MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 27/1987

Partielle Differentialgleichungen

21.6. bis 27.6. 1987

This conference was organized by S. Hildebrandt (Bonn), P. Lax (New York) and K. Uhlenbeck (Chicago). Taking part were 60 participants, of whom 31 gave talks. The lectures were scheduled so as to allow ample opportunity for informal scientific discussion.

The various contributions ranged over a wide spectrum with specific topics in current research being highlighted.

A particular emphasis was placed on the investigation of oscillations in sequences of solutions to nonlinear differential equations. Among the examples considered were equations with small dispersion, the Euler equations for incompressible fluid flow and quasilinear hyperbolic systems.

Another subject of interest was the study of the blow up of solutions to several classes of nonlinear parabolic and hyperbolic systems.

Further emphasis was placed upon special equations from mathematical physics, such as those arising from nonlinear σ-models and the Yang-Mills-Higgs equations. For the latter the relation to geometry and topology was also discussed.

A final important topic was the consideration of nonlinear equations of both elliptic and non-elliptic type associated with problems from differential geometry.

The participants would like to thank the organizers and also the management of the institute and all personnel for their excellent work.

Vortragsauszüge:

A. Bahri:

Pseudo-orbits of contact forms

We are considering a variational problem lacking compactness on a submanifold of the loop space of a three-dimensional compact and orientable manifold M. The objects studied are M, α, ξ and v, where α is a contact form on M, ξ its Reeb vector field, v a non-singular vector field in the kernel of α. Let $\beta = d\alpha(v,\cdot)$. The functional is $I(x) = \int_0^1 \alpha_x(\dot{x})dt$ on the space of variations $C_\beta = \{x \in H^1(S^1,M)$ s.th. $d\alpha(\dot{x},v) \equiv 0,\ \alpha_x(\dot{x}) \equiv C > 0\}$. The topology of C_β has been studied in some cases by S. Smale. The critical points of I on C_β are periodic orbits of ξ. Nevertheless, the problem is ill-posed: the condition (C) of Palais and Smale does not hold and the gradient of I has no Fredholm structure. We then introduce a new notion in variational theory: "Critical points at infinity". These geometrical objects are ends of flow-lines of a pseudo-gradient for the functional. They are curves which, when parametrized suitably, explain why the Palais-Smale condition fails. There are stable and unstable manifolds for the flow attached to these objects, as well as a Morse index. These curves are reached through a singular perturbation technique, involving the full pendulum equation with coefficients equal to functions on M. A full study of the convergence problem is completed, involving cancellation of oscillations and geometric convergence of some of them (convergence in graph). The study relies heavily on α being a contact form and on its behaviour along the vector field v. Depending on this behaviour, different situations are singled out. With adding these ends, it is possible to complete variational theory.

R. Böhme:

Plateau problems with many solutions of high genus spanning a simple boundary curve

For any $g \in \mathbb{N} \cup \{0\}$ we constructed a Jordan curve $\Gamma = \Gamma(g)$ in $\mathbb{R}^3$ such that Γ

is smooth and bounds 2^{2g} different minimal surfaces of genus $0 \leq \gamma \leq g$. Γ is given as a perturbation in an obvious monotonic explicit way from a double cover G_2 of a planar curve $G \subset \mathbb{R}^2$. G can be described easily, G_2 bounds easily very many pieces of different hyperelliptic surfaces, whose branch points could vary freely in the interior of G. For the perturbation argument we needed only 2(2g+1) points on G (or Γ) where the third coordinate has to obey inequalities. Then we can apply a version of Teichmüller theory which we presented at the conference on "calculus of variations" here last year.

R.J. DiPerna:

Concentration effects in the Euler equations

We discuss some work dealing with sequences of solutions to the inviscid Euler equations in two space dimensions having uniformly bounded energy and vorticity. One is given that the sequences converge weakly in L^2. The problem is to determine whether or not the sequences converge strongly in L^2. The deviation between weak and strong convergence is recorded by defects (concentrations) in the energy density. No oscillations can persist in the limit. We show that the defect set has Hausdorff dimension ≤ 1 in space-time $\mathbb{R}^2 \times \mathbb{R}$ locally. We investigate the response of the inertial terms to defects in the energy field. If the defect set has Hausdorff dimension strictly less than one we prove that the limiting field is a distributional solution. The results are part of a joint program with A. Majda.

K. Dressler:

The stationary Vlasov-Fokker-Planck equation

Steady states are of high interest in plasma physics. It is well known that generally the nonlinear Vlasov-Poisson equation, which describes the plasma evolution, has "many" stationary solutions (BGK-modes). In this talk we investigate the Vlasov-Fokker-Planck equation, a modification of the Vlasov equation, obtained by adding a diffusion term with respect to velocity. Physically it describes a plasma in thermal equilibrium. We prove a complete existence (in C^∞) and uniqueness theorem (in the space of probability measures) for stationary solutions of that equation. The uniqueness result in its most general form is obtained through the characteristic stochastic differential equations. This result is of special interest since it distinguishes one of the stationary solutions of the Vlasov-Poisson equation.

A. Floer:

Instantons and holomorphic curves

Morse theory of the symplectic action function on the loop space of a symplectic manifold can be done by analyzing the space of trajectories of the gradient flow between critical points. The equation defining such trajectories is up to lower order the equation for holomorphic maps from $\mathbb{R}\times S^1$ into P with respect to some almost complex structure on P. This p.d.e is very similar to the instanton equation on $\mathbb{R}\times M$, where M is a three-manifold.

L.S. Frank:

Coercive singular perturbations: reduction to regular perturbations and applications

The algebraic concept of coerciveness, introduced in 1976 as a necessary and sufficient condition for the stability of singular perturbations (validity of two-sided a priori estimates uniformly with respect to the small parameter), turns out to guarantee also the possibility of a reduction of a singular perturbation to a regular one. The following applications of this constructive reduction procedure are considered: 1. Simple derivation of asymptotic formulae for the solutions of coercive singular perturbations. 2. Asymptotics for their eigenvalues and eigenfunctions. 3. Bifurcation phenomenon for coercive singular perturbations. 4. Construction of efficient and robust algorithms for their numerical treatment. 5. Asymptotic analysis of some classes of singular perturbations of strictly hyperbolic operators, the Boussinesq's system in the theory of wave propagation being an example of such a singular perturbation.

G. Huisken:

Asymptotic behaviour for singularities of a geometric evolution equation

We study closed hypersurfaces of $\mathbb{R}^{n+1}$ which move in direction of their mean curvature vector. It is known that convex hypersurfaces contract smoothly to a point under this flow, whereas other singularities can occur if the initial data are not convex. These singularities are studied with a rescaling technique and it is shown that in certain cases the singularities behave like self-similar solutions of the mean curvature flow. Some results are obtained concerning the classification of these self-similar solutions.

N. Jacob:

On Gårding's inequality

An analysis of the proof of Gårding's inequality for strongly elliptic differential operators shows that a generalization of this inequality can be proved for certain non-elliptic operators, provided a generalized principal part is defined and the usual Sobolev-space is changed in an appropriate way. Moreover it turns out that we have to distinguish two classes of operators. In the most general class we cannot use a partition of unity in the proof. This implies that the coefficients of the generalized principal part are assumed to vary only slowly. The second class consists of operators which can be handled completely analogously to elliptic operators. These operators have to be formally hypoelliptic (in a certain sense). Examples are given for both classes.

W. Jäger:

Explosions in chemotaxis systems

The following equations are a simple model for the aggregation of microorganisms (amoeba; concentration u) caused by a chemical substance (acresin;

concentration v) produced by them:

$$\partial_t u = \Delta u - \chi \nabla(u\nabla v)$$

$$\text{in } \Omega \subset \mathbb{R}^n, \quad n = 2,3$$

$$\partial_t v = \gamma\Delta v - \mu v + \beta u$$

no flux on the boundary (Keller-Segel-model).

In case of large γ and β the system can be approximated by

$$\partial_t u = \Delta u - \chi\nabla(u\nabla v)$$

$$0 = \Delta v - \alpha(u-\bar{u}), \quad \bar{u} = \int_\Omega u_0 \, dx.$$

Result (S. Luckhaus, W. Jäger): There exists a $c(\Omega) > 0$, s.th. $\alpha \, ◻ \, \chi < c(\Omega)$ implies existence of smooth global solutions. If this condition is violated, there exists blow up in finite time. Quantitative criteria are given in the radial symmetric case. The results are reflected by the experimental observations (formation of fruiting bodies).

R. Illner:

A boundary value problem in the kinetic theory of gases

On the rectangle $[0,a] \times [0,b]$, we consider the boundary value problem

$$\partial_x f_1 + f_1 f_2 = f_3 f_4 \,, \quad f_1(0,y) = \varphi_1(y)$$

$$-\partial_x f_2 + f_1 f_2 = f_3 f_4 \,, \quad f_2(a,y) = \varphi_2(y)$$

$$\partial_y f_3 + f_3 f_4 = f_1 f_2 \,, \quad f_3(x,0) = \varphi_3(x)$$

$$-\partial_y f_4 + f_3 f_4 = f_1 f_2 \,, \quad f_4(x,b) = \varphi_4(x) \,,$$

where the $\varphi_1,\dots,\varphi_4$ are bounded, continuous and nonnegative on the intervals $[0,a]$ and $[0,b]$ respectively. By using a suitable fixed point theory (Schaefer's theorem), it is shown that this problem has a solution for any size of the data a,b and $\varphi_1,\dots,\varphi_4$. The solution is unique if the data are small, because the operator under consideration is then contractive. The problem is a model problem for a more fundamental question from rarefied gas dynamics.

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B. Kawohl:

Remarks on blow up, quenching and dead cores

Nonlinear parabolic differential equations can exhibit seemingly different phenomena such as the ones listed in the title. Nonetheless these "different" phenomena can be tackled by similar techniques. The purpose of my remarks is to point out that this convenience has a simple explanation. As an example I reduce the blow up problem

$$\begin{aligned} u_t - \Delta u &= u^p - |\nabla u|^2 , && t > 0,\ x \in \Omega , \\ u &= 0 && t > 0,\ x \in \partial\Omega , \\ u &= u_0 \gneq 0 && x \in \Omega , \end{aligned}$$

to a dead core problem

$$\begin{aligned} v_t - \Delta v &= -h(v) && t > 0,\ x \in \Omega , \\ v &= 1 && t > 0,\ x \in \partial\Omega , \\ v &= v_0 && x \in \Omega . \end{aligned}$$

There is no blow up for $p \leq 2$ but blow up for $p > 2$, Ω large and u_0 large. This and other results were obtained jointly with L. Peletier (Leiden) and A. Acker (Ames).

H. Kielhöfer:

Bifurcation of periodic solutions of Klein-Gordon equations

We consider a semilinear wave equation

$$u_{tt} - u_{xx} - c(\lambda)u - h(\lambda,x,u) = 0 , \quad h = o(|u|) , \quad \lambda \in \mathbb{R} ,$$

together with periodic Dirichlet or Neumann boundary conditions for x and/or t. We prove bifurcation of nontrivial solution at $\lambda = \lambda_0$ for a dense set of periods (intervals) provided $c(\lambda)$ is strictly monotone near λ_0. The main tool is a new bifurcation result for potential operators which is not proved by variational methods but uses Conley's bifurcation theory for bounded invariant sets.

W. Kirsch:

Schrödinger operators with stochastic potentials behaving differently in two half spaces

Suppose $V_\omega^{(1)}$, $V_\omega^{(2)}$ are random potentials both of which are ergodic. Let $V_\omega(x)$ be given by $V_\omega^{(1)}(x)$ for $x_1 \leq 0$ and by $V_\omega^{(2)}(x)$ for $x_1 > 0$, $x = (x_1,\ldots,x_d)$, $d > 1$. $V_\omega(x)$ is a random potential which is not stationary (in the sense of stochastic processes). Nevertheless one can carry through some of the "general nonsense" about ergodic operators. A significant and perhaps physically interesting difference is the occurence of "surface" states, i.e. of states (=solutions of the Schrödinger equation) that live near the surface $x_1 = 0$. We describe various ways to make this notion precise, including a density of states measure normalized by a surface rather than a volume (joint work with H. Englisch (Leipzig) and B. Simon (Pasadena)).

R.V. Kohn:

Blow up of semilinear heat equations

I discuss joint work with Y. Giga concerning the blow up of solutions of $u_t - \Delta u = |u|^{p-1}u$ (and related equations). One idea is to use the scaling property of the equation: that if u is a solution then so is

$$u_\lambda(x,t) = \lambda^{\frac{2}{p-1}} u(\lambda x, \lambda^2 t) \qquad \text{for any } \lambda > 0.$$

As it blows up, the solution is expected to become asymptotically scaling invariant (i.e. self-similar). This is proved (under appropriate hypotheses) by means of a change of variables that transforms the study of u as it blows up to the analysis of the large-time behaviour of a different semilinear parabolic equation.

H. Lange:

Collapse of periodic solutions to nonlinear Schrödinger equations

We consider sufficient conditions such that any classical solution of the periodic initial-boundary value problem

$$(1)\qquad \begin{cases} iu_t = -u_{xx} + f(|u|^2)u \\ u(x+2,t) = u(x,t),\ u(x,0) = u_0(x) \end{cases}$$

has a finite life-span, i.e. the solution exists only on a finite time interval $[0,T)$. The conditions to ensure that are some growing properties on the nonlinearity f of (1) and a relation of the type

$$E_1(0) < E^* \leq 0$$

where $E_1(0)$ is the initial energy of u_0, and E^* a certain constant depending on the data of (1). The main ingredient for the proof of the collapse result is to follow the evolution of appropriate moments of the solution $u(x,t)$ of (1), e.g.

$$G(t,\eta) = \int_0^\eta F(y,t)dy,\quad F(y,t) = \int_{y-1}^{y+1} \psi(x,y)|u(x,t)|^2\, dx \qquad (0 < \eta < 2)$$

with some quadratic function $\psi(x,y)$.

P.D. Lax:

Oscillatory solutions

Solutions of differential equations with nonlinearity, and a small dispersive term have a tendency to develop oscillations. Examples are the Korteweg-deVries equation and dispersive difference approximations to nonlinear hyperbolic equations. In the limit of zero dispersion, solutions converge weakly but not strongly. The weak limit of solutions of the KdV equation as dispersion tends to zero therefore does not satisfy the limiting equations in the integral sense. We conjecture - on the basis of analogy, analysis and numerical evidence - that the same is true for dispersive difference approximations to nonlinear conservation laws.

A. Majda:

Vortex dynamics: numerical analysis, scientific computing, and mathematical theory

Vortex dynamics dominates the behavior of incompressible fluid flow at high Reynolds numbers in diverse applications such as the accurate tracking of hurricane paths, the design of internal combustion engines, and the control of hazardous large vortices shed by landing jumbo jets. Here we present an overview of ideas involving the interaction of numerical analysis, large scale computing, and mathematical theory. First we describe the recent progress in the design and numerical analysis of vortex algorithms; then we report on numerical calculations for vortex sheets displaying incredible complexity. Finally we describe the recent progress in the mathematical theory geared toward understanding these complex phenomena.

D.W. McLaughlin:

Integrable geometry, coherence, and chaos for the Sine-Gordon pde

Numerical experiments are discussed which illustrate a quasi-periodic route to intermittent chaos in the damped, driven Sine-Gordon partial differential equation. This route to chaos seems typical for near-conservative, dispersive waves in one spatial dimension. This route has one temporal frequency, then two, then chaos; spatially coherent patterns; pattern competition and selection; low dimensional chaotic attractors. In the second part of the talk, the geometry of level sets for the invariants of the integrable, spatially periodic Sine-Gordon equation is developed. In particular, singular level sets and their associated homoclinic orbits are emphasized. The spectrum of a linear operator is used to detect these singular level sets - both theoretically and numerically. In the last part of the talk, this spectrum is measured numerically for chaotic data in order to detect and correlate "homoclinic crossings" with these chaotic solutions of the damped driven partial differential equation. The talk summarizes joint work with A. Bishop, N. Ercolani, M.G. Forest, and E. Overman.

J. Neu:

Topological vortices and electrodynamics

There is a perennial notion that matter may have a geometric-topological origin. There are nonlinear field theories originating in physics and differential geometry with topological vortex or monopole solutions. The topological invariants associated with these vortices and monopoles are referred to as "charge", as though these vortices and monopoles are analogous to material charged particles. Are these analogies meaningful dynamically? Do "charged" vortices and monopoles really interact like electromagnetic charged particles? We investigate the dynamics of topological vortices governed by the nonlinear wave equation $\psi_{tt} - \Delta\psi - (1 - |\psi|^2)\psi = 0$. Here, ψ is a complex scalar field defined on 2+1-D Minkowski space. The topological objects ("vortices") of the ψ field are zeros with winding numbers not equal to zero. From the full field dynamic we asymptotically derive a "reduced" dynamic for the phase gradient tensor $F^{ij} = \varepsilon^{ijk} \cdot \partial_k(\arg \psi)$ and the world lines l of the zeros of ψ. We discover that the reduced dynamic governing F^{ij} and the l's consists of Maxwell's equations for F^{ij} and a Lorentz equation giving the proper accelerations of the world lines l in response to the field F^{ij}. In this "electrodynamics of vortices" the "charge" of a zero is 2π x (winding number). 3+1-D models based on O(3) Yang-Mills-Higgs equations or Kaluza-Klein theory are presently being examined on a similar basis.

M. Plum:

Eigenvalue inclusions for elliptic differential operators by a numerical algorithm

Object of consideration is the eigenvalue problem for linear symmetric elliptic differential operators (mainly of second order) with a discrete spectrum $\lambda_1 \leq \lambda_2 \leq \ldots$. An algorithm and its theoretical background are presented which yield, for given n, guaranteed and close inclusion intervals for the first n eigenvalues. In particular, intervals containing <u>no</u> eigenvalue are computable. The algorithm is based on Hilbert space analysis and an appropriate homotopy method. For practical numerical computations, a projection method is needed in order to calculate approximate solutions for linear boundary value problems. Concrete examples are given to illustrate the method.

R. Racke:

<u>The Cauchy-problem in 3-D-thermoelasticity</u>

We consider the following Cauchy problem (homogeneous, initially isotropic 3-D-thermoelasticity):

$$\frac{\partial^2 U_i}{\partial t^2} = C_{imjk}(\nabla U,\vartheta)\frac{\partial^2 U_j}{\partial x_m \partial x_k} + \tilde{C}_{im}(\nabla U,\vartheta)\frac{\partial \vartheta}{\partial x_m},$$

$$a(\nabla U,\vartheta)\frac{\partial \vartheta}{\nabla t} = \frac{1}{f(\vartheta)}\operatorname{div} q(\nabla U,\vartheta,\nabla\vartheta) + \operatorname{tr}\{(\tilde{C}_{im}(\nabla U,\vartheta))^t_{i,m}(\frac{\partial}{\partial t}\frac{\partial U_i}{\partial x_m})_{i,m}\}$$

$$U(t=0) = U^0,\ \frac{\partial U}{\partial t}(t=0) = U^1,\ \vartheta(t=0) = \vartheta^0.$$

Global existence is shown -for small data- in the class of functions

$$\nabla U,\ \frac{\partial U}{\partial t} \in C_1(\mathbb{R}_0^+,W^{s-1,2}) \cap C_0(\mathbb{R}_0^+,W^{s,2}),\ \vartheta \in C_1(\mathbb{R}_0^+,W^{s-2,2}) \cap C_0(\mathbb{R}_0^+,W^{s,2}),$$

for some $s \in \mathbb{N}$, if the nonlinearity degenerates up to order 2, that is e.g. $|C_{imjk}(\nabla U,\vartheta) - C_{imjk}(0,0)| = O(|\nabla U|^2 + |\vartheta|^2)$ near the origin.

Moreover the time-decay is given as well as the scattering behaviour. The proof uses the following ingredients: a) Transformation to a suitable first-order system $V_t + AV = F(V,\nabla V,\nabla^2 V)$, $V(0) = V^0$, where $-A$ generates a contraction semigroup; b) L_p-L_q-time decay of solutions of the linearized problem; c) local existence result (foll. Kawashima); d) high energy estimate:

$$\|V(t)\|_{W^{s,2}} \leq C\cdot\|V^0\|_{W^{s,2}}\cdot\exp\int_0^t(|V|^2_{L^\infty} + |V_t|^2_{L^\infty} + |\nabla V|^2_{L^\infty} + |\nabla^2 V^3|^2_{L^\infty})(\tau)d\tau,$$

and weighted a priori estimate: $\sup\limits_{0\leq t\leq T}(1+t)^{2/3}\|V(t)\|_{W^{s_1,6}} \leq M_0 < \infty$

for some $s_1 \in \mathbb{N}$, M_0 being independent of T (foll. Klainerman & Ponce).

J. Shatah:

Hyperbolic harmonic mappings

We present an existence theorem for harmonic maps from Minkowski space-time into SU(2). The existence is done by using a penalization method. These solutions are shown to be only weak solutions of the equation. We also present an example where smooth initial data develop singularities in finite time.

L. Simon:

Nodal sets for solutions of elliptic equations

For solutions of second order equations

$$a_{ij}\, D_i\, D_j u + b_j\, D_j u + c\, u = 0\ ,$$

where (a_{ij}) is continuous and positive definite, and b_j, c are bounded, the talk described a method for bounding the (n-1)-dimensional measure of $u^{-1}\{0\}$ in the neighbourhood of any point x_0 at which the solution u has finite order of vanishing d. Specifically

$$\mathcal{H}^{n-1}(u^{-1}\{0\} \cap B_\varrho(x_0)) \leq cd\varrho^{n-1}\ ,\ \varrho \leq \varrho_0 = \varrho_0(x_0,u,\gamma,\mu)\ ,\ c = c(n).$$

In addition it was shown that the singular part $u^{-1}\{0\} \cap |Du|^{-1}\{0\}$ has dimension $\leq$ n-2, generalizing a result of Caffarelli and Friedmann, and that, if the coefficients are of class C^d, $u^{-1}\{0\} \cap |Du|^{-1}\{0\}$ is countably (n-2)-rectifiable. This is joint work with Prof. R. Hardt (University of Minnesota).

L. Tartar:

Oscillations of nonlinear partial differential equations and their propagation

Oscillations in a sequence of solutions of a nonlinear partial differential equation are described by means of L.C. Young's measures which give the weak limits of functions of the solutions. Strong convergence corresponds to the case where the corresponding Young's measures are Dirac masses. The compensated compactness lemma gives information on weak limits of quadratic functions in the solution and uses the information on combinations of derivatives for the solutions. Using "entropy" conditions one can then write some constraints on the Young's measures and study how these Young's measures vary with respect to x. A few examples are described, corresponding to semilinear hyperbolic systems and quasilinear hyperbolic systems having some linearly degenerate fields.

K.K. Uhlenbeck:

Yang-Mills equations as partial differential equations

A brief history of the origin of the Yang-Mills equations in physics, starting with Maxwell's equations, was given. Originally defined in space-time, they become Euclidean in standard quantum calculations. The instanton equation for

$$A = \sum_j A_j \, dx^j \text{ is (in } \mathbb{R}^4) \qquad F = \sum_{i,j} F_{ij} \, dx^i \wedge dx^j$$

$$F_{ij} = \frac{\partial}{\partial x^i} A_j - \frac{\partial}{\partial x^j} A_i + [A_i, A_j]$$

$$F = * F \quad \text{or} \quad \begin{pmatrix} F_{12} = F_{34} \\ F_{13} = -F_{24} \\ F_{14} = F_{23} \end{pmatrix} .$$

The t'Hooft solutions

$$A = \operatorname{Im} \frac{\partial}{\partial q} (\ln f) dq \qquad q = x^1 + ix^2 + jx^3 + kx^4 .$$

for $\Delta f + \lambda f^3 = 0$ provide a lot of intuition. The gauge theory on manifolds has many applications in 3 and 4 dimensional topology and algebraic geometry.

J. Urbas:

Regularity of generalized solutions of Monge-Ampère equations

I will show that sufficiently smooth generalized convex solutions of the equation $\det D^2 u = f$, where f is a C^∞ positive function, are of class C^∞. An example of Pogorelov shows that the hypotheses on the initial regularity of the solution u cannot be weakened.

S. Venakides:

Small dispersion limit of the Korteweg-deVries equation

The space periodic KdV equation $u_t - 6uu_x + \varepsilon^2 u_{xxx} = 0$ can be solved explicitly in terms of a theta function whose period matrix is derived from a hyperelliptic function. The interest in the small dispersion limit $\varepsilon \to 0$ arises from the fact that fast oscillations are generated out of nonoscillatory initial data. As $\varepsilon \to 0$ the genus N of the associated Riemann surface is of order $1/\varepsilon$. I show that the theta function, expressed as a sum of exponentials over the lattice $\mathbb{Z}^N$, is dominated by its largest term. This leads to the computation of the weak limit of the solution by a procedure similar to the one derived by P.D. Lax and C.D. Levermore. An averaging assumption allows me to describe the local oscillations up to phase shifts. Further results point toward the possibility of a rigorous proof of the averaging assumption.

P. Werner:

Asympototic phenomena in wave propagation: resonances and instabilities

We study the propagation of sound waves in domains with noncompact boundaries ("waveguides") and show that resonances occurring in certain unperturbed waveguides are deleted by small perturbations of the boundary.

W. von Wahl:

Estimates for the pressure in the Navier-Stokes equations in exterior domains and their consequences

Let u be any weak solution of the Navier-Stokes system $u' - \Delta u + u \cdot \nabla u + \nabla \pi = f$, $\nabla \cdot u = 0$, $u|\partial\Omega = 0$, $u(0) = \varphi$, on $(0,T) \times \Omega$, Ω an exterior domain of $\mathbb{R}^n$. Then we show that

$$\nabla\pi \in L^\sigma((0,T), L^p(\Omega)) \ , \quad \pi \in L^\sigma((0,T), L^{p*}(\Omega))$$

provided $n+1 \le \frac{2}{\sigma} + \frac{n}{p}$, $1/p* = 1/p - 1/n$. In particular we obtain $\pi \in L^{(n+2)/n}((0,T) \times \Omega)$. The consequences are as follows: We can construct a weak solution which is bounded in (t,x) if $|x|$ is sufficiently large and fulfils at the same time the energy inequality for almost every $s > 0$, for $s = 0$ and for all $t \geq s$.

As we show, this implies $\|u(t)\|_{L^2} \to 0$ if $t \to \infty$ (if $T = +\infty$). The work reported here is joint one with H. Sohr and with H. Sohr and M. Wiegner.

D. Yang:

Local solvability of nonlinear partial differential equations of real principal type and applications to geometry

(Joint work with Jonathan Goodman, NYU).

Local solvability of a linear PDE of real principal type was first proved by L. Hörmander. We prove local solvability of nonlinear PDE's of real principal type. Using the Nash-Moser implicit function theorem, the proof reduces to proving so-called Moser-type estimates for solutions to linear PDE's of real principal type. One extremely tedious approach would be a detailed examination of Duistermaat-Hörmander's microlocal Fourier integral operator construction of a parametrix to a linear differential operator. We avoid this by showing that given a differential operator P_0 of real principal type, every differential operator P sufficiently "close" to P_0 is conjugate, via classical Fourier integral operators, to P_0. We will also have the following applications of the result:

(1) Transonic air flow; nonlinear Tricomi equation. (2) Local existence of isometric embeddings of a Riemannian manifold in Euclidean space. (3) Local existence of Riemannian metric with prescribed curvature tensor. (4) Towards a C^∞ Cartan-Kähler theorem: local solvability of nonlinear overdetermined systems of PDE's.

R. Ye:

Hyperspheres of constant mean curvature in Riemannian manifolds

We first consider the volume-partitioning problem: minimize area among hypersurfaces dividing a given Riemannian manifold into two parts with given volume ratio. We show that minimizing hypersurfaces for small volume ratio must be regular hyperspheres. We also locate solutions. In case of manifolds with boundary we obtain instead hemispheres as solutions. Some a priori estimates are the main point of the proof. Then we consider the problem of perturbing geodesic spheres into spheres of constant mean curvature. A crucial condition about Ricci curvature arises here. In dimension 2 this condition reads: the center of geodesic spheres be a nondegenerate critical point of Gaussian curvature. The argument relies on a device to treat the kernel of a linearized operator on S^n which is motivated by the recent construction of R. Schoen about singular solutions of the Yamabe equation.

<u>Berichterstatter</u>: M. Meier und R. Racke (Bonn)

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