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MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 30/1985

Topological Methods in Nonlinear Analysis
8.7. bis 12.7.1985

Die Tagung fand unter der Leitung von Herrn J. Smoller (Ann Arbor) und Herrn H.-O. Peitgen (Bremen) statt. Im Mittelpunkt des Interesses stand die Diskussion topologischer Methoden (Fixpunktindex, Conley-Index, Gruppeninvarianz, Theorie kritischer Punkte, Maslov Index, Kegelinvariante Operatoren, Homokline Strukturen) in der Theorie gewöhnlicher, retardierter und partieller Differentialgleichungen.

Vortragsauszüge

K. ALLIGOOD:

Why Period-Doubling Cascades Occur.

We show that if a horseshoe is created in a natural manner as a parameter is varied, then the process of creation involves the appearance of attracting periodic orbits of all periods. Each of these orbits will period double repeatedly, with those periods going to infinity. (With J. Yorke) Furthermore, we show that the orbits in the horseshoe can be naturally partitioned into sequences containing a saddle (period k) and infinitely many twisted saddles (one of each period $2^n k$, $n>0$). Each such sequence lies on one component of orbits (in full parameter space).

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A. AMBROSETTI:

We discuss the existence of multiple critical points for functionals $f + \varepsilon g$ in the case in which:

(i) f is invariant under the action of a symmetry group and possesses a compact, connected manifold of critical points;

(ii) g possibly breaks such symmetry.

Applications to the existence of forced oscillations of Hamiltonian systems are given.

S. ANGENENT:

Banach sheaves ... .

A rather general trick is presented that allows one to construct "global inverses" for linear operators that are "locally invertible". This has several applications:

(1) An easy proof that certain elliptic operators generate analytic semigroups.
(2) Exponential decay of eigenfunctions of elliptic operators on unbounded domains.
(3) A generalization of the shadowing lemma from dynamical systems which is related to a construction of P. Fife in singular perturbation theory.

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O. DIEKMANN:

Slowly oscillating solutions are stable.

Let F denote the translation invariant nonlinear Volterra integral operator defined by $(Fx)(t)=\int_0^\infty A(\tau)f(x(t-\tau))d\tau$, let R denote the reflection operator $(Rx)(t)=x(-t)$ and T_s the translation operator $(T_s x)(t)= x(t+s)$. Under the assumption that $A(1-\tau)=A(1+\tau)$ we have $RF=T_2FR$. As a consequence one can easily obtain branches of (Hopf) bifurcating solutions of $x=Fx$ which are even and have fixed period 2. If we take $A(\tau)=\frac{1}{2\varepsilon}\chi[1-\varepsilon,1+\varepsilon]$ and f odd, monotone decreasing and convex for $x>0$, F maps the cone $K=\{x|T_1x=-x,Rx=x,x(t)\geq 0$ on $[-\frac{1}{2},+\frac{1}{2}]\}$ into itself. Hence concave operator techniques yield detailed results on existence, multiplicity, continuation and limiting behaviour for $\varepsilon\downarrow 0$ (note that the limiting equation is the difference equation $x(t)=f(x(t-1))$). Finally, the stability of these periodic solutions is determined through estimates for the Floquet multipliers. Here the basic idea is that slowly oscillating solutions of the linear variational equation correspond to dominant Floquet multipliers. This lecture is based on joint work with Shui-Nee Chow and John Mallet-Paret.

B. Fiedler (& P. BRUNOVSKY) :

Connecting orbits in scalar reaction diffusion equations.

For the scalar equation

$$u_t = u_{xx} + f(u) \quad , \qquad f \in C^2$$

with Dirichlet boundary conditions $u = 0$ at $x = 0,1$ we consider the problem of connecting orbits $u(t,\cdot)$, $t \in \mathbb{R}$, running from a hyperbolic stationary solution v to a stationary w. For given v, we describe which w occur in terms of the unstable dimensions of v,w and the number of zeroes of $v(x)$, $w(x)$. All previous results by Conley and Smoller, Henry are recovered and generalized to arbitrary f.

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R. GARDNER:

A non-local conservation law arising in combustion theory.

The non-local conservation law

$$u_t + R\,u\,u_x + (R-1)\left[\int_0^\infty u(x+\beta s)\,u_x(x+s)\,ds\right]_x = 0$$

$$u(x,0) = u_0(x) \qquad (*)$$

arises as an asymptotic approximation which governs the growth of 2-dimensional perturbations in planar detonation wave solutions of the equations of reactive gas dynamics. The formation of shocks in smooth, rapidly decaying solutions of (*) signals the onset of Mach stem formation in solutions of the full system.
We study the local existence of smooth solutions of (*) in a scale of Sobolev spaces based on L^1 . We also prove that such solutions develop shocks in finite time for certain values of the parameters β and R .

J. HERNÁNDEZ :

Existence of positive solutions for some reaction-diffusion systems.

We study the existence of (strictly) positive solutions for some stationary reaction-diffusion systems. The problem is reduced to a single equation with a non-local, maybe nonlinear, perturbation, and then we show that the method of sub- and supersolutions can be applied and that the bifurcation results are very similar to the non-perturbated problem. We also prove the uniqueness of nontrivial positive solutions and then use a continuation argument to prove they form a smooth (e.g. C^2) branch. We apply this method to some examples arising in morphogenesis and predator-prey systems.

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G. HETZER:

A diffusion problem from climate modelling.

The parameter dependent diffusion equation

$$(*) \quad c(x)\frac{\partial u}{\partial t}(x,t) - \operatorname{div}(k \operatorname{grad} u)(x,t) = \mu R_a(x,u(x,t)) - R_e(x,u(x,t))$$

$(x \in S^2, t \in \mathbb{R}_+)$ arises from 2-dimensional energy balance climate models, if the horizontal heat transport is parameterised by a diffusion operator, the so-called eddy diffusive approximation. Here S^2 replaces the earth's surface, u is the annually averaged sea level temperature, c is the heat capacity, R_a is the absorbed, R_e the emitted radiation and k the diffusion coefficient. The earth's global climates appear in this model as the steady state solutions of $(*)$. Their sensitivity can be qualitatively understood from the asymptotic behavior of the solutions of $(*)$ and the "μ-dependence" of the steady state solutions. Some results are presented.

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H. HOFER:

Critical Point Theory and Hamiltonian Systems.

Since the seminal work by Rabinowitz in 77 one knows how to use the classical variational principle for Hamiltonian systems in order to establish existence of solutions satisfying so called canonical boundary conditions for problems in $\mathbb{R}^{2n}$. There are severe technical problems if one tries to carry over the critical point theory to study Hamiltonian systems on cotangent bundles.
Nevertheless it is possible and one has for example the following Theorem:

Theorem. Let M be a compact manifold, T^*M its cotangent bundle and $\Sigma \subset T^*M$ the subset of zero-vectors. Assume $h: [0,1] \times T^*M \to \mathbb{R}$ is a smooth map with compact support. Then the corresponding Hamiltonian system $\dot{x} = X_t(x)$ has at least $c(M)$ solutions $x: [0,1] \to T^*M$ with $x(0), x(1) \in \Sigma$, where $c(M)$ is the cohomological category of M. This result implies a conjecture in Lagrangean intersection theory due to Arnold. The particular case $M=T^n$ has been previously solved by M. Chaperon using a different approach strongly influenced by work of Conley and Zehnder.

L. HSIAO:

Analysis of a coupled system of a conservation law and a diffusion equation.

The following system arises, for instance, in the modelling of nucleation and the growth of chemical or biological populations.

(1) $\partial_t f(R,t,\varkappa) + \partial_R [f(R,t,\varkappa) \cdot V(C(t,\varkappa),R)] = 0$

(2) $\partial_t C(t,\varkappa) - \Delta C(t,\varkappa) = -\int_0^\infty V(C(t,\varkappa),R) f(R,t,\varkappa) \rho(R)\, dR$

where $V(C,R), \rho(R)$ are given functions.

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In case of nucleation, f describes the density of nuclei of radius R at a space-time point, C is the space-time density of the substance forming the growing nuclei. The function V describes the growth law, ρ relates growth of the nuclei and consumption of the substance.
We consider (1), (2) with

$$(3) \qquad f|_{t=0} = f_0(R,\varkappa) > 0 \quad \text{for} \quad R > 0 \, , \quad \varkappa \in \Omega$$

$$(4) \qquad c|_{t=0} = c_0(\varkappa) > 0 \quad \text{for} \quad \varkappa \in \Omega$$

$$(5) \qquad \frac{\partial c}{\partial n}\Big|_{\partial\Omega} = 0 \quad .$$

We prove the global existence of the solution of (1) - (5) and uniqueness. We discuss the asymptotic behavior of the solution also. This is a joint work with Prof. W. Jäger.

Chr. JONES:

The Maslov Index and Stability Problems.

A natural approach to understanding the stability of standing or travelling waves relative to their respective PDE's is to consider the linearisation of the PDE at the wave. The wave will, for instance, be unstable if the linearised operator has a positive eigenvalue. Such positive eigenvalues may be determined by a shooting argument in the space of Lagrangian planes. It is explained how this shooting argument works with reference to an instability problem for a nonlinear Schrödinger type equation. The key is a topological description of the space of Lagrangian 2-planes as an S^2 fiber bundle over S^1. An application to a problem that arises in optical waveguides is given.

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S. KICHENASSAMY:

Positive solutions of quasilinear equations.

I report on a joint work with J. Smoller.
We prove the following results for quasilinear equations of the following kind:

$$(*)\quad \begin{cases} -\mathrm{div}\left(j'(|\nabla u|)\dfrac{\nabla u}{|\nabla u|}\right) - f(u) = 0 & \text{on } B \quad (N\text{-ball}) \\ u = 0 & \text{on } \partial B \end{cases}$$

1) If $j'' > c > 0$ and $f(p_o) = 0$, $p_o > 0$, then i $\Leftrightarrow$ ii

i) $\forall p \in [0,p_o]$: $\int_0^p f(\zeta)\,d\zeta < \int_0^{p_o} f(\zeta)\,d\zeta$

ii) (*) has a (radial positive) solution in some ball, so that its value at 0 be arbitrarily close to p_o.

2) In case $N = 1$ and $f(u) = -u(u-a)(u-1)$, $0<a<\frac{1}{2}$, (*) has at most 3 solutions over $[-L,L]$ such that

$u(0) > \frac{a+1}{2}$.

This makes an index theory for such equations possible.

G. KLAASEN:

Existence and Stability of Steady State Solutions of a Reaction-Diffusion System.

Sufficient conditions are given for the existence of two non-trivial solutions of the Dirichlet problem for $-\Delta u = -\sigma g(u) - \beta v$, $-\Delta v = \lambda u - \delta v$ where $\sigma, \beta, \lambda, \delta$ are positive constants, $g(0) = 0$, $g'(0) > 0$ and g is of cubic type. Existence is proved using variational priciples. We also prove the stability of these solutions as steady state solutions of the system $u_t - \Delta u = -\sigma g(u) - \beta v$, $v_t - D\Delta v = -\lambda u - \delta v$.

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J. MAWHIN:

Some remarks on the Ambrosetti-Prodi problem.

We show that the Ambrosetti-Prodi conclusions for the problem

$$(1) \quad \begin{array}{l} u'' + f(u)u' + g(x,u) = s \\ u(o) - u(2\pi) = u'(0) - u'(2\pi) = 0 \end{array}$$

(i.e. the existence of s_o such that there are 0 , at least one or at least two solutions for (1) according to $s < s_o$, $s = s_o$, $s > s_o$) which are classically proved when $f \equiv 0$ and

$$(2) \quad \limsup_{u \to -\infty} \frac{g(x,u)}{u} < 0 < \liminf_{u \to +\infty} \frac{g(x,u)}{u} \quad (\text{unif. in } x \in [0,2\pi])$$

survive for arbitrary continuous f and (2) replaced by

$$(3) \quad \lim_{|u| \to \infty} g(x,u) = +\infty \qquad (\text{unif. in } x \in [0,2\pi]).$$

Thus the shape of g leads to the phenomenon and not the asymptotic derivatives at $\pm\infty$. This is a joint work with Fabry and Nkashama.

Condition (2) implies the same conclusions for the Dirichlet problem

$$u'' + g(x,u) = s \sin x$$

$$x(0) = 0 = x(\pi)$$

joint work with Chiappinelli and Nugavi). Extensions to higher order differential equations and to elliptic partial differential equations are briefly discussed.

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R. NUSSBAUM:

Iterated Nonlinear Maps and Hilbert's Projective Metric.

Let K denote the cone of nonnegative vectors in $\mathbb{R}^n$ and suppose that $f: \mathring{K} \to \mathring{K}$ is a continuous map which is homogeneous of degree 1 and preserves the partial ordering induced by K (call this assumption H).

Theorem 1. Assume that $f: \mathring{K} \to \mathring{K}$ satisfies H, that f is C^1 on $\mathring{K}$ and that $df(x)$ is a nonnegative, irreducible matrix for all $x \in \mathring{K}$. Then there exists at most one $u \in \mathring{K}$ such that $|u|=1$ and $f(u)=\lambda u$ for some $\lambda \in \mathbb{R}$.

We do not assume $f(K-\{0\}) \subset \mathring{K}$, so existence of such a u is a non-trivial question which I do not have space to discuss here.

Theorem 2. Assume that f is as in Theorem 1 and that $df(x)$ is primitive for all x in $\mathring{K}$. Assume that there exists $u \in \mathring{K}$ as in Theorem 1. Then for any $x \in \mathring{K}$,

$\lim_{m\to\infty} \frac{f^m(x)}{|f^m(x)|} = u$. If $f(u) = u$, $\lim_{m\to\infty} f^m(x) = \alpha(x)u$, where $\alpha(x)$ is a positive scalar depending on x.

K. RYBAKOWSKI:

A homotopy index continuation principle and periodic solutions of second order gradient systems

We consider certain boundary value problems of the type

$$Au = f(u)$$

with A selfadjoint and noninvertible and f gradient. Using an extension (due to this author) of Conley's homotopy index theory to semiflows satisfying a Palais-Smale type condition, we establish an analogue of the coincidence degree method of Mawhin for the homotopy index. This is then applied to proving the existence of periodic solutions of second order gradient systems with mean values lying in a prescribed set. It is shown that, when applicable, the homotopy index approach has several advantages over the degree method.

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D. SALAMON:

Index theory for discrete dynamical systems.

Conley's index theory generalizes the classical Morse inequalities for critical points in gradient flows on manifolds to isolated invariant sets in flows on metric spaces. A crucial role in this theory plays the concept of an index pair consisting of an isolating neighborhood and an exit set. The Conley index is the homotopy type of the pointed space which is obtained from the isolating neighborhood by collapsing the exit set to a single point. An index theory for discrete dynamical systems can be developed along the same lines. In this case the index is an equivalence class of inverse systems whose terms are the afore mentioned pointed spaces and whose bonding morphisms are induced by the dynamical system. The index is invariant under continuation. Furthermore, generalizing results by Franks, one obtains Morse inequalities involving the homology zeta function for a decompsition of an isolated invariant set, which satisfies the "no cycle" condition.

J. SCHEURLE:

Chaotic motion in almost-periodically forced systems.

It is well known that the chaotic behaviour of certain dynamical systems can be explained by the presence of homoclinic or heteroclinic orbits. In this lecture a two-dimensional system is considered which results from adding a small almost-periodic forcing term to an autonomous system which has a hyperbolic saddle point and a corresponding homoclinic orbit. It is shown that the existence of a transversal homoclinic orbit corresponding to the perturbed saddle point leads to a large number of solutions which exhibit chaotic behaviour. The Melnikov function provides a sufficient condition for the occurence of such chaos. This result generalizes known results for periodically forced systems.

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J. SMOLLER:

Symmetry-Breaking.

We study the bifurcation of radial solutions of $\Delta u + f(u) = 0$, (on bounded domains), into asymmetric ones.

M. STRUWE:

Functional analytic aspects of the Plateau problem.

Ljusternik-Schnirelman theory on closed convex sets in Banach spaces is presented as a natural tool for studying the Plateau problem.
In the Plateau problem for multiply connected minimal surfaces, moreover, a loss of compactness is encountered similar to many problems in geometry and mathematical physics: A Palais-Smale sequence may degenerate topologically through the process of "separation of spheres" on which non-compact groups of symmetries act.
For the Plateau problem for annulus-type minimal surfaces it is shown how Ljusternik-Schnirelman and Morse theories may be extended to such a degenerate situation.

A. J. TROMBA:

Weil-Petersson Metric.

Let M be a compact oriented Riemann surface of genus > 1 . Let A be the set of almost complex structures on M . Then A is a manifold with tangent space $T_J A = \{H \in C^\infty(T_1^1(M)) \mid HJ = -JH\}$. Then there exists a natural L^2 metric $\langle\langle\, ,\, \rangle\rangle$ on the tangent bundle TA , namely

$$\langle\langle H,K \rangle\rangle = \frac{1}{2} \int_M (\text{trace } HK)\, d\mu_{g(J)} , \quad \text{where } g(J)$$

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is the unique Poincaré metric associated with the almost complex structure J . Then $\mathcal{D}_o$, the group of diffeomorphisms homotopic to the identity acts as a group of isometries. The induced metric on the quotient is the Weil-Petersson metric. The curvature of this metric can be understood through the use of the algebraic connection ∇ on TA given by

$$\nabla_Y X = DX(Y) - \frac{1}{2} J(XY+YX)$$

for vector fields X and Y on A .

A. VANDERBAUWHEDE:

<u>Symmetry-breaking near positive solutions of semi-linear elliptic equations</u>

Consider the boundary value problem:

$$\Delta u + f(u) = 0 \ , \quad x\ B_R \ ; \quad u = 0 \ , \quad x \in \partial B_R \ , \qquad (1)$$

where $B_R = \{x \in \mathbb{R}^n \mid |x| < R\}$ and $f: \mathbb{R} \to \mathbb{R}$ is sufficiently smooth; we consider R as a parameter. It is well known from a theorem of Gidas, Ni and Nirenberg that if u_O is a positive solution for $R = R_O > 0$, then u_O is spherically symmetric. We prove that under the same conditions all solutions u of (1) near u_O and with R near R_O are axi-symmetric; if grad $u_O \neq 0$ on ∂B_{R_o} they are even sperically symmetric. The proof uses earlier work of Smoller and Wasserman. We explain a general method from equivariant bifurcation theory which gives sufficient conditions for symmetry breaking, and we apply this to (1) and to the same problem with Neumann boundary conditions. Finally we discuss a symmetry-breaking perturbation of the equation (1).

Berichterstatter: H.O. Peitgen

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