MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Function Theoretic Methods in Partial Differential and Integral Equations

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The conference was organized by R.P. Gilbert (Newark, Del. U.S.A.), E. Meister and W.L. Wendland (TH Darmstadt).

In proposing the subject of the meeting the organizers wanted to bring together persons whose common interests cover applications of complex methods to partial differential equations as well as to singular integral equations.
The meeting was attended by 40 persons, of whom 32 presented talks.
A strong programme of lectures was offered and they have been grouped according to subject areas. Most significant topics were for partial differential equations:
Applications of complex methods to problems in elasticity and acoustics, representations of solutions and transmutations, expansions and constructive solution methods, boundary value problems, higher dimensional problems in particular in connection with Clifford algebras and
for integral equations: Canonical problems and Wiener-Hopf problems, constructive and numerical methods, special types of equations, Fredholm theory.
The open discussion on Thursday evening revealed several open problems in connection with matrix factorization, the approximation of singular integrals and integral equations, the asymptotic behaviour of solutions, comparison theorems for solutions of different elliptic equations, spectral problems, representations of functionals by

analytic and generalized analytic functions, inverse scattering and conformal mapping. It became obvious that function theoretic methods are as powerful and essential as they have always been in partial differential and integral equations. This meeting was in some sense successor of a conference on similar topics 1976 in Darmstadt; the organizers are particularly grateful to G. Fichera and W. Haack for their sincere support. The participants expressed their conviction to have organized a corresponding subsequent meeting after a much shorter period.

The participants and organizers express their gratitude to the Oberwolfach Research Institute and all the staff for the outstanding facilities, accomodation and support and for the kind and friendly hospitality.

Vortragsauszüge

H. BART:

Spectral analysis of Wiener-Hopf factorization using concepts from systems theory

An expression of the form

$$(*) \qquad W(\lambda) = I + C(\lambda - A)^{-1}B$$

is called a realization. This notion has its origin in systems theory, where the right hand side of (*) appears as the transfer function of the linear dynamical system $\dot{x}(t) = Ax(t) + Bu(t)$, $y(t) = Cx(t) + u(t)$. In recent years it has become clear that realizations are an important tool in dealing with a variety of problems in operator and matrix theory. Results have been obtained exhibiting a striking degree of explicitness.

In the present talk (*) is used to obtain explicit formulas for Wiener-Hopf factors and Wiener-Hopf factorization indices in case $W(\lambda)$ is a rational (square) matrix function. In addition, necessary and sufficient conditions are given in order that the Wiener-Hopf factorization is canonical (i.e.,all its factorization indices vanish). The results are formulated in terms of the matrices A, B, C and certain spectral subspaces of A and $A - BC$.

Generalizations to infinite dimensional situations (even involving unbounded linear operators) are briefly discussed, too. In this context complicated problems of splitting of possibly connected spectra arise. These can be overcome for exponentially dichotomous operators , i.e., operators that are the direct sum of two infinitesimal generators of exponentially decaying C_0-semigroups, one acting on $[0,\infty)$, the other on $(-\infty,0]$. Another infinite dimensional case that can be handled (by Hilbert space methods) concerns the energy (or neutron) transport equation.

The lecture is a report on joint work with I. Gohberg (Tel Aviv University) and M.A. Kaashoek (Amsterdam, Free University).

H. BEGEHR:

Hele-Shaw Strömungen und Momentenproblem in $\mathbb{R}^n$

Die Momente eines beschränkten Gebietes D von $\mathbb{C}$ sind die Koeffizienten der Taylorentwicklung von

$$\int_D \frac{d\xi d\eta}{z-\zeta} \qquad (\zeta = \xi + i\eta \; , \; z \in C)$$

in ∞. Analog kann man im $\mathbb{R}^n$

$$\int_D \frac{x-y}{|x-y|^n} \, dy \qquad (D \subset \mathbb{R}^n \; , \; x \in \mathbb{R}^n)$$

betrachten und durch Heranziehung der harmonischen Polynome Momente definieren.

Es wird gezeigt, wie das Momentenproblem - Bestimmung von D zu vorgegebener Folge von Momenten - mit der Lösung von bewegten Rand-Problemen vom Hele-Shaw Typ zusammenhängt.
Dieses Ergebnis ist in einer gemeinsamen Arbeit mit R.P. Gilbert enthalten.

P. BERGLEZ:

Zur Darstellung von Lösungen partieller Differentialgleichungen in der Nähe isolierter Singularitäten

Den Ausgangspunkt bilden Darstellungen für Lösungen elliptischer bzw. formal hyperbolischer Differentialgleichungen unter Verwendung spezieller Differentialoperatoren , sogenannter BAUER-Operatoren, die auf holomorphe Funktionen wirken. Erst kürzlich konnte eine Charakterisierung der Gleichungen, zu denen es solche Operatoren gibt, angegeben werden, wobei es auch möglich war, diese Operatoren explizit anzugeben. Es werden verschiedene Zusammenhänge zwischen diesen Lösungen und ihren Erzeugenden aufgezeigt. Die Darstellung von Lösungen, die in mehrfach zusammenhängenden Gebieten definiert sind, bilden den Mittelpunkt dieses Berichtes. Dabei wird speziell auf die Darstellbarkeit derjenigen Lösungen, die nur eine Erzeugende besitzen, und der reellwertigen Lösungen elliptischer Differentialgleichungen eingegangen. Abschließend werden multiplikative und algebraisch verzweigte Lösungen diskutiert.

B. BOJARSKI:

Complex and real variable methods for p-harmonic functions

p-harmonic functions in a domain $\Omega \subset \mathbb{R}^n$ are weak solutions $u \in W^{1,p}_{loc}(\Omega)$, $1 < p < \infty$, $p \neq 2$ of the non-linear equation

$$(*) \qquad \operatorname{div}(|\nabla u|^{p-2} \nabla u) = 0 .$$

The set Σ of critical points of u , $\Sigma = \{x \in \Omega , \nabla u = 0\}$ is crucial for the study of local and global properties of p-harmonic functions. By classical Hopf-Schauder-Petrovsky results a p-harmonic function is real analytic in $\Omega \setminus \Sigma$. Uraltseva showed that u - p-harmonic $\Rightarrow u \in C^{1,\alpha}_{loc}(\Omega)$ for some $\alpha > 0$. In the lecture various properties of p-harmonic functions were discussed: interior regularity, comparison principle, Harnack property, Fatou type theorems, structure of quasiradial solutions of the form $u = r^{\beta}\varphi(\theta)$, $\varphi(\theta + 2\pi) = \varphi(\theta)$. The case $n = 2$ is intimately connected with the complex non-linear equation

$$(**) \qquad f_{\bar z} = \alpha\left(\frac{\bar f}{f} f_z + \frac{f}{\bar f} \overline{f_z}\right) , \quad \alpha = \frac{1}{p} - \frac{1}{2} , \quad f = u_x - iu_y .$$

In particular, any p-harmonic function u belongs to $W^{2,2+\varepsilon}_{loc}(\Omega)$ for some $\varepsilon > 0$, and the set Σ consists of isolated points $\{z_k\}$, each with an associated topological invariant n_k , the winding number of f at z_k . The following result generalizes H. Lewy's theorem on the gradient of real analytic p-harmonic functions. If a solution of $(**)$, for $|\alpha| < \frac{1}{2}$, vanishes at z_o then f cannot be C^{∞} smooth at z_o . Since zeros of f have topological character, the same holds for singularities of f . In particular the singularities of f are stable (solitons!). It is possible to construct solutions of $(**)$ in the classes $C^k_{loc}(\Omega)$ for arbitrary big but finite k . These results suggest, that, although in view of T. Wolff's recent counterexamples the Fatou theorem doesn't hold for p-harmonic functions, $p \neq 2$, the boundary behaviour of real analytic or C^{∞} p-harmonic functions is much more regular than the boundary behaviour of p-harmonic functions admitting interior singularties.

G. BRUHN:

Boundary value problems with characteristics in $\frac{1}{2}\mathbb{Z}$ for elliptic systems

As is well known, the operator F of a general regular elliptic BVP for a bounded region $G \subseteq \mathbb{C}$ has the Fredholm index $\nu(F) = n-2\gamma$, where n := system dimension and γ := characteristic of the boundary condition. For boundary data in $C^{1+\alpha}$ we have $\gamma \in \mathbb{Z}$ and thus $\nu = n-2\gamma$ has values only odd or only even. For filling these gaps we slightly weaken the $C^{1+\alpha}$-condition of the boundary data: They are said to be <u>essentially</u> in $C^{1+\alpha}$, if for every $z_o \in \partial G$ there is a neighborhood $U(z_o)$ and a <u>nonsingular real</u> matrix R (not continuous in general) such that the boundary data multiplied by R are in $C^{1+\alpha}(\partial G \cap U(z_o))$. Thus we get $\gamma \in \frac{1}{2}\mathbb{Z}$. As in the case $\gamma \in \mathbb{Z}$ we can construct a homotopy $F \sim F_o$ and compute $\nu(F) = \nu(F_o)$, using the stability theorems of KATO and ATKINSON, because we need only local arguments which are still available when the boundary data are only <u>essentially</u> in $C^{1+\alpha}$. F_o consists of n well known simple BVPs of systems of dimension 1 and we get $\nu(F) = \nu(F_o) = n-2\gamma$ by simple enumeration. Because of $\gamma \in \frac{1}{2}\mathbb{Z}$ now all gaps of ν are closed. Last not least as for $\gamma \in \mathbb{Z}$ there is an adjoint operator F^* to F with $\gamma^* = n-\gamma$ and $\nu(F^*) = -\nu(F)$ and the problems for F and F^* are normally solvable.

J. CHABROWSKI:

The Dirichlet problem with L^2-boundary data in a half-space for an elliptic linear operator

We discuss the Dirichlet problem $Lu = f$ in $\mathbb{R}_n^+$ and $u(x',0) = \varphi(x')$ in $\mathbb{R}_{n-1}$, where L is an elliptic linear operator and $\varphi \in L^2(\mathbb{R}_{n-1})$. The main result is the existence theorem under the assumption of Dini's continuity with respect to two variables on some leading coefficients of L.

C. CONSTANDA:

Complex variable treatment of bending of thin elastic plates

Kirchhoff's kinematic hypothesis concerning the displacements, that is

$$u_k = x_3 v_k(x_1,x_2), \qquad k = 1,2 ,$$

$$u_3 = v_3(x_1,x_2) ,$$

leads to a two-dimensional system of equations for the unknown functions v_i, $i = 1,2,3$. This system has been integrated previously in terms of real (Somigliana) potentials, and the existence of the solution has been established in certain classes of finite energy functions.

To solve the displacement boundary-value problem by means of complex variables, the v_i are represented in terms of three complex potentials, which are then determined from the boundary conditions.

The traction boundary-value problem is solved by first finding Airy-type potentials for the stresses and by deriving suitable expressions for the complex resultants of the forces and moments on the boundary, After mapping the domain conformally onto the unit disk, the problem reduces to the solution of a Fredholm integral equation of the second kind. It is shown that this equation is solvable in the case of both the interior and exterior region if and only if the total force and moment across the boundary are zero.

M. COSTABEL:

Spline collocation for boundary integral equations on corner domains

Collocation with piecewise linear trial functions is studied for the first kind integral equation with logarithmic kernel and for Cauchy singular integral equations. On curves with corners, one uses local Mellin transformation to derive stability results. For the first kind equation one finds stability in some weighted $H^{1/2}$ Sobolev spaces for weighted splines. For singular integral equations with smooth coefficients stability holds in H^1 provided a certain condition on the coefficients is satisfied that depends on the corner angles and reduces for smooth curves to a strong ellipticity condition well known to be necessary and sufficient for convergence in this case.

J.W. DETTMAN:

Construction of function theories for the Yukawa and Helmholtz equations using transmutations

Transmutation operators are used to transform the heat polynomials and associated functions of Widder and Rosenbloom into solutions of the Yukawa and Helmholtz equations. Generating functions are obtained leading to recurrence relations and generalized Cauchy-Riemann equations. Expansion theorems are proved including Fourier transform criteria for expansions in terms of associated functions.

R. DUDUCHAVA:

Singular integral equations on curves with corners with complex conjugation

Criteria for the Fredholm properties of the above mentioned equations and the index formulae are obtained; applications to the I,II and mixed boundary value problems of the elasticity theory are given.

G. FICHERA:

A physical interpretation of the brothers Riesz theorem

Analytic problems connected with plane strain in the classical elasticity have been extensively studied by complex methods, by the theory of the biharmonic equation or by a direct approach to the two-dimensional system of elasticity. Strangely enough very little is known on the analytic problems arising from plane stresses. These problems are of a quite different nature with respect to the ones concerning plane strain. A review of results obtained quite recently in this field is outlined and some new result added: in particular a physical interpretation of the classical "Brothers Riesz Theorem" is provided in the framework of the theory of plane stresses.

R.P. GILBERT and D.H. WOOD:

Function theoretic methods in underwater acoustics

We investigate solutions of the depth dependent, time harmonic, acoustic equation

$$\Delta u + k^2 n^2(z) u = 0 \quad , \quad 0 < z < z_b \quad ,$$

which models the propagation of sound underwater. Solution of this equation may be obtained using the transmutation $u = (\underset{\sim}{I} + \underset{\sim}{k})\, h$ where h is a Helmholtz function. The kernel $K(z,s)$ of the transmutation may be seen to satisfy the hyperbolic equation

$$\frac{\partial^2 K}{\partial \bar{z}^2} - \frac{\partial^2 K}{\partial z^2} + k^2 \left[n^2(z) - 1\right] K = 0$$

and certain Goursat conditions. Using this transmutation we are able to obtain many of the usual representations for the propagation of sound, namely the model, ray, and Hankel representations. It is also possible to investigate the case of a range dependent index of refraction $n(r,z)$ using a second transmutation.

W.K. HAYMAN:

On a class of integral inequalities of Hardy-Littlewood type

We shall give an analysis of the integral inequality

$$\left(\int_0^\infty \{|f'(x)|^2+(x^2-\tau)|f(x)|^2\}dx\right)^2$$
$$\leq K(\tau)\left(\int_0^\infty |f(x)|^2 dx \int_0^\infty |f''(x)-(x^2-\tau)f(x)|^2 dx\right) .$$

Subject to suitable conditions on f this is valid with a finite value of $K(\tau)$ if and only if $\tau = 2n+1$, $n = 0,1,2,\ldots$.

If n is even $K(\tau) = 4$, while $K(\tau)$ decreases strictly with τ and tends to 4 as $n \to \infty$ through odd values.

The work is joint with D.E. Evans, W.N. Everitt and S. Ruscheweyh. A general theory of Evans and Everitt shows that the existence and value of $K(\tau)$ depends on the behaviour of the argument of $\Gamma(-\frac{n}{2}+z)/\Gamma(-\frac{n-1}{2}+z)$ on rays through the origin.

A.E. HEINS:

Function theoretic aspects of the Sommerfeld half-plane problem

This talk will be devoted to a discussion of two function theoretic methods which have been developed to solve various problems associated with the "Sommerfeld Half-Plane Problem" of diffraction theory. One, which has been in development since 1943, is the Wiener-Hopf method and has provided solutions to numerous scalar and some vector problems. [The vector problems arise from fairly general boundary conditions.] In the scalar cases, the original ideas of Wiener and Hopf are applicable. In the vector cases, a method proposed by the speaker in 1948 is of some assistance. An overlapping method has been proposed by Hurd in 1978 which will solve the same scalar problems as the method of Wiener-Hopf but can also solve a problem which offers some difficulties by the method of Wiener and Hopf. The Hurd method depends on the fact that these problems may be reduced to a scalar or vector Hilbert problem. With both methods there is a strong dependence on function theoretic concepts which will be discussed in some detail.

S. KNECHT:

On the existence of a solution to the Riemann-Hilbert problem for a partial differential equation

A joint paper of A.S. Mshimba (Dar es Salaam) and myself having the same title was presented. We worked on nonlinear differential equations of the kind $w_{\bar{z}} = F(z,w)$, extending some recent results of von Wolfersdorf on nonlinear Riemann-Hilbert problems for holomorphic functions.

Using Schauder's continuity method, one part of the existence proof is based on the Kantorović - Cacciopoli principle, the second on an a-priori estimate which was derived from an integral representation formula for solutions of the boundary value problem. This integral representation formula follows from Vekua's theory and an explicit solution to the linear problem of directional derivative for harmonic functions.

O. LIESS:

Propagation of singularities of solutions of linear partial differential equations

Let $p(D)$ be some linear p.d.o. with constant coefficients of order m , denote by p_m its principal part and consider $x^o \in R^n$, $|x^o| = 1$. We shall say that we have propagation of singularities for $(p(D),x^o)$ if for all convex $\omega \subset R^n$ and all solutions $u \in \mathcal{D}'(\omega)$ of $p(D)u = 0$ it follows from $x \in$ sing $\mathrm{supp}_a u$ that $x + \lambda x^o \in$ sing $\mathrm{supp}_a u$ for all $\lambda > 0$ for which $[x,x+\lambda x^o] \subset \omega$. Propagation of analyticity (and its microlocal variants) is well-understood when p is of principal type. When p is not of principal type, and $m = 2$, it follows that p_m does not act in all variables from R^n_x . Assume then (no restriction on m) that $p_m = \sum a_{\alpha'} D_1^{\alpha_1} \ldots D_{n'}^{\alpha_{n'}}$ for some $n' < n$. Assume moreover that p_m is of principal type when restricted to $R^{n'}_{x'}$. The main result of the talk (not stated here) then gives: propagation of analytic singularities holds for $(p(D),x^o)$ precisely when $x^o \in \mathrm{span}\,\{\mathrm{Re}\,\nabla p_m(\eta^o),\ \mathrm{Im}\,\nabla p_m(\eta^o))$, $\forall \eta^o \in R^m$ with $p_m(\eta^o) = 0$, $\nabla p_m(\eta^o) \neq 0$. Among the other results stated: if propagation of analytic singularities holds for $(p(D),x^o)$, then it so does for $(p_m(D),x^o)$.

W. LIN :

Some second-order systems of PDEs of composite type

In the first part of this paper we consider the following composite system

$$\left[A\frac{\partial^2}{\partial x^2}+2B\frac{\partial^2}{\partial x\partial y}+C\frac{\partial^2}{\partial y^2}\right]\binom{u}{v}=0\ ,$$

where A,B,C are 2×2 constant matrices. First we reduce this system into two kinds of canonical form: The composite system of second kind

$$(C_2)\qquad \left[\begin{pmatrix}1&0\\0&-2\end{pmatrix}\frac{\partial^2}{\partial x^2}+\begin{bmatrix}0&2(1+\frac{k^2}{\lambda})\\(1-\lambda)&0\end{bmatrix}\frac{\partial^2}{\partial x\partial y}+\begin{bmatrix}\lambda&0\\0&\frac{2k^2}{\lambda}\end{bmatrix}\frac{\partial^2}{\partial y^2}\right]\binom{u}{v}=0$$

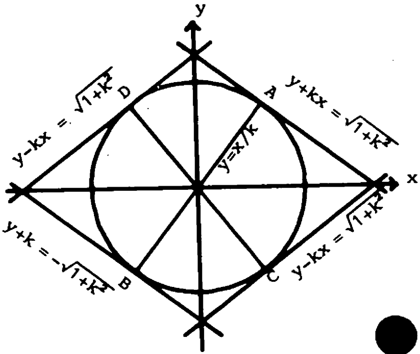
and the composite system of first kind

$$(C_1)\qquad \left[\begin{pmatrix}1&0\\0&1\end{pmatrix}\frac{\partial^2}{\partial x^2}+2\begin{bmatrix}0&1\\\frac{\lambda-1}{4}&0\end{bmatrix}\frac{\partial^2}{\partial x\partial y}+\begin{pmatrix}\lambda&0\\0&0\end{pmatrix}\frac{\partial^2}{\partial y^2}\right]\binom{u}{v}=0$$

We solve the following problems:

1. Problems posed on the unit disc:

(i) Dirichlet-Darboux Problem for (C_2)



Theorem 1: Assume $\chi(\zeta)$ to be a continuous function given on $|\zeta| = 1$, and $\psi(y),\phi(y)$ to be the functions differentiable up to second order and $|\psi(y)|\le M_1|y|$, $|\phi(y)|\le M_2|y|$, $|y|\le\frac{1}{\sqrt{1+k^2}}$,

then there exist solutions to systems (C_2) in $|z| < 1$ such that

$$\begin{cases}\left(\frac{\partial u}{\partial x}+\frac{2k^2}{\lambda}\frac{\partial v}{\partial y}\right)\Big|_{|z|=1}=\chi(\zeta)\ ,\\ u\big|_{x=ky}=\psi(y)\ ,\\ v\big|_{x=-ky}=\phi(y)\ .\end{cases}\qquad |y|\le\frac{1}{\sqrt{1+k^2}}$$

They are determined uniquely up to u,v with an arbitrary term of first degree.

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(ii) Dirichlet - Darboux problem for (C_1).

Find out the solutions u, v to (C_1) in $|z| < 1$ such that

$$u_x\big|_{|z|=1} = \chi(\zeta) ,$$

$$u\big|_{x=0} = \psi(y) , \quad v\big|_{x=0} = \phi(y) , \quad |y| \le 1 .$$

2. Problems posed on the upper half plane

(i) Dirichlet-Cauchy problem for (C_2). Find out solutions u, v to (C_2) in $\mathrm{Im}(z) \ge 0$ such that

$$\left(\frac{\partial u}{\partial x} + \frac{2k^2}{\lambda}\frac{\partial v}{\partial y}\right)\bigg|_{y=0} = \chi(x) , \qquad -\infty < x < +\infty$$

$$u\big|_{y=0} = \psi(x) , \quad v\big|_{y=0} = \phi(x) , \quad -\infty < x < +\infty .$$

(ii) Cauchy problems for (C_2). Find out solutions u, v to (C_2) in $\mathrm{Im}(z) > 0$ such that

$$u\big|_{y=0} = \psi(x) , \quad \frac{\partial u}{\partial y}\bigg|_{y=0} = g_1(x) , \quad \frac{\partial v}{\partial y}\bigg|_{y=0} = g_2(x) ,$$

$$-\infty < x < +\infty .$$

3. In the second part of this paper we generalize the Dirichlet-Cauchy problem to the composite system with $2m$ unknown functions.

<u>Theorem 3:</u> Assume $\chi_\ell(x)$ $(\ell = 1,2,\dots,m)$ be Hölder continuous on the real axis with $\chi_\ell(x) = O(|x|^{\alpha+1})$ $(\alpha>0)$ as $|x| \to \infty$, and $\psi_j(x)$ $(j = 1,2,\dots,2m)$ be differentiable up to second order on the real axis. Then there exist the solutions

$$u_k(x,y) = \psi(x+\mu_k y) - 2\,\mathrm{Re}\Big\{\sum_{\beta=1}^{2m}\sum_{j=1}^{m}\Omega_{k\beta}\,\varepsilon_{\beta j}\Big[\frac{1}{2\pi i}\int_{-\infty}^{+\infty}\sum_{\ell=1}^{m}\tilde{t}_{\ell_j}\,\chi_\ell(t)$$

$$\ln\frac{z-t}{x+\mu_j y-t}\,dt + i\sum_{\ell=1}^{m}\tilde{t}_{\ell_j}\,C_\ell((x+\mu_\beta y)-z)\Big]\Big\} ,$$

to the system

$$\Big[E\frac{\partial}{\partial y} - B\frac{\partial}{\partial x}\Big]\Big[E\frac{\partial}{\partial y} - D\frac{\partial}{\partial x}\Big]u = 0 , \quad u = (u_1, u_2, \dots, u_{2m})^T$$

such that

$$\begin{cases} \sum\limits_{j=1}^{2m}\Big[\Big(\delta_{i_\ell j}\dfrac{\partial}{\partial y} - d_{i_\ell j}\dfrac{\partial}{\partial x}\Big)U_j\Big]\bigg|_{y=0} = \chi_{i_\ell}(x) , & \ell = 1,2,\dots,m , \quad -\infty < x < +\infty , \\ u_j\big|_{y=0} = \psi_j(x) , & j = 1,2,\dots,2m , \quad -\infty < x < +\infty . \end{cases}$$

A. MAMOURIAN

On a non-uniformly elliptic equation with linear boundary condition

The Riemann-Hilbert boundary value problem for the first order nonlinear non-uniformly elliptic system of equations

$$\Psi_1(x, y, u, v, u_x, u_y, v_x, v_y) = 0$$

$$\Psi_2(x, y, u, v, u_x, u_y, v_x, v_y) = 0$$

in multiply-connected domains and with linear boundary conditions of non-zero index is solved by the use of a successive approximation method. This procedure is based on an a-priori estimate which arises from an integral representation formula developed by Haack and Wendland.

P.A. McCOY

Singularities of Jacobi series on $\mathbb{C}^2$ and the Poisson process equation

Poisson processes are modelled as analytic solutions of a hyperbolic partial differential equation. These solutions expand as Jacobi series on $\mathbb{C}^2$ and are associated with unique analytic functions of one complex variable through reciprocal integral equations. By applying the envelope method to this pair, the singularities of a Poisson process are identified with those of its associates. Thus, the singularities are determined from the Jacobi series coefficients by a link with analytic function theory. As a corollary, classical theorems of Szegö and Nehari relating to the singularities of zonal harmonic and Legendre series with analytic functions appear in a functiontheoretic setting on characteristic subspaces of $\mathbb{C}^2$.

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E. MEISTER

Some solved and unsolved canonical transmission problems of diffraction theory

Assume that the $\mathbb{R}^3$-space is divided into four wedges with the x_3-axis as common edge and filled by different dielectric materials. A line- or point-source is situated within the first quadrant Q_1 , the cross section of the wedge with positive x_1- and x_2-co-ordinates. Two of the semi-infinite boundary faces of the four wedges may consist of metallic plates forming a right-angled wedge, or a Sommerfeld half-plane.

The mathematical problem is to find four wave functions $\Phi_j(\underline{x})$ solving Helmholtz' equations in the quadrants Q_j with some Dirichlet, Neumann or mixed boundary conditions on the metallic half-planes and transmission conditions across the remaining parts of the coordinate planes. Near the common edge and the origin the gradients of the wave-functions should be locally square integrable and at large distances fulfill Sommerfeld's radiation conditions.

A four-part Wiener-Hopf equation for the unknown double Fourier tranforms $\hat{\Phi}_j(\underline{\xi})$ of the four wave-functions is established and uniquely solved in case of absorptive materials. In an alternate formulation, the problems are reduced to a 4 by 4-system of integral equations for the Fourier-Cosine-transforms of the normal derivatives $\hat{g}_\ell$ of the wave-functions on the coordinate semi-axes.

R.F. MILLAR

Application of the Schwarz function to boundary problems for Laplace's equation

A method is described for studying analytic boundary value problems for the Laplace equation in a simply-connected domain D bounded by an analytic curve C with Schwarz function G . The basis for the procedure is that (i) a functional F of the boundary Cauchy data is analytic in D and (ii) there exists a further integral relation between the Cauchy data on C, involving the analytic function G. Elimination of the unknown data from this relation by means of (i) and the boundary condition leads to an integral relation on C between F and prescribed data. The relationship of (ii) is first obtained. Its application to the Dirichlet and other linear boundary value problems is outlined, and a nonlinear Riemann-Hilbert problem is solved. Some possible generalizations are briefly mentioned.

F. PENZEL:

Systeme von verallgemeinerten Abelschen Integralgleichungen auf der Halbachse

Betrachtet werden Integralgleichungen vom Typ

$$(1) \qquad \underbrace{A(x)\, x^{-\alpha} \int_0^x \frac{f(t)}{(x-t)^{1-\alpha}}\, dt}_{=:\, I_\alpha f} + \underbrace{B(x)\, x^{-\alpha} \int_x^\infty \frac{f(t)}{(t-x)^{1-\alpha}}\, dt}_{=:\, J_\alpha f} = g(x)\,, \quad x \in (0,\infty)$$

mit Hölder-stetigen differenzierbaren n×n-Matrizen A und B, $0 < \alpha < \frac{\mu}{p}$, $1 < p < \infty$, $f,g \in (L_{\mu,p}(\mathbb{R}^+))^n$, mit

$$f \in L_{\mu,p}(\mathbb{R}^+) :\Leftrightarrow \int_0^\infty t^{\mu-1}\, |f(t)|^p dt < \infty$$

Die Operatoren I_α und J_α sind stetig von $L_{\mu,p}$ in sich (Roomey, 1972). Es zeigt sich, daß notwendig für die Lösbarkeit von (1) die Lösbarkeit eines singulären Integralgleichungssystems mit gesuchter Funktion $I_\alpha f \in (L_{\mu,p}(\mathbb{R}^+))^n$ ist.

Es wird gezeigt, daß für $g \in I_\alpha(L_{\mu,p}(\mathbb{R}^+))^n$ und unter gewissen Voraussetzungen an A und B die Lösung des singulären Integralgleichungssystems Element von $I_\alpha(L_{\mu,p}(\mathbb{R}^+))^n$ ist.

V.A. POPOV

Estimation of the error of numerical methods for solving differential and integral equations

1) The average modulus of smoothness is given as follows: The local modulus of f of k-th order at the point $x \in [a,b]$ is defined by

$$\omega_k(f,x;\delta) = \sup \{ |\Delta_h^k f(t)| : t + kh \in [x - k\delta/z, x + k\delta/z]\} .$$

Then we define the k-th average modulus of smoothness of f by

$$\tau_k(f;\delta)_p = \| \omega_k(f,\cdot,\delta) \|_p .$$

By means of the τ_k it is possible to obtain estimates of the error in numerical methods for differential and integral equations (for example, for collocation methods) without any additional assumptions for the solution.

2) Some inequalities for the best spline approximation in L_p (with free knots) are given. The Besov spaces $B^{\alpha}_{\sigma\sigma}$ with $0 < \sigma < 1$ play an essential role here.

3.) The analogous problem for rational best L_p-approximation is also considered.

S. PRÖSSDORF

Strongly elliptic singular integral equations with piecewise continuous coefficients

Consider the singular integral operator of the form $A = aI+bS_\Gamma$ with the Cauchy operator $S_\Gamma u(t)=(\pi i)^{-1}\int_\Gamma (t-s)^{-1}u(s)ds$ and piecewise continuous $m \times m$ matrix functions a and b on a Ljapunov curve Γ. The operator $A \in \mathcal{L}(L_2(\Gamma))$ is called strongly elliptic, if there exist a compact operator $T \in L(L_2(\Gamma))$ and an invertible piecewise continuous matrix function θ on Γ such that the operator $\theta A - T$ has a positive definite real part. Necessary and sufficient conditions in terms of the coefficients a,b for the strong ellipticity of A are given. It is proved that the Galerkin method with piecewise smooth polynomial splines on arbitrary partitions for the approximate solution of the system Au=f converges if and only if A is strongly elliptic. An optimal order of convergence of Galerkin's method can be achieved for special nonuniform partitions using the complete asymptotics of the solution in the neighborhood of the end points of Γ and of the points of discontinuity of the coefficients. Generalizations to the case of weighted L_2 spaces and connections with collocation methods are discussed.

N. RADZABOV:

Integral representations for certain elliptic and hyperbolic equations with regular and singular coefficients

For general linear elliptic and hyperbolic equations of the second order with regular and singular coefficients and corresponding systems, a series of integral representations depending on the coefficients of the equations has been obtained. These integral representations are used for the solution of a number of boundary value problems.

L. REICHEL:

Numerical aspects of the boundary collocation method for solving some elliptic boundary value problems

One of the oldest methods for solving boundary value problems for linear partial differential equations is to approximate the solution by a linear combination of finitely many particular solutions to the differential equation. In the boundary collocation method this linear combination is determined by requiring the boundary condition be satisfied in a least squares sense in a finite number of boundary points, so-called collocation points. Using function theoretic methods, we develop guidelines for the selection of subspaces and collocation points.

J. RYAN:

Boundary value problems in complex Clifford analysis

Let Ω be a domain in $\mathbb{R}^n \subset \mathbb{C}^n$ and Harm (Ω, C) be the set of complex valued harmonic functions defined on Ω. Then in 1954 P. Lelong showed that each element of Harm (Ω, C) may be holomorphically extended to a covering of the domain in $\mathbb{C}^n \backslash X(\Omega)$ containing Ω, where

$$X(\Omega) = \bigcup_{\underline{x} \in \partial\Omega} N(\underline{x}) \quad \text{and} \quad N(\underline{o}) = \{(z_1, \ldots, z_n) \in \mathbb{C}^n : z_1^2 + \ldots z_n^2 = 0\} .$$

In our lecture we use a complexification of the complex numbers (i.e. $\mathbb{C} \otimes_{\mathbb{R}} \mathbb{C} \cong \mathbb{C} \oplus \mathbb{C}$) to use Cauchy's integral formula for regular functions to obtain a similar result for regular functions. We then proceed to use complex Clifford algebras and a generalization of the Cauchy integral formula to obtain similar results in higher even dimensions (we then describe the case where n is odd). The arguments given here differ from those given by Lelong, they are shorter than those given before and rely on the intrinsic geometry of $\mathbb{C}^n$.

A Runge approximation theorem is briefly introduced to construct holomorphic harmonic functions which may not be extended beyond any point of its boundary. Classes of real n-dimensional manifolds lying in $\mathbb{C}^n$, and such that they and their tangent spaces contain no non-zero null vectors with respect to the inner product

$$z_1^2 + \ldots + z_n^2$$

are introduced, and Lelong's results are extended to domains constructed from these manifolds.

In the last part of the talk results of Riesz on integral representations of solutions to the wave equation in even dimensions are used to show that the Cauchy integral formula used here differs from the ones used in the Euclidean setting, where the integral gives the value of the function inside the domain, and zero outside. In the $\mathbb{C}^{2m}$, $(m \geq 2)$, setting the integral formula gives a continuous interphase between these two extremes. The one variable residue calculus is used to describe this boundary related problem.

H. H. SNYDER:

Effective analytic methods for the Dirichlet-Neumann problem for Laplace's and Poisson's equations

Let G be a bounded simply-connected domain in $\mathbb{R}^2 (\mathbb{R}^3)$ with polygonal (polyhedral) boundary ∂G. For such domains, Snyder and Wilkerson have developed a computer-assisted analytic method for the solution of the Dirichlet and Neumann problems for Laplace's equation. In this paper, it will be shown how that method may be adapted to mixed boundary conditions and also how they may be used to solve the same problems for Poisson's equation.

F. SOMMEN:

Plane waves and singular integral operators in Clifford analysis

Let D be the Dirac operator in $\mathbb{R}^m$; $\mathbb{C}_m$ the complex Clifford algebra over $\mathbb{R}^m$ and $\tilde{\Omega} \subseteq \mathbb{R}^{m+1}$ open. Then $f \in C_1(\tilde{\Omega};\mathbb{C}_m)$ is left monogenic in $\tilde{\Omega}$ if $(\frac{\partial}{\partial x_o} + D)f = 0$ in $\tilde{\Omega}$. For $\Omega \subseteq \mathbb{R}^m$ open we choose $\tilde{\Omega} \subseteq \mathbb{R}^{m+1}$ such that $\Omega \subset \tilde{\Omega}$ is relatively closed. Then the space $\mathcal{B}(\Omega_1, \mathbb{C}_m)$ of hyperfunctions in Ω admits the monogenic representation

$$\mathcal{B}(\Omega;\mathbb{C}_m) = M_{(r)}(\tilde{\Omega}\backslash\Omega;\mathbb{C}_m) / M_{(r)}(\tilde{\Omega};\mathbb{C}_m) ,$$

$M_{(r)}(\tilde{\Omega};\mathbb{C}_m)$ being the right module of left monogenic functions in $\tilde{\Omega}$.

Let $\mathcal{C}(\Omega;\mathbb{C}_m) = \mathcal{B}(\Omega;\mathbb{C}_m) \,/\, \mathcal{A}(\Omega;\mathbb{C}_m)$ be the space of microfunctions, then

$$\mathcal{C}(\Omega\,;\mathbb{C}_m) = \mathcal{C}_+(\Omega;\mathbb{C}_m) \oplus \mathcal{C}_-(\Omega;\mathbb{C}^m) ,$$

where

$$\mathcal{C}_\pm(\Omega;\mathbb{C}_m) = M_{(\Gamma)}(\tilde\Omega_\pm;\mathbb{C}_m)/M_{(\Gamma),\pm}(\tilde\Omega,\Omega) ,$$

$\tilde\Omega_\pm = \{\vec{x} + x_0 \in \tilde\Omega : x_0 \geq 0\}$ and $M_{(\Gamma),+}(\tilde\Omega,\Omega)$ is the space of monogenic functions in $\tilde\Omega_+$, which are extendable about each point of Ω .

This leads to boundary value operators

$$BV_\pm : \mathcal{C}(\Omega;\mathbb{C}_m) \to \mathcal{C}_\pm(\Omega;\mathbb{C}_m) ,$$

$BV_\pm = \frac{1}{2}(1 \pm H)$, H being the Hilbert-Riesz transform.

For $\varphi,\phi \in \mathcal{C}(\Omega;\mathbb{C}_m)$, we define $\varphi * \phi$ in the case $\operatorname{supp}\varphi = \operatorname{supp}\phi = \{0\}$.

Furthermore, if $\operatorname{supp}\varphi = K$, K compact and $\varphi = f(\vec{x}+0)$, f left monogenic, we define the Fourier transform of φ by

$$F\varphi(\vec{t}) = \int_\Sigma E_+(\vec{t},x)d\sigma_n f(x) , \quad d\sigma_n = e_n dS ,$$

where Σ is an m-surface in $\mathbb{R}^{m+1}$, covering the singularity support K and

$$E_+(\vec{t},x) = \frac{1}{2}\left(1 - \frac{i\vec{t}}{|\vec{t}|}\right) e^{i<\vec{t},\vec{x}>+x_0|\vec{t}|} .$$

It is proved that F is injective and asymptotically defined modulo terms of exponential decay.

Furthermore, consider the Cauchy transform

$$\Lambda_k^+(f)(\vec{x}+x_0) = \frac{1}{\omega_{m+1}} \int_0^{+\infty} \frac{x_0+t-\vec{x}}{|x_0+t-\vec{x}|^{m+2k+1}} f(t)dt ;$$

then $\lambda_k^+(f)(\vec{x}) = \Lambda_k^+(f)(\vec{x}+0)$ and if $P_k(\vec{x})$ is inner spherical monogenic of degree k , $\lambda_k^+(f)P_k \in \mathcal{C}_+(\Omega,\mathbb{C}_m)$. Furthermore the Fourier transform is given by

$$F(\lambda_k^+(f)P_k)(\vec{t}) = \frac{\Gamma(\frac{m+1}{2})}{2^{k+1}\Gamma(k+\frac{m+1}{2})}\left(1 - \frac{i\vec{t}}{|\vec{t}|}\right)P_k(i\vec{t})L(f)(|\vec{t}|) ,$$

where $L(f)(t) = \int_0^{+\infty} e^{-st}f(s)ds$.

L.H. SON:

Extension problem in mathematical physics

By considering a physical phenomenon there is the following situation: We want to know the physical appearance in a domain of space ($\mathbb{R}^3$), but we can only measure and observe this appearance in a subset of this domain. Hence, for understanding this phenomenon in the whole of this domain we must use the physical laws which have to be fulfilled for the phenomenon. From that we can derive the necessary informations about the phenomenon in the whole of the observed domain. In most cases the physical laws are described by partial differential equations. For this reason the above problem leads to the extension problem for partial differential equations. In this report we prove extension theorems for the electro-magnetic vector field which satisfies the Maxwell equations and for the zero- divergence- and irrotational vector field, which is a solution of the Riesz system.

H. WALLNER:

Integraloperatoren bei zugeordneten Differentialgleichungen

Jeder formalhyperbolischen Differentialgleichung läßt sich mittels eines Differentialoperators erster Ordnung eine weitere derartige Differentialgleichung zuordnen. Es werden notwendige und hinreichende Bedingungen für einen solchen Operator angegeben. In Spezialfällen ist auch eine explizite Charakterisierung möglich. Im Zusammenhang mit formalhyperbolischen Differentialgleichungen sind die Integraloperatoren von Bergman and Vekua von gewissem Interesse. Es zeigt sich, daß die Kerne dieser Operatoren für die zugeordneten Differentialgleichungen sich in relativ einfacher Weise aus jenen der ursprünglichen Gleichungen konstruieren lassen.

W.L. WENDLAND:

Exterior boundary value problems and boundary integral equations

The lecture is a report on a joint paper with G.C. Hsiao: On a boundary integral method for some exterior problems in elasticity (1983), to appear in the special issue of Dokl. Akad. Nauk, dedicated to the late Academician Prof. Dr. V.D. Kupradze on the occasion of his 80th birthday.

We consider the exterior two- and three-dimensional first and second boundary value problems for the Navier equations governing an ideal elastic medium. The problems are formulated via the "direct method" in terms of boundary integral equations of the first kind with weakly singular or with hypersingular kernels, respectively, as well as in terms of the classical Cauchy singular integral equations of elasticity. In order to formulate uniquely solvable equations we incorporate rigid motions at infinity finding a unified treatment of the generalized exterior problems. All the boundary integral operators are analyzed as strongly elliptic pseudodifferential operators. The corresponding Fourier analysis is indicated as well as its constructive aspects for corresponding spectral methods. Finally we give existence and regularity results as well as a constructive solution procedure in the scale of Sobolev spaces on the boundary.

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