MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 29/1985

Riesz Spaces and Operator Theory

1.7. bis 5.7.1985

Die Tagung fand unter Leitung von Herrn Prof. W. A. J. Luxemburg (Pasadena) und Herrn Prof. H. H. Schaefer (Tübingen) statt. Es wurden 32 Vorträge über neue Forschungsergebnisse gehalten. Ferner wurde die Tagung zu regem Gedankenaustausch und zur Zusammenarbeit genutzt.

Der überwiegende Teil der Vorträge beschäftigte sich mit (positiven) Operatoren auf Riesz-Räumen oder Banachverbänden. Dabei wurden Beiträge zur Spektraltheorie, Ergodentheorie und zur Theorie der einparametrigen Halbgruppen geleistet. Ferner wurde die Ordnungstheorie zur Gewinnung neuer Resultate über Spektraloperatoren auf lokal konvexen Räumen genutzt. Mehrere Vorträge beschäftigten sich mit Fortsetzungsproblemen für positive Operatoren

Weiterhin wurden ordnungstheoretische Fragen aus der Maßtheorie, Choquettheorie, der Theorie der Banachalgebren und der Theorie der Laplacetransformationen behandelt. Schließlich wurden Anwendungen in Ökonomie und Numerik besprochen.

Vortragsauszüge:

C. D. ALIPRANTIS:

Banach lattices in Economics

Recently, Banach lattices have been found useful in Economics. The talk will summarize this progress and outline the problems that lie ahead.

T. ANDO:

Reflexivity and bicommutant property of contractions

This is to introduce some recent results of my collaborator, K. Takahashi.

Let T be a contraction on a Hilbert space. T is said to have REF (res. BCP) if $\operatorname{Alg}\operatorname{Lat}(T) = \operatorname{Alg}(T)$ (resp. $\operatorname{Alg}(T) = \{T\}''$).

A principal result is that there is a non-zero operator X such that $XT = SX$ or $XT^* = SX$, S being a unilateral shift, then T has REF as well as BCP. An effective condition which guarantees the existence of such X is that the defect operator $D_T = (I - T^*T)^{1/2}$ is of Hilbert-Schmidt and the spectrum of T covers the whole unit disc.

If T is a C_{11}-contraction, i. e. quasi-similar to a unitary and if the characteristic function θ_T has a scalar multiple, then REF and BCP of T can be well characterized.

W. ARENDT:

Bernstein's theorem and resolvent positive operators

Let A be an operator on a Banach lattice such that $(\lambda-A)^{-1}$ exists and is positive for $\lambda > \lambda_0$. We observe, that then the function $\lambda \longrightarrow (\lambda-A)^{-1}$ is automatically completely monotonic. Applying Bernstein's theorem one obtains that $(\lambda-A)^{-1}$ is a Laplace Stieltjes transform (if in addition A has dense domain or E is order continuous). This yields a theorem of existence and uniqueness of the Cauchy problem associated with A. Conversely, if A admits unique positive solutions of the Cauchy problem for sufficiently many positive initial values, then A is resolvent positive.

R. BECKER:

Representation of positive linear forms on spaces of functions

On donne des conditions nécessaires et suffisantes pour que toute forme ≥ 0 sur un espace vectoriel réticulé de fonctions soit representêe par une mesure de Radon, ou soit une intégrale de Daniell. On dégage aussi des conditions suffisantes simples et constructives, fournissant donc des examples. Le cas des mesures de Radon concerne surtout les espaces héréditairs de fonctions continues sur un espace localement compact. Le cas des intégrales de Daniell concerne des espaces héréditairs de fonctions continues sur un espace topologique quelconque.

S. BERNAU:

Extension of Vector Lattice Homomorphisms

Suppose B is a complete Boolean algebra, D a distributive lattice and φ a lattice homomorphism from a sublattice, D_0, of D, into B, then φ can be extended to a lattice homomorphism of D into B. This generalizes Sikorski's extension theorem for Boolean algebras. It also leads to a new proof that if N is a complete vector lattice, L a vector lattice, M a sub-vector lattice of L, and f a vector lattice homomorphism of M into N, then f can be extended to a vector lattice homomorphism, g, of L into N. The proof of this is constructive, after applying the lattice homomorphism extension theorem, and leads to the previously unknown result that g is uniquely determined by the collection of polar subspaces, $g(x)^{\perp\perp}$ $(x \in L)$. The paper concludes with a new approach to the known result that the vector lattice homomorphic extensions of f are precisely the extreme points of the set of positive extensions of f.

G. BUSKES:

Extension of Riesz homomorphisms

Some theorems on extension of Riesz homomorphisms are presented. In particular, we will formulate a cardinal version of the Luxemburg-Schep-Lipecki theorem on majorizing injective Riesz spaces, a char-

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acterization of ideal-injective Banach lattices (i. e. continuous Riesz homomorphisms from ideals into such a Banach lattice extend), and a characterization of the injective elements in the class of spaces of the form C(K).
Furthermore we will discuss some connections with simultaneous extension operators which are Riesz homomorphisms.

P. G. DODDS:

Riesz spaces and scalar type spectral operators

Let X be a quasi-complete l.c.t.v.s. and denote by L(X) the space of continuous linear operators on X . A closed subalgebra N of L(X) is called reflexive if N consists precisely of those operators $T \in L(X)$ which leave invariant each closed N-invariant subspace of X . If $T \in L(X)$, then T is called reflexive if the strongly closed algebra generated by the identity and T is reflexive. The principal result discussed is the following: if L(X) is sequentially complete for the topology of pointwise convergence on X , then each closed unital subalgebra of the strongly closed algebra generated by an equicontinuous, complete (in the sence of Bade) Boolean algebra of projections in X , is reflexive. It follows, in particular, that each scalar type spectral operator (in the sense of Dunford) is reflexive. This is joint work with Ben de Pagter and Werner Ricker.

K. DONNER:

Extension of positive operators in spaces of continuous functions

Using isotone sublinear operators with values in the cone S of all lower semicontinuous, extended real-valued functions on a metrizable compact space X an extension theorem of Hahn-Banach type for positive linear operators in C(X) is proved. Uniqueness results can also be formulated in terms of sublinear S-valued maps.

P. VAN ELDIK:

Equimeasurable sets in (normed) Riesz spaces

We introduce Grothendieck's notion of equimeasurable sets in the setting of Riesz spaces and develop the basic properties of these sets. In the second part of the talk we discuss the application of these results in obtaining characterizations of abstract kernel operators and Carleman kernel operators.

Our main results are

Theorem 1. Let E and F be Dedekind complete spaces such that for every $0 \le f \in F$ the ideal F_f generated by F is separated by $(F_f)^{\sim}_n$.

If $T : E \to F$ is a linear operator such that $T \in L_b(E, F^u)$ (F^u the universal completion of F), then the following statements are equivalent

(a) $T \in (E^{\sim}_n \otimes F^u)^{dd}$ (an abstract kernel operator)

(b) T is order continuous from E into F^u and T maps order intervals into equimeasurable sets.

Theorem 2. Under the same conditions as stated in theorem 1 with E a Dedekind complete Banach lattice such that E' has order continuous norm, the following statements are equivalent.

(a) T is a Carleman operator and $T \in (E^{\sim}_n \otimes F^u)^{dd}$

(b) T is order continuous from E into F^u and T maps norm bounded sets into equimeasurable sets.

W. FILTER:

Representations of Riesz spaces as spaces of measures

It is shown that exactly those Riesz spaces E which are separated by the set E^{π} of their order continuous linear forms (such spaces are called π-spaces) can be represented as spaces of measures. More exactly: If E is a π-space, if e is a weak unit of the extended order dual E^{ρ} of E, and if $R \subset E^{\pi}$ is a set of components of e with $e = \bigvee_{g \in R} g$ (such pairs (e, R) always exist), then there exists a locally compact hyperstonian space Y (unique up to a canonical homeomorphism) with the following properties: There exist an injective Riesz homomorphism $v : E \to M(Y) := \{\mu \mid \mu$ is a normal Radon measure on $Y\}$ and a Riesz isomorphism $\hat{u} : E^{\rho} \to C_{\infty}(Y) := \{f \in \mathbb{R}^Y \mid f$ is continuous, $\overline{\{|f| < \infty\}} = Y\}$ such that vE is order dense in $M(Y)$, $\hat{u}e = 1_Y$, $\operatorname{supp} \hat{u}g$ is compact for all $g \in R$, $Y = \bigcup_{g \in R} \operatorname{supp} \hat{u}g$, and $\xi(x) = \int \hat{u}\xi \, d\, vx$ for all $(x,\xi) \in E \times E^{\rho}$ with $\xi(x) \in \mathbb{R}$.

S. GOLDBERG:

Factorizations and extensions of Hilbert-Schmidt operators along totally ordered sets of orthogonal projections

Let $\mathcal{P}$ be a set of orthogonal projections on a separable Hilbert space H with the following properties:

1) $0, I \in \mathcal{P}$ 2) $P, Q \in \mathcal{P} \Longrightarrow P \leq Q$ or $Q \leq P$

3) If $\{P_n\} \subset \mathcal{P}$ and $P_n x \longrightarrow P_x$ $\forall x \in H$, then $P \in \mathcal{P}$

4) If P, Q are in $\mathcal{P}$ and $P < Q$, then there exists a $P_1 \in \mathcal{P}$ such that $P < P_1 < Q$.

Let S_2 denote the Hilbert-Schmidt operators on H . Define $\mathcal{Q}_+ = \{A \in S_2 : AR(P) \subset R(P)\}$, $\mathcal{Q}_- = \{B \in S_2 : BR(I-P) \subset R(I-P)$ for all $P \in \mathcal{P}$. Then $S_2 = \mathcal{Q}_+ \oplus \mathcal{Q}_-$. Therefore, given $T \in S_2$, $T = A_+ A_-$, $A_\pm \in \mathcal{Q}_+$. The operators $A_+ + A_-$ are called the upper and lower triangular parts, resp., of T . Motivated by a problem in scattering theory, we (Gohberg-Goldberg) solve the following Given $F_\pm \in \mathcal{Q}_\pm$, give necessary and sufficient conditions for the existence of a Hilbert-Schmidt operator K with the following properties.

a) $I + K$ is invertible b) K has upper triangular part F_+

c) $(I+K)^{-1} - I$ has lower triangular part F_- and

d) $I + K = (I+X_+)(I+X_-) = (I+Y_-)(I+Y_+)$, when $(I + X_\pm)^{\pm 1}$, $(I+Y_\pm)^{\pm 1}$ exist and are in $I + \mathcal{Q}_\pm$.

G. GREINER:

Domination for semigroups

A C_0-semigroup $(S(t))_{t\geq 0}$ on a Banach lattice E is dominated by the positive semigroup $(T(t))_{t\geq 0}$ provided that $|S(t)f| \leq T(t)|f|$ for all $t \leq 0$, $f \in E$. If B and A are the generators of $(S(t))$ and $(T(t))$ resp., then domination can be characterized as follows:

$$\mathrm{Re} < \mathrm{sign}\, \bar{f} \cdot Bf\ , \Phi > \ \leq \ <|f|, A'\Phi> \text{ for all } f \in D(B),\ \Phi \in D(A')_+$$

The question arises whether for a given semigroup $(S(t))$ a dominating semigroup $(T(t))$ exists and, if so, when does a minimal one exist (so to say an 'absolute value' of $(S(t))$. For example, for semigroups on L^p-spaces which are contractions for the regular norm there exists a dominating semigroup . Furthermore, for a semigroup on a Banach lattice with order continuous norm which is dominated there exists an absolute value. For special semigroups (e.g. those arising in the theory of retarded linear differential equations) one can describe the generators of the absolute value.

J. J. GROBLER:

The Fremlin cone in the Riesz tensor product

Let E and F be Archimedean Riesz spaces with order units e and f. We construct the Riesz tensor product $E \bar{\otimes} F$ of E and F by operator methods and characterize the Archimedean cone $\mathcal{F}$ induced by $E \bar{\otimes} F$ on $E \bar{\otimes} F$. This cone turns out to be the smallest Archimedean cone which contains the semi-Archimedean cone

$E_+ \otimes F_+ := \{\Sigma_{i=1}^n x_i \otimes y_i : n \in \mathbb{N}, x_i \in E_+, y_i \in F_+\}$.

Explicitly $\mathcal{F} = \{u \in E \bar{\otimes} F : u + \varepsilon (e \otimes f) \in E_+ \otimes F_+ \ \forall \varepsilon > 0\}$.

R. GRZASLEWICZ:

Structure of positive operators on L^p-spaces

I. Theorem. Let $1 < p < \infty$, and let $0 \le T \in \mathcal{L}(L^p(m), L^p(n))$. Then a set

$$M(T) = \{f \in L^p(m) : \|Tf\| = \|T\| \|f\| \}$$

is a closed linear sublattice of $L^p(m)$.

Moreover, in the class of Orlicz spaces L^ϕ only L^p-spaces have the above property.

II. Some faces of the positive part of the unit ball of the space of operators on L^p are affinely isomorphic to the set of doubly stochastic operators with respect to appropriate measures. Therefore we can restrict the problem of characterizing extreme positive contraction on L^p to the problem of characterizing extreme doubly stochastic operators (measures).

W. HACKENBROCH:

Characterizations of extremal measure extensions

If μ is a probability measure on some σ-algebra $\mathfrak{A}$ contained in a larger σ-algebra $\mathfrak{A}'$, the set of extensions μ' of μ to $\mathfrak{A}'$ is convex (possibly empty). Many constructions of extensions naturally lead to extremal points of this set. The aim of the talk is to "dilate" the underlying measure space in such a way that, in this dilation, the extremal extensions are exactly the natural ones occuring in the classical constructions, thereby avoiding all additional assumptions on the measure space otherwise needed to obtain a similar characterization of extremal extensions.

D. HART:

An abstract Banach-Stone type theorem

Let A and B be semi-simple commutative Banach algebras. An operator $T : A \to B$ is called disjointness preserving if $f \cdot g = 0 \ (f, g \in A) \implies T(f) \cdot T(g) = 0$.

Theorem. Suppose A and B satisfy the following condition: $f \cdot g = 0 \iff$ for every extreme point of the dual unit ball $\varphi(f)\varphi(g) = 0$.

Then every isometry from A onto B is disjointness preserving. We discuss applications of this theorem to spaces of continuously differentiable functions.

C. B. HUIJSMANS:

The Arens multiplication in the second order dual of lattice ordered algebras

Throughout A is an Archimedean lattice ordered algebra with point separating order dual A'.

Theorem 1. A'' and $(A')'_n$ (the band of all order continuous linear functionals on A') are lattice ordered algebras with respect to the Arens multiplication.

Theorem 2. If A is an f-algebra, then $(A')'_n$ is an f-algebra. If A has a unit element, then $A'' = (A')'_n$ is an f-algebra.

Now, let A be semiprime and suppose that A satisfies the Stone condition (so if we embed A in orth (A), then $a \in A^+ \implies a \wedge I \in A^+$) .

Theorem 3. $(A')'_n$ is semiprime $\iff \exists\ 0 \leq b_\tau \uparrow$ in A^+ such that $f(a) = \sup_\tau f(a b_\tau)$ for all $a \in A^+$, $f \in (A')^+$.

Theorem 4. $(A')_n'$ has a unit element $\Longleftrightarrow$ $\sup\{f(a) : a \in A^+ , a^2 \leq a\} < \infty \quad \forall f \in (A')^+ \Longleftrightarrow$ every $f \in (A')^+$ has a positive extension to orth (A)

Theorems 3 and 4 have the advantage that one can draw conclusions about the Arens multiplication in $(A')_n'$ without calculating $(A')_n'$ explicitly.

H. KÖNIG:

On the basic extension theorem in measure theory

The talk extends and unifies the known results on the extension of set functions with values in $[0,\infty]$ defined on lattices of subsets to measures. One modifies the known sufficient conditions (tightness and the like) to become necessary and sufficient conditions which are of the same simple type. To achieve this one introduces new versions of the pertinent outer and inner measures.

I. LABUDA:

Domains of integral transformations

Let (X, dx) and (Y, dy) be positive measure spaces. I will consider an integral transformation K between $L^0(X)$ and $L^0(Y)$, with kernel $k = k(x, y) \in L^0(X \times Y, dx \otimes dy)$, defined by the usual formula:

$$(Ku)(y) = \int_X k(x, y)\, u(x)\, dx$$

Then the natural domain $\mathcal{D}$ of K is $\{u \in L^0(X) : Ku \in L^0(Y)\}$ together with the natural topology on $\mathcal{D}$ which is defined as the graph topology of the sublinear transformation $u \longrightarrow |K|\,|u|$.
The extended domain $\mathcal{D}^{\#}$ of K is the largest solid vector subspace in $L^0(X)$ together with a locally solid topology (in fact, the weakest one) such that

$$K : \mathcal{D}^{\#} \longrightarrow L^0(Y)$$

is continuous. Some properties of $\mathcal{D}$ and $\mathcal{D}^{\#}$ will be presented.

Z. LIPECKI:

Maximal-valued extensions of positive operators

PROPOSITION. Let X be a vector lattice and let $V \subset X$ be a cone.
(a) $V-V$ is a vector subspace of X. If V is, moreover, closed under finite suprema, then $V-V$ is a sublattice of X.

(b) Let Z be a majorizing vector sublattice of X with $Z \subset V$ and let Y be an order complete vector lattice. A positive operator $T : Z \longrightarrow Y$ extends to a positive operator $S : V-V \longrightarrow Y$ such that $S(v) = T_e(v)$ for all $v \in V$ (*) if and only if T_e is additive on V. If, moreover, V is closed under finite suprema and T is a vector-lattice homomorphism, then so is S.
This proposition is applied to extending vector-lattice homomorphisms defined on Z to the whole of X and to representing positive operators on $C_b(\Omega)$.

(*) By definition, $T_e(v) = \inf \{T(z) : x \leq z \in Z\}$.

H. P. LOTZ:

The strong ergodic theorem for positive operators

We will prove the following theorem:
Let E be an ordered Banach space with normal generating cone and let $T \in L(E)$ be positive. If for every $x \in E$ the sequence $(T_n x)$ is relatively weakly compact then the operators T_n converge in strong operator topology.

A. MARQUINA:

Stability and positivity of finite difference operators

In the context of theory of discretizations of Stetter [1] and the extension of Sanz-Senna and Palencia [2], it is studied, from the point of view of positive operator theory, the necessary conditions for stability of abstract difference operators, obtaining, as a consequence, the Godunov-Ryabenki theorem.

[1] Analysis of Discretizations Methods for Ordinary Differential Equations. Springer 1973.
[2] Sanz Serna, J. and C. Palencia: A General Equivalence theorem. (Preprint).

P. MEYER-NIEBERG:

Some interpolation properties for the spectral radius

Let E be an order complete vector lattice and let $T : E \longrightarrow E$ be a positive linear operator. We define a local spectral radius for T and derive some interpolation results. One of the results

is the following:
Let E be a Banach lattice with order continuous norm and let F be a Banach lattice "close to E" with $\nu(T|_F) < \lambda < \nu(T|_E)$ then there exists an intermediate space G with $\nu(T|_G) = \lambda$.

B. DE PAGTER:

Positive projections and averaging operators

We discuss some properties of positive projections in lattice ordered algebras, in particular in f-algebras.
Theorem A. Any positive contractive projection T in an (unitary) f-algebra satisfies $T(a\ Tb) = T(Ta\ Tb)$ for all a, b.
Theorem B. The range of the positive projection T in an (unitary) f-algebra is a subalgebra iff T is averaging, i.e., $T(a\ Tb) = Ta \cdot Tb$ for all a, b.
These theorems extend results of Kelley and Seever for positive projections in $C(K)$-spaces.

A. VAN ROOIJ:

Spaces of regular operators

For (Archimedean) Riesz spaces E and F, the regular linear operators $E \longrightarrow F$ form an ordered vector space $L^r(E,F)$.
Question: when is $L^r(E,F)$ a Riesz space? It is in case F is Dedekind complete; moreover, in that situation the positive part of an element T of $L^r(E,F)$ is known to be given by
(*) $T^+f = \sup\{Tg : g \in E^+,\ g \le f\}$. $(f \in E^+)$.
There are many other situations in which $L^r(E,F)$ is Riesz (e.g., F is any Banach lattice, E is the space of all A-measurable functions, where A is any σ-algebra of sets). In each case I know of, (*) is true for all T in $L^r(E,F)$.
Second question: does (*) hold whenever $L^r(E,F)$ is Riesz?

E. SCHEFFOLD:

Banachverbandsalgebren mit multiplikativer Zerlegungseigenschaft

Der Vortrag befaßt sich mit den komplexen Homomorphismen, den zentralen Homomorphismen und den Strukturidealen in den genannten Algebren.
Die wichtigsten Resultate sind:

1. Die Menge der komplexen Homomorphismen ist "signumsinvariant".
2. Die zentralen Homomorphismen bilden eine betrags- und konjugationsinvariante Halbgruppe.
3. Jede solche endlich dimensionale kommutative, halbeinfache und halbstruktureinfache Algebra mit Einselement ist eine direkte Summe von Gruppenalgebren.

A. R. SCHEP:

Nuclearity and compositions of kernel operators

A classical theorem of Lalesco and Chang states that an operator is nuclear on L_2 if and only if it is a product of 2 Hilbert-Schmidt operators. We study generalizations of this result for kernel operators on Banach function spaces. As a consequence of the obtained result we obtain for Banach lattices with order continuous norm (optimal) summability properties of eigenvalues of nuclear operators.

A. R. SOUROUR:

Spectrum preserving linear maps

Theorem 1. (joint work with A. Jafarian). For two complex Banach spaces X , Y , we show that every linear map of $\mathcal{B}(X)$ onto $\mathcal{B}(Y)$ which preserve the spectrum is of the form $\varphi(T) = ATA^{-1}$ or the form $\varphi(T) = BT^*B^{-1}$ where A is an isomorphism of X onto Y , and B is an isomorphism of X^* onto Y . It follows that such a map is a Jordan isomorphism and it is either an algebra isomorphism or anti-isomorphism.

Theorem 2. Let E and F be Banach lattices and $\varphi: \mathcal{L}^r(E) \longrightarrow \mathcal{L}^r($
a surjective linear map. The following conditions are equivalent.

(i) φ preserves the sprectrum

(ii) φ is a Jordan isomorphism

(iii) φ is an algebra isomorphism or anti-isomorphism.

F. TAKEO:

Simplex homomorphism

It is well known that the spectrum and the point spectrum of a lattice homomorphism are cyclic. As for a simplex homomorphism whose second adjoint operator is a lattice homomorphism, its

point spectrum is not necessarily cyclic.
In this report, by investigating the behavior of the point transformation induced by a simplex homomorphism T, we shall show that there is a T-invariant ideal I to which the restriction T_I of T is uniformly ergodic with $P_\sigma(T) \cap \Gamma = P_\sigma(T_I) \cap \Gamma$ and there is a T-invariant simplex subspace A to which the restriction T_A of T is a simplex isomorphism with $P_\sigma(T) \cap \Gamma = P_\sigma(T_A) \cap \Gamma$ under some conditions. By using the result, we show that $P_\sigma(T) \cap \Gamma$ is cyclic if T is a simplex homomorphism and satisfies some conditions.

L. WEIS:

The range of operators on $C(K)$ and their representing stochastic kernel

Let $T : C(X) \longrightarrow C(Y)$ be a bounded linear operator in the spaces of continuous functions on compact, metric sets. With $\mu_y := T'\delta_y$ we can represent T in the form

$$Tf(y) = \int f \, d\mu_y \quad , \quad y \in Y .$$

Denote by $w(\cdot)$ the oscillation of the function

$$y \in Y \longrightarrow \mu_y \in (M(Y), \| \ \|) .$$

Theorem: If $w(y) \neq 0$ for a continuity point of w then there is a perfect $K \subset Y$ s. th. $\chi_k T : C(X) \longrightarrow C(K)$ is surjective. Moreover, there is a subspace $E \subset C(X)$, isometric to $C(X)$, such that $\chi_k \, T|_E : E \longrightarrow C(K)$ is an isomorphism.

This extends a result of H. P. Rosenthal and shows, that T is either an integral operator or behaves 'locally' like a Riesz-homomorphism. The possible 'form' of K is discussed for examples such as the Radon transform and certain random walks.

A. WICKSTEAD:

The injective hull of an Archimedean f-algebra

We consider injectivity in the category of A-modules, where A is a semi-prime Archimedean f-algebra. A is itself a A-module and we show that it is an injective A-module if and only if it is laterally complete and von Neumann regular. The class of weak orthomorphisms, order bounded disjointness preserving linear operators defined on order dense sublattices of A, is introduced and is shown to have the structure of a semi-prime Archimedean f-algebra. It is shown, by proving that it coincides with the complete ring of quotients of A , to be an injective hull of A (as an A-module). In particular the injective hull of A may be given an Archimedean f-algebra structure extending that on A . This property characterises, amongst the class of Archimedean f-algebras, those which are semi-prime.

G. WITTSTOCK:

Integral operators on non-commutative L^p-spaces

Measurable kernels k affiliated with the tensor product $M \bar{\otimes} N$ of two finite von Neumann algebras M and N are defined. To k there corresponds an integral operator $K : L^p(M) \longrightarrow L^q(N)$, $\langle Kf, g\rangle = \mu \bar{\otimes} \nu(f^{1/2} \otimes g\, k\, f^{1/2} \otimes f^{1/2})$ for $0 \leq f \in L^p(M)$, $0 \leq g \in L^{q'}(N)$. K is completely positive if and only if the corresponding kernel k is a positive self-adjoint operator. A completely positive map $T : L^p \longrightarrow L^q$ has a unique decomposition into a completely positive integral operator and a completely positive singular operator, which dominates no integral operator. This decomposition is additiv. Denote by $\varphi_T(f \otimes g) = \langle Tf, g\rangle$ the corresponding linear form on the algebraic tensor product $L^p(M) \odot L^q(N)$. T is an integral operator if and only if φ_T has the following continuity property: Let $0 \leq h_n \leq h$ in $L^p \odot L^q$ and $h_n \longrightarrow 0$ a measure then $\varphi_T(h_n) \longrightarrow 0$.

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Gefördert durch
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