MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 36/1985

Singularitäten

18.8. bis 24.8.1985

Die Tagung fand unter der Leitung der Herren E. Brieskorn (Bonn), H. Knörrer (Bonn) und E. Looijenga (Nijmegen) statt. In insgesamt 25 Vorträgen wurden neuere Ergebnisse bei der Untersuchung von Singularitäten holomorpher Abbildungen und komplexer (bzw. algebraischer) Varietäten dargestellt. Einen besonderen Schwerpunkt bildete das Studium der Deformationen von Singularitäten mit topologischen und algebraisch - geometrischen Methoden. Eine Reihe von Beiträgen beschäftigte sich auch mit der Untersuchung der Auflösung und der Klassifikation von Singularitäten. Darüber hinaus wurden Vorträge über Stratifikationen, Automorphismen von Singularitäten und über die Darstellungstheorie ihrer lokalen Ringe gehalten. An mehreren Abenden wurden Diskussionen über speziellere Fragen organisiert.

Vortragsauszüge

K. BEHNKE:

Infinitesimal deformations of rational surface singularities

Let X be a rational surface singularity with minimal good resolution $\pi:(\tilde{X},E)\longrightarrow(X,x)$. It has been conjectured by O. Riemenschneider, that $\dim T^1_X \geq \dim H^1(\tilde{X},\Theta)$ + embedding-dimension(X) - 4 with equality for nonhypersurface quotient singularities. The inequality has been proved by J. Wahl, the formula for a quotient singularity is contained in papers of Behnke - Riemenschneider, Pinkham and C.Kahn, based on work of Behnke - Kahn - Riemenschneider. The problem we consider here is to find a larger class of rational singularities for which the formula above holds.

Theorem (Behnke-Knörrer):
Let $E = \bigcup_{i=1}^{r} E_i$ be the decomposition of the exceptional set into irreducible components, let $b_i = -E_i \cdot E_i$, let t_i be the number of curves adjacent to E_i, and let s_i be the number of (-2) curves E_j with $t_j = 2$, which meet the curve E_i. Assume that

(a) $b_i \geq t_i + 1$ if $b_i > 2$, $b_i \geq t_i$ if $b_i = 2$

(b) $s_i \leq b_i - t_i - 2$ if $b_i - t_i \geq 2$

(c) $s_i = 0$ if $b_i = t_i + 1$

and assume moreover one of the inequalities (b) is strict. Then

$$\dim T^1_X = \dim H^1(\tilde{X}, \Theta_{\tilde{X}}) + \operatorname{embdim}(X) - 4.$$

This result applies to almost all twodimensional quotient singularities.

R.O. BUCHWEITZ:

Maximal Cohen-Macaulay Modules

Let R be a local Cohen-Macaulay ring. A finite generated R-module M is maximal Cohen-Macaulay (MCM) iff depth M = dim R. R is of finite (MCM-) representation type (f.r.t.) iff there are only finitely many isomorphism classes of MCM's over R. We report on recent work on the classification of rings which are of f.r.t.. Among the results are:

(M. Auslander,'84):

R f.r.t. $\Longrightarrow$ R has an isolated singularity

(Artin-Verdier,'83; J.Herzog '78; M.Auslander; H.Esnault):

$R = k[[\underline{x}]] / I$, dim R = 2, R normal. Then R of f.r.t. iff R is a quotient singularity.

(Greuel-Knörrer '84; Kiyek - Steinke 84/85):

$R = k[[\underline{x}]] / I$, dim R = 1, R reduced. Then R is of f.r.t. iff R dominates a simple plane curve singularity iff for a generic map $k[[\underline{x}, y]] \longrightarrow R$ the image defines a simple plane curve singularity.

(J. Herzog '78):

R Gorenstein and f.r.t. $\Longrightarrow$ R is an abstract hyperplane singularity.

(H. Knörrer '84):

$R = k[\![\underline{x}]\!] / (f)$, char $k \neq 2$, $f \in \mathfrak{m} \setminus \{0\}$. Then

(i) R is of f.r.t. iff $R' = k[\![\underline{x}, y]\!] / (f+y^2)$ is of f.r.t..

(ii) there exists a natural bijection between isomorphism classes of MCM's over R and isomorphism classes of MCM's over $R'' = k[\![\underline{x}, y, z]\!] / (f+yz)$.

Cor.: The simple hypersurface singularities (in the sense of V.I.Arnold) are of f.r.t..

(R.O. Buchweitz; G.M. Greuel; F.O. Schreyer; '85):

Under the same assumptions

(i) R of f.r.t. $\Longleftrightarrow$ R is a simple hypersurface singul.

(ii) R has countably infinitely many isomorphism classes of MCM's (indecomposable) $\Longleftrightarrow$ R is an A_∞ or D_∞ singularity.

(M. Auslander - I. Reiten 84/85):

In dimension 3, the cone over the Veronese embedding $\mathbb{P}^2 \longrightarrow \mathbb{P}^5$ is of f.r.t..

The cone over the scroll $\mathbb{P}_{\mathbb{P}_1}(\mathcal{O}(1) \oplus \mathcal{O}(2))$ is of f.r.t..

Except the above mentioned ones there are so far no other rings of f.r.t. known.

J. DAMON:

Deformations of Complete Intersections and the Versality Discriminant

If $f: (\mathbb{C}^{s+q}, 0) \longrightarrow (\mathbb{C}^{t+q}, 0)$ is an unfolding of $f_0: (\mathbb{C}^s, 0) \longrightarrow (\mathbb{C}^t, 0)$ which defines an isolated complete intersection singularity, then it is not true that "μ const $\Longrightarrow$ topological triviality of the family of germs". Additional information about the unfolding is needed to ensure that topological triviality holds. This is provided by the local structure of the family near the versality discriminant. The versality discriminant is where the germs $f_u: (\mathbb{C}^s, 0) \longrightarrow (\mathbb{C}^t, 0)$ (for $f(x,u) = (f_u(x), u)$)

fail to be infinitesimally stable. The theorem we describe states that if the restriction of f to a "conical neighbourhood" of the versality discriminant is topologically trivial in a certain stratified sense then f itself is topologically trivial. We also describe the use of this theorem in joint work with Andre Galligo in determining the universal topological stratification for the Pham example.

W. EBELING

Dynkin diagrams and monodromy of complete intersections

We discuss invariants related to the Milnor lattice $L = H_n(X_s, \mathbb{Z})$ of an isolated complete intersection singularity (ICIS). We can prove that the following characterizations are true for all ICIS's of even dimension n with the exceptions listed below:

(i) The monodromy group is the subgroup $O^*(L)$ of all $g \in O(L)$ with spinor norm 1, inducing the identity on $L^{\#}/L$.

(ii) The set of vanishing cycles is the set of all $v \in L$ with $\langle v,v\rangle = (-1)^{n/2}\,2$ and $\langle v,L\rangle = \mathbb{Z}$.

The exceptions are all among the class of hyperbolic singularities ($\mu_+ = 1$): These are classified.
We also have results on the computation of monodromy groups and monodromy operators for ICIS's. We generalize the notion of distinguished bases to ICIS's, and generalize a method of Gabrielov for computing Dynkin diagrams for these bases. We indicate such diagrams for some classes of ICIS's. Among them we find examples of different ICIS's with conjugate (over $\mathbb{C}$) generic monodromy operators, but with different Milnor lattices.

F. ELZEIN

Variations of mixed Hodge structures (MHS)

Let $f\colon X \longrightarrow D$ be a morphism over a disk and suppose f quasi-projektive. Then $R^n f_* \mathbb{C}_X|_{D^*}$ is a local system and the weight-filtration W_i on $H^n(X_t, \mathbb{Q})$, the cohomology of a generic fibre at $t \in D^*$, is a filtration by sub-local systems. The Hodge filtration F is a filtration by sub-bundles, and we have $\nabla F^p \subset \Omega^1_{D^*} \otimes F^{p-1}$. Let $\tilde{X}^* = \tilde{D}^* \times_D X$, where $\tilde{D}^*$ is the universal cover of D^*. Then the filtration W lifted on $\tilde{D}^*$ is trivial and defines a filtration W^f on $H^n(\tilde{X}^*,\mathbb{Q})$. Then we prove, if the monodromy T is unipotent, the following theorem:

There exists a MHS on $H^n(\tilde{X}^*,\mathbb{C})$ with filtrations W and F such that

(i) W^f is a filtration by sub - MHS.

(ii) The MHS induced on $Gr_i^{W^f} H^n(X^*,\mathbb{C})$ is the limit of variations of HS induced on $Gr_i^W H^n(X_t,\mathbb{C})$ for $t \in D^*$.

(iii) $\forall$ i,b, let $N = \mathrm{Log}\, T$, then $N W_i \subset W_{i-2}$ and $N^b: Gr^W_{i+b}\, Gr_i^{W^f} H^n(\tilde{X}^*,\mathbb{C}) \simeq Gr^W_{i-b}\, Gr_i^{W^f} H^n(\tilde{X}^*,\mathbb{C})$

H. ESNAULT:

Deformation of 2 - dimensional quotient singularities

Theorem (H. Esnault, E. Viehweg):
2 - dimensional quotient singularities are stable under deformations.
This answers positively a question known as the Riemenschneider conjecture (1974). The method relies on techniques of cyclic coverings and vanishing theorems.

M. GIUSTI:

Effective computations in algebraic geometry

Let K be a field of characteristic O. Consider a homogeneous ideal I in $R = K[x_o,\dots,x_n]$ generated by polynomials of degree not greater than d, and the set $\mathfrak{J}(n,d)$ of such ideals.

1.) Hilbert function of R/I:
Define the regularity H(I) as the smallest integer where the Hilbert function and the Hilbert polynomial coincide.
What is $H(n,d) = \sup \{H(I) : I \in \mathfrak{J}(n,d)\}$?

2.) Free resolution of R/I:

Let $L^\cdot$ be a R - free resolution of R/I; if e^i is a basis of L^i, define $S_i(I,L^\cdot,e^\cdot) = \sup \{ \text{degree}(h) - i + 1 \mid h \in e^i \}$, then $S_i(I) = \inf \{ S_i(I,L^\cdot,e^\cdot) \mid L^\cdot,e^\cdot \text{ as above} \}$ and $S(I) = \sup_i S_i(I)$. What is $S(n,d) = \sup \{ S(I) \mid I \in \mathfrak{J}(n,d) \}$?

3.) Standard basis of I:

Let D(I) be the maximal degree of the elements of a standard basis of I, relative to generic coordinates and lexicographic ordering. What is $D(n,d) = \sup \{ D(I) \mid I \in \mathfrak{J}(n,d) \}$?

Theorem (Angéniol, Giusti, Lazard):

$$S(n,d) = D(n,d) = H(n,d).$$

Then an asymptotic behaviour of this common bound is given, and proved to be of doubly exponential nature. But in several particular cases this function is much more pleasant.

G.M. GREUEL:

The dimensions of smoothing components

Let (X,o) be an isolated singularity of a complex space and $f: (\mathfrak{X},o) \longrightarrow (\Delta,o)$ be a smoothing of (X,o), i.e. a 1 - parameter deformation of (X,o) such that the generic fibre is smooth. Let S(f) denote an irreducible component of the semiuniversal base over which the smoothing occurs. S(f) is then a smoothing component. After reviewing previous work on the dimension of S(f), mostly due to J. Wahl (Topology 1981) using a recent result of E. Looijenga about the globalizability of smoothings, we indicated the proof of the following theorem which was conjectured by J. Wahl and constitutes joint work with E. Looijenga (Duke Math. J. '85):

$$\dim S(f) = \dim_{\mathbb{C}} \operatorname{coker} (\Theta_{\mathfrak{X}/\Delta,o} \longrightarrow \Theta_{X,o})$$

where $\Theta_{\mathfrak{X}/\Delta}$ respectively Θ_X denotes vectorfields relative to f respectively on X.

H. HAUSER

Characterizing singularities

(X,o) complex analytic hypersurface defined by f = 0, Sing (X,o) the singular subspace of the local ring $\mathcal{O}_n / (f)+j(f)$, where $j(f)=(\partial_1 f,\ldots,\partial_n f)$ denotes the Jacobian ideal, Sing*(X,o) strict singular subspace of the local ring $\mathcal{O}_n / (f)+\mathfrak{m}j(f)$. (X,o) is called of <u>isolated singularity type</u> if Sing(X,o) $\not\approx$ Sing(X,a) for a close to o.

<u>Theorem</u> (T. Gaffney, H. Hauser):

(A) (X,o), (Y,o) arbitrary hypersurfaces. Then (X,o) is stably equivalent to (Y,o) iff Sing*(X,o) $\approx$ Sing*(Y,o).

(B) Let (X,o), (Y,o) be hypersurfaces of isolated singularity type. Then (X,o) is stably equivalent to (Y,o) iff Sing(X,o) $\approx$ Sing(Y,o).

The statement (B) is false in general.

U. KARRAS:

<u>On the μ - const. stratum of 2 - dimensional hypersurface singularities</u>

Let Δ_μ denote the μ - constant stratum of the semiuniversal deformation δ of an isolated hypersurface singularity. Then one knows by work of Zariski, Lê and Wahl that for plane curve singularities Δ_μ is smooth and δ is equimultiple along Δ_μ. This arises the question whether or not this also holds in higher dimensions. In 1982 Varchenko gave an affirmative answer if the singularities are quasihomogeneous. In this talk I introduce quite different techniques which turn out to be helpful to handle the question for rather general 2-dimensional hypersurface singularities. The first key point is that by work of Wahl, Laufer, Perron, Bingmer and Kosarew I can show that there exists a deformation functor which describes the μ - constant deformations.

<u>Theorem</u>:

dim (V,o) = 2, π: M $\longrightarrow$ V minimal good resolution. Then there exists a semiuniversal deformation space (Σ,o) for the equisingular functor ES(M,-) such that $\Sigma_{red} = \Delta_\mu$.
As a corollary one gets that Σ is smooth if the functor ES(M,-) is non-obstructed. Except some easy cases computations of obstructions are very difficult. So the idea is to look for

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a smooth functor F together with a smooth transformation $F \longrightarrow ES$. In his recent thesis K. Altmann introduces a smooth functor ESE which works well in the cases where the defining equation f is given by a polynomial which is "non - degenerated" on the Newtonboundary of the Newtonpolyhedron $\Gamma_+(f)$.

Theorem:

Γ_μ is smooth if there exists a "nice" subdivision of the complex of rational cones one gets from $\Gamma_+(f)$.

P. KLUITMANN:

Braidgroup and singularities

Let L be a $\mathbb{Z}$ - lattice generated by the set Λ of its vectors of length 2. Then the Artin braid group Br_n with n strings operates on Λ^n. Restrict this operation to the set $\mathcal{B}_L$ of ordered bases of L consisting of vectors of length 2. Then one has:

1.) Let L be the lattice of an A, D, E root - system. Then the Br orbits in $\mathcal{B}_L$ are exactly those sets of bases, for which the product of the corresponding reflections is a fixed quasi - Coxeter - element. (Deligne - Voigt)

2.) For L of type: $E_k \oplus 0 \oplus 0$, $k = 6,7,8$: The sets of bases with reflection product a fixed Coxeter - element in the sense of K. Saito, form Br_{k+2} orbits.

Case 1.) seems to yield interesting epimorphisms from braid groups onto permutation groups, e.g. $Br_4 \longrightarrow \mathcal{S}_{25}$
$Br_5 \longrightarrow \mathcal{O}_{216}$

J. LIPMANN:

Topology of quasi - ordinary singularities

Quasi - ordinary (q.o.) surface singularities $P \in F \subset \mathbb{C}^3$ are those admitting a finite (germ-) map $\pi : (F,P) \longrightarrow (\mathbb{C}^2, o)$ with normal crossing discriminant in $\mathbb{C}^2$. π is called q.o. projection. There exists then a local parametrization $z = H(x^{1/n}, y^{1/n})$, H convergent series, such that for any n-th roots of unity ν, ω

we have

$$H(x^{1/n},y^{1/n}) - H(\nu x^{1/n},\omega y^{1/n}) = x^{\lambda}y^{\mu}\varepsilon(x^{1/n},y^{1/n})$$

where $\lambda,\mu \in \frac{1}{n}\mathbb{Z}$, $\varepsilon(o,o) \neq o$ (unless $\varepsilon \equiv o$). Such pairs (λ,μ) (for $\varepsilon \not\equiv o$) are called "distinguished pairs" of π. (Motivation: compare with the Puiseux parametrizations for plane curves). They determine the saturation hence by Zariski the local topology of (F,P). [Reference: Proc. Symp. Pure Math. 40, part 2 (Arcata)]. Conversely:

Theorem (Y.-N. Gau):

If P,P' are q.o., with respective q.o. projections π,π', then the distinguished pairs of π and π' coincide (mod. switching of λ and μ).

The statement must be technically elaborated if some λ_i or $\mu_i = 0$.

Corollary:

Then P and P' have the same multiplicity, and even isomorphic Zariski tangent cones (non-reduced). Also the Galois groups of the abelian covers π and π' are isomorphic.

The proof uses the invariance of topological type of plane slices $\perp$ components of Sing(F).

Theorem:

Galois group of $\pi \cong H_1$(link of (F,P)).

Problems: 1.) Find a topological interpretation of $\{(\lambda_i,\mu_i)\}$.

2.) Generalize theorem to higher dimensions.

J.Y. MERINDOL:

Déformation verselle du cône sur l'intersection lisse de deux quadriques

Notons $X \subset P^{2n+1}$ cette intersection et C_X le cône affine sur X. Soit S la base de la déformation verselle négative et $\tilde{S} \longrightarrow S$ le revêtement associé à la représentation monodromique du groupe $\pi_1(S-D)$ (D est le discriminant). Le groupe de ce revêtement est $W(D_{2n+3})$. Les fibres F_Δ de la déformation sont des variétés affines dont la structure de Hodge mixte est très simple: c'est un 1-motif. L'étude détailée du morphisme des périodes permet de prouver que $\tilde{S}/\mathbb{C}^*$ est une sous-variété (que l'on peut expliciter) de $\mathcal{A} := [\mathrm{Jac}\ X]^{2n+3}$. La polarisation de la structure de Hodge mixte permet de construire une polarisation ample sur $\mathcal{A}$ et de reconstruire le cône $\tilde{S} \dashrightarrow \tilde{S}/\mathbb{C}^*$. On retrouve donc en particulier les résultats dus à H. Knörrer. La démonstration utilise une analyse assez fine des projections des intersections projectives de deux quadriques par rapport aux sous - espaces projectifs de dimension maximum.

M. MERLE:

Regular Stratifications and Lipschitz Conditions

Let $X \subset (\mathbb{C}^n, o)$ be a germ of an analytic space of pure dimension d and Y a smooth subspace of X (of dimension t). We (J.P. Henry and the author) look for the following property of the couple (X,Y):
Given $\pi: X \longrightarrow \mathbb{C}^{n-1}$ a generic linear projection and Δ the closure of the image by π of the critical locus of π restricted to the smooth part of X. Given a vector field on $\mathbb{C}^{n-1}$ tangent to Δ which is Lipschitz nearby $\pi(Y)$, non zero at the origin if $Y \neq 0$, then it can be lifted to X in such a way it remains Lipschitz nearby Y.
We can give a condition on X and Y which implies this previous one and is constructed on the model of what happens for Whitney conditions (Tessier) or Thom conditions (Henry, Merle, Sabbah).
The point in the proof is the following: If $\pi: B_Y X \longrightarrow X$ is a projective morphism which is smooth above the smooth points of $X \setminus Y$, and such that $\pi^{-1}(Y)$ is a divisor on $B_Y X$, then the constancy of the fibres-dimension of the morphism $|\pi^{-1}(Y)| \longrightarrow Y$ can be expressed by the fact that some section of a fibre bundle on $B_Y X$ is bounded.

G. MÜLLER:

Reductive automorphism groups of analytic $\mathbb{C}$ - algebras

Let R be an analytic or formal $\mathbb{C}$ - algebra, and let $\mathcal{L}$ be the image of the automorphism group $\mathcal{G}$ = Aut R in the general linear group of the cotangent space. It was shown that two reductive

subgroups of $\mathcal{G}$ are conjugate in $\mathcal{G}$ if and only if their images in $\mathcal{L}$ are conjugate in $\mathcal{L}$.
Since $\mathcal{L}$ is an algebraic group it has a reductive subgroup which contains any reductive subgroup of $\mathcal{L}$ up to conjugacy.
Now it is reasonable to ask whether also $\mathcal{G}$ (which is not algebraic in general) has a reductive subgroup with the corresponding property.
The answer was shown to be yes for any formal R, and for any analytic R which are homogeneous. This generalizes results of Jänich, Wall and Wahl for isolated singularities.

K. SAITO:

Regular systems of weights and associated singularities

We discussed about 18 families of algebraic surfaces. 6 of them are elliptic K3 - surfaces, 7 of them are of Kodaira-dimension 1 with an elliptic fibration ober $\mathbb{P}_1$, and the remaining are families of surfaces of general type with $(p_g, c_1^2) = (4,5)$, $(3,3)$, $(3,2)$, $(3,2)$ and $(2,1)$.
The families are obtained as compactifications of smoothings of 18 singularities with $\mathbb{C}^*$ - actions, whose generality is due to Pinkham. The singularities are hypersurface singularities whose weights $(a,b,c;h)$ are listed with the help of regular systems of weights as follows:
Let Γ be a discrete subgroup of $SL_2(\mathbb{R})$ with $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \notin \Gamma$ and $\mathbb{H}/\Gamma$ compact. Then the one point partial compactification $\{0\} \cup \widetilde{\mathbb{H}}/\Gamma$ for $\widetilde{\mathbb{H}} := \{(u,v) \in \mathbb{C}^2 : \operatorname{Im}(u/v) > 0\}$ is an affine surface with an isolated singular point at 0. Then there are exactly 18 families of such Γ such that $\{0\} \cup \widetilde{\mathbb{H}}/\Gamma$ are hypersurfaces whose weights are characterized by the following two conditions:

(i) $(T^h - T^a)(T^h - T^b)(T^h - T^c) \,/\, (T^a - 1)(T^b - 1)(T^c - 1)$ is a polynomial in T

(ii) $a + b + c - h = -2$

J. SCHERK:

Locally homogeneous links of surface singularities

Result (with F. Ehlers):
The surface singularities with locally homogeneous links (as CR manifolds) are

(i) simple singularities
(ii) simple elliptic singularities
(iii) cusps
(iv) quasihomogeneous Dolgachev singularities
(v) non-quasihomogeneous Dolgachev singularities

D. SIERSMA:

Non - isolated singularites

We consider holomorphic functions $f: (\mathbb{C}^{n+1},o) \longrightarrow \mathbb{C}$ with a 1-dim. critical locus Σ. On every irreducible branch of the singular locus there is a well defined class of isolated singularities, which we call the transversal type of the branch.
Let Σ be defined by an ideal I. We define the primitive ideal $\int I = \{f \in \mathcal{O}: (f)+J(f) \subset I\}$ where $J(f)$ is generated by the partial derivates of f. In general $I^2 \subset \int I \subset I$ and $I^2 = \int I$ if I is a reduced complete intersection.
The group $\mathcal{D}_I = \{h \in \mathcal{D}: h^*(I) = I\}$ acts on $\int I$. We get orbits in $\int I$. The tangent space to the orbit is denoted by $\tau(f)$, and $\tau(f) = \mathfrak{m}J(f) \cap \int I$ in case I is radical.
Theorem (Pellikaan):
Let I be a radical ideal. Then $c_f := \dim \int I / \tau(f) < \infty \iff$ $j_f := \dim I / J(f) < \infty \iff$ transversal type A_1 on $\Sigma - \{O\}$
There exists a theory of finite determinacy. As an application: f transversal A_1 on $\Sigma - \{O\} \Longrightarrow f \sim$ polynomial.
The above subject is treated in more generality in the thesis of Pellikaan.
We now restrict to $\dim \Sigma = 1$ and transversal type A_1 on $\Sigma - \{o\}$, and consider 1 - parameter deformations f_t, satisfying

(1) f_t has only D_∞ and A_∞ singularities on $\Sigma - \{O\}$
(2) f_t has only A_1 singularities outside Σ
(3) The Milnor fibrations over $S = \partial\Delta$ are equivalent.

Theorem (Pellikaan):

Deformations satisfying (1), (2), (3) have the property

$$j_f = {}^{\#}A_1 + {}^{\#}D_\infty + j_{f'}$$

where f' is the germ at o of f_t, $t \neq 0$.

For the proof Pellikaan shows: $I/J(f_t)$ is a free $\mathbb{C}\{t\}$-module with a finite free resolution.

Next assume also Σ is a plane curve singularity, and given by $g(x,y)=0$, $z=0$, where $z=z_3,\ldots,z_{n+1}$. We call $f=(g(x,y))^2+\Sigma z_i^2$ central singularity.

Lemma: Let f have a Σ-locus-singularity, with transversal A_1-type then there is a deformation with (1) - (3) and moreover

(4) f_t has the central singularity type at o.

Theorem:

Let Σ be a plane curve singularity with isolated singularity with transversal A_1 - type and a positive number of D_∞- points in the above deformation, then the Milnor fibre of f is homotopy equivalent to a wedge of n-spheres with

$$\mu_{top} = \dim H_n(F) = 2\mu(g) + 2\,{}^{\#}D_\infty + {}^{\#}A_1 - 1$$

If ${}^{\#}D_\infty = 0$: $b_{n-1}(F) = 1$ and $b_n = 2\mu(g) + {}^{\#}A_1$

Theorem (van Straten):

Let $\Omega_f^\bullet = \Omega^\bullet / df \wedge \Omega^{\bullet -1}$ the relative De Rham complex. $H^n(\Omega_f^\bullet)$ and $H^{n-1}(\Omega_f^\bullet)$ are free $\mathbb{C}\{f\}$-modules of finite rank $b_n(F)=\mu_{top}$ and $b_{n-1}(F)=0$ or 1. Assumption on Σ: Σ isolated complete 1-dimensional intersection with transversal A_1- type.

P. SLODOWY:

Lie groups and singularities; how far can we get?

The purpose of this talk is to give a survey of the constructions of semiuniversal deformations for some specific classes of singularities in a Lie group theoretic context. So we review Brieskorn's theorem relating the simple surface singularities of type A, D, E to the corresponding simple Lie groups and our (partial) extension of this result to simply elliptic and cusp singularities (of degree ≤ 5) which relies on previous work by Looijenga and Pinkham. Here, groups attached to Kac Moody Lie algebras come into play. Guided by work of Sekiguchi and

Shimizu on the deformations of simple plane curve singularities and infinitesimally symmetric spaces, we finally present some conjectures relating the results of Wirthmüller on semiuniversal deformations of certain space curves (as presented by him during this conference) to adjoint quotients of cones over compactified symmetric varieties (in the sense of De Concini - Procesi, Luna - Vust).

J.H.M. STEENBRINK:

Invariants of isolated singularities

Some invariants of isolated singularities have a strange jumping behaviour under μ-constant deformations. Yau introduced the irregularity $q = \dim H^0(\Omega_U^{n-1}) / H^0(\Omega_{\tilde{X}}^{n-1})$, where $(\tilde{X}, D) \longrightarrow (X,x)$ is a good resolution of the n-dimensional isolated singularity, $n \geq 2$, and $U = X \setminus \{x\}$. As the differentiation map $d_1 : H^0(\Omega_U^{n-1}) / H^0(\Omega_{\tilde{X}}^{n-1}) \longrightarrow H^0(\Omega_U^n) / H^0(\Omega_{\tilde{X}}^n(D))$ is injective, $p_g = h^{n-1}(\mathcal{O}_D) + q + a_1$, where p_g is the geometric genus and $a_1 = \dim \operatorname{coker}(d_1)$

Related invariants are $a_2 = \dim H^0(\Omega_{\tilde{X}}^n) / dH^0(\Omega_{\tilde{X}}^{n-1})$ and $a_3 = \dim H^0(\Omega_{\tilde{D}}^{n-1})$ / image of closed (n-1) forms on U . Wahl (for smoothable Gorenstein surfaces) and Looijenga and the speaker showed that for isolated complete intersections with Milnor number μ and Tjurina number τ:

$$\mu - \tau = h^{n-1}(\mathcal{O}_D) - h^0(\Omega_{\tilde{D}}^{n-1}) + a_1 + a_2 + a_3$$

moreover $h^{n-1}(\mathcal{O}_D) \geq h^0(\Omega_{\tilde{D}}^{n-1})$ by Hodge theory, so $\mu \geq \tau$; equality holds iff (X,x) is weighted quasihomogeneous, if $n = 2$ (Wahl). Also $\mu = \tau$ and $p_g = h^{n-1}(\mathcal{O}_D) = 1$ imply quasi-homogeneity for $n > 2$. Work of van Straten enables one to compute q for hypersurfaces. It appears that q does not behave semicontinously on the μ - constant stratum and need not to be zero at its general point.

J. STEVENS:

A class of surface singularities with their hyperplane sections

By blowing up regular points in the special fibre of a family $f: W \longrightarrow S$ of (degenerating) curves of genus g over a disc, the strict transform of the special fibre becomes the exceptional set of a surface singularity V (called a Kulikov singularity) and f induces a general hyperplane section X on it. For $g \leq 2$ there is a classification of curves that might occur in this way; these are the curves with $\delta - r + 1 = g$. This classification of curves is used in determining the analytic type of X from the points which are blown up. All curve singularities of the list occur. For $g = 2$ the hyperelliptic involution on W plays an important role. As a corollary we obtain a new proof of the fact that all curve singularities with $\delta - r + 1 \leq 2$ are smoothable.

D. VAN STRATEN:

Non isolated surface singularities

A basic question about non isolated surface singularities is: "What is a good class to study (in analogy with normal surfaces)?" It turns out that the weakly normal Cohen - Macaulay (WNCM) surfaces behave similar as the normal ones. For example:

<u>Theorem</u>: $\begin{array}{ccc} X & \hookrightarrow & \mathcal{X} \\ \downarrow & & \downarrow \\ 0 & \hookrightarrow & S \end{array}$ smoothing of a weakly-normal X. Then $b_1(X_t) \leq \#$ irr. comp.(X) - 1 . Equality holds if $\mathcal{X}$ is smooth.

This is generalizing $b_1(X_t) = 0$ for normal surfaces.
Because the Gorenstein WNCM's have generic transversal A_1-type, it seems wise to study the transversal A_1 CM's first. For those one can define an invarinat p_g as

$$p_g = \dim (R^1\pi_* \mathcal{O}_Y)_p$$

with $Y \xrightarrow{\pi} X$ an <u>improvement</u> replacing the resolution in the normal case. The notion of improvement is due to Shepard - Barron. One can prove several theorems using this notion of improvement and p_g.

<u>Theorem</u>: Let (X,p) a Gorenstein Du Bois singularity. Then either

(i) (X,p) isolated $\Longrightarrow$ X is a rational double point or simple elliptic or a cusp singularity (well known) or

(ii). (X,p) non-isolated $\Longrightarrow$ X is A_∞, D_∞ or a degenerate cusp.

The invariant p_g has really the properties one expect:

- independent of the improvement
- $p_g = \dim H^0(\omega_{Y-E}) / H^0(\omega_Y)$
- p_g is semicontinous under deformation

It is an invariant that is "easy" to compute.

Theorem: Let X be transversal A_1, CM, and Σ its singular locus. Let $\tilde{X} \xrightarrow{\pi} X$ be the normalization of X and $\tilde{\Sigma} = \pi^{-1}(\Sigma)$.

Then $p_g(X) = p_g(\tilde{X}) + \delta_{\tilde{\Sigma}} - \delta_\Sigma$

One can make a beginning of a classification: (non-isolated)

- $p_g = 0$ and Gorenstein $\Longrightarrow$ A_∞ or D_∞
- $p_g = 1$ and Gorenstein $\Longrightarrow$ minimal elliptic

Consider the following two singularities:

$z^2 = y(y-x^6)^2$ with $p_g = 3$ and $z^2 = y(y^6 - x^2)$ with $p_g = 2$.

Their improvements look in both cases like:

J. WAHL:

Smoothing surface singularities

One would like information on the set $\mathcal{C}$ of smoothing components in the semiuniversal deformation of an isolated surface singularity X, especially if X is "basic" - e.g. simple elliptic, $D_{p,q,r}$ or cusp. (For example, is $\mathcal{C} \neq \emptyset$?). For quasihomogeneous X, Pinkham's method of deformation of negative weight gives many examples of smoothings. One should study invariants of the Milnor fibre M of a smoothing. In the Gorenstein case, the invariants of the real homology have been computed via work of Laufer, Durfee, Wahl, Greuel, Steenbrink. The integral structure of the (even) lattice $H_2(M)$ is described in a recent paper of Looijenga-Wahl (to appear, Topology). The key point is the construction of a natural quadratic form on $H_1(L)_{tor}$ (L = link) inducing the linking form. One can associate to each smoothing component of X an element of a set $\mathfrak{Z}(X)$ of algebraic data (an even lattice of a certain discriminant quadratic form and signature, plus an isotropic subgroup of $H_1(L)_{tor}$, plus some more, satisfying some conditions). $\overline{\mathfrak{Z}}(X)$ is computable explicitly in basic cases from a resolution of X.

__Theorem__: The map $\mathfrak{a} \longrightarrow \overline{\mathfrak{Z}}(X)$ is a bijection for simple elliptics and $D_{p,q,r}$'s, and all cusps of embedding-dimension ≤ 5 (perhaps for all cusps).

C.T.C. WALL:

__Classification of invariant functions__

It is sufficient to consider germs (at o) of functions on V invariant by a linear action of G (G compact - or in the complex case - reductive). These are just the functions on the geometric quotient $V/G = \mathrm{Spec}\,(\mathcal{O}_V^G)$ (also, by a theorem of Schwarz, in the C^∞-case). In many cases, right equivalence of invariant functions on V also translates directly to equivalence of functions on V/G under selfequivalences respecting the orbit type stratification, by another theorem of Schwarz (Publ. Math. IHES *51): this holds if G acts orthogonally (e.g. in the real case) or is finite, but not in general.

There are fewer quotients V/G than pairs (V,G), but still rather many. If dim V/G = 2, then if G is finite, we have a quotient singularity (Prill); if G is a torus, a cyclic quotient (easy); if G is semisimple, V/G is smooth (Kempf). I conjecture that G reductive, dim V/G = 2 implies V/G is a quotient singularity.

Now suppose that G is finite, and the action on $V \cong \mathbb{C}^2$ is free outside o. Then for any invariant f on V with isolated singularity at o, $\mathcal{O}(V)/J_f$ is the sum of a free $\mathbb{C}G$ - module and a copy of $\lambda^2 V^*$. The proof uses the deformation theory of f. The dimension of $\mathcal{O}(V)/J_f$ is the Milnor number $\mu(f)$; the dimension of the invariant part is denoted by $\mu^G(f)$.

__Theorem__: For a generic invariant f we have

$$\mu^G(f) = \begin{cases} \max(e,4) - 3 & \text{if G is cyclic or } b \geq 3 \\ \max(e,4) - 2 & \text{if G is non-cyclic and } b = 2 \end{cases}$$

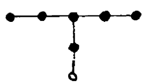
where e is the embedding dimension of V/G and (-b) the self-intersection of the central curve in the resolution.

The modality of such a function is $\mu^G(f) - 1$, so o-modal functions exist only if $e \leq 4$ and G cyclic or $b \geq 3$. The proof is by explicit calculation; the meaning of "generic" is made precise in each case.
A corresponding calculation can also be made (more easily) for a cyclic quotient $T(u,v) = (\xi u, \xi^q v)$, where the images of one or both of the lines $u = 0$, $v = 0$ are assigned to the stratification of V/G.

K.WIRTHMÜLLER:

Deformations of space curve singularities

Let X_o be an isolated complete intersection singularity, and let (S,D) be the pair (base, discriminant) of its semiuniversal deformation. We look for good descriptions of (S,D) modelled on that given by E. Brieskorn for the case where X_o is a simple hypersurface singularity. Thus we try to construct a space $\mathfrak{X}$ with a discrete group Γ acting plus an isomorphism $S \simeq \mathfrak{X}/\Gamma$ that sends D to the branch locus consisting of irregular Γ-orbits. Such a description is achiewed for X_o certain (S_μ, T_μ) of the simple space curve singularites classified and labelled by M. Giusti. The whole set-up is described in terms of certain extensions of classical Dynkin diagrams - a typical one being

$E_6[*]$ 

for the T_7 - singularity. This allows to determine all configurations of singularities in fibres adjacent to X_o as well as some geometric data on the corresponding stratification of D. The method also yields a presentation for $\pi_1(S \setminus D)$. For the S_μ - singularities, the higher homotopy groups vanish, while this is an open question in the remaining cases.

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