MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 39/1985

Low Dimensional Topology

8.9. bis 14.9.1985

The organisers for the meeting were K. Johannson (Bielefeld) and P. Scott (Liverpool). The subject of the meeting was geometric topology in dimensions two and three. Lectures were given on a wide variety of topics including: knot theory and the new two variable polynomial invariants for knots, the classification of branched coverings of surfaces, compact and non-compact 3-manifolds, Teichmüller space for surfaces, hyperbolic structures on 3-manifolds and combinatorial group theory.

List of participants:

Boileau, M. (Genêva)
Bonahon, F. (Orsay)
Bowditch, B.H. (Coventry)
Brin, M.G. (Binghamton)
Burde, G. (Frankfurt)
Carrière, Y. (Villeneuve d'Ascq)
Dunwoody, M.J. (Brighton)
Epstein, D.B.A. (Coventry)
Fenn, R.A. (Brighton)
Gabai, D. (Berkeley)
Green, P. (Coventry)
Hendriks, H. (Nijmegen)
Howie, J. (Glasgow)
Jaco, W. (Oklahoma)
Kourouniotis, C. (Liverpool)
Lickorish, W.B.R. (Cambridge)
Long, D.D. (Southampton)
Millett, K. (Santa Barbara)
Morin, B. (Strasbourg)
Morton, H.R. (Liverpool)
Otal, J.P. (Orsay)
Papadopoulos, A. (Strasbourg)
Penner, R.C. (Princeton)
Przytycki, J.H. (Warszawa)
Repovs, D. (Ljubljana)
Riley, R.F. (Binghamton)
Scharlemann, M. (Santa Barbara)
Scott, P. (Liverpool)
Short, H. (Liverpool)
Traczyk, P. (Warszawa)
Tchangang Tambekov, R. (Strasbourg)
Zieschang, H. (Bochum)
Zimmermann, B. (Bochum)

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Vortragsauszüge

M. BOILEAU: On Heegaard decompositions of torus knot exteriors

This is joint work with Markus Rost and Heiner Zieschang.

A Heegaard decomposition of genus g of a knot-exterior is the union of a handlebody of genus g with (g-1) 2-handles, which are attached along curves on the boundary of the handlebody. The Heegaard genus of the knot is the minimal genus of the Heegaard decompositions of the knot exterior.

Two Heegaard decompositions of a knot exterior are homeomorphic if there exists a homeomorphism of the knot exterior sending one handlebody to the other.

For torus knot exteriors, which are of Heegaard genus 2 we have the following result:

Thm: Let $\tau_{(p,q)}$ a torus knot of type (p,q) $(2 \leq p < q)$. The exterior of $\tau_{(p,q)}$ admits exactly 3 distinct homeomorphism classes of Heegaard decompositions of genus 2, except in the following cases:

i) $q = np \pm 1$, $q > p+1$, where there exist only two distinct equivalence classes of such Heegaard decompositions.

ii) $q = p+1$, where the Heegaard decomposition of genus 2 is unique up to homeomorphism.

Moreover these genus 2 Heegaard decompositions become homeomorphic after one stabilisation.

B.H. BOWDITCH: A natural triangulation of Teichmüller space

Let S be a compact orientable surface of genus g, and $P \subset S$, $|P| = r \geq 1$, a finite set of points. Let J_g^r be the Teichmüller space of complete finite-area hyperbolic structures on $S \setminus P$. Using quadratic differential theory, Harer and Mumford have obtained a natural triangulation of $J_g^r \times \Delta^{r-1}$ (Δ^{r-1} is a (r-1)-dim. simplex), invariant under the action of the mapping class group. It has been used by the former to investigate the algebra of this group. David Epstein and I have a geometric construction of a combinatorially identical triangulation. The simplexes correspond to isotopy classes of spines on $S\backslash P$. At a point of such a simplex, the spine has a geometric realization which can be extended to a triangulation of S, by joining its vertices to the cusp by geodesic "ribs". The barycentric coordinates in the simplex are obtained by measuring the intersections of the ribs with certain standard horocycles (given by the second coordinate of the space $J_g^r \times \Delta^{r-1}$).

M.G. BRIN: Non compact 3-manifolds

We consider non compact 3-manifolds M that are "end 1-movable", (E1M). That is, for each compact K in M, there is a compact L in M with $K \subseteq L$ so that every map $l: S^1 \to (M-L)$ extends to a map $\bar{l}: S^1 \times [0,\infty) \to (M-K)$ with $\bar{l}(x,o) = l(x)$ for all x in S^1 and with $\bar{l}$ proper into M. We describe the ends of orientable 3-manifolds that are E1M. If in addition the manifold is open and irreducible, each end is homeomorphic to an open regular neighborhood of a collection (possibly infinite) of circles and closed oriented surfaces joined together in a chain by arcs. For arbitrary, orientable, E1M 3-manifolds, the description is more complicated. Our method of proof uses a very nicely behaved object called an end reduction associated to a pair (M,K) where M is a non compact 3-manifold and $K \subseteq M$ is compact. In the talk we describe the construction of end reductions, list some of their properties, and outline how they are used in analyzing arbitrary non compact 3-manifolds.

M.J. DUNWOODY: The Accessibility of Finitely Presented Groups

A finitely generated group G is said to have more than one end ($e(G) > 1$) if it acts freely on a topological space X with more than one end so that $G\backslash X$ is compact.

Theorem (Stallings). $e(G) > 1$ if and only if there is a G-tree T such that G_e is finite for each $e \in ET$ and $G \neq G_v$ for each $v \in VT$.

Defn. A f.g. group G is accessible if there is a G-tree T such that G_e is finite if $e \in ET$ and $e(G_v) \leq 1$ if $v \in VT$.

<u>Conjecture</u> (Wall 1971). Finitely generated groups are accessible

A proof of accessibility is given for a class of groups which includes finitely presented groups.

D.B.A. EPSTEIN: <u>Convex Hulls</u>

I reported on joint work with A. Marden. We have given a proof of the following result of Sullivan:

Let Ω be an open topological disk in S^2 such that $\Lambda = S^2 \setminus \Omega$ is not contained in a round circle. We regard S^2 as the boundary of hyperbolic 3-space $\mathbb{H}^3$, in the Poincaré ball model, and let $C\Lambda$ be the convex hull of Λ in $\mathbb{H}^3$. Let $\partial C\Lambda$ be the relative boundary in $\mathbb{H}^3$. Then $\partial C\Lambda$ can be metrized by using the lengths of rectifiable paths in $\partial C\Lambda$ and, with this metric, $\partial C\Lambda$ is isometric to $\mathbb{H}^2$. There is a homeomorphism $h: \Omega \to \partial C\Lambda$ with the following properties.

1) If Ω is given the Poincaré metric, then h is K-bilipschitz. K is a universal constant.
2) h extends to the identity map from the frontier of Ω in S^2 to the frontier of $\partial C\Lambda$ in $\mathbb{B}^3 = S^2 \cup \mathbb{H}^3$

We have shown that for the smallest possible constant K such that this is true,

$$2 \le K \le 63.$$

D. GABAI: <u>The Topology of Maps of Surfaces</u>

We will discuss what a map of closed oriented surfaces must look like. In particular we will discuss the following Theorem and Corollary which was worked out jointly with Will Kazez.

<u>Theorem</u>. Let $f,g: S \to T$ be generic branched coverings of closed oriented surfaces such that $g_*, f_*: \pi_1(S) \to \pi_1(T)$ are surjective and degree f = degree g. Then there exist homeomorphisms $h: S \to S$, $k: T \to T$ such that k is isotopic to the identity and $f \circ h = k \circ g$.

Corollary. Let f,g: $S \to T$ be maps of closed oriented surfaces such that $f_*(\pi_1(S)) = g_*(\pi_1(S))$ and degree f = degree g > 0. Then there exists a homeomorphism h: $S \to S$ such that $f \circ h$ is homotopic to g.

P. GREEN: Vector fields on S^1

Thurston has shown how to go from a homeomorphism of S^1 to an earthquake (that is a (possibly discontinuous) map of $\mathbb{H}^2 \cup S^1$ to itself which restricts to a hyperbolic isometry on each stratum of some geodesic lamination).

We show how to start with a continuous vector field on S^1, and derive an "earthquake vectorfield". The proof follows the strategy of Thurston, but is simpler in places.

This construction seems to give a tangent space to universal Teichmüller space. The construction also works for vector fields making a constant angle with S^1.

For more information contact Warwick and i'll send you something!

H. HENDRIKS: The diffeomorphism group of reducible compact 3-manifolds

Let M be a connected orientable compact 3-manifold, $D_0 \subset M$ a 3-disk. Let $K \subset M$ be a "decomposing" submanifold, i.e. 1) $D_0 \subset K$, 2) K is a punctured 3-sphere, 3) M-K consists of once punctured irreducible 3-manifolds $P_1 - D_1, \dots, P_n - D_n$ (P_i irreducible and D_i a 3-disk in P_i) and of g "handles" $]0,1[\times S^2$. Following is joint work with Darryl McCullough.

Theorem: The principal Diff(M rel $K \cup \partial M$)-fibration "restriction to K": Diff(M rel $D_0 \cup \partial M$) $\to$ Emb_e(K,M rel D_0) is a product fibration. (Emb_e stands for embeddings that extend to a diffeomorphism of M.)
The proof depends largely on the following one:
Theorem (Laudenbach, Hendriks): There are H-space morphisms

$\alpha: (F_g)^n \to \mathrm{Diff}(M \text{ rel } D_0 \cup \partial M)$
$\beta: \Omega C_1 \to \mathrm{Diff}(M \text{ rel } D_0 \cup \partial M)$
$\gamma: \mathrm{Diff}(M \text{ rel } K \cup \partial M) \to \mathrm{Diff}(M \text{ rel } D_0 \cup \partial M)$

such that $h(x,y,z) = \alpha(x) \circ \beta(y) \circ \gamma(z)$ defines a homotopy equivalence. Here F_g stands for the free group of rank g, $\alpha(x_1,\dots,x_n)$ stands for the composition of "slides" of P_i along a loop traced on the handles and representing x_i. C_1 is a Kan s.s. complex consisting of certain "configurations", where a configuration is understood as a way to get a diffeomorphism of M by connected sum operations using $P_1,\dots,P_n$, a 3-sphere P_0 containing D_0 and any number of additional 3-spheres. $\beta(y)$ stands for a "slide" corresponding to y.

We also derive:

Theorem: The composition
$(F_g)^n \times \partial C_1 \xrightarrow{\alpha\cdot\beta} \mathrm{Diff}(M \text{ rel } D_0 \cup \partial M) \xrightarrow{\text{restr}} \mathrm{Emb}_e(K,M \text{ rel } D_0)$ is a homotopy equivalence.

As an application we have.

Theorem: Suppose any compact subset of the universal cover of M may be embedded in $\mathbb{R}^3$. Then the inclusion

$$\mathrm{Diff}(M \bmod K \text{ rel } D_0 \cup \partial M) \to \mathrm{Diff}(M \text{ rel } D_0 \cup \partial M)$$

induces injective homomorphisms of homotopy groups.
($h \in \mathrm{Diff}(M \bmod K \text{ rel } D_0 \cup \partial M)$ if $h: M \to M$ is a diffeomorphism such that $h(K) = K$ and $h|_{D_0 \cup \partial M}$ is the identity.)

J. HOWIE: Singular surfaces in 3-manifolds

In 1983 John Stallings made the following conjecture. Let $M \subset N$ be a smooth embedding of connected, oriented 3-manifolds, such that $H_2(N,M) = 0$. Let S be a compact oriented surface and $f: (S,\partial S) \to (N,M)$ a continuous map. Then there is a continuous map $g: S \to M$ which agrees with f on ∂S.

This is known to hold when f is an embedding (Stallings) or even an embedding on $f^{-1}(\partial M)$ (Scharlemann).

Theorem 1. The conjecture is true when S is a disc or an annulus.

The proof is a standard tower argument.

However, the conjecture turns out to be false in general.

Theorem 2 (S.M. Gersten). There exists a counterexample, in which S is a planar surface with eleven boundary components, and M,N are handlebodies of genus 2.

Indeed N is formed from a regular neighbourhood of the dunce cap Δ in $\mathbb{R}^3$ by removing a regular neighbourhood of the unique vertex of Δ. Then M is taken to be a regular neighbourhood in N of the graph $\Delta \cap \partial N$.

W. JACO: Normal surface theory and decision problems

Given a triangulation T of a 3-manifold M a properly embedded surface F in M is a normal surface if it intersects each tetrahedron of T in a collection of distinguished disks in the tetrahedron. Normal surface theory associates to the triangulation T a system of linear equations and inequalities

$$b_{ij} \Sigma (a_{ik} - a_{jk})x_k = 0$$

$$(*) \qquad x_k \geq 0$$

$$\Sigma x_k = 1.$$

The compact, convex, linear cell P_T of solutions to (*) is called the projective solution space. Each normal isotopy class of normal surfaces corresponds to a rational tuple $(x_1,\ldots x_n)$, which is a solution to (*).

In 1928, H. Kneser proved if M contains a 2-sphere which does not bound a 3-cell, then for any T there is a normal 2-sphere which does not bound a 3-cell; and in 1961, W. Haken proved if M is irreducible and ∂-irreducible, then for any T each isotopy class of incompressible and ∂-incompressible surfaces in M is represented by a normal surface.

This lecture showed if M contains a 2-sphere which does not bound a 3-cell, then for any T there is a vertex of P_T which corresponds to the normal isotopy class of a 2-sphere which does not bound a 3-cell. A similar statement holds for injective surfaces.

An irreducible decomposition of M can be constructed from any maximal collection of pairwise disjoint normal 2-spheres represented by vertices of P_T. Since it can be decided if a normal surface is injective, an algorithm exists to decide if M contains an injective surface. Restricting to Euler characteristic zero incompressible surfaces, the characteristic submanifold of M can be constructed from tori and annuli at the vertices of P_T.

Several other properties of P_T were discussed.

C. KOUROUNIOTIS: Deformations of Hyperbolic Structures

It is well known by Mostow's Rigidity Theorem that, in contrast to the case of a surface, a compact hyperbolic manifold X of dimension greater than 2 admits no (non-trivial) deformations. It is possible however to obtain deformations of the conformal structure or, equivalently, of the hyperbolic structure on the manifold $X \times I$. I shall describe one class of such deformations obtained by bending along a totally geodesic submanifold. It is conjectured that these are the only deformations of hyperbolic structures for dimension greater than 2.

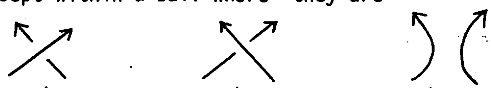
In the case of $\dim(X) = 2$, bending is a class of quasi-Fuchsian deformations of the structure on X, which are closely related to the Fenchel-Nielsen deformations. It can be extended to a class of deformations which act transitively on the space Q(X) of quasi-Fuchsian structures. It can also be extended to bending along intersecting geodesics. This gives a large enough class of paths in Q(X) which may be used to study the geometry of Q(X).

W.B.R. LICKORISH: <u>Values of the oriented link polynomial</u>

About a year ago the following result was proven:

<u>Theorem 0</u>. There exists a unique function
P: {oriented links in S^3} $\to \mathbb{Z}[l^{\pm1}, m^{\pm1}]$,
$L \longrightarrow PL(l,m)$, such that

(i) $P(\text{unknot}) = 1$,
(ii) $lP_{L_+} + l^{-1}P_{L_-} + mP_{L_0} = 0$ where $L_{\pm}$ are any three oriented links identical except within a ball where they are



L_+ L_- L_0

This polynomial (a Laurent polynomial in two variables) is related to previously known polynomials by

The Alexander polynomial	$\Delta_L(t) = P_L(i, i(t^{1/2} - t^{-1/2}))$
The Jones polynomial	$V_L(t) = P_L(it^{-1}, -i(t^{1/2} - t^{-1/2}))$.

This talk discussed the value of P_L for various substitutions of l and m.

It is elementary that Det $L = \Delta_L(-1) = V_L(-1) = P_L(i,-2)$. Also that $P_L(l,-(l+l^{-1})) \equiv 1$ and $P_L(l,m) = P_L(-l,m)$. More interesting is:

<u>Theorem 1</u>. For any oriented link L,

$$V_L(i) = P_L(1,\sqrt{2}) = \begin{cases} (-\sqrt{2})^{c-1}(-1)^{A(L)} & \text{if } A(L) \text{ is defined} \\ 0 \text{ otherwise} \end{cases}$$

Here $A(L)$ is the Arf , or Robertello, invariant of L. If L has components $c_1, c_2, \dots, c_c$ (so c is the number of components of L) $A(L) \in \mathbb{Z}_2$ is defined (as the Arf invariant of a mod 2 quadratic form) if and only if for <u>each</u> <u>i</u> $\sum_{j, j \neq i}$ linking number $(c_i, c_j) \equiv 0 \bmod 2$. The formula of the theorem not only evaluates $P_L(1,\sqrt{2})$ but it also finally resolves the unsatisfactory fact that A is defined only on <u>some</u> links.

The proof of this result uses the following facts about A and nothing more:

(i) $A(L_1 \# L_2) \equiv A(L_1) + A(L_2)$ modulo 2, if $A(L_1)$ and $A(L_2)$ are defined.
(ii) A (Trefoil knot) = 1.
(iii) If $A(L)$ is defined and $\hat{L}$ is obtained from L by banding together distinct components of L, then $A(\hat{L})$ is defined and $A(L) \doteq A(\hat{L})$.

Thus these properties, and knowledge of when $A(L)$ is defined, characterise A.

Theorem 2. $P_L(1,1) = (-1)^{(1/2)\dim H_1(T_L;\mathbb{Z}_2)}$ where T_L is the three-fold cyclic cover of S^3 branched over the oriented link L.

The proof here involved analysing T_{L_+}, T_{L_-} and T_{L_0} noting that each is a certain 3-manifold M to which a handlebody of genus 2 is glued in three different-ways. The manner these ways are related allows direct comparison of $(1/2)H_1(T_L;\mathbb{Z}_2)$ for L being L_+, L_-, L_0; the three values are n,n,n+1 in some order for some integer n.

Theorem 3. $V_L(e^{i\pi/3}) \doteq P_L(e^{i\pi/6},1) = \pm i^{c-1}(i\sqrt{3})^{\dim H_1(D_L;\mathbb{Z}_3)}$ where D_L is the double cover of S^3 branched over L.

A proof was not given.

These results are part of joint-work with K.C. Millett.

Theorem 1 and 3 were obtained in some form, without identification of the exponents, by V.F.R. Jones and J.S. Birman. Also A. Ocneanu conjectured that $P_L(1,1)$ be a power of -2.

D.D. LONG: Hyperbolic surface bundles

We discuss some joint work with U. Oertel which begins the examination of those hyperbolic 3-manifolds which are surface bundles over the circle. Our methods are largely elementary, using branched surfaces and a very little ergodic theory.

A bundle admits a hyperbolic structure if and only if its monodromy is pseudo- Anosov. This is a wellknown theorem of Thurston. The invariant foliations for the monodromy give rise to a pair of foliations for the bundle via the mapping torus construction. These foliations are shown to be in some sense "characteristic" over the relevant face of the Thurston norm. We are also able to obtain results about the variation of the topological entropy over the face, in particular, we have an elementary proof of a theorem of Fried, that this function is continuous.

K. MILLETT: Polynomial invariants of knots and links

Associated to a (often oriented) knot or link, K, there are Laurent poly-nomials with integer coefficients which are invariants of its isotopy type: the classical Alexander polynomial, $\Delta_K(t)$, the recently discovered Jones polynomial, $V_K(t)$, and the even more recent two variable polynomial discovered, independently, by Freyd & Yetter; Hoste; Lickorish & Millett; Ocneanu; and Przytycki & Traczyk, $P_K(l,m)$. During the spring of 1985, in joint work with Robert Brandt and W.B.R. Lickorish, the existence of a new polynomial invariant, $Q_K(b)$, was discovered. It, like the Alexander poly-nomial and a suitably normalized version of the Jones polynomial, is an unoriented knot or link invariant. It is independent of the previous in-variants in that it distinguishes between knots having all earlier poly-nomials equal. It satisfies a dependence relation analogous to those discovered for the earlier polynomials and which, thereby, provides a simple algorithmic method for its calculation:
$Q_{K^+}(b) + Q_{K^-}(b) = b(Q_{K^0}(b) + Q_{K^\infty}(b))$, where K^- and K^+ denote crossing changes and K^0 and K^∞ denote crossing eliminations; and $Q_U(b) = 1$, where U denotes the unknot.

In this lecture the existence and the fundamental nature of these polyno-mial invariants will be described as well as some of their implications and associated conjectures.

H.R. MORTON: Simple covers of S^3 and fibred links

A fibred link in M^3 will arise for any choice of cover $\pi_c: M^3 \to S^3$ branched over c by presenting c as a closed braid with some axis and considering the link $\pi_c^{-1}(L)$ in M^3.

Theorem. When the d-fold cover π_c is simple, i.e. a meridian of c is covered by d-1 curves with one projecting by degree 2, then any two different choices L, L' for axis yield fibred links $\pi_c^{-1}(L)$, $\pi_c^{-1}(L')$ whose fibres differ by a sequence of plumbing and deplumbing Hopf bands.

Other results using simple covers were described, showing that all fibred links can be constructed in this way, and giving restrictions which may be imposed on the branch set and choice of axis in constructing links which arise from the trivial knot by plumbing Hopf bands (without deplumbing).

A. PAPADOPOULOS: The twist flow on measured foliations space

Let S be a closed oriented surface, MF the space of measured foliations on S, equipped with its natural PL and symplectic structures defined by Thurston, and let γ be a homotopy class of simple closed curves on S. For each element F of MF satisfying $i(\gamma,F) \neq 0$, there exist representatives F^* and γ^* of the classes F and (resp.) γ such that γ^* is transverse to F^* and does not pass through singular points. For each real number t, define a measured foliation F_t^* by cutting the foliated surface (S,F^*) along γ^*, and gluing back again after an isometric twist whose amount is $t\cdot i(F,\gamma)$ (the sign of t being given by the sense of rotation). The equivalence class of F_t^* depends only on the equivalence classes F and γ, for each t. This defines a flow on the subset $N(\gamma)$ of MF defined by the equation $i(\gamma,\cdot) \neq 0$.

We prove the following:

(1) The flow extends continuously by the identity on the complement of $N(\gamma)$, and defines a flow h_t on the whole space MF. h_t is piecewise-linear.

(2) The flow h_γ is the symplectic dual of the square of the intersection function $i(\gamma,\cdot)$ on MF. This means that for each point $F \in MF$, one can find

a local coordinate chart around F s.t. the function and the flow are smooth in these coordinates, and s.t. the vectorfield tangent to the flow is in the usual sense dual to the differential of the function, w.r.t. the symplectic form in this coordinate chart.

(3) The flow h_γ induces a flow on PMF (the space of rays in MF) which is the extension at infinity of the parametrized Fenchel-Nielsen flow on Teichmüller space C. (Here, the Fenchel-Nielsen flow is parametrized in such a way that at time t, the amount of twisting is equal to $t \cdot$ length of γ).

These 2 flows associated to γ, resp. on C and PMF, interpolate the action of the Dehn twist along γ on the closed ball $C \cup PMF$.

R.C. PENNER: Moduli space and perturbative series

There is a natural parametrization of the Teichmüller space Y of a punctured surface on which the modular group MC acts algebraically. There is furthermore a natural cell decomposition of Y closely related to these parameters, and we compute cohomological invariants of MC using this decomposition. Using the technique of Feynmann Diagrams from Quantum Field Theory, we compute the virtual Euler characteristic of MC as a special case of a more general generating function. Our approach suggests that more delicate cohomological invariants should be computable in this manner; moreover, there is presumably an interesting connection between our computation and the string theory of particle physics.

J. PRZYTYCKI: Invariants of links of Conway type; algebras, polynomials and signatures (with P. Traczyk)

1. Conway algebras and partial algebras.

Def. 1.1. Conway algebra is an algebra A with a countable number of 0-argument operations $a_1, a_2, \dots$ and two 2-argument operations $|$ and $*$ which satisfy the following conditions:

$$\left.\begin{array}{ll} \text{C1.} & a_n | a_{n+1} = a_n \\ \text{C2.} & a_n * a_{n+1} = a_n \end{array}\right\} \text{ initial conditions property}$$

$$\left.\begin{array}{ll} \text{C3.} & (a|b)|(c|d) = (a|c)|(b|d) \\ \text{C4.} & (a|b)*(c|d) = (a*c)|(b*d) \\ \text{C5.} & (a*b)*(c*d) = (a*c)*(b*d) \end{array}\right\} \text{ transposition property}$$

$$\text{C6.} \quad (a|b)*b = a$$

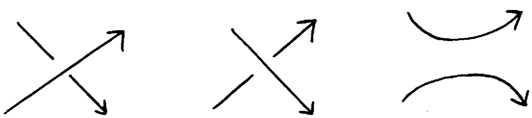
$$\text{C7.} \quad (a*b)|b = a$$

Every Conway algebra yields an invariant of links which is constant on skein equivalence classes (skein invariant). It is uniquely determined by the following conditions:

$$A_{T_n} = a_n \qquad \text{initial conditions}$$

$$\left.\begin{array}{l} A_{L_+} = A_{L_-} | A_{L_o} \\ A_{L_-} = A_{L_+} * A_{L_o} \end{array}\right\} \text{ Conway relations}$$

T_n denotes a trivial link of n components and L_+, L_- and L_o are diagrams of oriented links identical except near one crossing point, where they look like



L_+ $\qquad L_-$ $\qquad L_o$

It can be observed that in order to get a link invariant it is not necessary to have the operations $|$ and $*$ defined on the whole product $A \times A$ and relations C3 - C5 need not to be satisfied by all elements of $A \times A \times A \times A$. This observation is formalized in the notion of geometrically sufficient partial Conway algebra.

2. Polynomial invariants of Conway type

The two variable Jones polynomial (Homfly polynomial) is an example. A generalization into infinitely many variables polynomial is shown.

3. Signature and supersignature

We consider signature and Tristram-Levine signature from point of view of skein theory. We suggest further generalization of the signature (super-signature).

D. REPOVS: An exotic generalized 3-manifold

It has been conjectured that topological 3-manifolds are characterized by the amount of general position they possess and by their local homology and homotopy properties -- as is the case in higher dimensions (≥ 5). In 1983, R.C. Lacher and D. Repovs have shown that, indeed, topological 3-manifolds are precisely those generalized 3-manifolds (= ANR homology 3-manifolds) X with dim $S(X) \leq 0$, which satisfy the so-called map separation property (= finite collections of Dehn disks which intersect only at the interior points can be replaced by pairwise disjoint disks which agree with the old ones on the boundary). In order for such a theorem to be valid, one must, of course, assume the Poincaré conjecture. For we show that the existence of fake 3-cells (= compact 3-manifolds with boundary which are contractible but different from B^3) implies the existence of the following exotic space:

(1) X is a compact, homogeneous ANR;

(2) X is a $\mathbb{Z}$-homology 3-manifold (hence dim X = 3, so X is a generalized 3-manifold);

(3) X is not a cell-like image of any 3-manifold (hence X itself isn't a 3-manifold, so it is totally singular);

(4) X has the Dehn's lemma property;

(5) X has the map separation property;

(6) $X \times \mathbb{R} \approx S^3 \times \mathbb{R}$ (so, in particular, $X \hookrightarrow \mathbb{R}^4$ and $X \simeq S^3$).

This is a report on a joint work with W. Jakobsche.

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R.F. RILEY: NONFAITHFUL REPRESENTATIONS AMID ALL $SL_2(\mathbb{C})$-REPRESENTATIONS OF A HYPERBOLIC 3-MANIFOLD GROUP

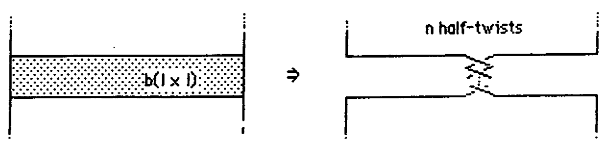
Let M be a noncompact complete hyperbolic 3-manifold of finite volume. Then the $SL_2(\mathbb{C})$-representations of $\pi_1 M$ correspond to points of a (generally reducible) algebraic variety V, and Thurston has shown that V has an algebraic component V_{f1} of dimension ≥ 4 whose generic representations are faithful. We show that points $p \in V_{f1}$ whose representations ϕ_p are not faithful are dense in V_{f1}. The proof actually shows that the image of the nonfaithful representations ϕ_p constructed have elements of large and varying finite orders. This is deduced from a general result about holomorphically parametrized families of groups in $SL_2(\mathbb{C})$, which is essentially a new conclusion to be drawn from Jørgensen's proof of his famous inequality

$$|tr(X)^2 - 4| + |tr(XYX^{-1}Y^{-1}) - 2| \geq 1,$$

when $gp(X,Y) \subset SL_2(\mathbb{C})$ is a discrete nonelementary group.

M. SCHARLEMANN: TWISTED BANDS ON THE UNKNOT

A band on a link $L \subset S^3$ is an embedding $b: I \times I \to S^3$ such that $b^{-1}(L) = \partial I \times I$. A band b is trivial if there is a disk in S^3 whose interior is disjoint from $L \cup \text{image}(b)$ and whose boundary is the union of $b(I \times \{0\})$ and a subarc of L. Say L_n is obtained by an n-twist banding of L if L_n is obtained by replacing the two arcs $b(\partial I \times I)$ of a band b on L with the following pair of arcs near b:



Theorem: If K and K_n are both the unknot, then the band is trivial. Applying this when n = 1, we have:

Corollary 1: An embedding of $\mathbb{R}P^2$ in $\mathbb{R}^4$ with three critical points is standard.

Corollary 2: Strongly invertible knots have property P.
In the notation of the Conway calculus we also have, using n = 2:

Corollary 3: If L_+ and L_- are the unknot, L_0 is the unlink.

R. TCHANGANG TAMBEKOV: Group of automorphisms of the modular group

I give a simpler and shorter proof of Ivanov's result, using not Thurston's classification theorem.

Theorem. - Let F_g be a closed orientable surface of genus g, $M_g = \pi_0 \mathrm{Diff}^+ F_g$ its mapping class group and $M_g^* = \pi_0 \mathrm{Diff}\, F_g$ its full mapping class group.
Then

$$\mathrm{Out}\, M_g \simeq \mathbb{Z}_2 \text{ and } \mathrm{Out}\, M_g^* = 1 \quad \text{for } g \geq 3 \;,$$
$$\mathrm{Out}\, M_2 \approx \mathbb{Z}_2 \times \mathbb{Z}_2 \simeq \mathrm{Out}\, M_2^* \;.$$

The key tool is hyperelliptic involutions (h.i.). We first prove that a given automorphism ϕ maps h.i. onto h.i., using the characterization that an h.i. is the 2g+1 power of an element of order 4g+2 of M_g.

Then, as all h.i. are conjugate, one supposes that ϕ fixes h, a given h.i., and so preserves the normalizer N(h) of h.

$F_g/h \simeq (S^2, A_{2g+2})$ where A_{2g+2} is the set of 2g+2 branch points of the projection $F_g \to F_g/h$; $N(h)/h \approx \pi_0 \mathrm{Diff}^+(S^2, A_{2g+2})$. A result of Dyer and Grossman implies that Out $N(h)/h \approx \mathbb{Z}_2$. One then deduces Out $N(h) \approx \mathbb{Z}_2 \times \mathbb{Z}_2$, Out $M_g \approx \mathbb{Z}_2$ and Out $M_g^* = 1$, for $g \geq 3$.

When g = 2, the knowledge of finite subgroup of M_2^* is relevant. The same proof extends to the case of surfaces with one puncture.

Theorem. - Out $M_{g,1} \simeq \mathbb{Z}_2$ and Out $M_{g,1}^* = 1$ for $g \geq 2$.

M. Ivanov has announced that this is true for an arbitrary number of punctures.

H. ZIESCHANG: 3-manifolds connected with Hamiltonian systems
(with A.T. Fomenko, Moskau)

Let M^4 be a symplectic manifold and v = sgrad H a Hamiltonian field on M^4 with a totally integrable Hamiltonian H. In a neighbourhood of a level surface $Q^3 = \{H = \text{const}\}$ there is a function commuting with H on Q having the Bott properties. Then Q can be decomposed in solid tori, cylinders and "trousers":

$$Q = m(D^2 \times S^1) + n(T^2 \times D^1) + q(N^2 \times S^1) \quad (T^2 \cong S^1 \times S^1).$$

Let (Q) denote the class of all orientable closed manifolds of this type.
Problems:
1) Which manifolds can appear?
2) Give a lower bound for the number of solid tori (which correspond to stable periodic solutions).

Main result:

Theorem 1: $3m \geq \varepsilon - 5\beta + 4$, $g \geq m-2$, where β is the first Betti number of Q, ε the minimal number of generators for Tor $H_1(Q,\mathbb{Z})$.
Let (Γ) denote the class of graph manifolds and (S) the class of S-manifolds which are defined as follows. There is a smooth function such that the critical points lie on circles and the regular points of the same value form a collection of disjoint tori.

Theorem 2: $(S) = (Q) = (\Gamma)$.

The first equality is due to Burmistrova-Matveev. - The proof of Theorem 1 is done by calculating a presentation of the fundamental group of a manifold of (Γ) and calculating bounds from it for the Betti numbers etc.

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