MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Combinatorial Optimization and its Relations to Other Mathematical Areas

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Die Tagung fand unter Leitung von R. E. Burkard (Technische Universität Graz) und M. Grötschel (Universität Augsburg) statt.
In 48 Vorträgen berichteten die Tagungsteilnehmer über neue Forschungsergebnisse auf dem Gebiet der Kombinatorischen Optimierung und ihren Anwendungen sowohl in anderen mathematischen Disziplinen als auch in Chemie, Physik, Informatik und Ingenieurwissenschaften.
Neben den vier täglichen Vortragssitzungen, einer Abendveranstaltung mit dem Thema VLSI-Design und einer abendlichen Problemsitzung wurde die exzellente Atmosphäre im Forschungsinstitut zu zahlreichen informellen Diskussionen mit viel Enthusiasmus genutzt.
Die folgenden Vortragszusammenfassungen geben einen Überblick über die in dieser Tagung behandelten Fragestellungen.

Vortragsauszüge

Achim Bachem

Matroids with a parallel hyperplane axiom

In this talk we show that Minty's Lemma can be used to prove the Hahn-Banach-Theorem as well as other theorems in this class such as Radon's and Helly's Theorem for oriented matroids having an intersection property which guarantees that every pair of flats intersects in some point extension $O \cup p$ of the oriented matroid O.
We relate this intersection property to Levi's intersection property as well as the so-called bundle condition and show how they can be derived from the Euclidean property of oriented matroids. Finally we discuss the importance of a lattice theoretic approach of oriented matroids in contrast to the well-known set theoretic duality.

Egon Balas

Projection and Inverse Projection of Combinatorial Polyhedra

Combinatorial structures can often be represented in different spaces. Sometimes a polyhedron $P \subset \mathbb{R}^n$ defined by a system of 2^n inequalities can be replaced by a polyhedron $P^* \subset \mathbb{R}^p$ defined by a system of q inequalities, with both p and q polynomial in n. At other times p or q may not be polynomial in n, but P^* may have a nicer structure than P. The ability to move from one representation to another is therefore often very helpful.
When the system defining P^* is given explicitly, projecting P^* into a subspace is a well-solved problem. We address the unsolved problem of projecting into a subspace a polyhedron P^* given as the convex hull of 0-1 points satisfying a system of linear inequalities. The solution we propose is a generalization to systems of linear inequalities in 0-1 variables of the well-known Fourier-Motzkin elimination as well as the resolution procedure of propositional calculus.

Michel Balinski

On the Core of the Assignment Game

There are two distinct sets of players, P and Q. Each player places a real value on each of the opposite members, this being a measure of the satisfaction a player of P (or Q) receives from being matched with a player of Q (of P). The analysis centers on the notion of a stable matching meaning some pairing of the players — with utility transfers between paired players — such that no two players not matched with each other could increase their utilities by being matched. The core is the set of stable matchings and is a convex polytope.
It is shown that the core can contain at most $\binom{2m}{m}$ extreme points, where $m = \min(|P|, |Q|)$. A subgame of a matching game is one obtained by considering subsets of players. A pair of players (p, q) are super-compatible if they must be matched

in the stable matchings of any subgame that contains them both. The core has the maximum number of vertices if and only if there are m super-compatible pairs and all players that do not belong to such are "negligible".

Francisco Barahona

Compositions of Graphs and Polyhedra

Given a graph G let $P(G)$ be a polytope associated with G. If G has a two-node cut set then G decomposes into G_1 and G_2. We shall give a technique to derive $P(G)$ provided that we know the two polytopes related to G_1 and G_2. We study the stable set polytope and the convex hull of incidence vectors of acyclic induced subgraphs of a directed graph.

Robert E. Bixby

A short proof of the Truemper-Tseng decomposition of the max-flow min-cut matroids

Seymour proved that a binary matroid M satisfies the strong max-flow min-cut equality with respect to a fixed element l if and only if M has no F_7^* (dual Fano) minor containing l. Truemper and Tseng have given a decomposition theorem for the 3-connected, binary, non-regular matroid with no F_7^* minor containing a fixed element. Together with other known results their result gives a complete characterization of the class singled-out by Seymour's Theorem.
Our proof uses splitter theory, a result on pulling elements close to connected minors, and a result on induced k-equations. The work is joint with Arvind Rajan.

Anders Björner

Argument complexity of the spanning property in greedoids

Let E be a finite set of cardinality n and suppose that $P \subseteq 2^E$. The complexity $c(P)$ is the minimum number of entries of the incidence vector χ_A that the best P-testing algorithm needs to inspect in the worst case $A \subseteq E$. Let $p(t) = \sum_{F \in P} t^{|F|}$. It is known (Rivest and Vuillemin, 1975) that $c(P) \leq k$ implies $(1+t)^{n-k} \mid p(t)$. Here we prove that the converse is true in case P is the family of spanning sets in a greedoid $G = (E, L)$.
With each greedoid $G = (E, L)$ is associated an invariant polynomial $\lambda_G(t)$, which can be recursively computed by a deletion-contraction type algorithm (Björner, Korte and Lovász, 1985). A reformulation of the result is: the lowest-degree nonzero term of $\lambda_G(t)$ has degree k, if and only if the complexity of the spanning property is $n-k$. This is also equivalent to that $n-k$ is the maximum size of a spanning and evasive subgreedoid. For instance, the complexity of the family of spanning arc-sets in a rooted digraph (i. e., branching greedoid) is equal to the maximum number of arcs in a spanning and acyclic subgraph.

Robert G. Bland and David F. Shallcross

Efficient sequencing on a four-circle diffractometer: Large TSP's from x-ray crystallography experiments

Experiments in x-ray crystallography often involve sequential collection of thousands of readings on a four-circle diffractometer. Sequencing the readings to minimize the time to complete the experiments is a very large, non-Euclidean traveling salesman problem. We have tested several TSP heuristics on twelve sample problems with an average of more than 8000 readings, and as many as 14,464 readings. Even some very simple heuristics outperformed the standard sequencing method for these problems by 25% or more on every test problem. The Lin-Kernighan heuristic achieved improvements of more than 34% on every problem, and was always within 1.7% of a computed lower bound on the optimal value.

Karl Heinz Borgwardt

Probabilistic analysis of optimization algorithms — utility and difficulties

Judging on an algorithm on behalf of its worst-case complexity gives a very pessimistic impression of its efficiency. For practical purposes the worst-case examples of problems may be exceptional or seldom.
So there is a need for other criteria — as the average behaviour of the algorithms. The derivation of theoretic results on this behaviour requires the introduction of a stochastic model, a characterization of the solution process and the evaluation of formulas for the desired probability or expectation values. In this talk we want to demonstrate with some examples of linear and combinatorial optimization how such an analysis could be done, which results could be obtained and which stochastic models were used.
In addition we shall discuss the difficulties to choose realistic models, to derive non-asymptotic results and to analyse complicated algorithms.

Peter Brucker

Scheduling irregular polygons with vertices on a circle line

Consider n irregular polygons with vertices on a circle line. How should the polygons be moved relative to each other such that the minimum (maximum) distance between the vertices is maximized (minimized)? Algorithms are given which solve these problems in polynomial time for fixed n. However, for general n the problems are $\mathcal{NP}$-hard.

Rainer E. Burkard

Saddle points in group and semigroup minimization

The group (semigroup) minimization problem, derived from integer programming, is discussed. A dual form of this problem is stated and weak and strong duality theorems together with complementarity conditions are shown. Moreover, a Lagrangean function is introduced and it is shown that the classical saddle point theorems still hold good. The objective function of the minimization problem is formed using elements shown from an ordered d-monoid, thereby treating sum, bottleneck and lexicographic objectives from a unified point of view. (Joint work with R. A. Cuninghame-Green).

William Cook

On cutting planes

Cutting-plane proofs, as introduced by Chvátal, provide a method for verifying that a given linear inequality is valid for all integral vectors in a given polyhedon. We discuss a variation of this method which arises by considering a simple version of the notion of a disjunctive cut developed by Balas and Jeroslow. This talk is based on joint work with R. Kannan and A. Schrijver.

Gérard Cornuéjols

General Factors in Graphs

Consider a graph $G = (N, E)$ and, for each node $i \in N$, let B_i be a subset of $\{0, 1, \ldots, d_G(i)\}$ where $d_G(i)$ denotes the degree of node i in G. The **general factor problem** asks whether there exists a subgraph of G, say $H = (N, F)$ where $F \subseteq E$, such that $d_H(i) \in B_i$ for every $i \in N$. This problem is $\mathcal{NP}$-complete. A set B_i is said to have a **gap of length** $p \geq 1$ if there exists an integer $k \in B_i$ such that $k+1, \ldots, k+p \notin B_i$ and $k+p+1 \in B_i$. Lovász conjectured that the general factor problem can be solved in polynomial time when, in each B_i, all the gaps (if any) have length one. We prove this conjecture. In cubic graphs, the result is obtained via a reduction to the edge and triangle partitioning problem. In general graphs, the proof uses an augmenting path theorem and an Edmonds-type algorithm.

R. A. Cuninghame-Green

Computational geometry using minimax algebra

Using the algebraic strcuture $(R, \max, +)$ one may give an algebraic formulation to any given planar shape, which may be irregular and non-convex, so long as it may be reasonably approximated by a simple closed polygonal curve. It is then possible by routine algebraic operations, to determine the parameters of all plane isometries which map the figure into one which does not overlap it. In this way, one is able to study regular arrangements of arbitrary shapes in the plane. The work has application to industrial cutting problems.

William H. Cunningham

The minimum 3-cut problem

Consider an undirected positively edge-weighted graph $G = (V, E)$, having three fixed **terminal** vertices. A **3-cut** of G is a set $A \subseteq E$ such that the terminals are in different components of $G - A$. The resulting minimum 3-cut problem is known to be $\mathcal{NP}$-hard. A polyhedral approach to this problem leads to some nice classes of facet-inducing inequalities for which good separation algorithms are known, as well as some nasty classes. An inequality is proved on the gap between the upper bound produced by a heuristic and the lower bound produced by a linear programming relaxation.

Reinhardt Euler

On minimal incomplete latin squares, which are not completable

A classical combinatorial problem is to decide for a given incomplete latin square of order n whether it is completable to a full one or not. This decision problem is $\mathcal{NP}$-complete. Nevertheless a characterization of all minimal incomplete latin squares, which are not completable, would solve this problem in a theoretical way. We present several classes of such latin squares. They may also be used to derive facet-defining inequalities for the polyhedron associated with the planar 3-index assignment problem, a combinatorial optimization problem whose solutions correspond exactly to all latin squares of a given order.

G. Finke

Combinatorial optimization problems reviewed as matrix approximation problems

The approach will be based on the Frobenius norm. This matrix norm is frequently used in numerical analysis, e. g. to characterize the pseudo-inverse, and also in combinatorial problems, e. g. in Barnes' algorithm for the node partitioning problem of a graph.
We shall consider the classical combinatorial optimization problems that are related to permutations: linear assignment problems, symmetric assignment or matching problems, travelling salesman problems, and quadratic assignment problems. These problems may be expressed in matrix trace form. Therefore, they are also equivalent to approximation or norm minimization problems. This formulation yields an easy access to algebraic manipulations and helps to establish new solvable cases and new lower bounds. In particular, the Hoffman-Wielandt inequality generates an eigenvalue bound which has a better asymptotic behaviour than the Gilmore-Lawler bound for quadratic assignment problems.

Jean Fonlupt

A polynomial algorithm to recognize perfect 3-chromatic $K_4 \setminus \{e\}$-free graphs

A perfect 3-chromatic $K_4 \setminus \{e\}$-free graph is a perfect graph which has no induced subgraph isomorphic to neither K_4, the clique induced by four vertices, nor $K_4 \setminus \{e\}$, the graph obtained from K_4 by removing an edge from it. We prove that the recognition problem of a perfect 3-chromatic $K_4 \setminus \{e\}$-free graph is polynomial. If G is such a graph, we show that at least one of the following properties is satisfied:

G is bipartite or a line graph of a bipartite graph.

G has a separating clique.

G has a separating stable set of cardinality two.

There exists in G a node z belonging to at least three maximal cliques, one at least of size three, such that the graph obtained from G by deleting the node z and the edges of all maximal cliques containing z, is not connected.

This is joint work with A. Zemirline.

András Frank

An optimization problem concerning supermodular functions

Let $p' : 2^S \to \mathbb{Z} \cup \{-\infty\}$ be an intersecting supermodular function, that is, $p'(X) + p'(Y) \leq p'(X \cap Y) + p'(X \cup Y)$ whenever $X, Y \subseteq S$, $X \cap Y \neq \emptyset$. Let $r : 2^S \to \mathbb{Z}$ be the rank-function of a matroid such that $r \geq p'$. We call a set $T \subseteq S$ **good** if $r(X \cap T) \geq p'(X)$ for $X \subseteq S$.

Theorem.

$$\min_{T \text{ good}} |T| = \max_{\mathcal{F}} \sum_{Y \in \mathcal{F}} (p'(Y) - r(\bigcup Z : Z \in \mathcal{F}, Z \subset Y)),$$

where $\mathcal{F}$ is a laminar family of subsets of S.
This is a generalization of a result of Lovász (1972) on supermodular functions and bipartite graphs. Another consequence is a theorem by Gröflin and Hoffman. We can derive a characterization, extending a theorem by Vidyasankar, for the existence of a family $\mathcal{F}$ of k spanning arborescences rooted at a node r of a digraph so that every edge occurs in at least $f(e)$ and at most $g(e)$ members of $\mathcal{F}$. The following consequence is a kind of counter-part of Tutte's 1-factor theorem: In a directed graph $G = (V, E)$ there is a branching meeting all directed cuts iff $c_{\text{out}}(X) \leq |X|$ from every $X \subseteq V$, where $c_{\text{out}}(X)$ denotes the number of components of $V - X$ having no entering edges. This is joint work with Éva Tardos.

A. M. H. Gerards

Stable sets and T-joins in graphs with no odd-K_4

We prove the following theorem:
Let G be a graph with no odd-K_4. Then the maximum cardinality of a stable set in G is equal to the minimum cost of a collection of edges and odd circuits. Here the cost of an edge is equal to 1, and the cost of a circuit of length $2k+1$ is equal to k.
An **odd-K_4** is a homeomorph of K_4 in which all triangles of K_4 have become odd circuits. The theorem above extends König's Theorem for stable sets in bipartite graphs. A similar result holds for node covers. Also we have extensions to the weighted versions of the stable set problem and the node cover number.
Next we give the following theorem:
Let G have no odd-K_4 and no odd-prism. Then for every even $T \subset V$ the minimal cardinality of a T-join is equal to the maximum number of pairwise disjoint cuts.
Here an **odd-prism** is a graph consisting of two node disjoint odd circuits, connected by three node disjoint paths.

Martin Grötschel

Master polytopes for cycles in binary matroids

The problem of finding a maximum weight cycle in a binary matroid generalizes the max-cut problem in graphs and the Eulerian subgraph problem. We investigate this problem from a polyhedral point of view. For notational convenience, we will only consider matroids without coloops and without coparallel elements. For $k \geq 1$ the complete binary matroid L_k of order k is the largest binary matroid of corank k. We prove that the cycle polytopes $P(L_k)$ can be viewed as master polytopes for the cycle polytopes of binary matroids. Namely, we first give a complete linear description of $P(L_k)$, $k \geq 1$, and then show that every other cycle polytope may be derived from some polytope $P(L_k)$ by projection. Moreover, we show that any maximal complete contraction minor L_k of a binary matroid M induces a large class of facets of the cycle polytope $P(M)$. As a corollary, we prove that the Hirsch conjecture holds for all cycle polytopes of binary matroids.

Horst W. Hamacher

Approximation algorithms for parametric network flow problems

Two algorithms to approximate the optimal cost function of a network flow problem with parametric capacities are presented.
The first one is based on an algebraisation of the negative cycle algorithm to solve min cost flow problems in which the flow variables are piecewise linear functions instead of real numbers.
The second procedure uses a "sandwich" strategy to bound the optimal cost function z from above and below by two piecewise linear functions z_u and z_l. The algorithm stops with ε-approximations of z after M computations of (non-parametric) min cost flows. An a-priori bound for M can be given. This procedure is also applicable to arbitrary convex functions z.

Peter Hammer

Order relations of variables in 0-1 programming

We present old and new results concerning two important preorders on the set of variables of a 0-1 (linear or nonlinear) program: a) the ordinary order relation $x_i \leq x_j$ which means "in every feasible vector x^*, if $x_i^* = 1$ then $x_j^* = 1$", and (b) the relation $x_i \leq\geq x_j$, which means "for every feasible vector, such that $x_i^* = 0$, $x_j^* = 1$, the vector obtained from x^* by replacing its i-th component by 1 and its j-th component by 0, is also feasible". Our new results include in particular: (1) a generalization of depth-first search trees, with an application to linearizing pseudo-Boolean functions using a minimum number of order relations; (2) a linear time algorithm for maximizing linear functions of 0-1 variables subject to "tree-like" order constraints; (3) an improved polynomial-time algorithm for the class of regular set covering problems.
This is joint work with Bruno Simeone.

Adalbert Kerber

Discrete Structures in Chemistry

The problem which lead to the development of Pólya's theory of enumeration is the isomerism problem of chemistry. It amounts to the construction of all the connected loop-free multigraphs with given degree sequence and (possibly) some prescribed subgraphs. There is no satisfactory solution yet, but we report here on our experiences with graph construction. The present situation allows the construction of cyclic and of acyclic substructures. Furthermore we discuss the generation of (unlabelled) graphs uniformly at random, following the Dixon/Wilf procedure (J. Algorithms 4), which opens an interesting field of experimental mathematics.

Peter Kleinschmidt

Toric varieties and combinatorics

Toric varieties are algebraic varieties which can be described in purely combinatorial and polyhedral terms. Recently, toric varieties have proved useful for various results concerning combinatorial properties of polyhedra. So far, these results could not be obtained by more elementary methods.

We will present some more applications of relations between combinatorial properties of polyhedra and properties of toric varieties:

1. A classification of toric varieties can be obtained using the matroid structure of cone systems.
2. Projection of a variety reduces to an LP-feasibility problem.
3. Upper bounds for the number of faces of special polyhedra can be computed by the cohomology of smooth varieties.

Eugene Lawler

Solving combinatorial optimization problems by distributed computation

A variety of branch-and-bound and dynamic programming procedures can be nicely implemented in homogeneous distributed processing environments, with automatic load balancing by randomization techniques. Computational results will be reported.

Thomas Lengauer

Linear time solutions of CMOS layout problems

We consider layout optimization problems occurring for a certain widely accepted design style for basic fractional cells in CMOS technologies. The problems take the combinatorial form of optimization on large sets of series parallel graphs. Here a series parallel graph represents the circuitry in the cell, with logical-and being represented by a series composition and logical-or being represented by a parallel composition. The optimization takes place on a set of series-parallel graphs implementing the same boolean function.

We define the following two combinatorial problems relevant for CMOS layout:

MNSP: Given a series-parallel graph G find the graph G' in the class of G that has the fewest odd-degree vertices.
MNDP: Given a series-parallel graph G find the graph G' in the class of G that can be made Eulerian by duplicating a minimum number of edges.
Both problems we solved in linear time in the size of G by extending a dynamic programming method of Bern, Lawler and Wong (presented in the talk by E. Lawler at this meeting).
This is joint work with R. Müller.

Jan Karel Lenstra

The parallel complexity of TSP heuristics

We investigate the computational complexity of a number of simple TSP heuristics on a PRAM. The nearest addition and double spanning tree heuristics require polylogarithmic time. In contrast, questions about the performance of the nearest neighbor, nearest merger, nearest insertion, cheapest insertion and farthest insertion heuristics are $\mathcal{P}$-complete. The parallel complexity of Christofides' heuristic, which combines a spanning tree with a perfect matching on its odd-degree vertices, remains open.
This is joint work with G. A. P. Kindervater.

Thomas M. Liebling

Polycrystal reconstruction: a new application of combinatorial optimization

Polycrystalline structures, when viewed on plane cuts through many materials (e. g. ceramics) are polygonal complexes. The problem of finding the edges of such complexes, when only the vertices are known is considered.
A combinatorial optimization model is proposed, whose solution yields an approximation of the complex. Several approaches that led to the successful one are discussed: The objective function may yield new insights into the physics of such materials. To find approximate solutions, simulated annealing was successfully applied.
Joint work with H. Telley and A. Mocellin.

László Lovász

Graph connectivity, rigidity, and algorithms

Various equivalent conditions on graph connectivity are presented. These involve representations of the vertex set by vectors in a linear space such that (a) each vertex is in the convex hull of its neighbors, or (b) non-adjacent vertices are orthogonal. These characterizations of k-connectivity can be used to design randomized connectivity tests whose running time is essentially that of a matrix inversion. This work is joint with Linial, Saks, Schrijver and Wigderson.

Francesco Maffioli

Multi-constrained matroidal knapsack problems

We consider multi-constrained knapsack problems where the sets of elements to be selected are subject to combinatorial constraints of matroidal nature. For this important class of $\mathcal{NP}$-hard combinatorial optimization problems we prove that Lagrangean relaxation techniques not only provide good bounds to the value of the optimum, but also yield approximate solutions, which are asymptotically optimal under mild probabilistic assumptions.
This is joint work with P. M. Camerini and C. Vercellis.

Thomas L. Magnanti

Modelling and solving some applications of fixed charge network flows

Capacitated fixed charge network flow problems arise in numerous applications including facility location, production planning and scheduling, and network design. Unfortunately, linear programming formulations (relaxations) of these problems often poorly approximate associated integer programming formulations. For several such application contexts (production planning with changeover costs, production lot sizing, and facility location), we describe new valid inequalities and facets that strengthen the linear programming formulations, and report on preliminary computational experience in using these inequalities in a cutting plane approach.

Kurt Mehlhorn

On routing for VLSI

We first review a theorem of Kaufmann and Mehlhorn. Let R be a subgraph of the planar grid, let B be the set of nodes of degree at most three, and let $N_1, \ldots, N_k$ be nets. A net is a path connecting two points in B. If the free capacity of every cut is even and nonnegative then there exists path p_i such that p_i is homotopic to N_i and p_i and p_j are edge-disjoint for $i \neq j$. Moreover, the p_i's can be found in time $O(n \log n)$. We then discuss the various shortcomings of this result: layer assignment, non-grid graphs, non-even problems, speed and multi-terminal nets. We indicate partial solutions for these shortcomings.

Rolf H. Möhring

Interval graphs: on-line recognition, applications, and a new data structure

The only known linear-time algorithm for recognizing interval graphs first tests whether the graph is triangulated, then computes its maximal cliques, and finally constructs a consecutive arrangement of the maximal cliques by using PQ-trees (Booth and Lueker, 1976). We present a much simpler algorithm which uses a related, but much more informative tree representation of interval graphs. This tree is constructed in an on-line fashion by traversing the graph in a lexicographic BFS and growing the tree gradually as the vertices are being traversed. The growth process takes $O(|\mathrm{Adj}(u)|+1)$ amortized time for visiting a vertex u. We apply the new tree structure to the "seriation problem with side constraints", in which one wants to find a transitive orientation of the complement $\overline{G}$ of an interval graph G that preserves certain already oriented edges.

James B. Orlin

A simple $O(nm + n^2 \log c_{\max})$ algorithm for the maximum flow problem

We present a variant of Goldberg's $O(n^3)$ maximum flow algorithm. Our algorithm reduces the number of "saturating pushes" to $n^2 \log c_{\max}$, so that the resulting running time of the algorithm is $O(nm + n^2 \log c_{\max})$. Our algorithm dominates all other maximum flow algorithms whenever $c_{\max} = O(n^{O(1)})$, and it strictly dominates the other algorithms if, in addition, the graph is neither very sparse nor very dense.
In the case that $c_{\max}$ is very large, the arithmetic model of computation in which each arithmetic operation is counted as one step is not as appropriate, and we consider the logarithmic model of computation in which we count the number of bit operations. Under this model of computation, our algorithm dominates all other proposed algorithms regardless of the density of the graph or the size of $c_{\max}$. In fact, if $c_{\max} > w^n$ for any fixed $w > 1$, then our algorithm dominates the best other algorithm (that of Goldberg and Tarjan) by a factor of $(m \log(n^2/m))/(n \log n)$; i. e., by a factor of m/n except when the graph is very dense.
This is joint work with R. K. Ahuja.

Manfred Padberg

Resolution of large-scale symmetric TSP's

We report the results of a computational study on symmetric travelling salesman problems (TSP's) done jointly with Giovanni Rinaldi (IASI-CNR, Rome) at New York University during 1985 – 1986. The study implements the results on the facial structure on the TSP polytope obtained by Grötschel & Padberg (1974-1979) and most recently by Grötschel & Pulleyblank. Problems with up to 2,392 cities were solved to optimality. this increases the problem size of problems of this class that can be considered solvable by a substantial margin.

William R. Pulleyblank

Projecting Combinatorial Polyhedra

We discuss the problem of obtaining a complete linear description of the dominant of the incidence vectors of the two-terminal Steiner trees in a directed graph. (A two terminal Steiner tree is a directed tree joining one source node to two terminal nodes in a directed graph.) We show how the standard dual formulation of a shortest path problem can be used to obtain a large linear system of which the desired polyhedron is a projection. Then we show how the "Benders" method can be used to obtain the desired system. This system has inequalities with very large coefficients.
This is joint work with Michael Ball and Liu Wei-guo.

Gerhard Reinelt

An algorithm for solving quadratic 0-1 problems

We consider the unconstrained quadratic 0-1 programming problem $\max\{x^T Qx + c^T x \mid x \in \{0,1\}^n\}$ where Q is an upper triangular matrix with zero diagonals. This problem is $\mathcal{NP}$-hard in general. It can be transformed into a max-cut problem in a graph, i. e., into a problem of the form $\max\{c(\delta(w)) \mid W \subseteq V\}$ for some graph $G = (V, E)$ with edge weights c_e for all $e \in E$. We present a cutting plane algorithm for the solution of this problem which is based on a partial linear description of the cut poytope $P_{\mathrm{CUT}}(G) := \mathrm{conv}\{\chi^F \in \{0,1\}^{|E|} \mid F \text{ cut in } G\}$. Our algorithm consists of a "branch-and-cut"-technique, where fractional LP solutions are exploited to derive lower bounds by reduced cost criteria and logical implications. We show that this approach compares favourably with existing algorithms.
This is joint work with Francisco Barahona and Michael Jünger.

Franz Rendl

On the Euclidean assignment problem

The Euclidean assignment problem is a special case of weighted bipartite matching and can be described as follows. For given sets R and B of n points in the plane find a bijection $p : R \to B$ that maximizes (minimizes)

$$\sum_{i=1}^{n} d(r_i, b_{p(i)})$$

the sum of (Euclidean) distances between pairs of points assigned to each other. We describe a linear time heuristic which solves the maximization case with relative error tending to zero, if R and B are uniformly distributed in the unit square. In this case it is further shown that the maximal value z satisfies $\frac{z}{n} \to (\sqrt{2} + \log(1 + \sqrt{2}))/3$ almost surely as $n \to \infty$. The heuristic can also be applied to the minimization case but the analysis seems much more complicated.

Günter Rote

A graphtheoretic proof of the Cayley-Hamilton Theorem

We give a graphtheoretic interpretation of the determinant and the characteristic equation of a matrix and a graphtheoretic proof of the theorem of Cayley and Hamilton stating that a matrix fulfills its own characteristic equation.
We show how the concept of a characteristic equation can be extended to matrices over commutative semirings, where subtraction is not defined. We give conditions when the eigenvalues of a matrix fulfill the characteristic equation.
For applications, an important instance of a semiring is the set of real numbers with the operations max and + ("path algebra", "schedule algebra"), to which these and related results of linear algebra can be applied.

Alexander Schrijver

A homotopic circulation theorem

We discuss the following theorem (which is related to a question asked by Kurt Mehlhorn concerning the automatic design of integrated circuits). Let $G = (V, E)$ be an undirected graph, embedded (without crossings) on a compact orientable surface S. Let $C_1, \ldots, C_k$ be cycles in G. Then there exist functions ("circulations") $f_1, \ldots, f_k$: $E \to \mathbb{R}$ so that: (i) f_i is a convex combination of incidence vectors of cycles in G homotopic to C_i ($i = 1, \ldots, k$), (ii) $f_1(e) + \ldots + f_k(e) \leq 1$ for each edge e, **if and only if** for each closed curve D on S not intersecting vertices of G, the number of edges intersected by D (counting multiplicities) is at least $\sum_{i=1}^{k}$ (minimum number of intersections of C and D among all closed curves C homotopic to C_i).

Éva Tardos

Approximation algorithms for unrelated parallel machines

The parallel machine scheduling problem is to schedule n jobs on m machines given the processing times p_{ij} (the time needed to process the jth job if it is scheduled on the ith machine) so as to minimize the completion time. The problem is $\mathcal{NP}$-complete even in the special case of 2 identical machines. Approximation algorithms for identical and related machines ($p_{ij} = p_i q_j$) were known with performance ratio $1 + \varepsilon$, (for any given $\varepsilon > 0$). The best polynomial time algorithm for the general case known had performance ration $O(\sqrt{m})$, (or 2 in case m, the number of machines, is fixed.
In this talk we give a polynomial time algorithm with performance ratio 2 for the general case, one with performance ratio $1 + \varepsilon$ for the case m, the number of machines and $\varepsilon > 0$ is fixed. Furthermore we prove that finding a schedule that is better than 1.5 times the optimal is $\mathcal{NP}$-hard. The results presented are joined work with Jan Karel Lenstra and David Shmoys.

Gottfried Tinhofer

Birkhoff graphs and linear programming for graph identification

Birkhoff graphs are undirected graphs having an adjacency matrix A such that the polytope $\{X \mid XA = AX, X \text{ doubly stochastic}\}$ has integral vertices only. It is shown that the isomorphism Problem of Birkhoff graphs is polynomial. Moreover Birkhoff graphs are exactly those graphs for which a certain backtracking isomorphism algorithm works in polynomial time. The recognition problem for Birkhoff graphs is more complicated. It will be shown which classes of Birkhoff graphs have been set up so far. A construction principle is presented which like a graph grammar allows to construct richer classes of Birkhoff graphs.

Leslie E. Trotter

On randomized stopping points and perfect graphs

Randomized stopping points form a convex compact set arising in the context of the optimal stopping problem for two-parameter processes. When this set is finite-dimensional, it can be identified with a bounded polyhedron defined by a $(0,1)$-matrix. Study of the extremal elements of this polytope motivates the definition of an apparently new class of perfectly orderable graphs. Properties of this class of graphs are examined. For this setting, it is shown that under a classical hypothesis on the probabilistic model, the extremal elements of the set of randomized stopping points are precisely ordinary stopping points.

Klaus Truemper

Testing total unimodularity

A new fast algorithm for testing total unimodularity of matrices is described. The method relies on induced decomposition results of earlier work on matroid decomposition.

Dominique de Werra

Some experiments in graph coloring

Various heuristic methods have been proposed for finding a partition of the node set of a graph into as few independent sets as possible. Such a problem occurs in many situations like clustering, timetabling or group technology in production planning. We shall describe some techniques based on combinations of classic methods together with simulated annealing. Computational results will show how the efficiency of these methods can be increased by appropriately mixing several strategies.

Laurence A. Wolsey

Single machine scheduling with release dates: A polyhedral view

We consider the problem of minimizing the weighted sum of start times $\sum w_j t_i$ with release dates r_j and processing times p_j for $j \in N$. Letting $\sigma = (j_1, \ldots, j_s)$ be an initial subsequence of jobs, $\tau(\sigma)$ denotes the complete sequence obtained by extending σ based on Smith's rule. This leads to the relaxation based on the enumeration of all initial sequences with $|\sigma| = s$

$$(R_s) \qquad \min_{|\sigma|=s} \{\sum_{j \in N} w_j t_j : t \equiv \text{ Short Times of } \tau(\sigma)\}.$$

The main result is a mixed integer programming formulation whose linear programming relaxation (P_s) solves (R_s).
By projecting (P_s) into the space of (t, S) variables ($\delta_j = 1$ if i proceeds j), we obtain other formulations whose relaxations solve (R_s). These are examined explicitly for $s = 0$, $s = 1$ gives facets for (R_s) and strong valid inequalities for (R). In a different vein we compare the strength of several of the lower bounds suggested in the literature.

Zaw Win

On the windy postman problem

Let $G = (V, E)$ be an undirected connected graph; with each edge $ij \in E$ two real numbers c_{ij} and c_{ji} are associated where c_{ij} resp. c_{ji} is the cost of traversing the edge ij from i to j resp. from j to i. A "windy postman tour" is a closed directed walk which is an orientation of a closed walk in G containing each edge of E at least once; and the "windy postman polyhedron" of G, denoted by $WP(G)$, is the convex hull of incidence vectors of windy postman tours. The "windy postman problem (WPP)" is to find a windy postman tour of minimum cost. It is known that WPP is $\mathcal{NP}$-hard.
In this talk we show that the LP relaxation of the canonical integer linear programming formulation of the WPP is a complete description of $WP(G)$ if and only if G is Eulerian. From this fact follows the polynomial time solvability of the WPP on Eulerian graphs.

Uwe Zimmermann

A strongly polynomial algorithm for submodular flows

The only known strongly polynomial algorithm for solving minimum cost submodular flow problems before the end of 1985 was due to Frank and Tardos [1985]. Based on the simultaneous approximation algorithm of Lenstra, Lenstra and Lovász [1982] they approximate the cost vector by a polynomially sized one. The corresponding equivalent submodular flow problem can then be solved by an algorithm of Cunningham and Frank in strongly polynomial time relying on a strongly polynomial algorithm for minimizing submodular set functions due to Grötschel, Lovász and Schrijver [1987]. We present a joint paper with Fujishige and Röck on a combinatorial algorithm which extends the approach of Tardos [1984] for circulation problems. It consists in solving a sequence of submodular flow problems which successively yield a description of the face of all optimal submodular flows in strongly polynomial time.

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