \subset N$ is parallel i.e. $D\alpha = 0$, then M is determined uniquely by $\alpha_p: T_pM \times T_pM \longrightarrow T_pM^{\perp}$ at one point $p \in M$. Necessary conditions for an arbitary (symmetric,

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bilinear) $\bar{\alpha}: U \times U \longrightarrow U^{\perp}$, $U \subset T_pN$, are given to ensure that $\bar{\alpha}$ is the second fundamental form of a submanifold with $D\alpha = 0$. These conditions are sufficient if N is analytic. An application to symmetric N is made.

M.Min-Oo

Some Remark on a theorem of Guillemin and Kazhdan.

V.Guillemin and D.Kazhdan proved that a compact 2-manifold with negative curvature is spectrally rigid, i.e. do not admit any non-trivial deformations of the metric leaving the spectrum of the Laplacian invariant. In higher dimensions they prove the corresponding theorem under the following pinching assumption on the sectional curvature: $-1-\frac{1}{n} < K < -1+\frac{1}{n}$. We improve this result by showing that spectral rigidity is still true for manifolds with negative definite curvature operator. A similar result concerning the spectrum of the Schrödinger operator $\Delta+q$ can also be obtained under this assumption. Our proof follows almost exactly the original proof except that we are able to estimate a quadratic expression of Bochner-Yano type involving the curvature in a different manner.

U.Abresch

Tori of constant mean curvature.

It has been conjectured that the round spheres are the only immersed surfaces $M^2 \longrightarrow \mathbb{E}^3$ with constant mean curvature. Alexandrov has proved this for embedded surface and H.Hopf showed that such a result holds for immersed spheres. However last year H.C.Wente found tori of constant mean curvature $H = \frac{1}{2}$ in $\mathbb{E}^3$ by solving the partial

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differential equation $\Delta\omega + \sinh\omega\cosh\omega = 0$, which is actually the Gauss equation in a conformal parameterization via lines of curvature. Parameters have to be adapted carefully to get a closed surface from a doubly periodic solution $\omega$. Wente's solution $\omega$ have large maximum $\omega_{max}$, and hence - the principal curvature vary by a factor of $\exp(2\omega_{max})$ - are not drawable. It has been possible to find a numericl solution with $2\omega_{max} \approx 3.5$ which gives rise to the simplest torus the construction can yield; its lattice is a sublattice of the curvature functions with index 3. Moreover this immersion is known to be not regularly homotopic to a standard torus.

J.Brüning

Some $L^2$-index theorem.

Consider a manifold M and hermitian bundles E, F $\longrightarrow$ M. Let D be a first order elliptic operator between the sections. Moreover assume that there is U M such that $\overline{M-U}$ is a compact manifold with boundary and there are isomorphisms of Hilbert spaces.

$$\varphi_1 : L^2(E|_U) \longrightarrow L^2(I,H), \quad I = (0,1), \quad H \text{ is Hilbertspace;}$$

$$\varphi_2 : L^2(F|_U) \longrightarrow L^2(I,H),$$

$$\varphi_1(C_0^\infty(E|_U)) = C_0^\infty(I,H_A)$$

$$= \varphi_1(C_0^\infty(F|_U)) \ , \quad H_A \quad H \text{ dense subspace,}$$

$$\varphi_2 \cdot D \cdot \varphi_1^{-1} = d_x + \frac{A(x)}{x} : C_0^\infty(I,H_A) \longrightarrow C_0^\infty(I,H_A)$$

where A(x) is a smooth family of some operators in H with common domain $H_A$ each having a discrete spectrum (and satisfying more technical conditions modelling elliptic operators on compact manifold). Then we prove the following result (joint work with R.Seeley)

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<u>Theorem</u>. The closed extensions of D as operators $L^2(E) \longrightarrow L^2(F)$ are classified by the subspaces V of $\bigoplus_{\substack{\lambda\in spec.A(o)\\ |\lambda|<1}} Ker(A-\lambda\cdot id)$. If the operator corresponding to V is denoted by $D_V$ then $D_V^* = D_{V^\perp}$.

Moreover

$$\text{index } D_V = \int_M \omega_D - \frac{1}{2}\eta_{A(O)} + C_V$$

where $\omega_D$ is the Atiyah-Singer-indexform of D (whose integral must be possibly regularized). $\eta_{A(O)}$ is the $\eta$-invariant of Atiyah-Patodi -Singer, and $C_V$ is a function of the eigenvalues $\lambda$ of $A(O)$, $|\lambda|<\frac{1}{2}$. If there are no such eigenvalues, $C_V = 0$.

This result is shown to generalize Cheeger´s Gauß-Bonnet theorem and signature theorem on manifolds with cone-like singularities and Atiyah-Patodi-Singer $L^2$-index theorem for manifolds with cylindrical ends. In particular, the Gauß-Bonnet theorem for manifolds with boundary arises from a nontrivial $\eta$-invariant.

Berichterstatter: X.W.Peng

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TAGUNGSTEILNEHMER:

| | |
|---|---|
| U.Abresch | Mathematisches Institut der Universität Bonn<br>Wegelerstr.10, 5300 Bonn 1 |
| W.Ballmann | Department of Mathematics,University of Maryland<br>College Park, MD 20742, USA |
| V.Bangert | Mathematisches Institut der Universität Bonn<br>Wegelerstr.10, 5300 Bonn 1 |
| E.Binz | Lehrstuhl für Mathematik I, Universität Mannheim<br>Seminargebäude A5 68 Mannheim |
| J.P.Bourguignon | Centre de Mathématiques, Ecole Polytechnique,<br>F-91128 Palaiseau Cedex, France |
| J.Brüning | Institut für Mathematik, Memmingerstr.6<br>8900 Augsburg |
| E.Calabi | Department of Mathematics, University of<br>Pennsylvania, Philadelphia,PA 19104, USA |
| D.DeTurck | Department of Mathematics E1, University of<br>Pennsylvania, Philadelphia, PA 19104, USA |
| P.Dombrowski | Mathematisches Institut der Universität Köln<br>Weyertal86-90, 5000 Köln 41 |

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| | |
|---|---|
| C.L.Epstein | Department of Mathematics, Fine Hall, Washington Rd. Princeton, NY 08544, USA |
| J.M.Eschenburg | Mathematisches Institut der WWU, Einsteinstr.62, 4400 Münster |
| D.Ferus | Mathematisches Institut, Straße des 17. Juni 135 1000 Berlin 12 |
| C.Gerhardt | Institut für Angewandte Mathematik, Im Neuenheimer Feld 294, 6900 Heidelberg |
| D.Gromoll | Department of Mathematics, S.U.N.Y. , Stony Brook,N.Y. 11794, USA |
| K.Grove | Department of Mathematics, University of Maryland College Park, MD 20742, USA |
| H.Hagenguth | Mathematisches Institut der Universität Bonn Beringstr. 1, 5300 Bonn 1 |
| U.Hamenstädt | Mathematisches Institut der Universität Bonn Beringstr.1, 5300 Bonn 1 |
| E.Heintze | Mathematisches Institut der Universität Münster Einsteinstr.62 , 4400 Münster |
| N.Hingston | Department of Mathematics E1, University of Pennsylvania, Philadelphia PA 19104, USA |

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| | |
|---|---|
| D.Hulin | Université de Paris VII,U.E.R.des Math.<br>Tour45-55, 2 place Jussieu F-75251 Paris-Cedex 05 |
| H.Karcher | Mathematisches Institut der Universität Bonn<br>Beringstr.4, 5300 Bonn 1 |
| W.Klingenberg | Mathematisches Institut der Universität Bonn<br>Beringstr.1, 5300 Bonn 1 |
| N.H.Kuiper | IHES Route de Chartres 35<br>91440 Bures-sur-Yvette,France |
| J.Lafontaine | U.E.R de Mathématique de l'Université Paris VII<br>F-75251 Paris Cedex 05, France |
| L.Lemaire | CP 218 Campus Plaine,Université Libre de Bruxelles,<br>BD.du Triomphe,1050 Bruxelles, Belgique |
| C.Margerin | Centre de Mathématiques,Ecole Polytechnique<br>F-91128 Palaiseau Cedex, France |
| W.Meyer | Mathematisches Institut der WWU<br>Einsteinstr.62 , 4400 Münster |
| M.Min-Oo | Mathematisches Institut der Universität Bonn<br>Wegelerstr.10 , 5300 Bonn 1 |
| K.Nomizu | Department of Mathematics, Brown University,<br>Providence, RI 02912, USA |


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| | |
|---|---|
| X.W.Peng | Mathematisches Institut der Universität Bonn<br>Beringstr.1, 5300 Bonn 1 |
| U.Pinkall | Max-Planck-Institut für Mathematik,<br>Gottfried-Claren-Str.26, 5300 Bonn 1 |
| H.B.Rademacher | Mathematisches Institut der Universität Bonn<br>Beringstr.1 5300 Bonn 1 |
| H.Reckziegel | Mathematisches Institut der Universität Köln<br>Weyertal 86-90, 5000 Köln 41 |
| E.Ruh | Mathematisches Institut<br>Universitätsstr.1, 4000 Düsseldorf |
| Y.Shikata | Nagoya University,Mathematics Institute,<br>Fac. of Science ,Chikusa-ku, 464 Nagoya, Japan |
| L.Simon | Department of Matnematics, Australian National<br>University, Canberra,A.C.T. 2600, Australia |
| U.Simon | TU Berlin, FB Mathematik, MA8-3,<br>Straße 17. Juni 135, 1000 Berlin 12 |
| G.Thorbergsson | Mathematisches Institut der Universität Bonn<br>Wegelerstr.10, 5300 Bonn 1 |
| F.Tricerri | Dipartimento di Matematica,Universitá via<br>Principe Amedeo 8, 10123 Torino, Italy |


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M.Ville 219 boulevard Raspail
75014 Paris,France

R.Walter Abteilung Mathematik-Lehrstuhl VII
Universität Dortmund,Postfach 500500
4600 Dortmund 50

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MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Differentialgeometrie im Großen

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D.Gromoll

The low dimensional metric foliations of euclidean spheres

A metric foliation $F^k$ in $S^{n+k}$ is a ( local or global ) riemannian foliation which is bundlelike with respect to the euclidean metric, i.e. any two leaves are locally everywhere equidistant. The basic examples are:

(i) The "flat" foliations $F^k = \{\exp_\nu X\}$, where X is a parallel section over any submanifold $N^k \subset S^{n+k}$ with flat normal bundle $\nu$, so in particular always for k=1 or n=1.

(ii) "Homogeneous" foliations $F = \{$principal orbits $G/H\}$ on an open dense subset of $S^{n+k}$, for any $G \subset O(n+k+1)$.

iii) "Spherical sums" of the above.

We call F an "isoparametric family" if the mean curvature form $\kappa$ is basic, or equivalently $d\kappa = 0$, or all principal curvatures of the second fundamental form $S_X$ are constant along leaves for any basic normal field X. F is isoparametric if O'Neill's integrability tensor A is "substantial" at some point, i.e. $A_X$ is onto for some X. We then prove in that case that F is locally homogeneous for $k \leq 3$. In the global (non-singular) situation it is shown that A must be subatantial everywhere, $k \leq 3$. A main application is the following rigidity result:

Theorem: Let $F^k$ be a global metric foliation of $S^{n+k}$, $k \leq 3$.

k=1: F is generated by a non-vanishing Killing field. If F is a fibration, then it is congruent to the Hopf fibration $S^{2m+1} \to \mathbb{C}P^m$.

k=2: Is not possible.

k=3: F is a fibration, congruent to the Hopf fibration $S^{4l+3} \to \mathbb{H}P^l$.

This is joint work with K.Grove.

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