MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 45/1985

Mathematische Logik

12.10. bis 26.10.1985

Die Tagung fand unter der Leitung von Herrn W. Felscher (Tübingen) und Herrn H. Schwichtenberg (München) statt. An ihr nahmen 43 Wissenschaftler aus 12 Ländern teil, darunter 23 aus Deutschland.

Es wurden 28 Vorträge aus allen wichtigen Teilgebieten der mathematischen Logik gehalten. Einen Schwerpunkt der Tagung bildete die Beweistheorie und konstruktive Mathematik mit 9 Vorträgen und einer Abendveranstaltung, in der Herr Girard (Paris) über seine Modelle des λ-Kalküls vortrug.

Weitere Schwerpunkte der Tagung waren die Modelltheorie mit 5 Vorträgen und die Verbindungen der Logik zur Informatik, insbesondere die Komplexitätstheorie, mit 7 Vorträgen.

Vortragsauszüge

W. ALEXI: Extraktion und Verifikation von Programmen aus formalen Beweisen.

Konstruktive Beweise für $\bigwedge x \bigvee y A(x,y)$ enthalten implizit Algorithmen, um zu jedem x_o ein y_o mit $A(x_o,y_o)$ zu berechnen. Wir beschreiben ein neues, effizientes Verfahren, diese Algorithmen explizit in Termen einer einfachen, kompilationsfähigen Programmiersprache ohne höhere Typen anzugeben. Diese Programme benutzen nur die elementaren Daten des Beweises.

Wir gehen von einem Beweis in der natürlichen Deduktion (mit +,·, Funktionszeichen und voller Induktionsregel) aus und fassen zunächst gleichgelegene Zeichen durch untere Indizes zusammen. Die $\bigvee$-Vorkommen mit gleichem Index werden in natürlicher Weise als Knoten eines gerichteten Graphen aufgefaßt. Eine Kante von ν_1 nach ν_2 bedeutet, daß eine Realisierung von ν_2 für die von ν_1 erforderlich ist. Durch Entfernen geeigneter Kanten erhält man zyklenfreie Komponenten, die sich als Bäume erweisen. Jedem Baum entspricht ein Funktionsunterprogramm. An der Wurzel wird die definierende Gleichung abgelesen, die Blätter kennzeichnen die Stellen, an denen die Funktion benutzt wird. Das endgültige Programm ergibt sich durch natürliche Zusammensetzung. Das Extraktionsverfahren benötigt höchstens O (Beweisgröße^3) viele Rechenschritte.

Für die so extrahierten Programme P wird Terminierung und Korrektheit (d.h. das zur Eingabe x_o berechnete $P(x_o)$ erfüllt $A(x_o,P(x_o))$) global mathematisch bewiesen, indem eine Korrespondenz des Programmablaufs mit der Normalisierung der wesentlichen Teile des Beweises hergestellt wird.

G. ASSER: Logische Abhängigkeiten im Prädikatenkalkül der zweiten Stufe.

Es werden verschiedene in der Sprache des Prädikatenkalküls der zweiten Stufe formalisierbare Endlichkeitsdefinitionen sowie Auswahlprinzipien und verwandte Aussagen (Wohlordnungssatz, Ordnungssatz, Vergleichbarkeitssatz, Zornsches Lemma) auf ihre gegenseitigen logischen Abhängigkeiten in diesem Kalkül untersucht.

E. BÖRGER: Logical decision problems and complexity of logic programs.

We report on recent developments on logical characterizations of (recursive) complexity classes. We mention the approach of restricting logic semantically to finite structures and looking for natural syntactical enrichments of pure logic by new expressive means like least fixed points, transitive closure operators etc.; examples are recent ACM-STOC, IEEE-FOCS, JSL, JC SS papers by Fagin, Immerman, Vardi, Gurevich, Shelah, Lewis and forthcoming work by Gurevich and myself (Ω-series book in preparation). Then we discuss the approach of restricting logic purely syntactically to capture given complexity classes. We report on our recent solution (together with U. Löwen) of the complexity of subcases of the Bernays-Schönfinkel class and the relation to the computational complexity of the corresponding PROLOG programs. For details we refer to our book "Berechenbarkeit, Komplexität, Logik", Vieweg 1985.

A. CANTINI: A Forcing-free Proof of a Theorem of Harrington.

D. van DALEN: Apartness relations on free abelian groups.

In intuitionistic mathematics we can carry out the construction of free abelian groups F_X over arbitrary sets X. Equality between elements $\vec{a}$, $\vec{b}$, considered as words, is defined by $\bigvee_{\sigma \in S_n} \bigwedge_{i \leq n} a_i = b_{\sigma i}$

Facts: (i) subgroups of free abelian groups need not be free
(ii) projective $\not\Rightarrow$ free abelian $\not\Rightarrow$ projective
(iii) (free abelian $\Rightarrow$ projective) $\Rightarrow$ tertium non datur.

Under certain conditions one can extend an apartness relation on X (canonically) to an apartness relation on F_X.

Let $B_n := \neg \bigwedge_\sigma \bigvee_i a_i \# b_{\sigma i} \to \bigvee_\sigma \bigwedge_i a_i = b_{\sigma i}$, where $\sigma \in S_n$ and $1 \leq i \leq n$.
Then one can prove

Theorem. $\vec{a} \# \vec{b} := \bigwedge_\sigma \bigvee_i a_i \# b_{\sigma i}$ is an apartness relation on F_X iff $X \models B_n$ for all n. Moreover this is the weakest apartness relation on F that extends the apartness relation on X.

Put $\underset{\sim}{AP}^n$ the theory of equality plus B_n, and $\underset{\sim}{AB}^\omega = \bigcup \underset{\sim}{AP}^n$; then

Theorem. $\vdash B_{n+1} \to B_n$, $\nvdash B_n \to B_{n+1}$ and $\underset{\sim}{AP}^\omega$ is not finitely axiomatizable.

The sequence of axioms B_n could give rise to a hierarchy of subsets of $\mathbb{R}$. So far we know that $\mathbb{N} \models \underset{\sim}{AP}^\omega$, $\mathbb{N}^\mathbb{N} \models \underset{\sim}{AP}^\omega$, $\mathbb{R} \not\models \underset{\sim}{AP}^2$. It would be interesting to see natural examples of sequences of sets corresponding to the increasing sequence $\underset{\sim}{AP}^n$. Note that $\underset{\sim}{AP}^\omega$ still remains below decidable equality.

U. FELGNER: Defining K in K(t).

Let K be a field and let t be transcendental over K and consider the problem whether K is first-order definable (without parameters) in the field of rational functions, K(t). If K is finite, real-closed or algebraically closed, then K is obviously definable in K(t). If K is of the form

F(x), where F is a field and x transcendental over F, then K is clearly not definable in K(t). The result we proved is the following: there is a first-order formula $\Phi(v)$ such that for any finite field K, $\Phi(v)$ defines K in K(t). The proof uses a combinatorial lemma about finite abelian groups. It follows that each pseudo-finite field K (in the sense of Serre and Ax) is also definable by $\Phi(v)$ in K(t). These results are joint work with W.D. Geyer (Erlangen).

J.Y. GIRARD: Equalization and Π^1_1-determinacy.

This is a joint work with A.S Kechris. The axiom of equalization is stated for regular flowers, i.e. dilators F of the form $\int_{1+D}$.

(AE) $\forall F \ \forall F' \ \exists F'' \quad F \circ F'' = F' \circ F''$.

The basic properties of (AE) are given by

Theorem 1: assuming (AE), then any set X of regular flowers has an equalizer $X^\natural$, i.e.

$\forall F \in X \quad \forall F' \in X \quad F \circ X^\natural = F' \circ X^\natural$.

Moreover, if H is any other equalizer of X, then there is a unique morphism $T : X^\natural \to H$ s.t.

$\forall F \in X \quad \forall F' \in X \quad F' \circ T = F' \circ T$.

The axiom of equalization is plausible, i.e. it can be derived from standard set-theoretic assumptions such as measurability etc. Typically

Theorem 2: given $a \subset \mathbb{N}$, then

$a^\#$ exists $\to$ every family of regular flowers recursive in a have an equalizer.

The problem is the converse of Theorem 2.

Theorem 3: equalization implies $\underset{\sim}{\Pi}^1_1$-determinacy, (with obvious "lightface" improvements.)

Essentially the proof uses, when G is a $\underset{\sim}{\Pi}^1_1$-game auxiliary games $G^*(\alpha)$, $G^{**}(D)$ (α ordinal, D dilator).

These games have the property that:

- if I has a winning strategy in some $G^*(\alpha)$ then it has a winning strategy in G.

- if II has a winning strategy in G**(D) for some D, then it has a winning strategy in G.

So the theorem reduces to the fact that, if I has no winning strategy in any G*(α), than II has a winning strategy in some G**(D). The dilator D is indeed obtained by equalizing dilators D_t, where t ranges through all "bad positions" in G, i.e. finite sequences of moves from which I has no winning strategy, using the fact that "t is a bad position" is $\underset{\sim}{\Pi}^1_2$, so that one can associate a dilator D_t to this statement. Equalization of the D_t's is needed in order to replace an inequality between different D_t's (hence non functorial) by an inequality within D.

L. GORDEEV: <u>A Generalization of the Kruskal-Friedman Theorems</u>.

Recall that Kruskal Theorem (K) states that for each infinite sequence $T_1, T_2, \ldots T_n, \ldots$ of finite trees there are $i < j$ and an (infimum-preserving) embedding $f : T_i \overset{in}{\to} T_j$.

For every natural number $n > 0$, Kruskal + Friedman Theorem (KF_n) states that for each infinite sequence of n-labeled finite trees $(T_1, l_1), \ldots, (T_m, l_m), \ldots$ (where $l_m : T_m \to n$ just assigns labels $< n$ to the nodes in T_m) there are $i < j$ and an F-embedding $f : ((_i, l_i) \to (T_j, l_j)$ - the latter means that the following additional conditions are satisfied for all $a, b \in T_i$:

(1) $l_i(a) \leq l_j(f(a))$,

(2) if b is placed just over a (in T_i), and c between f(a) and f(b) (in T_j), then always $l_j(c) \geq l_j(f(b))$.

Friedman proved that the statement $KF := \forall n KF_n$ is not provable in (Π^1_1-CA) (but is provable in Π^1_1-CA).

By replacing Friedman's gap-condition (2) by

(2^M) if b is placed just over a (in T_i), and c between f(a) and f(b) (in T_j), then always $l_j(c) \geq \min(l_i(a), l_i(b))$,

we get the modified notion of M-embeddability, which is suitable also for infinite labels.

That is, given a constructive ordinal $\alpha > 0$ define the modified statement M_α as follows: for each infinite sequence of α-labeled finite intervals (= unary trees) $(I_1,l_1),\ldots,(I_m,l_m),\ldots$ there are $i<j$ and an M-embedding $f:(I_i,l_i)\to(I_j,l_j)$.

Thm. For every limit constructive ordinal $\alpha = \lim_n \alpha[n]$, the statement $M_{<\alpha} := \forall n M_{\alpha[n]}$ is not provable in $ID_{<\alpha}$.

Cor. $M_{\omega+5}$ is not provable in $(\Pi^1_1-CA)+BI$,
$M_{<\epsilon_0}$ is not provable in Δ^1_2-CA.

Finally, by using Friedman's method of reducing the Π^1_1-statements KF_n to their appropriate Π^0_2-versions we obtain the corresponding purely combinatory Π^0_2-statements not provable in $(\Pi^1_1$-CA$)+$BI or even Δ^1_2-CA respectively.

E. GRÄDEL: Über die Komplexität von Teilmengen der Presburger-Arithmetik.

In diesem Zwischenbericht werden obere und untere Schranken für die Entscheidungskomplexität gewisser Formelklassen mit fester Variabelnzahl aus der Presburger-Arithemtik (PA) präsentiert.

(Σ^p_n bzw. Π^p_n sind dabei die Klassen der 'polynomial-time-hierarchy')
Notation: $[Q_1\ldots Q_s]\cap PA$ sei die Menge der Formeln der Form $Q_1x_1\ldots Q_sx_s\,F(x_1,\ldots,x_s)$, (F quantorenfrei) die in PA wahr sind.

1. Für alle m, $s\geq m$ und alle Listen von Quantoren $Q_1,\ldots,Q_s$ mit höchstens m Q-Alternationen gilt
   a) $Q_1 = \exists \Rightarrow [Q_1\ldots Q_s]\cap PA \in \Sigma^p_{m-1}$
   b) $Q_1 = \forall \Rightarrow [Q_1\ldots Q_s]\cap PA \in \Pi^p_{m-1}$
2. a) Es gibt ein t sodaß für jedes m eine Liste von Quantoren $Q_1,\ldots Q_{s_m}$ mit höchstens $t+m$ Alternationen und mit $Q_1 = \exists$ existiert, sodaß: $[Q_1\ldots Q_{s_m}]\cap PA$ ist Σ^p_m-hart.
   b) analog mit $Q_1 = \forall$ und $[Q_1\ldots Q_{s_m}]\cap PA$ ist Σ^p_m-hart.
3. $[\exists\forall\forall\exists]\cap PA$ ist NP-hart.
   $[\exists\forall\forall\exists]\cap PA$ liegt nach 1. in Σ^p_2. Auch hier, besonders aber im allgemeinen Fall sind also noch Lücken zwischen oberen und unteren Schranken zu schließen.

E. GRIFFOR: Π^1_1-Det implies AE_{den}.

The implication Π^1_1-Det $\Rightarrow AE_{den}$ was proved first by J.Y. Girard, using the equivalence between Π^1_1-Det and $\forall a \in {}^{\omega}\omega$($a^{\#}$ exists), i.e., indiscernibles. Here I give a direct proof building an equalizer for two non-constant flowers from a winning strategy for the player trying to unicarize terms built from these two, i.e., if one can order them uniformly on initial segments of the countable ordinals then one can do it uniformly on all. Thus functoriality of the ordinal map is given by the strategy's uniformity. The result is a flower equalizing the given two and hence proving AE_{den}.

P. HAJEK: On logic in subsystems of arithmetic.

$I\Sigma_n$ is the fragment of Peano arithmetic with induction for Σ_n-formulas: $B\Sigma_m$ is $I\Sigma_o$ plus collection for Σ_m formulas. Syntax of logic can be developed in $I\Sigma_1$ and for each n, partial satisfaction for Σ_n formulas can be defined. This can be used to code $\Sigma_n(\Pi_n,\Delta_n)$ sets, i.e. in $I\Sigma_1$ we may quantify over them. This relativizes: in $B\Sigma_m (m > 1)$, we may code over $\Sigma_n(\Delta_m)$ sets for each n. In particular, in $B\Sigma_m$ we may define low Δ_m sets (X is low Δ_m iff each Y Σ_1 definable from X Δ_m). In the sequel, Γ is either Δ_m or low Δ_m. $I\Sigma_1(\Gamma)$ proves that if a model M is in Γ and has a Γ-definable satisfaction for $\Sigma_n \cup \Pi_n$ formulas then $\{\gamma \in \Pi_{n+2} \mid M \models \gamma\}$ is consistent. $B\Sigma_2(\Gamma)$ proves the low basis theorem, and thus the low arithmetized completeness theorem: If T is consistent and Γ-definable then it has a low $\Delta_2(\Gamma)$ model with low $\Delta_2(\Gamma)$ total satisfaction. A tricky use of the low basis theorem shows that $B\Sigma_2$ proves: each low Δ_2 model M of $I\Sigma_n$ with low Δ_2 satisfaction for all formulas has an $(n+1)$-elementary extension K with low Δ_2 satisfaction for $\Pi_{n+1} \cup \Sigma_{n+1}$ formulas and such that K satisfies $B\Sigma_{n+1}$. Thus $B\Sigma_2$ (and hence $I\Sigma_1$) proves that for each n, $B\Sigma_{n+1}$ is a Π_{n+2} conservative over $I\Sigma_n$.

Corollary: $I\Sigma_{n+1}$ proves consistency of $B\Sigma_{n+1}$. Various of the main results were proved independently by Clote, Paris, and Sieg. A joint paper with Clote is in preparation.

C.W. HENSON: Computational Complexity of First Order Theories.

Joint work with Kevin Compton is described, giving a new general method for obtaining lower bounds on the computational complexity of logical theories, as well as several new results which are obtained using the method. In the longer work from which this talk is excerpted, are given new and simpler proofs of the known complexity lower bounds (such as those found in the book of Ferrante and Rackoff and in the PhD thesis od Larry Stockmeyer), many of which have never before been published.

Here will be emphasised the methods and new results which pertain to various complexity classes of elementary recursive problems, esp. the classes $NTIME(exp_k(n))$ and $ATIME(exp_k(n),n)$, where exp_k denotes the k-fold iterated exponential function. By using inseparability results we obtain complexity lower bounds which are hereditary (apply to all subtheories.) The methods eliminate the need to code machine computations into models, and are based on much simpler definability considerations, namely: what kinds of binary relations on large finite sets can one define in models of the theory, using short formulas.

W. JUST: Boolean algebras of the form $P(\omega)/I$.

Shelah (see S. Shelah, "Proper forcing") proved that it is relatively consistent with ZFC that $\frac{P(\omega)}{Fin}$ has only trivial automorphisms, i.e. induced by some bijection $\pi : \omega \setminus A \to \omega \setminus B$ where A and B are finite sets.

We explain the idea of his proof and give two examples of theorems which can be proved by exactly the same method.

Theorem 1: It is relatively consistent with ZFC that there is no continuous $f : \omega^* \overset{onto}{\longrightarrow} \omega^* \times \omega^*$, where ω^* denotes $\beta\omega \setminus \omega$.

Theorem 2: Let I be an ideal containing Fin defined both by a $\underset{\sim}{\Sigma}^1_2$ and a $\underset{\sim}{\Pi}^1_2$ formula. Denote
$Tr(I) = \{A \subset \omega : \exists B \subset A$ "$I \cap P(A)$ is generated by $Fin \cup \{B\}$"$\}$
If ZFC $\vdash$ "$\frac{P(\omega)}{I}$ is embeddable in $\frac{P(\omega)}{Fin}$" then either $\omega \in Tr(I)$ or ZFC $\nvdash$ "Tr(I) has the Baire property".

H. KOTLARSKI: Some versions of ω-logic.

Let M be a model of PA (= Peano Arithmetic). Let L be the language of PA in the sense of M(via arithmetisation). We define the semi-non standard system of ω-logic. Axioms: all true atomic and all negated false atomic and all sentences of the form $A \vee \neg A$. Rules of proofs: as usual, but include

the ω-rule: $\dfrac{A \vee B(x) \,:\, x \in M}{A \vee \forall x\, B(x)}$.

Proof trees are defined by induction on the ordinals, outside of M(so ranks of such trees are standard).

THM (Lachlan, Krajewski, Kotlarski). If M is countable then this logic is consistent iff M is either standard or recursively saturated.

In fact, if M is recursively saturated then for any sentence A, provable in this logic, A is provable by means of a proof tree of finite rank.

We define another system of ω-logic. Work in PA. Define a sequence $\Gamma_n(\cdot)$ of formulas. $\Gamma_o(\varphi)$ is $PA \vdash \varphi$. $\Gamma'_{n+1}(\varphi)$ is "φ is of the form $\eta \vee \forall x \Psi(x)$ and $\forall z\ \Gamma_n(\eta \vee \ (x))$" and finally $\Gamma_{n+1}(\varphi)$ means that φ is provable (in the usual logic) from the axioms of PA and formulas having the property Γ'_{n+1}.

Remark: each Γ_n is a formula of PA, indeed, Γ_n is Σ_{2n+1}. The first system of ω-logic is used to construct full satisfaction classes on models of arithmetic; the second corresponds to full satisfaction classes S on M such that $(M,S) \models \Delta_o$-induction.

THM. (in PA). If $\neg\Gamma_n(0 = 1)$ then $\neg\Gamma_n(\neg\Gamma_n(0 = 1))$.
TMM. $PA \cup \{\neg\Gamma_n(0 = 1) : n \in \omega\} \vdash RFN$

[RFN is the reflection scheme: $(PA \vdash \varphi) \rightarrow \varphi$, φ a formula]
PROBLEM. Does $PA \cup RFN \vdash \neg\Gamma_n(0 = 1)$ for all n?
Presumably the answer is negative.

G. KREISEL: Zu zwei Tagungsthemen.

H. LUCKHARDT: Herbrand's Theorem heute.

A previously unexpected method, combining logical and mathematical elements, is shown to yield substantial numerical improvements in the area of diophantine approximations. Kreisel illustrated the method abstractly by noting that effective bounds on the number of elements are answered of Herbrand terms from ineffective proofs of Σ_2-finiteness theorems satisfy certain simple growth conditions. Here several efficient growth conditions for the same purpose are presented that are actually satisfied in practice, in particular, by the proofs of Roth's due to Roth himself and to Esnault/Viehweg. The analysis of the former improves mildly (by a factor e^{215}) the exponential bound (in ε^{-2}, d^{2}) given by Davenport/Roth in 1955, for (real) algebraic α of degree d and $|\alpha - P|q| < q^{-2-\varepsilon}$. The new bounds extracted from the other proof are polynomial of low degree (in ε^{-1}, log d). -

Corollaries: Apart from essentially the same bound for the number of solutions of the corresponding diophantine equations and inequalities, $\log \log q_{\nu} < C_{\alpha,\varepsilon} \nu^{5/6+\varepsilon}$, where q_{ν} are the denominators of the convergents to the continued fraction of α.

A.R.D. MATHIAS: Parametrized Galvin-Prikry.

G.P. MONRO: The interpretation of logic in categories.

This talk concerns the interpretation of logic (both first-order and higher-order) in categories. The usual interpretation (as in Makkai and Reyes "First Order Categorical Logic", Springer Lecture Notes vol. 611) requires predicates to be interpreted as arbitrary monomorphisms.

Here a generalization is obtained by imposing a factorization system (E,M) on the category and interpreting predicates as arrows in M. In the case of higher-order logic this enables the interpretation of the logic of elementary topoi in categories which are not topoi. An example is the category of topological spaces. Another example is obtained from an elementary topos with a (Lawvere-Tierney) topology. This leads to a proof "by logic" of the associated sheaf theorem for such topologies.

D. MUNDICI: Applications of abstract model theory to algebra and functional analysis.

THEOREM There is a categorial equivalence Γ between Lindenbaum algebras of the infinite-valued sentential calculus of Lukasiewicz and abelian lattice-ordered groups with strong unit.

THEOREM Every AF C*-algebra A can be embedded into a unique AF C*-algebra B such that $(K_o(B), [1_B]) \cong (K_o(A)_\ell, [1_A]_\ell)$, where $K_o(A)_\ell$ is the free ℓ-group over the Riesz group $K_o(A)$, and $[1_A]_\ell$ is the image of $[1_A]$ in the Weinberg free embedding.

Let now S be the set of sentences in the infinite-valued sentential calculus. Further, let correspond via K_o an Γ to the free Lindenbaum algebra of the infinite-valued sentential calculus, with denumerably many free generators.

THEOREM Let θ map each AF C*-algebra A with lattice-ordered K_o into the set $\theta(A) = \{T \subset S \mid$ the Lindenbaum algebra of T corresponds to A via K_o and $\Gamma\}$. Then $\theta(A) \neq \emptyset$. Moreover, for any two such C*-algebras A and B we have $A \cong B$ iff $\theta(A) = \theta(B)$. For every set $T \subset S$ there is a unique A such that $T \in \theta(A)$.

THEOREM The AF C*-algebras with comparability of projections in the sense of Murray, von Neumann are precisely the quotients of by primitive essential ideals of .

H. OSSWALD: Pettis Integrierbarkeit auf Loeb-Räumen.

Die Pettis-meßbaren und Pettis-integrierbaren Funktionen werden durch lifting-Eigenschaften charakterisiert. Als Anwendungen erhält man

(a) Lösungen für spezielle Peano-Caratheodory Integralgleichungen bzgl. des Pettis Integrals nach der Keisler-Methode,
(b) Existenz der bedingten Erwartungen für beschränkte Pettis integrierbare Funktionen,
(c) eine vektorwertige Version von Keislers Fubini Theorem.

W. RAUTENBERG: The Cantor Bernstein theorem for weak theories.

This theorem may be stated as follows

CBt. If $A \supseteq B$ are sets and $F : A \quad B$ is injective then there is a bijection $G : A \to B$.

If A,B are supposed to be classes definable by formulas and F is a definable mapping, we obtain the class form of CBt, denoted by CBT.

Fact 1. CBt holds in the system of Kripke-Platek. The proof essentially uses foundation.

Fact 2. There is a 1^{st} order theory T such that CBT does not hold for the definable classes over T.

Fact 3. Let T be a theory in which we can prove: $\emptyset$ is a set; if a is a set, b an individual, then $a \cup \{b\}$ is a set. Then CBT holds for the classes over T.

Corollary. CBT holds for weak 2^{nd} order logic.
Clearly, CBT holds for other logics slightly stronger then 1^{st} order logic as well. In fact, it holds for most of the natural 1^{st} order logics.

M.M. RICHTER: Algorithmen in Gruppen polynomialen Wachstums mit kleinem Grad.

Für den Fall endlicher präsentierter Gruppen wird der Zusammenhang zwischen folgenden Stichworten diskutiert: Vollständige Systeme von Reduktionen, welche Worte auf eine kanonische Normalform reduzieren; der Vervollständigungsalgorithmus nach Knuth und Bendix und sein Verhalten für gewichtsdefinierte Ordnungen; die Zykelstruktur des minimalen Automaten der irreduziblen Worte; die Wachstumsfunktion der Gruppe und konstruktive Aspekte des Satzes von Gromov. Dabei steht hier der Fall polynomialen Wachstums im Vordergrund und es werden die Fälle mit Wachstumsgrad ≤ 4 diskutiert. Für den Fall der nilpotenten Gruppe von zwei Erzeugenden wird B. Benningshofens Satz erklärt, daß der Vervollständigungsalgorithmus für keine Gewichtsordnung ein reguläres System generiert, auch wenn beliebig (endlich) viele neue Konstanten eingeführt werden. Der Beweis benutzt eine von v.d. Dries und Wilkie in den Bereich des Gromov'schen Satzes eingeführte Technik aus dem Bereich der Nichtstandard-Analysis.

U.R. SCHMERL, Über ein Entscheidungsproblem für ein Fragment der Arithmetik.

Folgendes Problem wird untersucht: Ist für eine gegebene diophantische Gleichung $f(x_1 \ldots x_n) = 0$, $f \in \mathbb{Z}[x_1 \ldots x_n]$, entscheidbar, ob sie im Fragment Z_0 der Arithmetik widerlegbar ist, d.h. ob $Z_0 \vdash \forall x_1 \ldots x_n \; f(x_1 \ldots x_n) \neq 0$? Dabei ist Z_0 das Arithmetikfragment mit den Symbolen 0,N,+,·, den üblichen Axiomen für diese Symbole und vollständiger Induktion für quantorenfreie Formeln. Bei diesem System kann die Herleitbarkeitsrelation für quantorenfreie Formeln übersetzt werden in eine idealtheoretische Beziehung für Polynome. Damit kann das Problem äquivalent umformuliert werden zu: Gegeben $f \in \mathbb{Z}[x_1 \ldots x_n]$, gibt es $c \in \mathbb{N}$ und $q \in \mathbb{Z}[x_1 \ldots x_n]$, so daß das Polynom $f(x_1 + c, \ldots, x_n + c) \cdot q(x_1 \ldots x_n)$ nur positive Koeffizienten hat? Mit Hilfe eines Satzes von Potya über positiv definite Formeln erhält man so die Entscheidbarkeit des Problems für verschiedene Teilklassen von Polynomen. Außerdem ergibt sich daraus die volle Entscheidbarkeit der Frage, ob eine beliebiges vorgelegtes Polynom eine Nullstelle in einem Ring M-M hat, der durch Vervollständigung eines Modells M von z_0 entsteht.

P.H. SCHMITT: Mathematische Logik und Künstliche Intelligenz.

In dem Vortrag wurden einige Probleme und Problembereiche aus der Künstlichen Intelligenz aufgezeigt, die in engem Zusammenhang stehen mit Theoremen, Methoden oder Fragestellungen der Mathematischen Logik. Im einzelnen wurde dabei auf automatische Deduktion, natürlichsprachige Systeme und logisches Programmieren eingegengen. Zum ersten Thema wurden verschiedene Anwendungen der Modallogik in der Künstlichen Intelligenz und ein offenes Problem aus der mehrwertigen Logik vorgestellt. Zum zweiten Thema wurde schwerpunktmäßig auf die Klassifikation verallgemeinerter Quantoren eingegangen. Zum Thema "logisches Programmieren" wurde auf die Einordnung des Unifikationsproblems in die Entscheidbarkeitsfrage für positiv-existentielle Formeln in freien Algebren und auf Theoreme aus der universellen Algebra mit unendlichen Termen hingewiesen.

A.S. TROELSTRA: The syntax of Martin-Löf's theory of types.

A precise syntactical description of Martin-Löf's type theory is presented; it is shown how a number of features of the formalisations are forced on us by the wish to conform to Martin-Löf's explanations of the meaning of the rules. Some critical remarks can be made concerning the merits of the system.

A. VISSER: Provability in Heyting Arithmetic meets Propositional Logic.

It is well known that often the question of provability of an arithmetical substitution instance of a propositional formula in Heyting Arithmetic reduces to the question of provability of an in some sense simpler instance. For example for $A \in \Sigma^0_1$ $(\neg\neg A \to A)$ is provable iff $(A \vee \neg A)$ is provable. I develop a general framework to study this phenomenon. Some results are: a characterization of the closed fragment of the provability logic of HA and a proof of We Tangh's Completeness Theorem for arithmetical interpretations of Proportional Logic; this proof can be formalized in HA + Con HA.

S.S. WAINER: Subrecursive Ordinals.

An attemptis made, to classify computable functions in terms of the complexity (size) of well-orderings needed to define them. An old stumblingblock (Myhill, Routledge 1953) is that every computable function can be defined by recursion over a well ordering with an elementary characteristic function. However, using basic ideas of Π^1_2-logic one can overcome this stumblingblock by coding well-orderings as direct systems $\langle g_{xy}\rangle_{x<y<\omega}$ in the category of integers and hence measuring the complexity of well-orderings in terms of the functors which "construct" them. By these means one can generate a large class of "accessible" recursive functions - these turn out to be provable ones of $(\Pi^1_1 CA)\upharpoonright$.

V. WEISPFENNIG: Efficient decision Algorithms for locally finite theories.

We find uniform, efficient decision and quantifier elimination procedures for the theory T' of existentially closed models of a locally finite universal theory T, whose class of models has the amalgamation property. Upper bounds on the complexity of these procedures are obtained in terms of the size of n-generated T-models. Applications include the theories T of linear and partial orders, graphs, semilattices, boolean algebras, Stone algebras, distributive p-algebras in general and in the Lee class B_2, abelian m-groups, and m-rings for a fixed positive integer m.

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