MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 35/1985

Abelsche Gruppen

11.8. bis 17.8.1985

Unter der Leitung von Herrn Professor Gobel (Essen) und Herrn Professor Walker (Las Cruces) fand die dritte Tagung in Oberwolfach über Abelsche Gruppen das Interesse einer stark angewachsenen Zahl von Mathematikern aus vier Kontinenten.

Auf dem Gebiet der Abelschen p-Gruppen wurde erstmalig über Computereinsatz berichtet, aber auch der Einfluß der Mengenlehre wurde deutlich, der bisher nur bei torsionsfreien Abelschen Gruppen in Erscheinung getreten war. Letzteren widmeten sich zahlreiche Vortrage, in denen Fortschritte sowohl für torsionsfreie abelsche Gruppen endlichen als auch unendlichen Ranges erläutert wurden. Bemerkenswert war auch die Übertragung von für Abelsche Gruppen entwickelten Methoden auf andere Gebiete der Algebra, wie z.B. Moduln, Körper und Ringe.

Der lebendigen Entwicklung der Theorie der Abelschen Gruppen Rechnung tragend, wird die nächste Tagung 1987 in Perth, Australien, stattfinden.

Vortragsauszüge

F. Richman:

Ulm's theorem for simply presented valuated p-groups

Let G be a valuated p-group and $x \in G$. The valuated height of x is (the equivalence class of) the valuated tree

$$\{y \in G : p^n y = x \text{ for some } n\}$$

If B is a valued tree, then

$$G(B) = \{x \in G : \text{vh } x \geqslant B\}.$$

where $B_1 \leqslant B_2$ if there is a map from B_1 to B_2 that respects predecessors and weakly increases values. The B^{th} valuated Ulm invariant of G is

$$\frac{G(B)[p]}{G(B^*) \cap G(B)[p]}$$

where $G(B^*) = \Sigma_{C \not\leqslant B} G(C)$. Ulm's theorem holds for simply presented valuated p-groups. The proof uses ideas of Laurel Rogers concerning stripping functions in the classical case.

Roger Hunter

Computing Valuated Trees

There is a 1-1 correspondence between finite irretractable valuated trees and indecomposable simply presented valuated p-groups. Let TREES_n be the (finite distributive) lattice of irretractable valuated trees with maximum value n. An algorithm for computing TREES_n was developed and implemented on a computer. The sizes are $|\text{TREES}_0| = 1$ $|\text{TREES}_2| = 3$, $|\text{TREES}_2| = 7$, $|\text{TREES}_3| = 16$, $|\text{TREES}_4| = 43$, $|\text{TREES}_5| = 217$, $|\text{TREES}_6| = 59049$, $|\text{TREES}_7| > 2^{517}$. The lattice structure reveals an unexpected duality for finite valuated trees.

Adolf MADER

Heinz Prüfer's papers on abelian groups

Some information on Heinz Prufer himself and the status of abelian group theory in 1920 will be offered. The main part of the talk will discuss the content of Prufer's papers on abelian groups and its implications.

Kin-ya HONDA

Global bases of abelian p-groups

In order to study the structure of abelian p-groups more concretely and more in details, Prof. Benabdallah and I introduced the notion of a straight basis into any abelian p-group A. Though we believe that it is of much value in the researches of abelian p-groups, it has a certain weak point that there is no explicit relation between it and the generalized heights of elements of A. Thus we introduce here the new notion of a global basis B^* into any abelian p-groups A. The family of elements B^* is composed of three types of elements - of B-type, of $\bar{B}$-type, and of C-type. The relations between the elements of B-type and those of C-type are relatively simple. The relations between the elements of $\bar{B}$-type and those of B-type are much complicated, and seem to comprise the true secret of the structure of the p-groups A.

Wolfgang LIEBERT

Isomorphic automorphism groups of abelian p-groups

Let G and H be abelian p-groups with automorphism groups Aut G and Aut H, respectively. H. Leptin proved in 1960 that Aut G $\cong$ Aut H implies G $\cong$ H, provided $p > 5$. We offer a proof of Leptin's theorem which works for all $p > 3$, by generalizing the method of involutions known from classical groups. In particular, transvections are used. The key lemma is a group-theoretical characterization of transvections within Aut G.

Jutta HAUSEN

On strongly irreducible torsion-free abelian groups

Let C denote the center of the endomorphism ring of a strongly irreducible torsion-free abelian group G. It is shown that, if every C-submodule of G of C-rank one is cyclic, then G is an $\aleph_1$-free C-module. This provides unified proofs of some structure theorems for strongly homogeneous and E-uniserial groups.

Sheila BRENNER

Some almost split sequences in torsion-free abelian group theory (Work of M.C.R. Butler).

A t.f. abelian group is called diagrammatic if it can be generated by finitely many rank-1 subgroups. Let Π be a finite sublattice of the lattice of all types and Σ be the subposet of max-irreducible elements of Π. Let

D_Π be the category of diagrammatic groups with typeset in Π and quasihomomorphisms. There is an isomorphism between D_Π and V_Σ, the category of subspace representations of Σ over Q. This allows us to transfer the many known results on V_Σ to D_Π. From the Kiev school we have, for example, criteria for deciding representation type. Results from representation theory of algebras may also be adapted to V_Σ. For example every indecomposable non-injective (non-projective) in V_Σ is the left (right) hand end of an almost split sequence in V_Σ.

Samir KHABBAZ

Submodules of Direct Products (Work of J. Irwin and S. Khabbaz)

We consider an arbitrary product Π of euclidean rings. Let ω be any subring of Π containing all characteristic functions. We give a complete determination of the ω-direct summands of the ω-module ω. As a special case it is shown in the case of a countable product of rings that an ω-submodule of ω is pure and countably generated if and only if it is a direct sum of direct summands of ω.

Anthony J. GIOVANITTI

Projective and Injective Butler groups

Necessary and sufficient conditions are given on the type of a rank-1 torsion free Abelian group to be projective or injective in the category of regular homomorphisms of the class of T-Butler groups for a finite sublattice T of the lattice of types. Using this it is shown that a rank-1 is projective in the category of Butler groups if and only if it is isomorphic to the integers, and it is injective if and only if it is either the integers localized at a prime of the group of rationals. Thus a nonzero Butler groups has a projective resolution if and only if it is a free, and it has an injecitive resolution if and only if it is semi-local (i.e., divisible by all but a finite number of primes).

B. CHARLES

The condition $H \oplus G/H \cong G$ for abelian groups

Let G be an abelian group and H a subgroup of G such that $H \oplus G/H \cong G$. We give sufficient conditions for H to be a direct summand of G. This problem is related to the matrix equation $AX-XB = C$.

Alexander SOIFER

Abelian Groups of Cofinality ω.

1. Let λ be a cardinal. A group G is called λ-indecomposable, if it cannot be decomposed into a direct sum of λ non-zero summands.
THEOREM: Let G be an uncountable group of cardinality λ and $\mathrm{cf}\ \lambda = \omega$. If G is λ-indecomposable, then G is σ-indecomposale for some $\sigma < \lambda$.
2. The Irwin Conjecture (1965) states: if every subgroups of a p-group G has a minimal system of generators, then G has a direct summand G_1, which decomposes into a direct sum of cyclics and has the same final rank as G.

D. Cutler (1984) showed that the Irwin Conjecture fails if fin $r(G) = \lambda^{\aleph_0}$ for some cardinal λ. We show here that the statement stronger than the Irwin Conjecture holds if fin $r(G)$ is of cofinality ω. This completes the problem if to assume the Generalized Continuum Hypothesis.

Khalid BENABDALLAH

Locally rectifiable modules

A module M over an associative ring is said to be locally rectifiable if for every pair U,V of uniserial submodules of a homomorphic image K of M there exists in K a uniserial summand of U+V. Abelian groups are all locally rectifiable $\mathbb{Z}$-modules. (Here the word uniserial is used in the sense that the family of submodules forms a finite chain). A notion of height of uniserial submodules is introduced giving rise to submodules $H_n(M)$ generated by those uniserial submodules of M of height $\geq n$, $n \in \mathbb{N}$. Various theorems of abelian p-groups can then be interpreted in locally rectifiable modules and shown to hold in this more general setting. If further M is generated by uniserial submodules and the associative ring is commutative M satisfies Ulm's theorem for its countably generated submodules. The methods used are however different from the usual development of torsion abelian group theory. Special care has been taken to use global methods so that the results can be transferred to appropriate objects in abelian categories and to the theory of lattices. This is a joint work with Saadia Hattab-Ibrahimi.

Phillip SCHULTZ

Finite extensions of torsion-free abelian groups

Let C be a finite rank, torsion-free group and k a positive integer. Let Δ be the set of groups between C and $k^{-1}C$, and Γ the set of subgroups of C/kC. $\Theta : A \to kA/kC$ is a lattice isomorphism between Δ and Γ, so we have an explicit enumeration of Δ. Furthermore, with an appropriate definition of morphisms, Θ is a category equivalence. Aut C is a group acting on Δ, Aut C/kC is a group acting on Γ. The kernels of these groups can be identified, yielding in certain cases a classification of the isomorphism classes in Δ.

James D. REID

Duality for Irreducible Groups

In seeking to develop analogies between irreducible groups and central simple algebras we want to construct something like the opposite algebra - in the theory of algebras - for these groups. There ought to be a product defined so that the product of the groups and its opposite is "split". The groups analogous to split algebras are the form $G = A \oplus H$ where A is rank 1 over the center, D, of $End(G)$ and H is strongly irreducible (cf. the authors's paper in the Oberwolfach Conference, 1981).. We first extend Warfield's results on duality to the case of modules over Dedekind rings, obtaining, we believe, a treatment simpler than the standard one even in the case of Z-modules. This extension is necessary for our purposes. Then the opposite of G is $G' = Hom_D(G,A)$, where A is any rank 1 D-image of G and the product is $G*G' = (G \otimes G')/K$ whwere $K = \cap\{\ker f \mid f \in Hom_D(G,A)\}$. Then, under mild assumptions on the irreducible G, $G*G'$ is split.

Ladislav BICAN

Two remarks on Butler groups

In the first part the Butler groups of finite rank were characterized by means of upper subsets (a subset M of a torsionfree group G is said an upper subset, if for each type the set $G(t) \cap M$ contains a basis of $G(t)$). Let $\mathfrak{M}$ be the class of all torsionfree groups G for which there is a partition $\Pi = \Pi_1 \cup \Pi_2 \cup \dots \cup \Pi_n$ of the set Π of all primes such that the group $G \otimes Z_{\Pi_j}$ is completely decomposable with ordered type set for each $j = 1,2,\dots,n$. In the second part the subclass of $\mathfrak{M}$ closed under pure subgroups was characterized.

Katsuya EDA

Abelianizations of free Σ-products and Σ-products

I introduce free Σ-products of groups. This notion is a countable version of usual free products of groups and defined by words with countable linear ordering. It is related to the fundamental group of the so-called Hawaiian ear ring and so the abelianization of it is related to the first integral singular homology group. There are a few old results of H.B. Griffith's on this topic. The abelianization of it is similar to that of Σ-products. Hence I state some results of both. One of them is the following: Let G be a group of order greater than 2. Then, the abelianization of the free Σ-product $\tilde{*}_{\mathbb{N}} G$ and the unrestricted direct product $(G * G)^{N}$ contains a divisible torsionfree group of the continuum cardinality, i. e. $\bigoplus_{2^{\aleph_0}} Q$.

John D. O'NEILL

On Decompositions of Modules and Submodules

Let M be an R-module. If X is a subset of M, then RX denotes the submodule generated by X. Suppose $M = \bigoplus_I RX_i = \bigoplus_J RY_j$ with $|\bigcup_I X_i| \leq |\bigcup_J Y_j| \leq \alpha$. Our main results are the following. First R has a subring S, inheriting most algebraic properties of R, such that: $|S| \leq \alpha+\omega$, $\bigoplus_I SX_i = \bigoplus_J SY_j$ and each component $SX_i (SY_j)$ is S-indecomposable if RX_i (RY_j) is R-indecomposable. Secondly, if is finite, R is an algebra over a commutative domain of characteristic 0 and the modules ${}_C R$ and ${}_C M$ are torsion-free and finite dimensional, then R has an $E \cap C$-essential subring E such that $\bigoplus_I EX_i = \bigoplus_J EY_j$ where each component is cotorsion-free and any EX_i (EY_j) is E-indecomposable if RX_i (RY_j) is R-indecomposable. These results are applied to decompositions of groups.

Radoslav DIMITRIC

On chains of free modules over valuation domains

As the title suggests, the paper deals with smooth ascending chains of modules over valuation domains. Using the definition of κ-free modules given in my earlier paper, introduced is a notion of an F_2-module: such a module is the union of a pure continuous chain of length ω_2 of free modules of rank $\aleph_1$; such that every quotient of consecutive terms of the chain is an $\aleph_1$-free module. It is proved that the union of a continuous chain of length ω_1 of free modules, with a property that its every quotient of successive terms is an F_2-module is a free module. The existence of non free F_2-modules is exhibited as well as some constructions to build new F_2-modules out of the given ones.

A.L.S. Corner

Fully rigid systems of modules

A fully rigid system for an algebra A over a commutative ring R is a family G_X $(X \subseteq I)$ of faithful A-modules such that $G_X \leqslant G_Y$ whenever $X \subseteq Y$ and

$$\mathrm{Hom}_R(G_X, G_Y) = \begin{cases} A & (X \subseteq Y) \\ 0 & (X \nsubseteq Y) \end{cases}$$

When I is infinite a fully rigid system G_X $(X \subseteq I)$ clearly contains a rigid system of size $2^{|I|}$.

Theorem. Suppose A admits a fully rigid system H_X $(X \_ I)$ where $|I| \geqslant 5$. Then for every infinite cardinal $\lambda \geqslant |H_I|$ there exists a fully rigid system G_X $(X \subseteq \lambda)$ such that $|G_X| = \lambda$ $(X \subseteq \lambda)$.

This theorem fills in cardinal gaps left by certain applications of Shelah's Black Box and like a related result obtained by B. Franzen and R. Göbel relies on a combinatorial idea of Shelah's dating back to 1974.

Adalberto ORSATTI

Torsion free abelian groups in topology

Let λ be an infinite cardinal number. It is proved that, for every positive integer r, there exists a compact, connected, homogeneous topological space X of weight λ such that $X^m \approx X^n \Longleftrightarrow m \equiv n(\mathrm{mod}\ r)$. For a given λ the cardinality of the homomorphism classes of these spaces is exactly 2^λ.
These results rely mostly on old and new powerful results on torsion free abelian groups obtained by T. Corner.

Laszlo FUCHS

Divisible modules over valuation domains

After reviewing the recent results on divisible modules over valuation domains R, the following two results are motivated. (Uniserial = submodules form a chain).

1. The torsion divisible uniserial R-modules form an abelian groups under the operation Tor_1. Using $\diamondsuit$, the structure of this group can be given more explicitly.
2. There are valuation domains R over which there exist arbitrarily large indecomposable torsion divisible modules.

Luigi SALCE

A duality for finitely generated modules over valuation domains

A classification by means of matrix invariants for a certain class of finitely genrerated torsion modules over a valuation domain is given, which resembles the classification of the torsion-free Z_p-modules of finite rank given by Kurosch-Malcev-Derry. It is also discussed a duality for a suitable subclass, which resembles the Arnold's duality.

George KOLETTIS

Almost p-maps with an application to p-groups

The notion of a p-map of abelian groups originated in algebraic topology. A slight relaxation of the conditions yields the notion of an almost p-map. Using exact sequences, necessary and sufficient conditions for an almost p-map are derived. The result for p-groups is then the following: A homomorphism $\phi : A \to B$ of abelian groups is an isomorphism if and only if the induced map $A/pA \to B/pB$ and $A[p] \to B[p]$ are, respectively, a monomorphism and an epimorphism.

Barbara L. OSOFSKY

Doubly infinite chain conditions

Definition: An object M in an AB5 category has dicc if and only if M has no poset of submodules of order type $\mathbb{Z}$.

Theorem: M has dicc if and only if there is a $K \subseteq M$ invariant under all endomorphisms of M such that (a) K has dcc

(b) M/K has acc

(c) $B \subseteq M$, $B \subseteq K \Rightarrow K/B \cap K$ has acc

In the proof, K is the union of the ascending socle series of M.

An object with dicc^+ (= dicc buth neither acc nor dcc) is a finite direct sum of an indecomposable dicc^+ object and one of finite length unique up to isomorphism . There are no dicc^+ modules over a Noetherian ring such as the Dicc^+ implies M is essential over its socle and finitely generated, and if M is a module over a commutative ring R. M. Contessa has given additional information about $R/\mathrm{Ann}(M)$.

Paul EKLOF

On reflexive and non-reflexive groups

We survey recent results on reflexive and non-reflexive groups. Define $A^* = \mathrm{Hom}(A,Z)$; $A^{*0} = A$; $A^{*(n+1)} = (A^{*n})^*$. A is said to be non-reflexive (resp. strongly non-reflexive) if $\sigma : A \to A^{**}$ is not an isomorphism (resp. $A \cong A^{**}$ by an isomorphism). Among the results discussed: The Eda-Ohta group G which is of cardinality $2^{\aleph_0}$ and such that G^* is not reflexive; the construction (due to Eklof-Mekler-Shelah), assuming , of a group A of cardinality $\aleph_1$ such that for all $n \in \omega$, A^{*n} is weakly $\aleph_1$-separable of cardinality $\aleph_1$ and strongly non-reflexive.

Alan MEKLER

The Solution to Crawley's Problem

Let A be a separable p-group. A uniquely elongates $Z(p)$ iff $Z(p) = p^\omega H \hookrightarrow H \twoheadrightarrow A$ and $Z(p) = p^\omega H' \hookrightarrow H' \twoheadrightarrow A$ implies $H \cong H'$. Crawley asked whether any such A is a direct sum of cyclic groups. Megibben [Pac. J. Math. 1983] showed that the answer to Crawley's Problem is independent of ZFC for groups of cardinality λ_1'. In joint work with Shelah, we show if $V = L$ then any group which uniquely elongates $Z(p)$ is a direct sum of cyclic groups. So Crawley's problem is independent of ZFC. In fact it is independent of both CH and $2^{\aleph_0} = 2^{\aleph_1}$.

Burkhard WALD

Slender fields

We work in a set theory, where we replace the axiom of choice by the assertion : "Every set of reals has the Baire-property". We can show that the following fields become slender rings: all finite, all countable subfields of $\mathbb{R}$. (For a slender ring we have duality between $\prod_{\mathbb{N}} R$ and $\bigoplus_{\mathbb{N}} R$ and $\prod_{\mathbb{N}} R = \bigoplus_{i \in I} A_i$ only if I is finite). Another result is that the group $B = \{x \in \prod_{\mathbb{N}} \mathbb{Z} \; : \; x$ is bounded$\}$ becomes "product-like" in the sense that $B = \bigoplus_{I} A_i$ only if I is finite. (Under the axiom of choice we know from Specker and Nöbeling that B is a free abelian group.)

K.M. RANGASWAMY

Torsion-free Abelian k-groups

It is shown that the knice subgroup S of a completely decomposable group C is $\aleph_1$-separable. If, in addition, S has cardinality $< \aleph_1$, then S is again completely decomposable. The k-groups of Hill and Megibben are just the torsion-free weakly separable abelian groups and have balanced projective dimension $< n$ if their cardinality is $< \aleph_n$.

M. DUGAS

Fields with prescribed group of automorphisms

Let κ be a regular cardinal and $E = \{\alpha < \kappa \mid cf(\alpha) = \omega\}$. We prove the following

Theorem (M. Dugas and R. Göbel). Assume $\diamondsuit_\kappa(E)$ holds. Let G be a group, K a field such that $char(K) \neq 2$ and $|G|, |K| < \kappa$. Then there exists a field F containing K such that $Aut(F) = G$. This solves a problem of E. Fried.

R. GÖBEL

Torsion-splitting

The Baer-Fomin theorem for torsion-splitting of abelian groups and Griffith's theorem on "Baer groups" has been extended to arbitrary torsion-theories over Dedekind domains R. In the first case we require that the quotient field of R is countably generated over R.

B. FRANZEN

The Brenner-Butler-Corner-Theorem

This is joint work with R. Göbel.

Given an infinite domain R, an uncountable regular cardinal λ, a torsion-free R-algebra A generated by at most λ many elements and a free R-module F of rank λ there are for every torsion-free A-module M five summands $U_i(M)$ $(i=1,\ldots,5)$ of $F \otimes M$ such that

$\{\phi \in \mathrm{Hom}_R(F \otimes M, F \otimes N) \mid U_i(M)\phi \subseteq U_i(N)\ (i=1,\ldots,5)\} = 1_F \otimes \mathrm{Hom}_A(M,N)$

for any other torsion-free A-module N.

This extends earlier results by S. Brenner, M.C.R. Butler adn A.L.S. Corner to arbitrary large cardinals. The proof is briefly sketched, especially its dependance on a method used by S. Shelah in 1974 to construct indecomposable abelian groups of arbitrary cardinal. This method was also employed by A.L.S. Corner who announced a related result on this conference.

Ulrich ALBRECHT

Abelian Groups with Self-Injective Endomorphism Rings

Abelian Groups having a right or left self-injective endomorphism ring are characterized. It is shown that an abelian group A has a right self-injective endomorphism ring if and only if i) $TA_p \cong \oplus_{I(p)} \mathbb{Z}/p^{n(p)}\mathbb{Z}$ where $n(p)$ is a positive integer, and ii) $A = (\oplus_I \mathbb{Q}) \oplus TA$, or A is pure and fully invariant in its cotorsion hull $\Pi_p TA_p$. A similar description can be obtained for abelian groups with left self-injective endomorphism rings. It is only necessary to add that I and I_p have to be finite. In particular, every left self-injective endomorphism ring is right self-injective. Finally, we give an existence theorem for groups with self-injective endomorphism ring which allows the construction of numerous examples and yields a complete set of invariants for these groups.

Paul HILL and Charles MEGIBBEN

Knice subgroups of mixed groups

The fundamental properties of knice subgroups of mixed groups are established, thereby generalizing our earlier treatmens of these subgroups for p-local and torsion free abelian groups. As in those treatments, an appropriate formulation of the notion of a primitive element and a *-valuated coproduct is crucial. One further novelty is our introduction of a new and intrinsic definition of niceness that does not require localization techniques. The results established here lay the foundations for the Axiom 3 characterization of global Warfield groups to appear in a subsequent paper.

R.S. PIERCE

The semigroup of isomorphism types of abelian groups

Let $\mathfrak{A}$ be a category of abelian groups that is closed under finite products. For $A \in \mathfrak{A}$, let $[A]$ be the isomorphism class of A. Denote $\Upsilon(\mathfrak{A}) = \{[A] : A \in \mathfrak{A}\}$. Then $\Upsilon(\mathfrak{A})$ is a commutative semigroup with addition defined by $[A] + [B] = [A \oplus B]$. The structure of $\Upsilon(\mathfrak{A})$ is discussed for various choices of $\mathfrak{A}$.

Temple H. FAY

Radicals and Torsion Theories

In this talk we concern ourselves with the question of when an annihilator class (a class closed under formation of products and subgroups) of abelian groups is cogenerated by a single group. We show that the class of p^{λ}-reduced groups (where p is a prime and λ is an ordinal greater than ω) is not singly cogenerated. If G is a cotorsion-free group, we show that the torsion-free class cogenerated by G is not singly cogenerated as an annihilator class. This result permits the identification of all singly cogenerated annihilator classes which are also closed under formation of extensions. Thus we characterize those singly cogenerated radicals which are idempotent; they are precisely the radicals determined by annihilator classes singly cogenerated by a pure injective. This represents joint work done with M. Dugas and S. Shelah.

R. VERGOHSEN

Non-isometric p-groups with isometric socles

We define for an abelian p-group A of regular, uncountable cardinality κ, a limit ordinal λ and $n \in \mathbb{N}$ the invariant $\Gamma^{n+1}_{\kappa,\lambda}(A)$ as follows. Let $A = \bigcup_{\alpha<\kappa} A_\alpha$ be a κ-filtration of A. We say that A_α is not (p^{n+1},p^λ)-closed if the p^λ-closure of A_α in A contains an element a of order p^{n+1} such that $\langle a\rangle \cap (A_\alpha + p^\lambda A) = 0$. Then $\Gamma^{n+1}_{\kappa,\lambda}(A)$ is the image of the set $E = \{\alpha < \kappa \mid \alpha$ is a limit and A_α is not (p^{n+1},p^λ)-closed$\}$ in the Boolean algebra $P(\kappa)$ modulo the ideal of non-stationary subsets of κ.

Assuming $\diamondsuit(E)$ for each stationary subset E of κ and $\Gamma^{n+1}_{\kappa,\lambda}(A) \neq 0$ we prove: If A is an I_λ-group of limit length λ and A is not complete in the p^λ-topology, then there exists a p-group A' such that $A \not\cong A'$ but $A[p^n]$ and $A'[p^n]$ are isometric. In the non-limit-case we show: If $n \neq 0$ and A is of length $\lambda+n+1$ then there exists a p-group A' such that $A \not\cong A'$ but $A[p]$ and $A'[p]$ are isometric.

Frank OKOH

Submodules of the torsion-free indecomposable divisible module

Let $K(X)$ be the field of rational functions in one variable X over an algebraically-closed field, K. Let $K[Y]$ be the polynomial ring. The submodules of $K(X)$ as a $K[X]$-module are precisely the rank one torsion-free $K[Y]$ modules. We can view $(K(X), K(X))$ as a Kronecker module. (A Kronecker module M is a pair of K-vector spaces (M_1,M_2) together with a K-bilinear map from $K^2 \times M_1$ to M_2, usually specified on a fixed basis (a,b) of K^2. Every $K[X]$-module M may be viewed as a Kronecker module (M,M) with $am = m$, $bm = Xm$ for all m in M.) As a Kronecker module $K(X)$ has a much richer structure. Among other things we shall show it has an infinite rigid family of indecomposable submodules of infinite rank. The proofs rely on techniques from abelian group theory-specifically torsion-free rank on abelian groups.

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Toshiko KOYAMA

On some torsion-free classes

The original problem is
"What does G look like if $Hom(G,Z(p^\infty))$ is a torsion group for any prime p?"
If a group G satisfies above condition we call G T-group. In case G is a torsion group, it is very interesting in connection with torsion-free class and with Grothendick group.
Main results are following.
Let G be a rank 1 torsion-free group. Then G is a T-group if and only if there is no ∞ in the height-sequences which belong to the type (G).
A torsion-free group is a T-group if and only if it is a subgroup of a finite direct sum of rank 1 T-groups.

Alfred W. HALES

Abelian Groups as Brauer and Character Groups

Let F be a field. Associated to F are two abelian (torsion) groups of classical interest, namely the Brauer group of F (which classifies finite dimensional central division algebras over F) and the character group of F (which classifies abelian Galois extensions of F). It is natural to ask which torsion abelian groups can occur in each case. We discuss recent results on these problems, including the distinguishability of such groups by Ulm invariants and the possible Ulm lengths that can occur.

Claudia METELLI

Dual Features in Torsionfree Abelian Groups

In the talk the t-local properties $\mathcal{C}_t$ and $\mathcal{C}'_t$, which were crucial to prove theorems on separable and coseparable groups, were analized and revealed the interest of a notion of t-balancedness and of a "dual" notion of t-cobalancedness. General cobalancedness was considered, and some of the ensuing problems outlined.

Ladislav PROCHAZKA

Concerning P-basic submodules

Let R be a commutative domain, P a prime ideal in R and A an R-module. A submodule $B \leq A$ is called a P-basic submodule of A if

(i) $B = \bigoplus_{x \in X} Rx$ where for every $x \in X$ it is either $o(x) = 0$ or $o(x) = P^k$ for some $1 \leq k \in \mathbb{Z}$ ($o(x)$ denotes the order of x),

(ii) B is P-pure in A and

(iii) A/B is P-divisible.

Some sufficient conditions are given under which a module A contains a P-basic submodule. Especially, if R is a Dedekind domain and A an R-module then every maximal P-independent set of A generates a P-basic submodule in A.

Bernhard AMBERG

Soluble products of groups of finite rank

The following theorem was discussed

Theorem. Let the soluble group $G = AB$ be the product of two subgroups A and B such that the hypercenter factor group $A/H(A)$ is a torsion group

(a) If A and B have finite torsionfree rank $r_o(A)$ and $r_o(B)$, then G has finite torsionfree rank

$$r_o(G) \leq r_o(A) + r_o(B) - r_o(A \cap B)$$

(b) If A and B have finite abelian section rank , then G has finite abelian section rank and equality holds in (a).

Note. A soluble group has finite torsionfree rank (finite abelian section rank) if all its abelian sections have finite torsionfree rank (finite p-rank for $p=0$ or a prime). Note also that $AB = \{ab \mid a \in A, b \in B\}$.

Otto MUTZBAUER

Endomorphism rings of irreducible strongly indecomposable groups

For irreducible stronlgy indecomposable torsion-free abelian groups of finite rank a complete collection of necessary conditions of the endomorphism rings were presented, and for such rings R an irreducible strongly indecomposable torsion-free group A were constructed with endomorphism ring $R = \text{End } A$.

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