MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 28/1987

Harmonische Analyse und Darstellungstheorie

topologischer Gruppen

28.6. bis 4.7.1987

Die Tagung fand unter der Leitung von Herrn R. Goodman (New Brunswick), Herrn R. Howe (New Haven) und Herrn D. Poguntke (Bielefeld) statt.

Im Mittelpunkt des Interesses standen Fragen der harmonischen Analyse auf symmetrischen Räumen und auf exponentiellen (insbesondere nilpotenten) Lie'schen Gruppen. Die Untersuchung invarianter Differentialoperatoren und die Zerlegung von induzierten und eingeschränkten Darstellungen fanden dabei große Aufmerksamkeit. Ferner wurden Themen wie Topologie auf dem Dual einer lokalkompakten Gruppe (insbesondere einer exponentiellen Lie'schen Gruppe), harmonische Analyse auf freien Gruppen, K-Theorie, Gruppen mit polynomialem Wachstum, geometrische Quantisierung und Darstellungen topologischer Gruppen behandelt.

Vortragsauszüge

L. BAGGETT

Representations of the Mautner group

Irreducible unitary representations of $G \times H$ factor into products of irreducible representations of G and H if either group is of type I. If neither is of type I, this factorization may not occur. The Mautner group M is not of type I, and this paper deals with representations of $M \times M$. Mackey's theory asserts that if the restriction of an irreducible representation T of $M \times M$ to one of the factors is a multiple of an irreducible representation of M, then T factors. We show that if the restriction to one of the factors is supported on a certain class of representations of M, then T factors.

E.P. VAN DEN BAN

Eisenstein integrals for semisimple symmetric spaces

Let G be a real semisimple Lie group, σ an involution of G and H an open subgroup of G^{σ}. There exists a Cartan involution θ which commutes with σ.

In harmonic analysis on the semisimple symmetric space G/H, minimal $\sigma\theta$-stable parabolic subgroups of G play a role comparable to that of minimal parabolics in harmonic analysis on a semisimple group. One may for instance expect them to contribute to the "most continuous" part of a Plancherel decomposition for $L^2(G/H)$. In the talk we shall define Eisenstein integrals in the setting described above. The main result is that asymptotically these integrals behave like vector valued plane waves whose amplitudes have the same norm. We hope that this phenomenon is connected with the explicit determination of part of the Plancherel measure, just as in the group case.

M. BEKKA

Irreducible representations that cannot be separated from the identity

The cortex of a locally compact group G is defined to be the set of all irreducible representations of G which cannot be Hausdorff-separated from the trivial one dimensional representation of G. This set has been introduced by Guichardet and by Vershik and Karpushev in connection with cohomology with values in unitary representations. We give some examples and a few general results about the cortex of semi-simple and of nilpotent Lie groups. For IN-groups (i.e. groups with a compact invariant neighbourhood of the group unit) we have a complete description of the cortex. If G is such a group and if G_F denotes the (open) normal subgroup consisting of all relatively compact conjugacy classes, then the cortex of G is exactly $\widehat{G/G_F}$. This is a joint work with E. Kaniuth.

G. CARCANO

Gelfand pairs and generalized half-planes

Let G be a separable locally compact group and K a compact group of continuous automorphisms of G. It is known that $L^1_K(G)$, the algebra of the K-invariant integrable functions on G, is commutative if and only if the trivial one-dimensional representation of K occurs at most once in every irreducible representation of the semidirect product $K \ltimes G$ (then $(K \ltimes G, K)$ is a 'Gelfand pair'). I prove a characterization of the commutativity of $L^1_K(G)$ which concerns only representations of G and of certain subgroups of K, avoiding the induction process of $K \ltimes G$. I apply this result to prove in which cases the Šilov boundary of a classical symmetric Siegel domain of type II, with a compact group of automorphisms naturally associated to it, is a Gelfand pair.

J.M. CYGAN

Globally hypoelliptic systems of vector fields on nilmanifolds

A system of differential operators $D_1,\dots,D_m$ on a C^∞-manifold M is *globally hypoelliptic* (GH) if when $D_1 f = g_1,\dots,D_m f = g_m$ with $f \in \mathcal{D}'(M)$, $g_1,\dots,g_m \in C^\infty(M)$, then $f \in C^\infty(M)$. We show that a system $\mathbb{L}$ of real vector fields on a general compact nilmanifold $M = \Gamma\backslash N$ induced by the Lie algebra $\mathfrak{n}$ of N is (GH) iff 1°.

The symbols of the vector fields of $\mathbb{L}$ projected onto the associated torus $T^n = \Gamma[N,N]\backslash N$ as functions on the integral lattice $\hat{T}^n$ collectively decrease at infinity not faster than a reciprocal of a polynomial and 2^o. The Lie subalgebra of $\mathfrak{n}$ that $\mathbb{L}$ generates is not annihilated by any non-zero *integral* linear functional on any $\mathfrak{n}_j/\mathfrak{n}_{j+1}$, $j = 0,1,\ldots$ ($\mathfrak{n}_{j+1} = [\mathfrak{n},\mathfrak{n}_j]$, $\mathfrak{n}_0 = \mathfrak{n}$). It follows that (GH) is equivalent to injectivity of the system $\mathbb{L}$ on the dual of the space of C^∞ - vectors of all the non-trivial representations in the spectrum of $\Gamma\backslash N$ (a "Rockland type" condition) plus a number-theoretic condition on $\mathbb{L}$ on the associated torus (to avoid "small divisors"). (Joint work with L.F. Richardson.)

J. FOX

<u>G-Equivariant K-Theory for non-compact G</u>

The talk will begin by briefly describing Kasparov's bi-invariant formalism for equivariant K-homology and K-cohomology. We will then indicate why these are interesting objects. In particular we will show how Kasparov defines the representation ring for a non-compact group in terms of his KK groups. Finally, we will introduce the notion of K-amenability and discuss the proof of K-amenability for SU(n,1).

H. FUJIWARA

Some monomial representations of exponential groups II

Let $G = \exp\mathfrak{g}$ be an exponential group with Lie algebra $\mathfrak{g}$, f a linear form on $\mathfrak{g}$ and let $\mathfrak{h}$ be a subalgebra of $\mathfrak{g}$ which is subordinate to f.

Then the unitary character χ_f, $\chi_f(\exp X) = e^{\sqrt{-1}f(X)}$ $(X \in \mathfrak{h})$ of the subgroup $H = \exp\mathfrak{h}$ gives a monomial representation $\tau = \mathrm{ind}_H^G \chi_f$ of G. As the last time, we study a kind of reciprocity and a Plancherel formula for these monomial representations.

G.I. GAUDRY

Maximal functions on some solvable Lie groups

We consider left-invariant and right-invariant Hardy-Littlewood maximal functions, relative to left/right translations of various families $\{B_\rho\}_{\rho>0}$ of neighbourhoods of e:

$$\mathcal{M}f(x) = \sup_{\rho>0} \frac{1}{|xB_\rho|} \int_{xB_\rho} f \, d\mu_1$$

(resp. with right translations). The groups are of type AN. In the 'ax + b' group, for instance, we consider

(1) $\qquad B_\rho^{(1)} = \{(a,b): e^{-\rho} < a < e^{\rho}, \quad |b| < \rho^Q\}$

(2) $\qquad B_\rho^{(2)} = \{(a,b): e^{-\rho} < a < e^{\rho}, \quad -\rho a < b < \rho a\}$

(3) $\qquad B_\rho^{(3)} = \{(a,b): e^{-\rho} < a < e^{\rho}, \quad -e^{Q\rho} a < b < e^{Q\rho} a\}$

For left translates of (1), $\mathcal{M}$ is of weak type (1,1), but for right translates of (1), $\mathcal{M}$ is not bounded on any L^p, $p < +\infty$. In case (2), the right invariant $\mathcal{M}$ is not bounded on any L^p, while in (3), it is bounded on every L^p, $p > 1$; not of weak type (1,1) if $0 \le Q < 1$; and of weak type (1,1) if $Q > 1$. This is joint work with A. Hulanicki, S. Giulini, A. Mantero and P. Sjögren.

F.P. GREENLEAF

Spectrum and multiplicity for induced and restricted representations of nilpotent Lie groups

Let G be a connected nilpotent Lie group and K a connected subgroup, with Lie algebras $\underline{\underline{g}},\underline{\underline{k}}$. Let $p:\ \underline{\underline{g}}^* \to \underline{\underline{k}}^*$ be the natural map and let $\pi \in \hat{G}$, $\sigma \in \hat{K}$ be irreducible unitary representations with coadjoint orbits $O_\pi \subseteq \underline{\underline{g}}^*$, $O_\sigma \subseteq \underline{\underline{k}}^*$. We give a geometric characterization of the spectrum and multiplicities for induced representations $\mathrm{Ind}(K\uparrow G,\sigma)$ and restricted representations $\pi|K$. In either case the multiplicity is given by

$$m = \#\{K\text{-orbits in } O_\pi \cap p^{-1}(O_\sigma)\}.$$

Multiplicities are either infinite $m \equiv \infty$ or are all finite and bounded $m \leq N$. Define the invariant

$$\tau_o = \text{generic value of } \dim G\cdot 1 - 2\dim K\cdot 1 + \dim K\cdot p(1)$$
$$(1 \in p^{-1}(O_\sigma) \text{ for induction, } 1 \in O_\pi \text{ for restriction.})$$

Then $m \equiv \infty$ when $\tau_o > 0$, and multiplicities are finite when $\tau_o \leq 0$. By complexifying the Kirillov orbit picture, and applying results from complex algebraic geometry, one can prove

$$m \text{ has constant parity (value mod 2) when } \tau_o = 0.$$

If $\underline{\underline{g}},\underline{\underline{k}}$ are complex nilpotent Lie algebras, G,K the groups regarded as *real* Lie groups, and if $\sigma \in \hat{K}$, then $m \equiv$ constant (possibly ∞).

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A. GUICHARDET

Differential Geometry on the dual of a Lie group

Several notions are introduced and discussed, for a Lie group G, which generalize classical notions of differential geometry on $\mathbb{R}^n$: smooth functions; their jets of order 0, n, ∞ at a point $\pi \in \hat{G}$; the category $\mathrm{Ext}(G,\pi)$ and its Gabriel algebra C. The algebra of smooth functions, $\mathcal{S}(G)$, is known for nilpotent, motion and semi-simple groups, partially for solvable groups; it is conjectured that, if π is CCR, $\pi(\mathcal{S}(G))$ is always isomorphic to a universal algebra $\mathcal{W}$, the algebra of all infinite matrices with rapidly decreasing coefficients; this conjecture is proved by F. Du Cloux for nilpotent groups, and is also true for semi-simple groups (by Arthur's results on $\mathcal{S}(G)$). Another conjecture claims that the algebra of ∞-jets, $\varprojlim \mathcal{S}(G)/M_\pi^{n+1}$, has a tensor decomposition as $\pi(\mathcal{S}(G)) \otimes C$; this is proved in case G is nilpotent or π is trivial.

B. HELFFER

Schrödinger operators with magnetic fields and representation theory of nilpotent Lie groups

In this talk, we present results obtained in collaboration with A. Mohammed (Nantes). We give the following criterion for compactness of the resolvent for the Schrödinger operator with magnetic fields:

$$H(A,V) = \sum_{j=1}^{n} (D_{x_j} - A_j)^2 + \sum_{j=1}^{p} V_j(x)^2 ,$$

where $A_j \in C^\infty(\mathbb{R}^n, \mathbb{R})$, $V_j \in C^\infty(\mathbb{R}^n, \mathbb{R})$. Let us introduce

$$B_{jk} = \partial_{x_j} A_k - \partial_{x_k} A_j \quad , \quad j = 1, \ldots, n$$

and for each $l \in \mathbb{N}$

$$m_l(x) = \sum_{\substack{j \\ |\alpha|=l}} |\partial_x^\alpha V_j(x)| + \sum_{\substack{j,k \\ |\alpha|=l-1}} |\partial_x^\alpha B_{jk}(x)|$$

$$m^{(r)}(x) = 1 + \sum_{l=o}^{r} m_l(x) .$$

Then if for some r we have

(H1) $\qquad m^{(r)}(x) \to \infty , \quad m_{r+1}(x) \le C m^{(r)}(x)$

$H(A,V)$ is with compact resolvent. In the case where this condition is not satisfied, we study the essential spectrum using the representation theory of nilpotent Lie groups.

A. HULANICKI

<u>Semi-group generated by $-\sum_j^k (-1)^{n_j} x_j^{2n_j}$ on nilpotent Lie groups</u>

Let G be a nilpotent Lie group, $x_1, \ldots, x_k$ elements of the Lie algebra $\mathfrak{g}$ of G such that Lie $\{x_1, \ldots, x_k\} = \mathfrak{g}$. Let $L = \sum (-1)^{n_j} x_j^{2n_j}$. Then $-L$ is the generator of a one parameter semi-group of operators T_t on $C_o(G)$, $T_t f = f * p_t$.

The map $t \to p_t \in L^1(G)$ has a holomorphic extension to $\{z; \operatorname{Arg} z < \delta\}$, $z \to p_z \in L^1(G)$, and for every ∂ from the enveloping algebra of G we have

$$|\partial p_z(x)| \leq C_{\alpha,z} e^{-\alpha|x|}$$

for all $\alpha > 0$, where $|x|$ denotes the Riemann distance of x from e.

H.P. JAKOBSEN

Unitarizable highest weight representations of loop groups

Let $\mathfrak{g}$ be a simple complex Lie algebra and let $\hat{L}(\mathfrak{g}) = \mathbb{C}[z,z^{-1}] \otimes \mathfrak{g} \oplus \mathbb{C}c$ be the associated Kac-Moody algebra. We report here on joint work with V. Kac in which we determine the full set of unitarizable highest weight modules of $\hat{L}(\mathfrak{g})$. The answer is that one should add, to the list we gave in Springer Lecture Notes in Physics #226, the highest component of a $\otimes$-product of an elementary representation with an exceptional. Further work shows more generally how to decompose $\otimes$-products of these non-standard representations, and, using the uniqueness of highest weight modules, we integrate these to projective representations of loop groups. It is finally shown how one can replace $\mathbb{C}[z,z^{-1}]$ by either the algebraic part of the irrational rotation algebra or the set of finite rank operators, and still obtain unitarizable representations.

J. JENKINS

Bi-invariant Schwartz multipliers

Let N be a connected, simply connected nilpotent Lie group. Let $\mathcal{S}(N)$ denote the Schwartz space on N and $\mathcal{S}^*(N)$ the space of tempered distributions.

Theorem A. For $D \in \mathcal{S}^*(N)$, convolution by D is a continuous, bi-invariant endomorphism of $\mathcal{S}(N)$ iff $\hat{D}$ is an Ad^*-invariant, smooth function on $\mathfrak{n}^*$ with polynomial bounds on all its derivatives.

Theorem B. Let $D \in \mathcal{S}^*(N)$ that satisfied the conditions of Thm. A. For $\xi \in \mathfrak{n}^*$

$$\pi_\xi(D*f) = \hat{D}(\xi)\pi_\xi(f)$$

for each $f \in \mathcal{S}(N)$, where π_ξ is the irreducible unitary representation of N corresponding to the Ad^*-orbit of ξ.

P.E.T. JORGENSEN

Nilpotent groups and the spectrum of curved magnetic field Schrödinger operators

Let G be a nilpotent Lie group with Lie algebra $\mathfrak{g}$. Let $\Delta = \sum x_i^2$ be a sublaplacian, $x_i \in \mathfrak{g}$. Let $\{p(t,g); t \in \mathbb{R}_+, g \in G\}$ be the corresponding subgaussian. We discuss regularity properties of p_t, and apply the results to trace-formulas and spectral asymptotics of Schrödinger operators

$$H = (\vec{P} - \vec{A})^2 = dU(\Delta) ,$$

$\vec{P} = (P_j)$, $\vec{A} = (A_j)$ with $P_j = (-1)^{-\frac{1}{2}}\partial/\partial x_j$, $A_j(x)$ real polynomial. We show that the spectrum of H is continuous if the corresponding classical system has one or more conserved quantities, and discrete spectrum with e^{-tH} trace class otherwise. In the first case we derive spectral asymptotics from

$$\operatorname{trace}(e^{-tH(\xi)}) = \int_L dx\ \hat{p}_t(x,\xi-P_{L^\perp}\vec{A}(x),\vec{B}(x),DB(x)),$$

where L is a submanifold, $H = \int^{\oplus} H(\xi)\,d\xi$, and G , $\mathfrak{g}$ are obtained naturally from $\vec{A}$, and $\vec{B} = \operatorname{curl} \vec{A}$.

E. KANIUTH

<u>Topological Frobenius properties of locally compact groups</u>

A locally compact group G is said to have Fell's topological Frobenius property (FP) if for any closed subgroup H of G , $\tau \in \hat{H}$, and $\pi \in \hat{G}$, the induced representation $\operatorname{ind}_H^G \tau$ weakly contains π $(\operatorname{ind}_H^G\tau \succ \pi)$ if and only if τ is weakly contained in the restriction $\pi|H$ $(\tau \prec \pi|H)$. The if and the only if parts of (FP) are called (FP1) and (FP2), respectively. It had been proved by Felix, Henrichs, and Skudlarek that if G is an amenable group with open connected component satisfying (FP2), then G is an $[FC]^-$ group, i.e. the conjugacy classes in G are relatively compact. Conversely, we could show that every $[FC]^-$ group has property (FP). We present several results concerning property (FP1): (1) A (countable) discrete group G satisfies (FP1) iff G is amenable and has a T_1 primitive idealspace (2) Every 2-step nilpotent pro-Lie group has property (FP1); (3) Let G be a simply connected nilpotent Lie group. If all the Kirillov orbits are linear varieties, then (FP1) holds for G. The converse is open, but true if G is of the form $\mathbb{R} \ltimes \mathbb{R}^n$. (2) and (3) as well as some further results are joint work with M. Bekka.

G. KUHN

Random walks on locally infinite trees

We consider random walks (on a free group with infinitely many generators) for which the transition probability functions are finitely additive probabilities. We prove that all bounded sets are transient.

P. LEVY-BRUHL

Local solvability of linear PDE in connection with representation theory

Let G be a simply connected, nilpotent graded Lie group, and H a closed subgroup. Let P be an homogeneous left invariant operator on G, and denote by $\pi_{(0,\mathfrak{h})}$ the representation induced by the trivial character on H. Then we have the following conjecture:

Let Σ be the spectrum of $\pi_{(0,\mathfrak{h})}$. Then $\pi_{(0,\mathfrak{h})}(P)$ is locally solvable if
(i) $\pi(P^*)$ is injective in $\mathcal{S}(\pi)$ for all unitary irreducible representations π of G with $\pi \in \Sigma \cap \mathcal{E}$, where $\mathcal{E}$ is the set of all representations not in general position, or degenerated on $\exp \mathfrak{g}_r$ (r = rank of G),
(ii) There is no open subset $U \subset \Omega$ $(U \neq \emptyset)$ such that $\pi_\omega(P^*)$ is non injective for all $\omega \in U$, where $\Omega \subset \mathbb{R}^n$ parametrizes Σ.
We prove this conjecture in several cases, including nilpotent symmetric spaces, and Grušhin operators. The case $H = \{e\}$ has been analysed in my earlier papers.

R. LIPSMAN

Orbital parameters for induced representations

A general formula for the spectral decomposition of the quasi-regular representation is presented for connected Lie groups $H \subset G$. The formula - which describes the actual spectrum, the multiplicity, and the spectral measure - is in terms of the usual parameters in the so-called Orbit Method. The formula is proven in the nilpotent situation, and more generally when G is completely solvable. Indications are given for the general exponential solvable case. The situation of semisimple homogeneous spaces, especially symmetric spaces, is also discussed.

V. LOSERT

Groups of polynomial growth

We give a characterization of Lie groups of polynomial growth. Furthermore, it is shown that any compactly generated group G of polynomial growth contains a compact normal subgroup K such that G/K is a Lie group. This implies that such a compactly generated group of polynomial growth is built up of compact groups, a connected Lie group (of a certain type) and a discrete nilpotent group. In particular, $L^1(G)$ is symmetric.

J. LUDWIG

Dual topology of an exponential group

Let G be an exponential Lie group with Lie algebra $\mathfrak{g}$. If one tries to prove that the inverse of the Kirillov map K, which assigns to every G-orbit O in $\mathfrak{g}^*$ its representation $\pi \in \hat{G}$, one has to cope with the following situation. We are given a sequence $\pi_k \subset \hat{G}$, such that for every k there does not exist any *normal* connected subgroup H_k, with $\pi_k = \mathrm{ind}_{H_k}^G \tau_k$, for some $\tau_k \in \hat{H}_k$. Example: $\mathfrak{g} = \langle T,X,Y,Z \rangle, [T,X] = -X, [T,Y] = Y$; $[X,Y] = Z$; $\pi_k\ (\exp zZ) = e^{-i\lambda_k z}$, $\lambda_k \neq 0$. It is relatively easy to handle the case $\pi_\infty = \lim \pi_k$ and $d\pi_\infty(Y) \neq 0$, using variable group techniques. The case $d\pi_\infty(Y) = 0$ is more difficult. One must change the representations π_k into representations $\tilde{\pi}_k$, such that some control is conserved on $\tilde{\pi}_\infty = \lim_k \tilde{\pi}_k$, and such that $d\tilde{\pi}_\infty(Y) \neq 0$. This can again be done by using the method of variable groups. It is possible to extend these methods to general variable exponential groups and thus to prove that K is a homeomorphism.

G. MOCKENHAUPT

The restriction theorem for K/M

Let $\underline{\underline{g}}$ be a real simple Lie algebra, $\underline{\underline{g}} = \underline{\underline{p}} + \underline{\underline{k}}$ a Cartan decomposition and K a compact Lie group with Lie algebra $\underline{\underline{k}}$, which acts on $\underline{\underline{p}}$ by $\mathrm{Ad}_G k$, G the Lie group corresponding to $\underline{\underline{g}}$. Furthermore let $\underline{\underline{a}}$ be a maximal abelian subspace of $\underline{\underline{p}}$. Then we show for the Fourier transform on $\underline{\underline{p}}$ the following restriction Theorem: If $f \in L^q(\underline{\underline{p}})$, $1 \leq q < 2\,\frac{(N+\ell)}{(N+3\ell)}$, $\ell = \dim \underline{\underline{a}}$, $N = \dim \underline{\underline{p}}$, then for regular $X \in \underline{\underline{p}}$

$$\int_K |\hat{f}(\mathrm{Ad}k\, X)|^2 dk \leq C \| f \|_q^2$$

For $q > 2\,\frac{(N+\ell)}{(N+3\ell)}$ such an inequality fails.

D. MÜLLER

Asymptotics for some Green kernels on the Heisenberg group and the Martin boundary

This is a joint work with H. Hueber (Bielefeld):

Let Δ_K denote the sub- (or Kohn-) Laplacian on the 3-dimensional Heisenberg group H_1. In contrast to the operator Δ_K itself, for which Folland found an explicit fundamental solution, the corresponding heat operator is not so well-understood. Thanks to works of Gaveau and Cygan/Hulanicki, a formula for a fundamental solution of $\partial/\partial s - \Delta_K$ is known which is explicit only up to the partial Fourier transform along the center of the Heisenberg group, and it seems very unlikely that this Fourier transformation could be carried through explicitly. So, the best one might hope for is a description of the asymptotic behaviour of this fundamental solution. A partial solution to this problem had already been given by Gaveau, and we are able to complete it and give a complete description of these asymptotics.

Moreover, we can solve the analogous problem for the operator $\Delta_K - \mu$, $\mu \in]0,\infty[$, too. This enables us to determine the Martin boundary of H_1 . A consequence of these results is the following: If $h \geq 0$ is any solution of $(\Delta_K - \mu)h = 0$, then h factorizes to a function on H_1 modulo its center.

T. NOMURA

<u>Unitary representations of a solvable Lie group on $\bar{\partial}_b$ cohomology spaces</u>

Let $(\mathfrak{g}, j, \omega)$ be a normal j-algebra and G the connected and simply connected (completely solvable) Lie group corresponding to $\mathfrak{g}$. We give a unitary representation of G in which every irreducible (up to a set of Plancherel measure zero) occurs with multiplicity one, relating its construction to a certain geometric structure (CR structure) of a nilpotent subgroup $N(D)$ of G. This subgroup $N(D)$ is canonically diffeomorphic to the Šilov boundary $S(D)$ of a Siegel domain D of type II on which G acts simply transitively by affine automorphisms. We will define unitary representations of G on $\bar{\partial}_b$ cohomology spaces on $S(D) \approx N(D)$. Note that there is no G-invariant Riemannian metric on $S(D)$.

E.M. OPDAM

Hypergeometric functions associated with root systems

Let $R \subset \mathfrak{a}^*$ be a rank n root system, $\mathfrak{h} = \mathfrak{a} \oplus i\mathfrak{a}$ and $H = \exp \mathfrak{h}$ the complex torus with characterlattice P (the weightlattice of R). For a choice of a multiplicity function $k = (k_\alpha)_{\alpha \in R}$ we have the differential operator

$$L(k) = \sum_{j=1}^{n} \partial(X_j)^2 + \sum_{\alpha \in R_+} k_\alpha \operatorname{cth}(\tfrac{\alpha}{2}) \partial(X_\alpha)$$

on H. We define the family of hypergeometric functions associated with R as eigenfunctions of $L(k)$ which have some prescribed monodromy type (viewed as Nilsson-class functions on $W \backslash H^{reg}$) and are analytic in a neighbourhood of $e \in W \backslash H$. It turns out to be an analytic family of functions. From that we can conclude that the commutant of $L(k)$ in $(\mathcal{S} \otimes \mathfrak{U}(\mathfrak{h}))^W$ ($\mathcal{S}$ is the algebra of functions on H generated by h^λ, $\lambda \in P$ and $1/1 - h^\alpha$, $\alpha \in R_+$) is isomorphic to a polynomial algebra in n variables (joint work with G. Heckman).

N.V. PEDERSEN

On the symplectic structure of coadjoint orbits of (solvable) Lie groups

Let G be a connected Lie group with Lie algebra $\mathfrak{g}$, and let $O \subset \mathfrak{g}^*$ be a coadjoint orbit of G. Suppose that $\mathfrak{h}$ is a *real* polarization at $g \in \mathfrak{g}^*$. Set H_o to be the analytic subgroup corresponding to $\mathfrak{h}$, and set $H = G_g H_o$. Suppose further that g is *integral*, i.e. there exists a character $\chi : H \to T$ such that $\chi(\exp X) = e^{i<g,X>}$, $X \in \mathfrak{h}$. It follows from Kostants theory of geometric quantization that one can define a Lie algebra homomorphism

$$\delta_\chi : \mathcal{E}^1(O,g,\mathfrak{h}) \to \mathcal{B}^1(G,\chi)$$

from the space of quantizable functions on O defined by $\mathfrak{h}$ to the first-order differential operators $\mathcal{B}^1(G,\chi)$ in a certain line bundle associated with χ.

Theorem: $\delta_\chi : \mathcal{E}^1(O,g,\mathfrak{h}) \to \mathcal{B}^1(G,\chi)$ is a Lie algebra *isomorphism*. As an application we show that there exists on each coadjoint orbit of an exponential group coordinates $(p_1,\dots,p_{d/2},q_1,\dots,q_{d/2})$ such that $\{p_r,p_s\} = 0, \{q_r,q_s\} = 0$, $\{p_r,q_s\} = \delta_{rs}$ (canonical coordinates).

R. PENNEY

The Laplace Beltrami operator on unbounded homogeneous domains

Let $\Omega \subset \mathbb{C}^n$ be a domain. Let G be a Lie group which acts homogeneously on Ω in such a manner that the action is analytic in G and holomorphic in Ω. We also assume that Ω has a G-invariant volume and that the corresponding Koszul form is non-degenerate. In this case Ω is a pseudo-Kählerian domain. Let $\square$ be the Laplace Beltrami operator for Ω. Under additional assumptions on Ω and its boundary, we are able to give an explicit description of the spectrum of $\square$ as an unbounded operator on $L^2(\Omega)$. This then yields criterion for determining whether two domains are biholomorphic.

T. PYTLIK

Norms of free convolution operators

A short proof of a theorem by Akemann and Ostrand is given with some applications i.e. short proof that (result of Powers) the reduced C^*– algebra of the free group is simple. The Akemann and Ostrand theorem gives an explicit formula for the norm of the convolution operator $\lambda(f)$ for a function f on G whose support satisfies the following freedom condition

(*) for any $x_1, x_2, \ldots, x_{2n} \in \operatorname{supp} f$, $x_i \neq x_{i+1}$ we have $x_1 x_2^{-1} x_3 \ldots x_{2n-1} x_{2n}^{-1} \neq e$.

The formula for the norm is

$$\| \lambda(f) \|_{C^*_\lambda(G)} = \inf_{s \geq 0} \left(2s + \sum_x \left(\sqrt{s^2 + |f(x)|^2} - s\right)\right).$$

R. SCARAMUZZI

Asymptotics of matrix coefficients and orbit equivalence of actions

If G_1, G_2 are groups acting on measure spaces S_1 and S_2 respectively, leaving a measure quasi-invariant, an orbit equivalence between the two actions is a measure-class preserving Borel isomorphism between S_1 and S_2 that takes orbits to orbits.

Let G be a group of the form $Gl_n(F)$, $Sl_n(F)$ or $Sp_{n/2}(F)$ $(F = \mathbb{R}$ or $\mathbb{C})$, and let V_i $(i = 1,2)$ be finite-dimensional irreducible modules for G defined over F.

An integral invariant $r(V_i)$, $0 \le r(V_i) \le n/2$ of the module V_i is defined. The number $r(V_i)$ can be easily computed from the highest weight of V_i.
We prove the following
Theorem. Let G and V_i be as above, and suppose $r(V_1) < r(V_2)$, $r(V_1) \le \frac{n-2}{3}$ $\left(\frac{n}{3} - 2 \text{ if } G = Sp_{n/2}\right)$. Suppose $G \ltimes V_i$ acts essentially freely and properly ergodically on S_i, with finite invariant measure. Assume V_2 acts ergodically. Then the actions on S_1 and S_2 are not orbit equivalent. (Joint work with R. Zimmer.)

G. SCHLICHTING

On the periodicity of group operations

Given a topological space X, an abstract group Γ and a homomorphism $\lambda : \Gamma \to H(X)$ into the group of all homeomorphisms of X, we consider the following periodicity properties which the action of Γ on X may have or not

$P_1(\Gamma,X)$: $O_\Gamma(x) < \infty \forall x \in X$ (for short $\lambda(\gamma)x = \gamma x$)
$P_2(\Gamma,X)$: O_Γ bounded , $P_3(\Gamma,X)$: $\lambda(\Gamma)$ is finite.
The following results are given

(1) $P_2(\Gamma,X) \iff \exists \Gamma \leq \tilde{\Gamma} \leq G$ s.t. $[\tilde{\Gamma}:\Gamma] < \infty$ and $P_3(\Gamma,G/\tilde{\Gamma})$
$\iff \exists$ normal $N \trianglelefteq G$ s.t. $[\Gamma:\Gamma\cap N] < \infty$, $[N:\Gamma\cap N] < \infty$

(2) Given a group A, $\Gamma \leq \mathrm{Aut}\, A$: $P_2(\Gamma,A)$
$\iff \exists$ finite normal subgroup $E \trianglelefteq A$ s.t. $P_3(\Gamma,A/E)$

(3) $\mathfrak{A}$ Boolean algebra, $\Gamma \leq \mathrm{Aut}\,\mathfrak{A}$: $P_2(\Gamma,\mathfrak{A}) \iff \Gamma$ finite

(4) A Frechet space, $\Gamma \leq \mathrm{Gl}(A)$: $P_1(\Gamma,A) \iff \Gamma$ finite

(5) G l.c.gr., $G \geq \Gamma$ closed, $X = G/\Gamma$ compact or connected then $P_1(\Gamma,X) \iff P_2(\Gamma,X)$

(6) G compact or connected: $P_1(\Gamma,G/\Gamma) \iff P_3(\Gamma,G/\Gamma)$

(7) A compact group, $\Gamma \leq \mathrm{Aut}\, A$: $P_1(\Gamma,A) + P_1(\Gamma,\hat{A}) \iff \Gamma$ finite

(2) generalizes a result of R. Baer where A is supposed to be abelian.

H. SCHLICHTKRULL

Eigenfunctions on Riemannian symmetric spaces

Let G/K be a Riemannian symmetric space of the non-compact type with boundary $B = K/M$. Let $f \in C^\infty(G/K)$ be a joint eigenfunction of the invariant differential operators on G/K. If f has at most exponential growth, f has an asymptotic expansion, whose coefficients are distributions on B. It is shown that knowledge of these distributions on an open subset of B determines f uniquely. This result was obtained in collaboration with Professor E.P. v.d.Ban of Utrecht.

J. SOTO-ANDRADE

Some examples of generalized Weil representations

The classical construction of Weil representations for the group $Sl_2(k)$ ($k = \mathbb{R}$ or more generally a local or finite field) via the Heisenberg group may be extended to the groups $Sl_n(k)$ (n even) by introducing a generalized Heisenberg group $H(V)$ associated to any vector space V over k. One defined $H(V)$ as the subgroup $1 + \bigoplus_{i\geq 1} \overset{i}{\Lambda} V$ of the multiplicative group $(\Lambda V)^x$ of the Grassmann algebra ΛV of V. For even dimensional V the analogue of Stone - von Neumann theorem holds and since the natural action of $Sl(V)$ fixes the isomorphy type of the corresponding "Schrödinger representation" of $H(V)$, one obtains a projective representation of $Sl(V) \simeq Sl_n(k)$, of Gelfand-Kirillov dimension 2^{n-2}, which reduces to the usual Weil representation for $n = 2$. This construction admits a geometric version which may be adapted to the case of $Sl_n(k)$ for odd n.

T. STEGER

Principal Series of Fuchsian Groups

If T is a homogeneous, infinite tree, if $G = Aut(T)$ and if $\Gamma \subseteq G$ is discrete and cocompact, then the principal series of G restricts to *irreducible* representations of Γ. This is also true if $G = PSL(2,\mathbb{R})$ and the representations are in the complementary series. The next question is: does a Fuchsian group have a principal series? I.e., what happens if $G = PSL(2,\mathbb{R})$ and the representations are in the principal series?

R. SZWARC

<u>Completely bounded multipliers of the Fourier algebra of the free group</u>

Let G be a locally compact group. A function φ on G is called a multiplier of $A(G)$ if $\varphi\psi$ is in $A(G)$ for any ψ in $A(G)$. φ is called a completely bounded multiplier if the transposed operator M_φ to the mapping $A(G) \ni \psi \longmapsto \psi\varphi \in A(G)$ is completely bounded.

Let φ be a radial function on the free group $\mathbb{F}_N$ i.e. $\varphi = \Sigma\dot{\varphi}(n)\chi_n$ where $\dot{\varphi}(n) \in \mathbb{C}$ and χ_n is the characteristic function of words of length n. Then φ is a completely bounded multiplier of $A(G)$ iff the hankel matrix h with entries $h_{ij} = \dot{\varphi}(i+j) - \dot{\varphi}(i+j+2)$ $i,j = 0,1,2,\ldots$ is of trace class. The formula for the completely bounded norm is given explicitly. As a corollary we get that every radial completely bounded multiplier is represented by the integral $\int_{\{|z|<1\}} z^{|x|}d\mu(z)$ where μ is a complex measure which integrates $\frac{|1-z^2|}{1-|z|^2}$.

(This is a joint work with Uffe Haagerup.)

N. WILDBERGER

<u>Moment maps, dequantization and constellations</u>

We define the moment map of a representation (V,ρ) of a Lie group G, this is a G-map from the projective space PV to the dual $\mathfrak{g}^*$ of the Lie algebra. We apply it to the compact semisimple group G to show how to "geometrically dequantize" a representation, that is how to establish a coadjoint orbit $\mathcal{O}$, line bundle $L \to \mathcal{O}$, connection ∇, polarization P which recovers the usual ingredients of the geometric quantization program of Kostant und Souriau.

The notion of constellation as used by Bacry in S^2 is shown to occur naturally as the zeros of certain functions defined by the moment on the coadjoint orbits of $SU(2)$.

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