MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 6/1986

Gruppen und Geometrien

9.2. bis 15.2.1986

Die Tagung fand unter Leitung von M.Aschbacher (Pasadena), D.Higman (Ann Arbor), B.Fischer (Bielefeld) und F.G.Timmesfeld (Gießen) statt.

Ein Schwerpunkt dieser Tagung war, neben der Anwendung der Klassifikation der endlichen einfachen Gruppen auf geometrisch-kombinatorische bzw. Permutationsgruppenfragen, die Theorie der Tits- und allgemeiner Diagrammgeometrien mit fahnentransitiver Automorphismengruppe. Hierbei scheint die Theorie der lokal endlichen Tits-Geometrien mit fahnentransitiver Automorphismengruppe, obwohl erst einige Jahre alt, sich in einem guten Zustand zu befinden. Eine vollständige (lokale) Klassifikation scheint erreichbar zu sein. Eine Theorie allgemeiner Diagrammgeometrien ist jedoch erst in den Anfangsstadien. Hier wäre insbesondere ein Verständnis der Geometrien der sporadischen Gruppen wünschenswert.

Wie die Vorträge und die zahlreichen privaten Diskussionen zeigten, ergibt sich bei der Behandlung dieser Fragen ein Zusammenwirken von : Endlicher Gruppentheorie, kombinatorischer Geometrie, bis hin zu arithmetischen Gruppen und Topologie (affine Gebäude).

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Vortragsauszüge

M. Aschbacher:

Chromatic Geometry

A chromatic geometry or rainbow on a set X is a coloring Δ of $X \times X$ subject to a few weak axioms. In essence Δ is coherent configuration without the strong numerical axiom. Among other results, a Krull-Schmidt Theorem for edge-transitive rainbows and a Jordan-Hölder Theorem for symmetric edge-transitive rainbows are established. Such results can be used to study permutation groups.

B.Baumann:

Groups acting on p-groups

Let $F(p,n,m) = F_n/\phi_p^m(F_n)$ where F_n is the free group on n letters, $\phi_p^o(G) = G'G^p$ and $\phi_p^i(G) = \phi_p^o(\phi_p^{i-1}(G))$ (p a prime, G a group, $i \geq 1$).

Some properties of $F(p,n,m)$ had been discussed and the irreducible sections of the action of $Aut(F(2,n,2))$ on $F(2,n,2)$ had been determined. Furthermore, a result how to interpret automorphism groups of graphs as automorphism groups of 2-groups was mentioned. As an application a theorem on groups G with $G/O_2(G) \simeq GL(n,2)$ acting naturally on $O_2(G)/\phi(O_2(G))$ had been stated.

A.E. Brouwer:

Classification of near hexagons with lines of size 3

Theorem. Let (X,α) be a near hexagon with quads and lines of size three. Then we have one of eleven cases:

(i) $v = 759$, M_{24}, (ii) $v = 729$, $3^6 M_{12}$, (iii) $v = 891$, $U_6(2)$,

(iv) $v = 567$, $\Omega_6^-(3)$, (v) $v = 405$, $3^5 Sym(6)$,

(vi) $v = 243$, $(3 \times AGL(2,3) \times AGL(2,3)) \cdot 2$,

(vii) $v = 135$, $Sp(6,2)$, (viii) $v = 105$, $Sym(8)$,

(ix)-(xi) $v = 81, 45, 29$ direct products of a quad and a line.

F. Buekenhout:

Some small and thin incidences

A finite graph (or incidence) is thin if every comaximal complete subgraph is contained in exactly two complete subgraphs of maximal size. Classifying thin graphs with up to 8 elements, produces a unique little surprise Γ_8 which is characterized by the fact that it has 4 vertices of degree 4 and 4 vertices of degree 5. Its automorphism group is D_8. Looking for all connected graphs in which each element has a neighbourhood isomorphic to Γ_8, Nathalie Lefèvre has shown that there is a unique one: it has 13 vertices and an automorphism group which is a Frobenius group of order 13·4. There is no further extension.

A. Chermack:

Triangular Amalgams

A method is described for generalizing some work of David Goldschmidt (on automorphisms of trivalent graphs) to flag-transitive groups operating on chamber systems of rank 3. The results concern "track-stabilizers" and, in particular, we show that track-stabilizers are essentially trivial in the case of separated irreducible triangular amalgams.

A.M. Cohen:

Simple subgroups of $E_8(\mathbb{C})$

Report on work in progress, joint with R.L.Griess, Jr.

In determining the finite subgroups of a given Lie group E, one may, in view of induction w.r.t. containment of closed Lie subgroups, restrict attention to the case where the finite subgroup is a maximal closed Lie subgroup of E. If $E = E_8(\mathbb{C})$, there is little hope of determining all conjugacy classes of finite subgroups, but it may be feasible to determine the isomorphism type of all groups occurring. By maximality, a finite subgroup G has the shape

$N_E(A)$ for a char-simple subgroup A of E. If A is elementary abelian, Alekseevskii (Funct.Anal.and its Appl.) shows that A is either isomorphic to 5^3 (and $G \simeq 5^3 : L_3(5)$) or to 2^5 (and $G \simeq 2^{5+10}L_5(2)$). We have begun to study the case where A has a non-abelian simple subgroup N such that $A \simeq N^t$ ($t \geq 1$, usually 1). Among the results is $E \geq N \simeq Alt_n \Leftrightarrow n \leq 10$. Also, $E \geq N$ is a sporadic finite simple group if and only if $N \simeq M_{11}, M_{12}$. If $E \geq N \simeq L_2(q)$, then q = 2,3,4,5,7,8,9,11,13,16,17,19,25, 27,29,31,32,37,41,49 or 61. We are working on the converse of the last statement, especially q = 61.

A. Delandtsheer:
<u>Basis-homogeneous geometric lattices</u>

Li proved that any finite (unordered) basis-homogeneous geometric lattice is a direct product of point- and basis-homogeneous geometric lattices. We can prove that any finite point- and basis-homogeneous geometric lattice containing at least one line of size ≥ 3 is a "wreath product" $C \wr \mathfrak{J}$ where C is a truncation of a desarguesian affine or projective geometry with line size ≥ 3, or the Hermitian unital of order 4, or a rank 2 geometric lattice on more than 2 points, and $\mathfrak{J}$ is a point- and basis-homogeneous matroid (either a geometric lattice or a complete multipartite regular graph).

B. Fischer:
<u>Clifford-Matrices</u>

Let N be a nilpotent normal subgroup of a finite group G. The (complex) irreducible characters of G can be computed in terms of (projective) characters of T_φ/N, where T_φ is the inertial group of an irreducible character φ of N, and certain coefficients collected in "Clifford-Matrices".

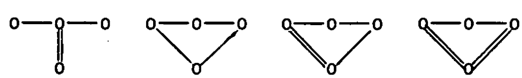
D. Gorenstein:
<u>Generic simple groups</u>

A simple group is called K-<u>local</u> if each of its local subgroups is a K-group. We shall discuss a proof of the following theorem.

<u>Theorem.</u> Let G be a K-local simple group in which $C_G(x)/O_{p'}(C_G(x))$ contains a group of Lie type of characteristic prime to p of Lie rank ≥ 3 or for $p = 2$ an alternating group of degree ≥ 13 for some element x of order p in G, p a prime. Then G is a group of Lie type of characteristic prime to p or an alternating group.

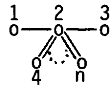
St. Heiss:
<u>Klassifikation einer Klasse von Tits Geometrien</u>

Es soll die Klassifikation einer Klasse von residual zusammenhängender Tits Geometrien Γ über I mit fahnentransitiver Automorphismengruppe skizziert werden. Die Residuen vom Rang 2 seien verallgemeinerte Zweiecke oder klassische verallgemeinerte Drei- bzw. Vierecke. Weiterhin existiere ein Residuum vom Rang 3 mit Diagramm o—o—o und induzierter Automorphismengruppe $\tilde{G} \simeq A_7$. Unter diesen Voraussetzungen werden alle möglichen Erweiterungen klassifiziert. Für derartige Geometrien vom Rang 4 erhält man die Diagramme



und die möglichen Stabilisatoren eines Elementes $v \in \Gamma$ werdem bestimmt.

Für Rang $\Gamma > 4$ erhält man nur noch das Diagramm



Ch. Hering:

Reflections in generalized groups of motions

We discuss finite groups G which contain a normal subgroup N and a normal set of involutions $I \leq G \setminus N$ such that $[G:N] = 2$ and $G = \langle I \rangle$. In particular, all solvable groups of this type are determined which are minimal with respect to not being a dihedral group. This result is applied to the following geometric situation: Let $\mathfrak{A}$ be a translation plane, $\mathcal{O}$ a point in $\mathfrak{A}$ and G a group of automorphisms of $\mathfrak{A}$ leaving invariant $\mathcal{O}$. Assume that

a) G contains a normal subgroup N of index 2,
b) G is generated by involutory perspectivities with affine axis in $G \setminus N$,
c) N does not contain any non-trivial elations.

Let T be the translation group of $\mathfrak{A}$. We call G a generalized orthogonal group and TG a generalized group of motions.

Theorem. If $\mathfrak{A}$ is finite; $2,3 \nmid |N|$ and N is nilpotent, then N is cyclic. Also we construct a new example. There $\mathfrak{A}$ has order 2^9 and N is the Blackburn group of order 3^4. Also, the translation complement of $\mathfrak{A}$ has two orbits on the line at infinity, of length 27 and 486 respectively.

D.G. Higman:

Quasiparallelisms

We define a quasiparallelism of a linear 2-design by dropping the assumption of transitivity (as a binary relation on the lines) from the definition of parallelism. A linear 2-design admitting an intransitive quasiparallelism is a $2\text{-}(S(\sigma(S-1)+1),S,1)$-design with $\sigma \geq 2$ such that $S+\sigma|\sigma(\sigma^2-1)(\sigma^2+\sigma-1)$, and either $\sigma \leq S-2$ or $\sigma = 2$ and $S = 3$. A unital admits an intransitive quasiparallelism if and only if it is defined by polarity of a projective plane. A linear 2-design admitting an intransitive quasiparallelism admits a parallelism corresponding to each of its points. The discussion is readily extended to quasisymmetric designs.

Z. Janko:

Symmetric block designs with small automorphism groups

Let D be a symmetric design for (v,k,λ) with $\lambda > 1$. Let G be the full collineation group of D. We say that G is "small" if the order of G is smaller than v.

It seems that all unknown symmetric designs with $\lambda > 1$ and $k-\lambda \leq 30$ would have small collineation groups. For example, it is shown that any unknown (79,13,2)-design (biplane) must have the full collineation group isomorphic to a subgroup of Z_6 or Z_5.

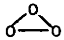
M.W. Liebeck:

The maximal local subgroups of the finite simple groups

Let X be a finite group such that $L \lhd X \leq \operatorname{Aut} L$ for some simple group L, and let M be a maximal subgroup of X. Then M is a maximal local subgroup if $M = N_X(E)$ for some elementary abelian subgroup E of X. If L is A_n or L a classical group the maximal local subgroups can be read off from results of O'Nan-Scott and Aschbacher. In joint work with J.Saxl and G.M.Seitz, we have determined the maximal local subgroups of X when L is an exceptional group of Lie type. Thus the maximal local subgroups are known now in all cases except when L is F_1, F_2 or Fi'_{24}.

H.v.Maldeghem:

Triangle buildings and PTR's with valuation

A triangle building is a building of type $\tilde{A}_2$ (diagram ). A PTR (planar ternary ring) is the algebraic structure coordinatizing a projective plane. A PTR with valutation is a PTR(R,T) together with a surjective map $v: R \times R \to \mathbb{Z} \cup \{\infty\}$ satisfying:

(v1) $v(a,b) = \infty$ if and only if $a = b$,

(v2) if $v(a,b) < v(b,c)$, then $v(a,c) = v(a,b)$,

(v3) if $T(a_1,b_1,c_1) = T(a_1,b_2,c_2)$ & $T(a_2,b_1,c_1) = T(a_2,b_2,c_3)$ then $v(a_1,a_2) + v(b_1,b_2) = v(c_2,c_3)$.

The theorem is: Any coordinatizing ring of the projective plane at infinity of a triangle building Δ is a PTR with valuation and conversely if a projective plane is coordinatized by at least <u>one</u> (complete) PTR with valuation, then it is the plane at infinity of a certain triangle building Δ. This is proven via the geometries V_n of the vertices at distance n ($n \in \mathbb{N}$ and fixed) from a given vertex 0, which appear to be n-uniform Hjelmslev-planes.

V.C. Mazurov:

Subgroups of finite groups and linear programming

(Joint work with N.P.Mazurova).

If H is a subgroup of a finite group G then the coefficients of the decomposition of the permutation character of G on G/H is the sum of irreducible characters of G satisfying a system of linear unequalities and some other conditions. These conditions permit to reduce the question about the existence of a subgroup of given index to the solving of a sequence of linear programming problems. Such approach gave the possibility to determine the proper subgroups of minimal index in all sporadic simple groups, except F_1, without using the Classification Theorem.

Th. Meixner:

Some Groups with Coxeter Diagram

The construction of groups acting chamber-transitively with finite chamber stabilizers on locally finite Tits chamber systems is discussed. For instance groups with diagram o—o—···—o< over GF(2), GF(3) or GF(7) in which the stabilizer of some cell of co-rank 1 is $Sp_{2n}(2)$, $U_{2n}(2)$, $U_{2n+1}(2)$, $PSp_{2n}(3)$ or $PSp_{2n}(7)$. In the first two cases there are infinite families of such finite groups. For the free products $P_1 *_B P_2$ with amalgamated subgroup B a faithful finite-dimensional representation is given, where $P_1 \leftarrow B \rightarrow P_2$ are the primitive 2-perfect amalgams G_4 resp. G_5 of index (3,3) from Goldschmidt's list, using Kantor's group with diagram o—o═o over GF(2).

U. Meierfrankenfeld:

Pushing up

Let p be a prime, M a finite group and $S \in Syl_p(M)$. Assume: (P) No non-trivial characteristic subgroup of S is normal in M. What can we say about the non-trivial chief factors of M in $O_p(M)$? Let $\widetilde{M} = M/O_p(M)$, $\overline{M} = \widetilde{M}/\Phi(M)$, $\overline{L} \lessdot O^p(\overline{M})$, $k \in \mathbb{N}$ and $q = p^k$.

Under the following assumption we can give a complete answer to the above question:

(A) (i) $p=2$, (ii) $O^2(\overline{M}) \cdot\Delta\, \overline{M}$, (iii) $\overline{L} \simeq Sp_{2n}(q)'$, $n \geq 1$, $\Omega^{\varepsilon}_{2n}(q)$, $n \geq 4$, $U_n(q)$, $n \geq 4$, $G_2(q)'$ or A_n, $n \geq 5$, $n \neq 8$, (iv) $O_{2'}(M) = 1$.

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<u>Theorem.</u> Suppose that (P) and (A) hold. There exist subnormal subgroups $L_1,\ldots,L_m$ with $[L_i,L_j] = 1$ for $i \neq j$, $O^2(L_i) = L_i$ and $O^2(M) = L_1\ldots L_m$, such that M operates transitively on $\{L_1,\ldots,L_m\}$. Furthermore let $L = L_1$, $V = [O_2(M),L]$, $\overline{V} = V/C_V(L)$ and $\tilde{L} = L/O_p(L)$. Then one of the following holds:

(1) $V = 1$,

(2) $\tilde{L} \simeq Sp_2(q)'$, $Sp_{2n}(2)'$ $Sp_{2n}(4)$, $\Omega^{\varepsilon}{}_{2n}(2)$, $G_2(2)'$, A_n, $\hat{A}_6$ or $\Omega^{+}{}_{10}(q)$, and $\overline{V}$ is a natural module for L.

(3) $\tilde{L} \simeq Sp_6(q)$, V contains exactly two non-trivial chief factors and $1 \neq \Phi(V) \leq Z(L)$.

A. Neumaier:

<u>Buekenhout geometries, involving semi-pentagons</u>

A partial linear space of girth 5 with the property that every path with 4 edges (in the incidence graph) is in some pentagon is called a semi-pentagon; diagram $\circ\overset{[5]}{\text{——}}\circ$. Examples are the ordinary pentagon, the Petersen graph, the Hoffman-Singleton graph, a triple cover of the generalized quadrangle of order 2, and the Glauberman geometry formed by triads and quartetts of the Steiner system, S(5,6,12). Probably no other flag-transitive example exists; it would have to be thick. Apart from the Hoffman-Singleton graph, all examples occur as rank 2 residues of higher rank geometries. The thick semi-pentagons extend to several minimal parabolic geometries discovered by Ronan and Stroth. The Petersen graph extends to a large number of higher rank geometries: A collection of Buekenhout contains among others J.Hall's locally Petersen graphs and some of Perkel's polygonal graphs. I found new examples related to the binary Golay codes and to minimal parabolic geometries. Recent examples and characterizations by Ivanov and Shpectorov were also mentioned, among them

$$\begin{array}{ccccccccl} \circ & \overset{[5]}{\text{——}} & \circ & \text{——} & \circ & \overset{(6)}{\text{——}} & \circ & (M_{11}) & \text{for O'N and 3O'N.} \\ 2 & & 1 & & |\,1 & & 1 & & \\ & & & & 1^{\circ}(J_1) & & & & \end{array}$$

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S. Norton:

Maximal subgroup and geometry

The progress of maximal subgroup determination since publication of the "Atlas of finite Groups" is summarized. In particular the completion of the subgroups of J_4 is announced with two new ones ($M_{22}\cdot 2$ and $L_3(3)$). It is shown how the 2-modular trick, already used for some of the Conway series of groups and the Rudvalis group, can be applied to show the completeness of the "Atlas" list for $F_4(2)$. Fianlly, it is shown that if $G \simeq J_1$ or $L_2(q)$, ($q = 19,29,31,41,49,59,71$) is in the monster, G has elements of class 2B, 3B, 5B in "Atlas" notation. Together with a proof that $L_2(41)$ must have 5A's and J_1 3C's this shows that these groups are not contained, or even involved, in the Monster. The J_1 result is due to R.A.Wilson.

A. Pasini:

C_3-Geometries

Let Γ be any geometry of type (C_3) $\underset{0}{\circ}\!\!-\!\!\!-\!\!\underset{1}{\circ}\!\!=\!\!=\!\!\underset{2}{\circ}$.
For every incident point-plane pair (a,u), let $\alpha(a,u)$ be the number of planes v incident with a, collinear with u and such that the line x through u and v does not pass through a. We show that $\alpha = \alpha(a,u)$ does not depend on the choice of (a,u), that it is equal to the total number of closed galleries of type 012012012 starting at a given chamber C, and that, for every non-incident point-plane pair (b,w), there are exactly $\alpha + 1$ planes incident with a and collinear with u.
Several consequences are drawn from these facts, both in the general case and in the case of finite geometries with thick lines or admitting parameters.

S. Rees:

Weak buildings of spherical type

I define a weak building of spherical type to be a building which is neither thick (every face of codimension one in a chamber is in at least 3 chambers) or thin (every face of codimension one in a chamber is in exactly 2 chambers). Both the thick and the thin buildings of spherical type are classified: I search for a classification of those which are neither thick nor thin. I exploit properties of the Coxeter complexes of spherical type to define a construction which produces weak buildings by amalgamating copies of buildings of lower rank: I claim everey weak building of spherical type arises in this way.

M.A. Ronan:

Building Buildings

A construction of all buildings was given assuming the existence of all rank 3 buildings of spherical type (i.e. A_3 and C_3). The talk represented joint work with J.Tits, and showed how it is possible to construct buildings of, say, type E_8 over any field k, without prior knowledge of the existence of the Chevalley group $E_8(k)$.

P. Rowley:

Pushing down parabolic systems

Suppose G is a group containing finite subgroups $P_1,\ldots,P_n$. Set $I = \{1,\ldots,n\}$ and assume the following:

(i) $P_i/O_2(P_i) \simeq S_3 \ \forall\ i \in I$

(ii) $S = P_1 \cap \ldots \cap P_n \in Syl_2(P_{ij}) \ \forall\ i,j \in I$ where $P_{ij} = \langle P_i, P_j \rangle$

(iii) $\langle P_i \mid i \in I \rangle = G \neq \langle P_i \mid i \in J \subset I \rangle$.

Such a collection of subgroups $\{P_i | i \in I\}$ we call a parabolic system for G. We impose further conditions upon P_{ij} describing these with a diagram. For $i,j \in I$, $i \neq j$ we write

$\underset{i}{o}\!\!-\!\!\!-\!\!\underset{j}{o}$ if $\overline{P}_{ij} \simeq L_3(2)$; $\underset{i}{o}\overset{\sim}{=\!=}\underset{j}{o}$ if $\overline{P}_{ij} \simeq \hat{S}_6$; and $\underset{i}{o}\quad\underset{j}{o}$ if $\overline{P}_{ij} \simeq S_3 \times S_3$

where $\overline{P}_{ij} = P_{ij}/O_2(P_{ij})$. In this talk we considered the problem of determining the structure of S/S_o (where $S_o = \mathrm{core}_G S$). Especially the followed two results were discussed:

<u>Theorem 1.</u> Suppose G possesses a parabolic system with diagram o—o$\overset{\sim}{=}$o. Then $|S/S_o| = 2^9$ or 2^{10}.

<u>Theorem 2.</u> Suppose G possesses a parabolic system with diagram o—o—o$\overset{\sim}{=}$o and that for the o—o$\overset{\sim}{=}$o part of the diagram the second alternative of Theorem 1 holds. Then $|S/S_o| \leq 2^{21}$ or one very specific configuration holds.

The method of proof (so-called pushing down) consists of examinating the subgroup lattice of S by looking at subgroups which are intersections of $O_2(P_i)$ and $O_2(P_{ij})$ and their conjugates. Relationships between these various subgroups are studied and the resulting accumulated information is needed to locate S_o.

J. Saxl:

<u>On multiplicity free permutation representation</u>

This talk is concerned with the following three conditions on a group G:

1) G acts distance transitively on a graph g

2) G acts transitively on a set Λ with all suborbits selfpaired

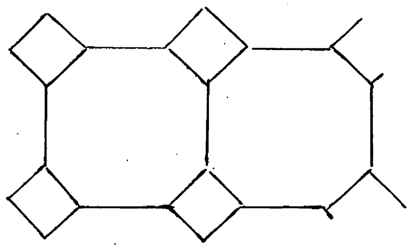
3) G has a multiplicity free character in its action on the set Λ.

Here 1) ⇒ 2) ⇒ 3). A programme for classification was outlined and some recent progress reported.

R. Scharlau:

Geometrical Realizations of Shadow Geometries

The apartments in buildings of spherical or affine type have a very classical interpretation as the (barycentric subdivisions of) the regular polytopes and tilings (space fillings) in Euclidean space. Examples show that some well known semi-regular (vertex transitive) polytopes and tilings, e.g. the truncated cube or the plane tiling by squares and octagons



can combinatorially be described by shadow geometries in the sense of Tits of the apartments. We introduce geometrical realizations E and shadow geometries S of chamber systems and represent the partially ordered set S by subsets ("cells") of a space E. It is shown that under certain assumptions, these cells form a tiling of the space E. This theorem includes the classical semi-regular polytopes and tilings (or rather their duals) and shows that the shadow geometries can be considered as an extension of what is known as "Wythoff's construction" for reflection groups.

J.J. Seidel:

Remark on Wielandt's Visibility Theorem

During the Conference on Groups and Geometries, Mai 1972 in Oberwolfach, Wielandt asked the audience of his lecture to provide a more direct geometric proof for his visibility theorem. This theorem had been proved by complicated arguments in Wielandt's Lecture Notes on Permutation Groups during the classification of groups of order p^2. In the present note we will give a short proof of Wielandt's result, using a theorem by Lovāsz and Schrijver which is equivalent to a theorem of Rêdei.

St.D. Smith:
Geometries and Representation Theory

a) Analogues of the Steinberg module.

It had been observed that, in contrast to the case of building, the highest dimensional homology of sporadic geometry Δ does not necessarily lead to a projective module (in the relevant characteristic) - but the failure seems to be "small". The phenomenon can now be explained, using a result of J. Thêvenaz (building on P.Webb's extension of Quillen's work). One finds that the Lefschetz module for Δ is projective relative to the set of p-subgroups P for which the fixed subcomplex Δ^P is not contractible.

b) Analogues of the adjoint module.

The adjoint module of a Chevalley group can be viewed also by means of the long-root geometry, or the large-extraspecial configuration. Since this configuration also appears in many sporadic simple groups, one is let to define an analogous geometry, and seek an embedding in an analogous module. Recent work of Ronan gives a convenient criterion for the existence of such embeddings, which has been verified for a number of groups, such as .1,Sz,$U_4(3)$, J_2,M_{12}. Naturally one conjectures there is a uniform analysis of such modules for the large-extraspecial class.

R. Solomon:
Small Simple Groups

In the classification of the finite simple groups of Lie type in odd characteristic of Lie rank at most 2, the initial data is 2-local and a key step is the transition from 2-local to p-local data where p is the appropriate odd prime. If z is a 2-central involution of G, one must in particular establish that $|C_G(z)|_p < |G|_p$. This is easy for the cases when the "target group" is $PSp(4,q)$, $G_2(q)$, ${}^3D_4(q)$, $L_4(q)$ or $U_4(q)$ with the exception of $G_2(3^r)$ and $L_4(5)$. When the "target group" is $PSU(3,q)$, or $PSL(3,q)$, the initial Bender dichotomy establishes $|C_G(z)|_p < |G|_p$ in the $L_3(q)$-case (i.e. when $C_G(z)$ is not maximal in G). We outline one of the key arguments in establishing $|C_G(z)|_p < |G|_p$ in the cases where $C_G(z)$ is maximal in G.

G. Stroth:
Classification of Tits geometries

Let Γ be a residually connected geometry and $G \leq Aut(\Gamma)$ a flag-transitive group. For any flag of corank two let the residue of F be a generalized m_F-gon which comes from a finite group of Lie type if $m_F > 2$. Associate with Γ a diagram Δ in the usual way. Let Δ be connected, then all Lie groups above are defined over fields with the same characteristic p. This denoted by char Γ=p. A proof of the following theorem has been sketched:

Theorem. Let Γ,G be as above and $p \neq 3$. Assume that for any flag of corank three whose residue possesses a connected diagram, this residue will be of type A_3 or C_3, then one of the following holds:

(i) Γ is a building of spherical type,

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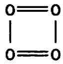
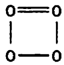
(ii) $p=2$ and Γ is of type $\tilde{D}_4$, $\tilde{A}_3$ or $\tilde{B}_3$. All residues of rank 3 with connected diagram are buildings.

(iii) $p=2$, $G \simeq U_3(5)$ and Δ is of type  or .

F.G. Timmesfeld:

<u>On groups satisfying P_n, $n \geq 2$</u>

A group $G = \langle P_i \mid i \in I \rangle$, $|I| = n$ satisfies P_n if and only if:

(1) $\overline{P}_i = O^{p'}(P_i)/M_i$ is a finite rank 1 Lie-type group in char p; where $M_i = O_p(P_i)$.

(2) There exists a $B \leq \cap P_i$ such that $B = N_{P_i}(S)$, $S \in Syl_p(P_i)$ for each $i \in I$.

G satisfies P_n^+ if in addition:

(3) $\overline{P}_{i,j} = O^{p'}(P_{i,j})/O_p(P_{i,j})$ is a rank 2 Lie-type group in char p and $S \in Syl_p(P_{i,j})$; where $P_{i,j} = \langle P_i, P_j \rangle$. (Also allowing products of two rank 1 groups!)

The classification of groups satisfying P_n^+ is in good shape, since in this case the corresponding chamber system is a classical locally finite Tits chamber system. If now G satisfies P_n define a graph $\Gamma(I)$ by: I is the set of vertices of $\Gamma(I)$ and (i,j) is an edge if and only if $M_i \cap M_j \ntriangleleft \tilde{P}_i$ and $\ntriangleleft \tilde{P}_j$; where $\tilde{P}_k = O^{p'}(P_k)$, $M_k = O_p(P_k)$, $k \in I$. If G satisfies P_n^+ then $\Gamma(I)$ is connected if and only if the diagram $\Delta(I)$ for G is connected. A possible proof of the following general theorem was discussed:

<u>Theorem.</u> Suppose G satisfies P_n. Then $\Gamma(I)$ contains no triangle.

T.van Trung:

Quasi-residual designs of Bhattacharya type

We say that a quasi-residual design (QRD) $\mathcal{D}$ with parameters 2-$(v,b,k+\lambda,k,\lambda)$, $k < \frac{1}{2}v$, is of Bhattacharya type if $\mathcal{D}$ has two blocks B_1 and B_2 with $|B_1 \cap B_2| > \lambda$. Of course, such a QRD is not embedded in a 2-$(v+k+\lambda, k+\lambda,\lambda)$ symmetric design. K.N. Bhattacharya (1944) has constructed for the first time a QRD with parameters 2-(16,24,9,6,3) having two blocks with 4 common points. Until now, this parameter set is the unique one, for which a QRD is known to exist. Then a natural question is: "Are there other QRD of Bhattacharya type?". Our purpose is to give an answer to this question. The answer is: There are infinitely many QRD of Bhattacharya type.

P.H. Zieschang:

The spectrum of the Cayley graph of certain finite groups

Let G be a finite group and T the union of a set of conjugacy classes of $G-\{1\}$ such that $t \in T$ implies $t^{-1} \in T$. Then by $g \sim h :\Leftrightarrow gh^{-1} \in T$ there there is defined a graph $\Gamma(G,T)$ on the elements of G. $\Gamma(G,T)$ is undirected and is called the cayley graph of G with respect to T.

We compute explicitely the spectrum of $\Gamma(G,T)$ in terms of complex character values.

Then we consider the following special case. Let W be a cyclic self-normalizing subgroup of G of order pq, where $p,q \in \mathbb{P}-\{2\}$, $p \neq q$. Further let W_o be the set of the elements of order pq of W and $T := \bigcup_{g\in G} W_o^{\,g}$. Then $\Gamma(G,T)$ has rank ≤ 5, and for each $\ell \in \mathbb{N}$ the number of paths of length ℓ between two adjacent vertices of $\Gamma(G,T)$ is independent of the choice of the two adjacent vertices.

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