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MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 28 / 1986

Graphentheorie

29.6. bis 5.7.1986

Die Tagung fand unter der Leitung von Herrn W. Mader (Hannover) und Herrn G. Ringel (Santa Cruz) statt. In 36 Vorträgen stellten die Tagungsteilnehmer ihre Forschungsergebnisse zur Diskussion. Die behandelten Probleme umfaßten die folgenden Themenbereiche: Zerlegungen von Graphen, Wege und Kreise in Graphen, Färbungsprobleme, Zusammenhangseigenschaften, minimale Graphen, Ramsey-Zahlen, unendliche Graphen, Knotentheorie, nichtkommutative Geometrie, kombinatorische Spiele und andere.

Eine Problemsitzung wurde von Herrn P. Erdös (Budapest) geleitet.

Die folgenden Vortragsauszüge sollen einen genaueren Überblick über die Beiträge geben.

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Vortragsauszüge

J. ANDRÉ

Graphen und nichtkommutative Geometrie

Eine Färbung des vollständigen Digraphen mit der Eckenmenge X ist eine Abbildung $\langle,\rangle: X^2 \to F$ mit $\langle x,y\rangle \neq \langle z,z\rangle$ für $x,y,z \in X$; $x \neq y$. Ein vollständiger Digraph $R=(X,\langle,\rangle,F)$ mit Färbung heißt (nichtkommutativer) P-Raum oder kurz Raum (J. Pfalzgraf). Die Elemente von X bzw. $R:=\{\langle x,y\rangle \mid x \neq y\}$ heißen eigentliche bzw. uneigentliche Punkte des Raumes. Die Verbindungslinie von x mit y $(x,y \in X)$ is definiert durch $x \square y := \{x\} \cup \{z \mid \langle x,y\rangle = \langle x,z\rangle\} \cup \{\langle x,y\rangle\}$; hierbei ist $\langle x,y\rangle$ der uneigentliche Punkt der Linie. Die Verknüpfung $\square$ ist im allgemeinen nicht kommutativ. Zwei Linien heißen parallel, wenn sie dieselben uneigentlichen Punkte haben. Einen Digraph kann man als einen Raum mit ausgezeichnetem uneigentlichen Punkt ansehen. Beziehungen zwischen Eigenschaften von Graphen und zugehörigen Räumen werden angegeben.

TH. ANDREAE

A ternary search problem on graphs

M. Aigner has studied the following search problem on graphs. For a graph G, let $e^* \in E(G)$ be an unknown edge. In order to find e^*, we choose a sequence of test-sets $A \subseteq V(G)$, where after every test we are told whether e^* has both end-vertices in A, one end-vertex, or none. Find the minimum c(G) of tests required. Since in this problem ternary tests

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are performed, we have the usual information theoretic bound $|\log_3|E(G)|| \leq c(G)$. Beside his main results, Aigner proved that each forest F with maximum degree at most two is optimal, i.e., the information theoretic bound is achieved. We consider the more general question, how close we can come to achieving the information theoretic bound for forests with maximum degree at most r, r=1,2,... Let $\underline{F}_r$ be the class of forests with non-empty edge-set and maximum degree at most r. We investigate the function $f(r)=\max\{c(f)-|\log_3|E(F)||: F\in\underline{F}_r\}$ and obtain that $f(r)=t+1-|\log_3(2^t+1)|$ for $2^t<r\leq 2^{t+1}$, t=0,1,.... In addition, we show that, with the exception of five small graphs, all members of $\underline{F}_3$ are optimal, and we conjecture that a similar result holds for $\underline{F}_r$, $r\geq 4$.

R. BODENDIEK

On minimal graphs

D. König has asked the following questions: a) How does the minimal basis $M_1(\Gamma_F)$ look like, and b) is the minimal basis finite for every orientable surface F? K. Wagner and R. Bodendiek have settled b) completely and a) only partially: $M_4(\Gamma_F) \supseteq \{G_1,\ldots, G_{23}\}$ for F=torus and $|M_4(\Gamma_F)|<\infty$ for every orientable surface F.

B. BOLLOBÁS

Partitions of graphs

Let G=(V,E) be a graph and let $\Lambda: V \to P(\mathbb{N})$. A Λ-colouring

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of G is a colouring $\phi:V \to \mathbb{N}$ s.t. $\phi(x) \in \Lambda(x)$ $\forall x \in V$. The list-chromatic number of G is $\chi_l(G)=\min\{k$: G has a Λ-colouring $\forall \Lambda : V \to \mathbb{N}^{(k)}\}$. The list-edge-chromatic number $\chi_l'(G)$ is defined analogously, $\chi'(G) \leq \chi_l'(G) \leq 2\Delta-1$. It was conjectured (Albertson-Tucker) that $\chi_l'(G)=\chi'(G)$; in particular, $\chi_l'(G) \leq \Delta+1$.

B. Bollobás and A.J.Harris proved (1) if $c>11/6$ and $\Delta(G)$ is sufficiently large then $\chi_l'(G)<c\Delta(G)$. Thomassen proved (2) $\exists\ \sigma:\mathbb{N} \to \mathbb{N}$ s.t. if $\delta(G) \geq \sigma(k)$, G is 2-connected and not bipartite then $\forall$ m, G contains a cycle of length $=2m \pmod k$. Both (1) and (2) are based on partitions of our graph which split the degrees: let $f(s)=\min\{r$: if $\delta(G) \geq r$ then $V=V_1 \cup V_2$, $\delta(G[V_i]) \geq s$, $i=1,2\}$. Improving results of Thomassen and Häggkvist it is proved that $\exists\ c > 0$ s.t. $2s+cs^2 \leq f(s) \leq 2s+2(s \log s)^2$, if s is sufficiently large.

K. CAMERON

Partitioning nodes into directed paths

For certain weightings w(P) of the simple dipaths P in a digraph G, we consider the problem of partitioning the node-set V of G by a set of dipaths whose weight-sum is minimum. In particular, where G is acyclic, k is a positive integer, and w(P) is the minimum of k and the number of nodes in P, we give a min-max equality and note structure of a dual optimum: It yields a sequence S=(S(1), S(2), ..., S(k)) of independent sets of nodes, such that each dipath P of every optimum partition intersects w(P) members of S, in order.

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For non-acyclic digraphs it may be true that each optimum partition intersects in this way some S, though there is no single S which will do for every optimum partition.

W. DEUBER

Ramsey's theorem for unary algebras

A unary algebra $\underline{A}$ is an (n+1)-tuple $(A,a_1,\ldots,a_n)$, where A is a set and $a_i:A \to A$ are operations. Subalgebras are defined as closed subsets.

Theorem (W. Deuber - B. Voigt). An algebra $\underline{A}$ has the partition property in the category of finite unary algebras if $\underline{A}$ is symmetric (i.e. every automorphism defined on a subalgebra may be extended to an automorphism of $\underline{A}$).

J. EDMONDS

Link-simple knots

A knot-movie is a sequence of knot-pictures related consecutively by Reidemeister moves.

Conjecture. If there is a movie from knot-picture K_1 with m crossings to knot-picture K_2 with n crossings, then there is a movie from K_1 to K_2 which is not longer than some fixed polynomial in m and n.

A simpler theory: (we use the natural terminology "knot having several (S') links" rather than the more usual terminology "link having several (S') knots"). A knot in $\mathbb{R}^3$ is called link-simple if each of its links is by itself

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trivial (unknotted). A knot-picture (in $\mathbb{R}^2$) is called link-simple if each of its links is by itself a simple closed curve. A knot-movie is called link-simple if each of its pictures is link-simple.

(1) Theorem. Every link-simple knot has a link-simple knot picture.

(2) We describe a theory of link-simple knot-movies.

A. FRANK

On circuit decomposition of Eulerian graphs

Generalizing earlier results of H. Fleischner and of P. Seymour, we prove the following theorem.

Theorem. Let G be a planar Eulerian graph. At every node v a partition $\underline{P}(v)$ of the incident edges is specified. Let $\underline{P} = \cup \underline{P}(v)$. There exists a circuit decomposition $\underline{D}$ of the edge set of G such that $|P \cap C| \leq 1$ for $P \in \underline{P}$, $C \in \underline{D}$ iff $|P \cap D| \leq |D|/2$ for every $P \in \underline{P}$ and for every cut D.

If $|P| \leq 2$ for $P \in \underline{P}$, we obtain Fleischner's theorem. If P consists of parallel edges for $P \in \underline{P}$, we obtain Seymour's ("integer sum of circuits") theorem.

M. GRÖTSCHEL

Adjacency characterizations and diameters of polytopal graphs

Let E be a finite set and $\underline{F} \subseteq 2^E$; for each $F \subseteq E$, let $x^F \in \mathbb{R}^E$ denote the incidence vector of F. Obiously, the vertices of

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$P(\underline{F}) := \text{conv}\,\{x^F \in \mathbb{R}^E \mid F \in \underline{F}\}$, i.e., the polytope associated with $\underline{F}$, are in 1-1-correspondence with the elements of $\underline{F}$. Define a graph $S(\underline{F})$, the so-called skeleton of $P(\underline{F})$, whose nodes are the vertices of $P(\underline{F})$ and where two nodes u,v are linked by an edge if and only if they are (geometrically) adjacent on $P(\underline{F})$. Polytopes of type $P(\underline{F})$ come up naturally in many applications (like the matching polytope, the travelling salesman polytope, the stable set polytope). Two questions of interest are: Is there a characterization of adjacency in $S(\underline{F})$ in terms of properties of $\underline{F}$? What is the diameter of $S(\underline{F})$? The latter question is related to the Hirsch conjecture of linear programming. In this talk we give a survey of some results of the literature and present new adjacency criteria and diameter estimations for the polytopes associated with clique partitionings, cycles in binary matriods, Eulerian subgraphs and cuts of a graph.

R. GUY

Graphs and combinatorial games

Conway's definition, $G = \{G^L \mid G^R\}$, of a (two-person combinatorial) game may be regarded as a digraph whose edges are colored Blue or Red of both. A game falls into one of 4 outcome classes. Games form a commutative group with the operation sum (disjunctive compound) and the outcome class "second player wins" as identity.

Not only are all games graphs, but graphs may be used in a

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variety of ways as the basis for games. For example, Fraenkel's annihilation games, Galvin's pawns-up-the-tree game, Conway's and von Neumann's versions of Hackenbush and games of pursuit.

Graphs have also been used to understand various puzzles, e.g. Sam Loyd's Tantalizer and Fifteen Puzzle, the latter leading to a remarkable generalization by Rick Wilson, which deserves to be better known.

A. HAJNAL

Ramsey - Turán numbers of graphs

Let $r, s>\nu+1$, $\nu\geq 1$. Let $RT_\nu(n,r,s)=\max\{e(G)$: G has n vertices, $K_r \not\subset G$ and $\alpha_\nu(G)\leq s\}$, where $\alpha_\nu(G)=\max\{|X|$: $X\subset V(G)$ and X does not contain a $K_{\nu+1}$ of G$\}$. We are looking for the constants a_r^ν satisfying $RT_\nu(n,r,o(n))=(a_r^\nu + o(1))n^2$. For $\nu=1$, $a_r^1 = \frac{1}{2}(\frac{r-3}{r-1})$ for r odd and $a_r^1 = \frac{1}{2}(\frac{3r-10}{3r-4})$ for r even.

Theorem 1. $a_r^\nu \leq \frac{1}{2}(\frac{r-\nu-2}{r-1})$, and this is best possible for $r\equiv 1 \mod(\nu+1)$.

Theorem 2. $a_r^2 \leq \frac{1}{12}$; $a_6^2 \leq \frac{1}{6}$

Theorem 3. $a_6^2 > 0$ iff $a_8^2 > \frac{1}{4}$

Problem: Is $a_5^2 > 0$? To establish $a_4^1 \geq \frac{1}{8}$, Bollobás and Erdös (1976) proved that there is a sequence G_n of graphs s.t. $|V(G_n)|=n$, $K_4 \not\subset G_n$, $\alpha(G_n)=o(n)$, $\lim e(G_n)/n^2=\frac{1}{8}$. Moreover, $V(G_n) = A_n \cup B_n$, $||A_n|-|B_n||\leq 1$, both A_n and B_n contain $o(n^2)$ edges and no triangles.

Theorem 4. The above sequence of graphs can be chosen so that the trace of G_n has large girth on A_n and B_n.

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(Joint work with P. Erdös, M. Simonovits, V.T. Sós and E. Szemerédi.)

R. HALIN

Unendliche maximal-ebene Graphen

Es werden die verschiedenen Möglichkeiten, den Begriff des maximal-ebenen Graphen vom endlichen auf den unendlichen Fall zu übertragen, diskutiert. Es zeigt sich, daß nur die Kuratowski-Charakterisierung der ebenen Graphen eine sinnvolle Erweiterung gestattet. Sodann werden die abzählbaren maximal-ebenen Graphen in Hinblick auf unendliche Wege und Enden untersucht. Jeder solche Graph besitzt einen $1,\infty$-Weg; es werden diejenigen maximal-ebenen Graphen charakterisiert, die keinen $2,\infty$ - Weg enthalten. Ein maximal-ebener Graph G hat eine ebene Darstellung ohne Häufungspunkte der Eckenmenge genau dann, wenn er höchstens ein Ende besitzt; G hat dann höchstens zwei Ecken unendlichen Grades.

H. HARBORTH

The Ramsey number of K_5-e

The Ramsey number $r=r(K_5-e)$ is defined to be the number of vertices of the smallest complete graph K_r, such that every 2-coloring of its edges contains a monochromatic K_5-e. After $r(K_5-e)\leq 23$ has been proved (J. Graph Theory 9 (1985), 483-485), we now use this result to prove $r(K_5-e)\leq 22$. A recent coloring of K_{21} without a monochromatic K_5-e, which was

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given by G. Exoo, finishes the proof of $r(K_5-e)=22$. - Now K_5 remains the only graph with five vertices, and with its Ramsey number unknown.

M.C. HEYDEMANN

Cycles in Orsay

A survey of recent results obtained by people working on cycles in Orsay (Paris XI - France) is given.

B. JACKSON

Alternating cycles in 2-edge coloured graphs with application to graphs with unique f-factors

We show that if a 2-connected graph is 2-edge coloured as that each vertex is incident with edges of both colours then it contains a cycle whose edges alternate in colour. We deduce that if a 2-edge connected graph G has a unique f-factor then some vertex x of G satisfies $d_G(x)=f(x)$.

H.A. JUNG

Longest circuits and paths in regular graphs of large degree

B. Jackson proved that a 2-connected d-regular graph on $n\geq 3d$ vertices has a hamiltonian circuit. As a "twin" tesult, G. Fan obtained that in a 3-connected d-regular graph a longest circuit has either n or at least 3d vertices. As a supplement one has that a non-bipartite 3-connected graph,

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which is d-regular, on at most 3d-2 vertices is hamiltonian connected; in a 4-connected d-regular graph any two vertices are joined by a path of length at least 3d-3 or n-1. Analogous results hold for dominant cycles and paths involving the factor 4 instead of 3.

D. JUNGNICKEL

Some applications of graph factoriszations in design theory

We report on general existence theorems for design like structures for which the construction part of the proof uses 1-factorizations. Required are

(i) 1-factorizations of complete graphs

(ii) 1-factorizations of regular bipartite graphs

(iii) 1-factorizations of cyclic graphs (the Stern-Lenz-Lemma).

Using these, the general existence conditions are established for

(i) Steiner triple systems with sub-triple systems (The Doyen-Wilson-Theorem, cf. G. Stern & H. Lenz, Bollet. U.M.I. (5) 17-A (1980), 109-114)

(ii) Resolvable designs $S_3(3,4,v)$ (cf. D.J. & J.A. Vanstone, JCTA)

(iii) Minimal linear spaces (cf. D.J. & H. Lenz, JCTA)

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W. KLOTZ

Clique covers and coloring problems of graphs

Classical theorems of Vizing and Shannon give upper bounds for the chromatic index $\chi'(G)$ of a graph, resp. multigraph G. The chromatic index of G coincides with the chromatic number of its line graph. If we characterize line graphs by clique covers, the theorems of Vizing and Shannon take the following form.

Vizing: Let the cliques $C_1,\ldots,C_m$ cover G, $k=\max |C_i|$. If $|C_i \cap C_j| \leq 1$ for every $i \neq j$ and every vertex is covered at most twice, then $\chi(G) \leq k+1$.

Shannon: Let the cliques $C_1,\ldots,C_m$ cover G, $k=\max |C_i|$. If every vertex is covered at most twice, then $\chi(G) \leq \frac{3}{2} k$.

We establish sharp upper bounds for the clique number $\omega(G)$, if the cliques covering G may cover the vertices of G more than twice. In some special cases these bounds turn out to be also upper bounds for $\chi(G)$.

A.V. KOSTOCHKA

Über die chromatische Zahl von Kreisgraphen und ihren Komplementen

Ist G ein Graph, so bezeichnet man mit $\omega(G)$ (bzw. $\chi(G)$)die Cliquezahl (bzw. die chromatische Zahl) von G. Mit X (bzw.$\overline{X}$) wollen wir die Menge aller Kreisgraphen (bzw. die Menge aller Komplemente von Kreisgraphen) bezeichnen. Wir interessieren uns fur die Größen $\phi(k,X)$ und $\phi(k,\overline{X})$, wobei

$$\phi(k,\underline{G}) := \max\{\chi(G) \mid G \in \underline{G} \ \& \ \omega(G) \leq k\}$$

fur eine Graphenmenge G.

A. Gyárfás hat gezeigt, daß $\phi(k,X) \leq 2^k(2^k-2)\cdot k^2$ gilt. Von

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I. Karapetjan, A. Gyárfás und J. Lehel wurde $\phi(k,\bar{X}) \leq \frac{k(k+1)}{2}$ und $\phi(2,X) \geq 4$ bewiesen, Karapetjan zeigte auch $\phi(2,X) \leq 8$. Gyarfas und Lehel stellten die Frage ob die Funktionen $\phi(k,X)$ und $\phi(k,\bar{X})$ linear sind. Wir können folgende Behauptungen beweisen.

Satz 1. $\phi(k,\bar{X}) = k \cdot \ln(k) \cdot (1+0(1))$.

Satz 2. $\frac{k}{2}(\ln(k) - 2) \leq \phi(k,X) \leq 2^k \cdot k(k+2)$

Satz 3. $\phi(2,X) \leq 5$.

W. MADER

On k-criticallly n-connected graphs

A graph G is k-critically n-connected or an (n,k)-graph if $\mu(G-V') = n - |V'|$ for every $V' \subset V(G)$ with $|V'| \leq k$, where $\mu(G)$ denotes the connectivity number of G. In 1977, P.J. Slater conjectured that every finite $(n, \lfloor \frac{n}{2} \rfloor + 1)$-graph is complete. In 1981, Y. O. Hamidoune proved that every non-complete (2k, k+1)-graph has at most 3k-2 vertices. Extending this result, we show that every non-complete (2k+1, k+1)-graph has at most 3k-1 vertices.

W. McCUAIG

Connectivity preserving edge reductions in cubic graphs.

We give a sharp lower bound on the number of edge reductions in a 3-connected (respectively, cyclically-4-connected) cubic graph which give a smaller 3-connected (respectively, cyclically-4-connected) cubic graph. For 3-connectivity the

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lower bound is $\frac{1}{2}|V(G)|+3$, and for cyclic-4-connectivity it is roughly $\frac{3}{10} \cdot |V(G)|$.
A method of generating all cyclically-5-connected cubic graphs is given.

H.M. MULDER

Amalgamation and expansion procedures in graphs

The amalgamation of two graphs G_1, G_2 is obtained by glueing them together along a (nonempty) common subgraph $G_1 \cap G_2$. Then the expansion is obtained by "pulling" G_1 and G_2 apart, which results in the disjoint union of G_1 and G_2 and a matching between corresponding vertices of the two copies of $G_1 \cap G_2$. Multiple expansion is obtained similarly from $G_1,\ldots,G_n$. We restrict ourselves to the case where $G_1 \cap G_2$ is (geodesically) convex or Δ-convex. (i.e. geodesically convex, and whenever $G_1 \cap G_2$ contains an edge of a triangle Δ, then it contains all of Δ).

Thm.1. G is obtainable by convex expansions from K_1 iff G is obtainable by convex amalgamations from hypercubes iff G is a median graph.

Thm.2. G is obtainable by Δ-convex multiple expansions from K_1 iff G is obtainable by Δ-convex amalgamations from Cartesian products of complete graphs iff G is a quasi-median graph.

Thm. 3. G is obtainable by Δ-convex amalgamations from snakes (i.e. Path-like 2-trees), wheels and complete graphs minus some matching, multiplied by some hypercube iff G is a

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pseudo-median graph. The same proof technique can be used to show that the geodesic convexity of the latter graphs satisfies the separation property S_4.

V. NEUMANN-LARA

On graphs which are locally homogeneous

Let G be a graph. For $u \in V(G)$ denote by $N(u,G)$ the subgraph of G induced by $\{w \in V(G) \mid uw \in E(G)\}$. G is said to be locally H if $N(u,G) \cong H$ for every $u \in V(G)$. Define $G_1 \times G_2$ by $V(G_1 \times G_2) = V(G_1) \times V(G_2)$; $(u_1,u_2)(v_1,v_2) \in E(G_1 \times G_2)$ iff $u_1v_1 \in E(G_1)$ and $u_2v_2 \in E(G_2)$.

Theorem. There exists a number n(r) such that $K_{m_1+1} \times K_{m_2+1} \times \ldots \times K_{m_r+1}$ is the only graph which is locally $K_{m_1} \times K_{m_2} \times \ldots \times K_{m_r}$, provided $m_i \geq n(r)$ for $i=1,2,\ldots,r$.

G. RINGEL

Numerierbare Graphen

Ein Graph G mit m Kanten heiße rigoros, wenn die Kanten mit 1,2,3,...,m so durchnumeriert und gleichzeitig gerichtet werden können, daß in jeder Ecke vom Grade $\neq 1$ das Kirchhoff'sche Gesetz gilt. Wenn G n Ecken besitzt und sich in t Hamilton'sche Kreise zerlegen läßt und t(n-1) eine gerade Zahl ist, so ist G rigoros. Aus diesem Satz und aus weiteren Konstruktionsprinzipien lassen sich folgende Graphen als rigoros erkennen: Der vollständige Graph K_n

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fur n ≠ 3; der vollständige paare Graph $K_{n,n}$ für n≡0 (mod 4); das Kantensystem Q_n des n-dimensionalen Würfels fur n ≠ 2; das Rad R_n mit n Speichen fur n≥3; jeder Baum, der nur Ecken vom Grade 1 oder 4 besitzt. Aber z.B. der paare Graph $K_{n,n}$ ist nicht rigoros, wenn n ungerade ist (n≠1).

N. ROBERTSON

On the proof of Wagner's conjecture

We discuss the recent proof (joint work with Paul Seymour) of a conjecture of K. Wagner dating from the 1960's. He conjectured that any set of finite graphs pairwise unrelated by minor inclusion, i.e. any antichain, is necessarily finite. This implies that for any surface the set of topologically minimal graphs not embedding in the surface is finite (a question raised by König in 1935), and that graphs with maximum valency 3 have no infinite antichain under topological inclusion (a conjecture of A. Vázsonyi from the 1940's).

The proof is based on a structure theory of the graphs G not including a fixed graph H as a minor. If $\underline{A}$ is an antichain and $H \in \underline{A}$ then all $G \in \underline{A}\setminus\{H\}$ will be of this type. After careful work with the structure the finiteness of $\underline{A}$ can be reduced to showing this for antichains of simpler graphs than in $\underline{A}$. Ultimately, after basically reducing the genus of H to being planar, the surface structure is removed and graphs of bounded size are left - a class where the finiteness property is easily seen to hold.

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H. SACHS

Über Fortschritte in der Theorie der färbungskritischen Graphen

Die betrachteten Graphen sind endlich, ungerichtet und schlicht. Ist G ein Graph, so sei $\chi(G)$ seine chromatische Zahl. G heißt k-kritisch, wenn $\chi(G) = k$ ist und $\chi(G') < k$ gilt für jeden echten Untergraphen G' von G. Ein hoher (niederer) Knotenpunkt - auch als Haupt- (Neben-) Knotenpunkt bezeichnet - eines k-kritischen Graphen ist ein Knotenpunkt der Valenz $\geq k$ $(=k-1)$.
Probleme, die mit niederen Knotenpunkten verbunden sind, tendieren dazu, weit leichter handhabbar zu sein als Fragen, die hohe Knotenpunkte betreffen.
Der Vortragende berichtete über gewisse Fortschritte in der Theorie der färbungskritischen Graphen, welche niedere Knotenpunkte besitzen, unter Hervorhebung von konstruktiven Methoden und Charakterisierungssätzen.

A. SCHRIJVER

Edge-disjoint paths in planar graphs

We discuss the following theorem, conjectured by K. Mehlhorn: Let $G = (V,E)$ be a planar graph embedded in the plane $\mathbb{R}^2$; let O denote the unbounded face, and let I be some other fixed face; let $C_1,\ldots,C_k$ be curves in $\mathbb{R}^2\setminus(I \cup O)$ with end points in V and on the boundary of $I \cup O$, so that for each $v \in V$: $\deg_G(v)$ + (# of C_i beginning or ending in v) is

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even. Then: there exist pairwise edge-disjoint paths $P_1,\ldots,P_k$ in G so that P_i is homotopic to C_i in the space $\mathbb{R}^2\setminus(I \cup O)$ for $i=1,\ldots,k$, if and only if for each dual path Q from $I \cup O$ to $I \cup O$ we have (# of edges intersected by Q) $\geq$ $\Sigma_{i=1}^{k}$ (number of necessary intersections of C_i and Q). The theorem generalizes a theorem of Okamura and Seymour and its proof yields a polynomial-time algorithm for finding the paths as required. The result is joint work with C. van Hoesel and M. Kaufmann.

M. SIMONOVITS

Supersaturated graphs, extremal graphs with large forbidden subgraphs

Let ex(n, L) denote the maximum number of edges a graph G^n (on n vertices) can have without containing L. (a) If $\chi(L)=p+1$ (χ is the chromatic number) and L has a critical edge e, i.e. $\chi(L-e)=p$, then the Lovász-Simonovits theorems on K_{p+1} generalize also to L: One can describe the structure of G^n with ex(n,L)+k edges, containing the least number of copies of L, assumed $k<cn$. This structure is stable in the sense that the graphs G^n with "almost" minimal number of copies of L are "very similar" to the corresponding Turán graph, for $k=o(n^2)$. (b) Some emphasis was put on the methods, e.g. application of sieve formulae, or of Szemerédi's uniformization lemma.

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J. SLATER

Labellings and numberings of infinite graphs

Some number theoretic questions, like those of M. Hall and R. Entringer involving various difference sets obtained from the set of natural numbers, can be generalized to the problem of gracefully labelling countably infinite graphs. Let $N_0=\{0,1,2,\ldots\}$, and let G be a graph with both V(G) and E(G) coutably infinite. Call G k-graceful if there is a one-to-one function $h:V(G)\to N_0$ such that $g_h:E(G)\to\{k,k+1,k+2,\ldots\}$ is a bijection where for uv in E(G) we have $g_h(uv)=|h(u)-h(v)|$. Call G bejectively-k-graceful if h can be chosen to be a bijection.

The countably infinite version of the Ringel-Kotzig conjecture that all finite trees are graceful is settled as follows. Let $B_1(T)$ denote the maximum number of independent edges in tree T. All countably infinite trees are k-graceful for each $k\geq 1$; any coutably infinite tree with $B_1(T)=\infty$ is bijectively-k-graceful for each $k\geq 1$; and a countably infinite tree with $B_1(T) < \infty$ is bijectively-k-graceful if and only if the number of vertices of infinite degree is one and $k=1$.

V.T. SÓS

On large induced trees and paths in graphs

Let t(G) (Resp. p(G)) be the maximal size of an induced tree (resp. path) in G_n. With M. Saks and P. Erdös, the

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relationship of t(G) with other parameters associated with G has been investigated (number od edges, radius, independence number, max clique size, connectivity). For example, the following result is proved.

Theorem. Let $\rho(G)=e(G)-n(G)+1$,

$l(n,\rho)=\{t(G) | G \text{ connected}, |V(G)|=n, |E(G)|=\rho+n-1\}$.

Then

$$l(n,\rho)=\frac{2n}{\rho+2} + o\left(\frac{n}{\rho+2}\right) \text{ if } \rho=o\left(\frac{n}{\log \log n}\right),$$

$$l(n,cn) = 2\log \log n + O(\log \log \log n) \text{ for all } c>0$$

$$l(n,n^{1+\delta}) = 2 \log\left(1+\frac{1}{\delta}\right) + \varepsilon \quad \text{for } n>40, \text{ where } |\varepsilon|>2.$$

So very sparse graphs can have relatively small t(G)!

M.M. SYSLO

<u>On some generalizations of outerplanar graphs</u>

Outerplanar graphs have been recently generalized in many directions. For instance: A graph is W-outerplanar, $W \subseteq V(G)$, if G can be embedded in the plane so that all vertices of W lie on the same face. G is k-outerplanar if it has a planar embedding with disjoint cycles properly nested at most k deep. A planar graph is of disk dimension $dd_0(G)$ if G admits an embedding on the spere (S_0) minus $dd_0(G)$ open disks $\underline{D}$ with every vertex in ∂D for some disk $D \in \underline{D}$, and $dd_0(G)$ is minimal. Note that the disk dimension $dd_k(G)$ can be defined for an arbitrary graph G embedded on the surface S_k.

Almost all generalizations of outerplanar graphs have been introduced to parametrize the family of planar graphs, so that in consequence some of the decision problems which are

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NP-complete for planar graphs and trivial for outerplanar graphs can be solved in polynomial time for every fixed value of a parameter.
In this talk, we review some generalizations of outerplanar graphs, present their characterizations and recognition algorithms, and discuss complexity of some of the problems which are NP-complete for arbitrary planar graphs.

B.VOIGT

On cycles in sparse graphs

Berichtet wurde über die Arbeit "On the sum of reciprocals of cycle lengths in sparse graphs" (A. Gyarfás, H.J. Prömel, E. Szemerédi, B. Voigt: Combinatorica 5 (1985), 41-52).

H.J. VOSS

Large cycles in graphs

A graph G has E_r ($r \geq 3$) iff G is 2-connected, has minimum degree $\geq r$ and $\geq 2r$ vertices. A well-known result of G. A. Dirac is: each graph with E_r has a cycle of length $\geq 2r$. C. Zuluaga and I proved: each non-bipartite graph with E_r contains both an odd and an even cycle of length $\geq 2r-1$. Taking into accout the girth, O. Ore showed: each graph of girth g, $g \geq 5$, with E_r, $r \geq 4$, contains a cycle of length $\geq (g-2)(r-2) + 5$. I improved and generalized this result. (I) Each graph (non-bipartite graph) of girth g, $g \geq 3$, with E_r contains a cycle (both an odd and an even cycle) of

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length $\geq 2^{c_1 g}$ with $\geq 2^{c_2 g}$ diagonals, where c_i is constant. (II) Let G be a k-connected graph with E_r, $r \geq k \geq 2$, or a cyclically k-vertex-connected graph (in the sense of C. Thomassen) with E_3. Then each set of k-1 edges of G lying on a cycle of length $\geq k$ is on a cycle of length $\geq 2^{c_1 g}$ with $\geq 2^{c_2 g}$ diagonals.

M.E. WATKINS

Distance sequences in infinite vertex-transitive graphs

Two recent results are presented:

1. Joint work with Carsten Thomassen.

We show that every vertex-transitive, edge-transitive infinite graph of odd valence and subexponential growth is 1-transitive, thus extending to infinite graphs a theorem of W. T. Tutte for finite graphs. We describe a number of counterexamples in the case of exponential growth.

2. Joint work with James B. Shearer.

It is shown that, contrary to a pair of well-known conjectures, there exist finite and infinite examples of (1) vertex-transitive graphs whose distance sequences are not unimodular and (2) graphs with primitive automorphism group whose distance sequences are not logarithmically convex. In particular, a family of finite graphs is presented whose automorphism groups are primitive and whose distance sequences are not unimodular.

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W. WOESS

Tree-like properties of infinite graphs

We consider locally finite, infinite connected graphs which have a tree-like structure. "Tree-like" may be defined in several ways:
(a) The graph has a "uniformly spanning tree",
(b) All ends of the graph have "finite diameter".
(c) The graph has a certain triangulation property.
In general, (a) and (c) are equivalent and imply (b). If the automorphism group of the graph has only finitely many orbits on the vertex-set, then (a), (b) and (c) are equivalent. Using results of Muller-Schupp and of Dunwoody, the finitely generated groups with tree-like Cayley graphs can be classified: they are just the finite extensions of finitely generated free groups. The interest in these problems originates from the study of random walks on gorups and graphs.

J. ZAKS

Decompositions of K_q into complete bipartite graphs and a solution to Bagemihl's conjecture on neighborly tetrahedra

A family of convex d-polytopes in E^d is called neighborly if every two of them intersect in a (d-1)-polytope. solving Bagemihl's long standing conjecture (1956), we have recently proved that there can be at most eight neighborly

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tetrahedra. The proof is quite long, and it uses Baston's (1965) result, that there can be at most nine such tetrahedra. In few parts of the proof we use computer to search all the combinatorial types of certain decompositions of K_9 into specific collections of complete bipartite graphs. In particular, we (me and my machine) had observed that $K_9 \neq 2K_{2,3} + 6K_{2,2}$ (see A. M. Monthly 1985), $K_9 \neq 3K_{2,3} + 4K_{2,2} + 2K_{1,1}$, $K_9 \neq 3K_{2,3} + 4K_{2,2} + K_{1,2}$, $K_9 \neq 4K_{2,3} + 2K_{2,2} + 2K_{1,2}$ and $K_9 \neq 5K_{2,3} + 3K_{1,2}$.

Let k be the maximum number of tetrahedra in E^3, such that every two are separated by (at least one) hyperplane which contains facets of both of them. Improving M. Perles' bound $k \leq 16$, we can prove that $k \leq 15$, by using the following: "the multigraph, obtained from K_{16} by taking the edges of some 1-factor as multiple edges, can be decomposed into not fewer than 9 complete bipartite graphs". It is conjectured that $k=8$.

Berichterstatter: Wolfgang Woess

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Tagungsteilnehmer

Prof. Dr. M. Aigner
II. Math. Institut
F. U. Berlin
Arnimallee 3
1000 B e r l i n 33

Prof. Dr. J. André
Fachbereich Mathematik der
Universität des Saarlandes
Bau 36
6600 S a a r b r ü c k e n

Dr. Th. Andreae
II. Math. Institut
F. U. Berlin
Arnimallee 3
1000 B e r l i n 33

Prof. Dr. R. Bodendiek
Mathematisches Seminar
PH Kiel
Olshausenstr. 75
2300 K i e l 1

Prof. Dr. B. Bollobás
University of Cambridge
Dept. of Pure Math. & Stat.
10 Mill Lane
C a m b r i d g e CB2
Grossbritannien

Prof. Dr. J. A. Bondy
Dept. of Combinatorics & Opt.
University of Waterloo
W a t e r l o o , Ontario N2L3G1
Canada

Prof. Dr. Kathie Cameron
Mathematical Sciences
Research Institute
1000 Centennial Drive
B e r k e l e y , Calif. 94720
USA

Prof. Dr. W. Deuber
Fakultät für Mathematik
Universität Bielefeld
Universitätsstr. 1
4800 B i e l e f e l d 1

Prof. Dr. J. Edmonds
Mathematical Sciences
Research Institute
1000 Centennial Drive
B e r k e l e y , Calif. 94720
USA

Prof. Dr. P. Erdös
Mathematical Institute of the
Hungarian Academy of Sciences
V. Reáltanoda u. 13-15, POB 127
H-1364 B u d a p e s t
Ungarn

Prof. Dr. A. Frank
Research Institute for
Telecommunication
Gábor A. utca 65
H-1026 B u d a p e s t II.
Ungarn

Prof. Dr. M. Grötschel
Lehrstuhl f. Angew. Math. II
Universität Augsburg
Memminger Str. 6
8900 A u g s b u r g

Gefördert durch DFG Deutsche Forschungsgemeinschaft

Prof. Dr. R. K. Guy

Dept. of Math. & Statistics
The University of Calgary
2500 University Drive N.W.

C a l g a r y , Alberta T2N1N4
Canada

Prof. Dr. A. Hajnal

Mathematical Institute of the
Hungarian Academy of Sciences
Pf. 127

H-1364 B u d a p e s t
Ungarn

Prof. Dr. R. Halin

Mathematisches Seminar der
Universität Hamburg
Bundesstr. 55

2000 H a m b u r g 13

Prof. Dr. H. Harborth

Institut B für Mathematik
T. U. Braunschweig
Pockelsstr. 14

3300 B r a u n s c h w e i g

Prof. Dr. R. Henn

Institut f. Stat. & Math.
Wirtschaftstheorie der T.H.
Postfach 6380, Zirkel 2

7500 K a r l s r u h e 1

Frau Professor
Dr. Marie-Claude Heydemann

99 rue de Paris
Bat. E

F-91400 O r s a y
Frankreich

Prof. Dr. B. Jackson

Dept. of Math. Sciences
Goldsmiths' College
New Cross

L o n d o n SE14 6NW
Grossbritannien

Prof. Dr. H. A. Jung

Technische Universität Berlin
Fachbereich 3 - Mathematik
Straße des 17. Juni 135

1000 B e r l i n 12

Prof. Dr. D. Jungnickel

Mathematisches Institut der
Justus-Liebig-Universität
Arndtstr. 2

6300 G i e s s e n

Prof. Dr. W. Klotz

Institut für Mathematik
Technische Universität
Erzstr. 1

3392 Clausthal-Zellerfeld

Prof. Dr. A. V. Kostochka

Institute of Mathematics of t
Siberian Branch of the Academy
of Sciences of the USSR
Universitetski pr. 4

USSR-630090 Novosibirsk-90

Prof. Dr. W. Mader

Universität Hannover
Institut für Mathematik
Welfengarten 1

3000 H a n n o v e r 1

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William McCuaig
Dept. of Combinatorics & Opt.
University of Waterloo
W a t e r l o o , Ontario N2L3G1
Canada

Prof. Dr. H. Sachs
Technische Hochschule Ilmenau
PSF 327
DDR - 6300 I l m e n a u

Dr. H. M. Mulder
Vrije Universiteit
Vakgroep A.V.,Faculteit Wisk.&N.
Postfach 7161
NL-1007 MC A m s t e r d a m
Niederlande

Alexander Schrijver
Department of Econometrics
Tilburg University
P.O.Box 90153
NL-5000 LE T i l b u r g
Niederlande

Prof. C.St.J.A. Nash-Williams
Department of Mathematics
University of Reading
P.O.Box 220, Whiteknights
R e a d i n g RG6 2AX
Berkshire - Grossbritannien

Prof. Dr. M. Simonovits
Department of Analysis I
Eötvös University
H-1088 B u d a p e s t
Ungarn

Prof. Dr. V. Neumann-Lara
Instituto de Matemáticas
U N A M
Circuito Exterior, C. U.
M e x i c o 04510 D.F.
Mexico

Prof. Dr. P. J. Slater
Dept. of Math. & Statistics
The University of Alabama
in Huntsville
Huntsville, Alabama 35899
USA

Prof. Dr. G. Ringel
Department of Mathematics
Division of Natural Sciences
University of California
S a n t a C r u z , Calif.95060
USA

Frau Professor
Dr. Vera Turán-Sós
Mathematical Institute of the
Hungarian Academy of Sciences
Reáltanoda 13-15
H-1053 B u d a p e s t

Prof. Dr. N. Robertson
Department of Mathematics
The Ohio State University
231 West 18 th Avenue
C o l u m b u s , Ohio 43210
USA

Prof. Dr. K. Steffens
Institut für Mathematik
Technische Universität Hannover
Welfengarten 1
3000 H a n n o v e r 1

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©

Prof. Dr. M. M. Syslo

Institute of Computer Science
University of Wroclaw
ul. Przesmyckiego 20

PL-51151 W r o c l a w
Polen

Dr. B. Voigt

Fakultät für Mathematik
Universität Bielefeld
Universitätsstr. 1

4800 B i e l e f e l d 1

Prof. Dr. H.-J. Voß

Pädagogische Hochschule Dresden
"Karl Friedrich Wilhelm Wander"
Wigardstr. 17, PSF 365

DDR - 8060 D r e s d e n

Prof. Dr. K. Wagner

Fachbereich 11 der
Universität Duisburg
Lotharstr. 61

4100 D u i s b u r g

Prof. Dr. M. E. Watkins

Mathematics Department
Syracuse University

S y r a c u s e , N.Y. 13244-1150

USA

Dr. W. Woess

Institut für Mathematik und
Angewandte Geometrie
Montanuniversität
Franz-Josef-Str. 18

A-8700 L e o b e n

Prof. Dr. J. Zaks

University of Haifa
Department of Mathematics

H a i f a 31999

Israel

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