MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 30/1987

Mathematische Probleme in Strömungen und Plasmen

12.7. bis 18.7.1987

Die Tagung stand unter der Leitung von Herrn Kirchgässner (Stuttgart) und Herrn Marsden (Berkeley). Es nahmen teil 46 Mathematiker und Physiker, davon 15 deutsche. Die wichtigsten Themen waren:

1. Hamiltonsche Beschreibung von Strömungen, insbesondere Wirbeltransport, nichtlineare Stabilität.
2. Dynamische Systeme und Turbulenz. Hier besonders: homokline Verzweigung durch exponentiell kleine Durchdringung von stabiler und instabiler Mannigfaltigkeit, nichtlineare Wellen, Stabilität und resonante äußere Kräfte.
3. Transonische Strömungen, eine neue variationelle Formulierung und Numerik hierzu.
4. Bifurkation und Symmetriebrechung, besonders am Beispiel der Couette-Taylor Instabilität. Im Mittelpunkt standen die Hopf-Bifurkationen bei vorhandener O(3)-Symmetrie. Ferner wurden behandelt neue Methoden zur Bestimmung von Normalformen nichtlinearer Vektorfelder.

Folgende Personen hielten Hauptvorträge (in zeitlicher Reihenfolge):

Busse (Bayreuth), Golubitsky (Houston), Bemelmans (Saarbrücken), Amick (Chicago), Pulvirenti (l'Aquila), Marsden (Berkeley), Newell (Tucson), Batt (München), Tasso (Garching), Marchioro (Rom), Nečas (Prag), Feistauer (Prag).

Seminare über Spezialthemen wurden veranstaltet von:

Iooss (Nizza), Stewart (Warwick): Symmetriebrechung und Bifurkation

Turner (Madison), Wan (Buffalo): Nichtlineare Wellen

Heywood (Vancouver), v. Wahl (Bayreuth): Navier-Stokes

Nicolaenko (Los Alamos), Scheurle (Fort Collins): Dynamische Systeme und Turbulenz

Binz (Mannheim), Ratiu (Berkeley): Hamiltonsche Strukturen

Cabannes (Paris VI), Cercignani (Mailand): Boltzmann-Gleichungen

Morrison (Houston), Horst (München): Plasma.

Die Spezialseminare wurden abends abgehalten, zum Teil in Parallelsitzungen . Alle Nicht-Hauptvorträge waren diesen Seminaren zugeordnet.

Vortragsauszüge

C.J. AMICK: Solitary water-waves in the presence of surface tension

Consider a wave moving from right to left without change of form on the surface of a two-dimensional, incompressible inviscid fluid. The density is constant, and the flow is irrotational. After a hodograph transformation, the problem becomes one of finding an analytic function $\tau + i\theta$ on the strip $I\!R \times (0,1)$ such that $\theta(x,0) = 0$, $x \in I\!R$, $\theta, \tau \to 0$ at infinity and

$$\nu\theta_y e^{2\tau} - e^{-\tau}\sin\theta + \gamma\theta_{xx}e^{\tau} + \gamma\theta_x\theta_y e^{\tau} = 0 \quad \text{on} \quad \{y=1\} \quad , \quad x \in I\!R,$$

where $\gamma = T/gh^2 \geq 0$ and $\nu = c^2/gh$. Here T is the surface tension constant, g is the gravitational constant, h is the asymptotic depth in the physical domain, and c is the speed of the wave. A reduction in the spirit of the centre-manifold theorem is performed, and all small solutions in a neighborhood of $\nu = 1$ are determined by a system of three nonlinear ordinary differential equations when $\gamma > \frac{1}{3}$ and five when $\gamma \in (0, \frac{1}{3}]$. A global branch of nontrivial solutions is shown to exist when $\gamma > \frac{1}{3}$, and only limited results found for $\gamma \in (0, \frac{1}{3}]$.
(joint with K. Kirchgässner)

C.J. AMICK: On steady Navier-Stokes flow past a body in the plane

A classical problem of Leray is considered: given an exterior domain $\Omega \subset I\!R^2$, $\nu > 0$, constant vector $\tilde{w}_\infty \in I\!R^2 - \{0\}$, find $w : \bar{\Omega} \to I\!R^2$, $p : \bar{\Omega} \to I\!R$ satisfying

$$-\nu\Delta w + (w \cdot \nabla)w = -\nabla p \quad , \quad \nabla \cdot w = 0 \quad \text{in } \Omega,$$

$$w = 0 \quad \text{on } \partial\Omega \quad , \quad w \to \tilde{w}_\infty \quad \text{at infinity}$$

Leray sought an approximate solution (w_R, p_R) in an annular domain $\Omega_R = \Omega \cap \{|z| < R\}$ by solving the first three equations with the final replaced by $w_R = \tilde{w}_\infty$ on $\{|z| = R\}$. An a priori bound on $\int_{\Omega_R} |\nabla w_R|^2$ was found, and this yields a solution $(\bar{w}, \bar{p})$ of the first three equations, but one cannot be sure about the boundary

condition at infinity. We extend the methods of Gilbarg & Weinberger and show that $(\tilde{w}, \tilde{p})$ is non-trivial, and has a pointwise limit w_∞ at infinity. However, it is still open whether $w_\infty = \tilde{w}_\infty$.

H. BABOSKY: A Convergent Simulation Scheme for the Boltzmann Equation

Monte Carlo schemes for the Boltzmann equation are usually derived by heuristic and intuitive arguments, and it is in most cases very hard to justify them rigorously. For Nantu's simulation scheme we are able to prove that the solutions of the simulation scheme converge almost sure to solutions of the Boltzmann equation if the number of simulated particles goes to infinity. The insights gained from the proof allow to construct a class of convergent simulation schemes - even completely deterministic schemes are possible. It turns out that the main problem to overcome is the following: Given an N-point distribution $\frac{1}{N}\sum \delta_{v_i}$ approximating a probability density $f_0(v)$, find a mapping $A(i) : \{1, \ldots, N\} \to \{1, \ldots, N\}$ such that $\frac{1}{N}\sum \delta_{v_i} \otimes \delta_{v_{A(i)}}$ approximates $f_0(v) \cdot f_0(w)$.

J. BATT: The present state of investigation of the VLASOV-POISSON system and the VLASOV-MAXWELL system in stellar dynamics and plasma physics

The goal of the talk was to give a survey over the mathematical investigation of the VLASOV-POISSON system

$$\frac{\partial f}{\partial t} + v\nabla_x f - \nabla_x U(t,x)\nabla_v f = 0$$

$$\Delta U(t,x) = 4\pi\rho(t,x) \quad \text{or} \quad U(t,x) = -\int \frac{\rho(t,y)}{|x-y|}dy$$

$$\rho(t,x) = \int f(t,x,v)dv,$$

of the VLASOV-MAXWELL system

$$\frac{\partial f}{\partial t} + v\nabla_x f + e(E(t,x) + \frac{1}{2}v \wedge B(t,x))\nabla_v f = 0$$

$$\begin{aligned}
\frac{1}{c}\frac{\partial E}{\partial t} &= \text{curl}B - \frac{4\pi}{c}j \quad , \quad j(t,x) = e\int vf(t,x,v)dv \\
\frac{1}{c}\frac{\partial B}{\partial t} &= -\text{curl}E \\
\text{div}E &= 4\pi u \quad [+4\pi u_i] \quad , \quad u(t,x) = e\int f(t,x,v)dv \\
\text{div}B &= 0
\end{aligned}$$

and of related equations, such as the relativistic systems and the VLASOV-FOKKER-PLANCK-POISSON system. Main topics of the treatment of the VLASOV-POISSON system were the following ones:

1. Global existence (in $t \geq 0$) of the initial value problem.
2. Construction of steady-state solutions.
3. Results for $t \to \infty$.
4. Stability questions.

J. BEMELMANS: **Free Boundary Problems for the Navier-Stokes Equations**

We consider the free boundary problem

$$\begin{aligned} -\nu\Delta v + Dp + (v \cdot D)v &= f \quad \text{in } \Omega \\ \operatorname{div} v &= 0 \quad \text{in } \Omega \\ v \cdot n = 0 \quad , \quad t_k \cdot T(v,p) \cdot n &= 0 \quad \text{on } \Sigma \quad , \quad k = 1,2 \\ n \cdot T(v,p) \cdot n &= 0 \quad \text{on } \Sigma. \end{aligned}$$

Under the assumption that f is of the form $f_0 + h$, where f_0 is the force of self-attraction, and $\|h\| \ll \|f_0\|$, the equation that governs the free boundary becomes the integral equation which has been studied in the theory of equilibrium figures of rotating liquids. The existence proof is based on Zehnder's version of the Moser/Nash implicit function theorem.

E. BINZ: **Natural Hamiltonian Systems on a Space of Embeddings**

Let $E(M, R^n)$ be the space of all smooth embeddings of a compact orientated smooth manifold M into a Euclidian space $(R^n, <,>)$. This Fréchet manifold is a principal bundle with Diff M as its structure group. The geodesics of the metric G given by

$$G(j)(h,k) = \int \rho(j) < h, k > \mu(j)$$

for all $j \in E(M, R^n)$ and all $h, k \in C^\infty(M, R)$ with $\rho(j)$ satisfying

$$D\rho(j)(h) = -\frac{\rho(j)}{2} tr_{m(j)} Div(j)(h) \qquad \forall h \in C^\infty(M, R^n)$$

and $m(j) = j^* <,>$ are lines. The geodesic equation on $j \circ \mathrm{Diff}_\mu M$ ($\mathrm{Diff}_\mu M$, the Fréchet manifold of all smooth $\mu(j)$ preserving diffeom.) is Euler's equation. Given next a potential $w : E(M, R^n) \to R$ with $w(j) = \int W(j)\mu(j)$ such that $W(j) = W_M m(j)$ where W_M is a smooth $C^\infty(M, R)$-valued map defined on $\mathcal{M}(M)$, the Fréchet manifold of all smooth Riemannian metrics on M, the Hamiltonian system on $j \circ \mathrm{Diff}_\mu M$ reads as:

$$\rho(\dot{X}(t) + \nabla(j)_{X(t)} X(t) + R(j)X(t) - \Delta(j)X(t) + \mathrm{grad}_{m(j)} I(j)) = F(X(t))$$

Here $\nabla(j)$ is the Levi-Cività connection of $m(j)$, $R(j)$ is the Ricci tensor of $m(j)$ represented as an operator by $m(j)$ and $\Delta(j)$ is the Laplacian given by $m(j)$. $X(t) \in \Gamma TR$ with $\mathrm{div}_{\mu(j)} X(t) = 0$. F is well defined.

F.H. BUSSE: **The Optimum Theory of Turbulence**

The main goals and methods of the optimum theory of turbulence are reviewed and a new application is outlined. Through the derivation of bounds on average properties of turbulent fluid flows rigorous results can be obtained in contrast to other theories of turbulence which must rely on additional assumptions. The extremalizing vector fields derived as solutions of the variational problems not only provide the upper bounds, but exhibit some interesting properties which can be compared with observed turbulent fluid flow. The newly treated case of an internally heated sphere is described as an application of the theory to a problem with a finite domain. The results of the Optimum Theory could be improved by the imposition of additional integralrelationships (moments) of the basic Navier-Stokes- Equations. Using additional constraints one could also introduce time-dependence into the formulation of the variational problems.

H. CABANNES: **Exact Solutions for some discrete models of the Boltzmann equation**

For the simplest model of the discrete Boltzmann equation the Broadwell model, exact solutions have been obtaind by Cornille, inspired by solutions obtained by Wick for the Carleman equations. We have obtained and presented exact solutions for more complex models. Those solutions exist globally even if the initial densities are not all positive, conversely of the cases considered before.

C. CERCIGNANI: **Existence of a steady solution for a nonlinear boundary value problem in kinetic theory**

The result proved in my talk was obtained in collaboration with R. Illner and M. Shinbrot (University of Victoria, Canada) and reads as follows: there is at least one solution of a steady kinetic model depending on just one space variable, x, and satisfying suitable boundary conditions at the endpoints of an interval. The only restrictions on the nonlinear kinetic models are that they satisfy the conditions of conservation of mass and momentum in a collision and zero velocity components along the x-axis are not allowed in the set of discrete velocities. The boundary conditions are of two types, mimicrying two situations: solid non porous plates and evaporating-condensing surfaces. The technique used is of the Leray-Schauder type. No uniqueness result is given except for short intervals.

F. CHEN: **A Model Study of Stability of Couette Flow**

A Galerkin method is used to derive a model system of evolutionary equations for axisymmetric Couette flows. The model equations are structurally similar to Lorenz equations for the case when the gap between the rotating cylinder is narrow. The model equations have an additional nonlinear term when the gap is not narrow. We are now doing some work on nonaxisymmetric Couette flow. Japanese Physician Yahata did the work. But we get the initial result different from his result. The paper of our work will be appeared.

P. CHOSSAT: **The instability of axisymmetric solutions in bifurcation problems with spherical symmetry**

Consider an equation

$$\frac{dx}{dt} = F(\mu, x) \quad , \quad x \in V(\text{real vector space}) \tag{1}$$

where $F(\mu, \gamma x) = \gamma F(\mu, x), \forall \gamma \in \Gamma^{(l)}$ irreducible natural representation of $O(3)$ in V (hence $\dim V = 2l+1$). It follows from irreducibiliy that $F(\mu, 0) = 0$ and $D_x F(\mu, 0) = c(\mu) Id_V$. Assume $\mu = 0$ is a stationnary bifurcation point: $c(0) = 0, c'(0) > 0$. Group theoretic methods permit to find bifurcated solutions of eq.(1) in subspace $V^\Sigma = \{x \in V | \sigma x = x, \forall \sigma \in \Sigma\}$, where Σ is a maximal isotropy subgroup of $O(3)$ (Ihrig - Golubitsky 1984). Among all possible Σ's, one always exists, which contains $SO(2)$, hence consisting of axisymmetric elements. When l is even, Ihrig and Golubitsky have shown that the bifurcated solutions are (generically) unstable. In their proof, a crucial hypothesis is $D^2F(0,0) \not\equiv 0$. For odd l however, there are no quadratic terms in the Taylor expansion of $F(\mu, .)$. When $l = 1$ it is well-known that the only bifurcated solutions are axisymmetric and the principle of exchange of stability holds. When $l \geq 3$, it can be shown that the linearized vectorfield at an axisymmetric solution is diagonal in the basis of V associated to spherical harmonics $Y_m^l (-l \leq m \leq l)$. Call σ_m the corresponding eigenvalues. By using the Lie algebra of $\Gamma^{(l)}$, we have computed the relevant cubic terms in the equivariant vector field F, which determine the principal part of σ_2 and σ_3. As a result, we get $\frac{\sigma_2}{\sigma_3} \sim \frac{9}{6-l(l+1)}$, which proves that one eigenvalue at least must be positive when $l \geq 3$.
(joint with R. Lauterbach)

W. ECKHAUS: **Non-classical bifurcations: Strong pattern selection or rejection**

We (joint work with G. Iooss) study bifurcations of periodic solutions in the case that coefficients of non-linear terms in the amplitude equations go through zero at near critical conditions. We give a full description of bifurcating solutions, derive the corresponding modulation equation and analyze the stability of the periodic solutions. The general result is that only solutions in a small neighbourhood of a curve in the R-k plane are stable (R a control parameter such as Reynolds number, k wave number of the spacial periodicity). Moreover this curve may stop at a value of R which is slightly higher than the critical value for linear instability.

M. FEISTAUER: **Mathematical and numerical study of transonic potential flows**

We study the solvability of transonic flow problems. The velocity potential equation of irrotational non-viscous transonic flow is nonlinear, second order and of mixed type. The velocity is discontinuous across the shocks. Numerical finite element experiments show that it is necessary to consider entropy condition across the shocks in order to get a physically admissible solution. We prove that some types of the entropy condition (e.g. the condition $\Delta u \leq k$ proposed by Glowinski et al.) have the compactification properties and allow to get some solvability results for potential transonic flow problems.

K. VAN GASTEL: **Surface marks of the sea bed**

Short waves on the sea surface can be modulated drastically by slowly varying currents. Under some circumstances a change in current of about 20 % can generate a change in the energy level of the waves of a factor 1000. This phenomenon has become generally known due to radar images of the sea surface: sometimes the sea bed could be seen on them, due to a modulation chain from sea bed via current via short surface waves to radar image. In recent years many attempts have been made to model this strong amplification using an energy balance for the short waves containing effects of refraction by current gradients, wind input and dissipation. None of these attempts have succeeded. It is shown here that by including the effects of nonlinear interactions between the short surface waves the large amplification can be found for the very short waves, i.e. for $k \geq 260m^{-1}$. In order to be able to include this nonlinear term in a numerical integration scheme, problems concerning, among others, a dependence of the results on grid side have to be solved. This is done by introducing a new concept, the multiwave space. This space gives a viewpoint at interactions from which it is easy to see the paths of the energy through the spectrum and to estimate the magnitude of the transfer.

R. GATIGNOL: The hydrodynamical description for a discrete velocity model of Gas

For a model of gas composed of identical particles with velocities restricted to a given finite set of vectors, the Boltzmann equation is replaced by a system of nonlinear coupled differential equations. The Chapman Enskog method can be applied and it gives the Navier-Stokes equations associated to the model. For the general model we show that the dissipative terms in the Navier-Stokes equations do not depend on the mean number density nor on its gradient. For a gas near a homogeneous state we give the transport coefficients that really correspond to dissipative phenomena. For some regular models we obtain Navier-Stokes equations similar to the classical Navier-Stokes equations.

M. GOLUBITSKY: Bifurcation and Symmetry in Taylor-Couette Systems

The nonlinear interaction of the time-independent Taylor Vortex mode with the time-dependent Spiral Vortex state produces a number of secondary states including Wavy Vortices, Twisted Vortices and Wavy Spirals. We use a combination of group theory and numerical computation via Liapunov-Schmidt reduction of the Navier-Stokes equation with periodic boundary conditions to make explicit predictions concerning this system. Our results are compared with experiments of Swinney and Tagg. This is joint work with Ian Stewart and Bill Langford.

J. HEYWOOD: On Various Definitions of Stability for the Navier-Stokes Equations

I report on stability results obtained jointly with Rolf Rannacher which were motivated by our investigation of long-time error estimates for numerical solutions. We showed that certain definitions of stability, exponential stability, and quasi-exponential stability, formulated to avoid reference to the initial behaviour of perturbations, are independent of the topologies in which they are expressed. The point is that the initial singular behaviour of perturbations, in strong norms, which occur whenever initial data fails to satisfy high order compatibility conditions, should not be confused with a lack of stability. Our proofs are based on continuous dependence estimates we have used previously in proving regularity theorems. The same methods yield simple proofs for the equivalence of linear and non-linear exponential stability. We proved the same for quasi-exponential stability, which is a definition of orbital stability (meant to apply to wavy Taylor-cells and von Karman vortex shedding) which includes a bound for phase shifts proportional to the initial value of a perturbation.

E. HORST: The Lagrange-Jacobi Identity for the Vlasov-Maxwell System

The relativistic Vlasov-Maxwell system is the following

$$\partial_t f + \frac{u}{\sqrt{1+u^2}}\partial_x f + \left(E + \frac{u}{\sqrt{1+u^2}} \wedge B\right)\partial_u f = 0 \quad \text{(Vlasov Eq.)}$$

$$\left.\begin{array}{ll} \partial_t E = \mathrm{curl}B - j \quad , & \mathrm{div}E = \rho \\ \partial_t B = -\mathrm{curl}E \quad , & \mathrm{div}B = 0 \end{array}\right\} \text{(Maxwell Eq.)}$$

where

$$\rho = \int f du \quad , \quad j = \int \frac{u}{\sqrt{1+u^2}} f du$$

The solutions satisfy the following identity, which has an analogue for the N-body problem used by Lagrange and Jacobi:

$$\frac{d^2}{dt^2}\left(\int x^2\sqrt{1+u^2}f dx du + \frac{1}{2}\int x^2(E^2+B^2)dx\right)$$

$$= 2\int \frac{u^2}{\sqrt{1+u^2}} f dx du + \int (E^2+B^2)dx$$

Similar equations are valid for similar systems of equations. They can be used to derive many qualitative results for these systems.

G. IOOSS: Hopf bifurcation with O(3) symmetry - Analytical results

We consider an $O(3)$ invariant system with a Hopf bifurcation. Let us denote by V the eigenspace belonging to the critical eigenvalue $i\omega$, and assume that the representation of $O(3)$ on V is absolutely irreducible, the dimension of V being $2l+1=5$. The first problem is to derive the form of the vector field $F(\mu, x)$ on the 10 dimensional center manifold, where $x \in V \oplus \overline{V}$, and F commutes with the action of $O(3)$ and with the group $\exp(L_o t)$ where L_o is the linear operator on $V \oplus \overline{V}$ with eigenvalues $\pm i\omega$. The general form of F can be obtained, using Lie groups techniques: F is odd and there are only 3 terms at cubic order, 9 terms at order 5, 23 terms at order 7, ... It is then possible to derive explicitely the 5 periodic bifurcating solutions, whose symmetries were predicted by Golubitsky and Stewart (1986): 1) axisymmetric solution, 2) rotating waves of two different types (one rotates twice faster than the other), 3) standing wave with a D_4 symmetry, 4) tetraedral wave. We study the domain of stability for all these solutions, in function of the 3 coefficients occuring in cubic terms of F, and of a combination of 3 coefficients occuring at order 5.One of the rotating waves (the fastest and less symmetric) is always unstable, and several solutions may be stable at the same time.
(joint with M. Rossi)

G. IOOSS: **Simple global characterization of normal forms**

We consider a vector field $\mathcal{F}(z)$ in a vector space E (finite dim. for example) and assume that $\mathcal{F}(0) = 0$ and $D_z\mathcal{F}(0) = \mathcal{L}$ has eigenvalues on the imaginary axis, the others being on the left side of the complex plane. Denoting by L_0 the restriction of $\mathcal{L}$ to the subspace E_0 belonging to the critical eigenvalues, we show that there is a normal form of the vector field on the center manifold such that it commutes with the group generated by the adjoint L_0^* in E_0. This characterization is extended

1. to systems depending on parameters,
2. to Hamiltonian systems,
3. to vector fields near closed orbits,
4. to mappings.

This characterization is stable under the symmetries of the system.

B.L. KEYFITZ: **Application of Bifurcation and Group Theory to Stability of Laminar Flames**

We (Chossàt, Golubitsky, Gorman, Keyfitz) interpret some experimental studies of a circular porous plug burner flame as a mode interaction between two Hopf bifurcations, one symmetry-preserving (simple eigenvalue) and one symmetry-breaking (double eigenvalue). On the six-dimensional eigenspace, the Birkhoff normal form equations can be formulated and solved, and conditions for stability of primary and secondary bifurcation and the existence of a tertiary Hopf bifurcation given. Furthermore, linear conditions for this mode interaction to occur are found in a classical model for the flame. For this, we make standard assumptions of a single exothermic reaction in the flame, constant density in the flow and large activation energy in the reaction kinetics, and we can show that a simple and a double pure imaginary eigenvalue occur at the same value of the bifurcation parameter. (In this case, the Lewis number is the bifurcation parameter, so that this is an example of a "double-diffusive" instability.) There is even the possibility in the equations of a three-mode interaction, for which there is some evidence in the experiments.

K. KIRCHGÄSSNER: **Resonantly forced nonlinear surface waves**

Consider a pressure wave on the surface of an inviscid fluid subjected to gravity and surface tension. The nonlinear interaction with resonant solitary waves is studied. Two cases are studied: the local case, when the external pressure p has compact support and the global case, when p is periodic. All steady solutions of the full Euler equations having moderate amplitudes are determined. In the global case it is shown that, if the period is sufficiently large, transverse homoclinic points exist. Therefore, the spatial structure of the response is chaotic.

C. MARCHIORO: An Example of Absence of Turbulence for any Reynolds Number

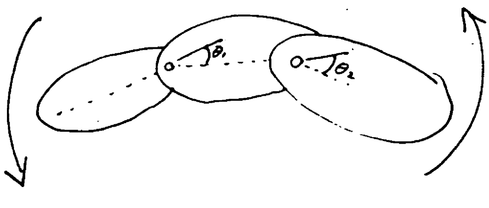
We consider a viscuous incompressible fluid moving in a two dimensional flat torus. We show a set of external forces for which the stationary state is attractive for any Reynolds number.

C. MARCHIORO: Euler Evolution for Singular Initial Data, Vortex Theory and Zero Viscosity Limit

We study the evolution of a two dimensional, incompressible, ideal fluid in a case in which the vorticity is concentrated in small disjoint regions and we prove its connection with the vortex model. The same problem is studied for a viscous fluid and we prove that the solutions converge in the zero viscosity limit to the corresponding solutions of the Euler equations.

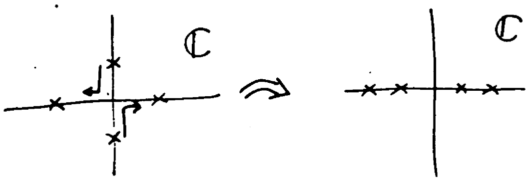
J. MARSDEN: Dynamics of Coupled Rigid Bodies

We study the equilibria, stability and chaos in the dynamics of three coupled planar rigid bodies.

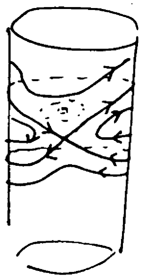


We prove that:

1. The number of equilibria lies between 4 and 6.
2. As system parameters change the bifurcations that change solutions from 4 to 6 are of this Hamiltonian type:



3. The dynamics is generally chaotic (proven by Melnikov's method) - the two body problem has (reduced) phase space a cylinder with a homoclinic orbit.
4. Near the stable equilibrium $\Theta_1 = \Theta_2 = 0$, there are two periodic solutions distinguished by symmetry type (proved by the Stewart-Montaldi-Roberts version of Moser-Weinstein).



(joint with Krishnaprasad, Sreenath & Oh)

J. MARSDEN & J. SCHEURLE : **Exponentially Small Splitting of Separatrices**

We consider as an example, the equation

$$\ddot{\varphi} + \sin\varphi = \delta\epsilon^p \sin(t/\epsilon) \quad , \quad p \geq 0$$

We prove that the separatrices for the Poincaré map split by an amount bounded above as follows: let $0 < \epsilon \leq 1$, $0 < \delta \leq \delta_0$ and $\eta > 0$ be given. Then there is a constant $C(\eta, \delta_0)$ such that

$$\text{splitting} \leq C(\eta, \delta_0) \times e^{-(\frac{\pi}{2}-\eta)/\epsilon} \times \delta\epsilon^p$$

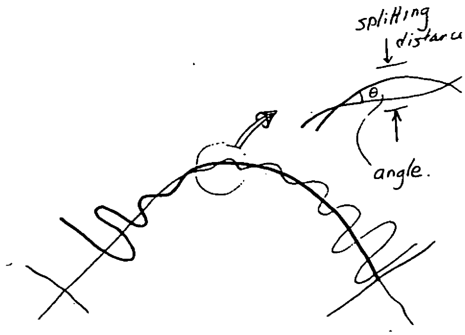
If $p \geq 8$ we prove the sharper estimates

$$C_1\delta\epsilon^p e^{-\pi/2\epsilon} \leq \text{splitting} \leq C_2\delta\epsilon^p e^{-\pi/2\epsilon}$$

There are similar estimates for the splitting angle. The result generalizes to systems of the form

$$\dot{u} = g(u,\epsilon) + \delta\epsilon^p h(u,\epsilon,t/\epsilon)$$

where g and h are analytic in u, ϵ ; h is H^1 in t/ϵ (H^2 for angle estimates) and $g(u,\epsilon)$ has a homoclinic orbit.



The results can be applied to estimate the thickness of stochastic layers in KAM theory and in the unfolding of degenerate singularities. The proof depends on a special extension of the iterates in the Liapunov Perron method to the complex t-plane.
(joint with P. Holmes)

T. MIYAKAWA: **On L^2 Decay for weak solutions of the Navier-Stokes equations and strong energy inequality**

Existence of weak solutions of the Navier-Stokes equations in unbounded domains in three dimensions is an open problem except for the case of the whole space. The writer announces the affirmative answer to this question in the case of halfspaces and exterior domains. The same proof applies also to the four-dimensional case. Using strong energy inequality one can show the large time regularity as well as decay in L^2 of the weak solutions. However, the decay property in L^2 of weak solutions is obtained also in the case of higher dimensions without appealing to the strong energy inequality (which is open for dimension ≥ 5). Indeed, one can show existence of a weak solution decaying in L^2 in any dimensions for the Navier-Stokes problem in the whole spaces and halfspaces.

P. J. MORRISON: **Sufficient and "Necessary" Free Energy Conditions for Stability**

There exist a large number of sufficient conditions for the stability of ideal fluid and plasma systems, which depend upon the positive definiteness of some quadratic form. Usually it is believed that these conditions yield no information when indefinite. I argue the contrary. Equilibria away from thermodynamic equilibrium, such as those with mean flow in fluids or bumpy distribution functions in kinetic theory, possess free energy that may not be tapped by the ideal linearized equations. Expressions for the change in free energy upon perturbation from equilibrium can be obtained. If such an expression is definite, stability is proved by Liapunov's theorem. If the expression is indefinite then there are two possibilities: 1) spectral instability or 2) spectral stability with negative energy modes. We argue via examples that the later case is generically unstable to infinitesimal perturbations, due to nonlinear effects, or to the inclusion of appropriate dissipation mechanisms. In this sense the free energy condition is necessary and sufficient for stability. This very general idea we call the free energy principle.

P. DE MOTTONI: **Travelling waves in a fluidized bed combustion model**

This is a preliminary report on a joint work with R. Dal Passo (Rome) and S. Luckhaus (Heidelberg). The combustion of coal particles in a fluidized bed is modelled assuming that the surface on which combustion takes place advances with a speed which is determined by the local degree of enfeeblement of the mechanical structure of the char, which in turn depends on the carbon conversion (= burning) rate. Upon certain physical assumptions (implying, among other things, the validity of the planar geometry for the particle's surface), one arrives at a system of two differential equations: an elliptic equation for the oxygen concentration balance, and an evolution equation for the carbon conversion rate. The first equation is supplemented by boundary conditions at the particle's surface and at its centre, the second equation by an appropriate initial condition. An additional equation relates the speed of advancement of the (moving!) particle surface to the current carbon conversion rate. For this system global existence and uniqueness results can be established. Here we focus on travelling wave solutions, and prove that there is a unique choice of initial conditions for which a travelling wave solution exists. The proof involves (elementary) shooting methods.

J. NEČAS: **On the solution of the transonic flow by viscosity method**

The goal of the investigation is to solve the Neumann problem for the isentropic, potential flow, governed by the equation

$$-\frac{\partial}{\partial x_i}\left(\rho(|\nabla u|^2)\frac{\partial u}{\partial x_i}\right) = 0 \quad , \quad \text{with} \quad \rho(|\nabla u|^2) = \rho_0\left(1 - \frac{\kappa - 1}{2a_0^2}|\nabla u|^2\right)^{\frac{1}{\kappa-1}}.$$

We consider the complete system of equations for a steady state of a compressible, perfect, viscous, conductive gas and we control the solutions for the viscosity $\mu \to 0$. The main assumption is the control of the entropy S on the boundary by

$$\left| \int_{\partial\Omega} S g dS \right| \le k\mu \quad , \quad \text{where} \quad g = \rho(|\nabla u|^2)\frac{\partial u}{\partial n} \quad \text{on } \partial\Omega$$

This gives from the energy equation estimates for $\ln T$ and $\frac{\nabla k}{\sqrt{T}}$, which allow to pass to the limit. In general, the solution is rotational and the presence of strong or weak shocks depends on the amount of cavitation of the density ρ. If the limit density is bounded from above and from below, the velocity field is from the space $W^{1,\frac{4}{3}}(\Omega)$.

A. C. NEWELL: **Turbulence and Finite Dimensional Dynamics**

It is reasonable to suggest that the number of degrees of freedom (closely related to the Hausdorff dimension of the attractor) which are active in turbulent shear flows at Reynolds numbers of 10^3 and higher is very large, probably varying like $R^{9/4}$. Therefore it is unlikely that replacing the Navier Stokes equations by a system of o.d.e.'s will lead to much new insight because of this large number. In this lecture I suggest an alternative point of view in which one does not demand that the approximation used is sufficiently accurate to keep the phase point which represents the state of the system in the attractor for all asymptotic time. Rather we simply ask that the approximation makes the phase point move parallel to the "important" orbits of the real attractor. These important orbits are homoclinic excursions which reflect the random occurence of coherent events in the real physical system. To make the point in a concrete way, I consider the collapse of Langmuir waves in a turbulent plasma. The idea is that the dissipation rate (once the energy is at sufficiently high k, Landau damping converts wave energy to electron energy) is controlled principally by the formations, growth and collapse of singular filaments which are the two dimensional analogue of envelope solutions. Wave-wave interactions contribute little to the process. In the context of Langmuir turbulence, this idea is not new. It has been long suggested by Zakharov, Suydcew, Rubenchick and colleagues in the Soviet Union. What is interesting is that this kind of behavior may be much more widespread. For example, it is plausible to suggest that the main momentum transfer across turbulent boundary layers is carried by coherent eddies (a composite of long, 3D, Tollmien-Schlichting waves and short wave infleximal instability pockets) which occur at random intervals. The transport of heat at very high Rayleigh numbers appears to be caused by well defined plumes. The suggestion, then, is that in a wide variety of situations, the field structures which do the transport are relatively coherent objects which are well approximated by (unstable) homoclinic orbits in the phase space. In physical space, these homoclinic orbits are related to exact, albeit singular perhaps, solutions of the field equations - the filaments in Langmuir turbulence, vortex sheets formations in Euler turbulence, coupled long wave, short wave bunts in boundary layers. If this picture is correct, it may be possible to approximate high dimensional turbulent fields by a low order system of o.d.e.'s for the parameters of the coherent eddies with stochastic coefficients containing information about the other active degrees of freedom which play little role in the transport properties.

C. PASSOV: **General Methodology for Plasma Modeling**

From a philosophical point of view a mathematical model is as valuable as a technical model, called experiment. Both are reconstructions of natural physical scenarios, worked out in order to discover hidden informations. However it is necessary to model in agreement with all known mathematical principles, in order to get not only effects, but a useful description of a real system.

A. PISKOREK: **Über die Anwendungen der symmetrischen partiellen Differentialgleichungssysteme in Fluid Mechanik**

Es wird eine Klasse der Anfangs-Randwertaufgaben für symmetrische lineare Differentialgleichungssysteme der Form

$$A^0(t,x)\partial_t u - A_j(t,x)\partial_{x_j} u - H(t,x)u = f$$

beschrieben. Mit Hilfe der Existenz- und Eindeutigkeitssätze für die glatten Lösungen solcher Anfangs-Randwertaufgaben beweist man die lokale in t Existenz der glatten Lösungen dieser Anfangs-Randwertaufgaben für symmetrische quasilineare Differentialgleichungssysteme. Die Anwendungen dieser Resultate kann man in der Fluid Mechanik (z.B. Eulersche Bewegungsgleichungen) benutzen.

M. PULVIRENTI: **Boundary Layer Problems for Incompressible Flows**

The Vortex Dynamics is a useful practical algorithm for incompressible flows. In presence of a boundary vorticity is created so that large gradients of the velocity field are concentrated in small regions. This makes difficult both the analytical and the numerical analysis of the problem. The vortex algorithms seem to be efficient in this situation even for reasonably large Reynolds number. My talk concerns convergence problems and a mathematical description of the vortex generation. In particular the convergence of the Chorin-Marsden product formula is discussed. References: G. Benfatto & M. Pulvirenti, Comm. Math. Phys. 96 p. 59 (1984), 106 p. 427 (1986).

T. RATIU: **Stability and Bifurcation of a 2-dimensional Euler liquiddrop**

Consider the incompressible, homogeneous Euler equations in $\mathbb{R}^2$ with a free boundary. The boundary condition is: the pressure equals the surface tension coefficient times the curvature of the contour. Take a rigidly rotating disk, it is a stationary solution of these equations. A stability criterion is found, under which this solution is Liapunov (conditionally) stable. Taking the angular velocity of the drop as bifurcation parameter, a branch of stable $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetric solutions is shown to exist. The same type of stability result holds in the compressible two-dimensional

case and for the situation of the MacLaurin spheroids; both problems have as potential the self-gravitating potential of rotating liquid masses.

R. RAUTMANN: **A convergent product formula for linearized Navier-Stokes problems**

Let ν_* denote a prescribed approximation in a velocity field $u(t,\cdot)$ on a domain $\Omega \subset R^3$. The linearization

$$rcl\frac{\partial}{\partial t}u + \nu Au + Pv_*\nabla u = 0 \quad , \quad \nabla \cdot u = 0 \quad , \quad (\nu > 0) \qquad (2)$$
$$u_{|\partial\Omega} = 0 \quad , \quad u(0,\cdot) = u_0$$

of the full Navier-Stokes problem leads to useful numerical schemas even at higher Reynolds numbers. Similar to Chorin and Marsden's product scheme, we solve (2) approximately by alternating application of a "Stokes resolvent step" H_h : $v \to (1+h\nu A)^{-1}v$ and an "Euler step" $E_h : v \to v \circ (I - hv_*)$, "$\circ$" denoting the composition of two maps.

Theorem: Assume $v_* \in D_A \cap H^3$, v_* real valued, $\partial\Omega$ a 2-dimensional compact $C^{2+\alpha}$ submanifold of R^3. Then the discrete semigroups $(U_h^k) = ((E_h H_h)^k)$, $k = 0, 1, \ldots$, are defined for any sufficiently small $h > 0$ and converge strongly in $L^2(\Omega)$ with $h \searrow 0$ to the contracting holomorphic semigroup e^{-tC} uniformly on any compact time interval in $t \geq 0$ $(C = vA + Pv_*\nabla)$.

J. SCHEURLE: **On a surface wave model**

We consider a singularly perturbed KdV equation. It models surface waves in a two dimensional incompressible fluid of constant density under the influence of gravity. The flow is assumed to be irrotational and the Bond number to be slightly below the criticality at one third. We prove the existence of elevation waves of permanent form, which look like solitary waves riding on a small amplitude oscillatory wave. (joint with John Hunter)

J. SCHEURLE: **Exponentially small splitting of separatrices (cont.)**

See the abstract of Jerry Marsden.

I. STEWART: **Hopf Bifurcation with O(3) Symmetry**

The existence of Hopf bifurcations in systems with symmetry Γ can be detected by finding isotropy subgroups of $\Gamma \times S'$ that have 2-dimensional fixed-point subspaces. Such subgroups are of the form $H^\Theta = \{(h, \Theta(h)) | h \in H\}$ where $H \subset \Gamma$ and $\Theta : H \to S'$ is a homomorphism, the twist. When $\Gamma = O(3)$ the classification

proceeds in stages. 1) List closed subgroups of $O(3)$. 2) List the possible twists. 3) Use trace formulas to compute the dimension of Fix (H^{Θ}). 4) Eliminate non-maximal groups and repetitions due to conjugacy. For example when $O(3)$ acts via its 5-dimensional irreducible representation there are 5 such isotropy subgroups. Pictures of the corresponding solutions will be shown.
(joint with M. Golubitsky)

H. TASSO: **Hamiltonians, Phase Space and Statistics of Continua**

The main part of the talk is devoted to the question of equilibrium statistics of ideal continua like fluids and plasmas. Hamiltonian formalism including generalized Poisson brackets and Lie-Poisson brackets is presented. Gyroviscous magnetohydrodynamics is taken as a relevant example in Euler and Clebsch variables. Degeneracy and redundancy in phase space are discussed and their effect on Gibbs disributions is considered. Phase space integrals are functional integrals whose ambiguous definition and evaluation lead to major problems in the calculation of statistical averages. The most impressive result is the exact calculation of the fluctuation spectrum of a class of evolution equations including the Korteweg-de Vries equation. This k-spectrum is essentially a Lorentz spectrum whose general shape is independent upon changes in the steepening term. The "ultraviolet catastrophe" is avoided in a purely classical way.

H. TASSO: **Linear Stability Analysis of Dissipative Fluid and Plasma Systems**

The general linear stability of dissipative fluids and plasmas is reduced to the equation

$$N\ddot{\underline{\xi}} + (P + M)\dot{\underline{\xi}} + (Q_s + Q_a)\underline{\xi} = 0$$

where $\underline{\xi}$ is the linearized Lagrangean displacement vector. N, M and Q_s are symmetric operators while P and Q_a are antisymmetric and N and M are positive definite. For 3-d gravitating plasmas at rest Q_a vanishes. In this case it is found using Lyapunov methods that the necessary and sufficient condition for stability is $(\xi, Q_s\xi) > 0$ with (ξ, ξ) finite. For 2-d equilibria in resistive magnetohydrodynamics and in the relevant "Tokamak scaling" limit Q_a vanishes too so the criterion reduces again to $(\xi, Q_s\xi) > 0$. Without this reduction a numerical investigation of the "tearing mode" stability in 2-d would not have been possible. Other unexpected results are also derived.

R. E. L. TURNER: **Internal Surges & Center Manifolds**

Consider two-dimensional motion in a system of two fluids of differing densities occupying a closed channel of finite vertical extent and infinite horizontal extent.

Gravity waves governed by the Euler equations are sought. Our earlier analytical work on solitary waves and computations by Turner and Vanden-Broeck led to the conjecture that there existed internal surges; i.e. waves approaching different limiting parallel flows as the horizontal coordinate approaches $-\infty$ and $+\infty$. Here we prove the existence of surges by using the dynamical systems approach to the elliptic problems arising. In that context the surge corresponds to a heteroclinic orbit in a center manifold.
(joint with C. J. Amick)

W. von WAHL: **Estimates for the pressure in the Navier-Stokes equations and their consequences**

Let u be any weak solution of the Navier-Stokes equations $u'-\Delta u+u\cdot\nabla u+\nabla\pi = f$, $\nabla\cdot u = 0$, $u|\partial\Omega = 0$, $u(0) = \varphi$, in $(0,T)\times\Omega$, Ω an exterior domain of R^n, $n\geq 3$. Then we show that

$$\nabla\pi \in L^{\sigma}((0,T),L^{p}(\Omega))$$

$$\pi \in L^{\sigma}((0,T),L^{p_*}(\Omega))$$

provided $n+1 \leq \frac{2}{\sigma}+\frac{n}{p}$, $\frac{1}{p_*} = \frac{1}{p}-\frac{1}{n}$. In particular we obtain

$$\pi \in L^{(n+2)/n}((0,T)\times\Omega).$$

The consequences are as follows: Set $n = 3$. We can construct a weak solution which is bounded in (t,x) if $|x|$ is sufficiently large and fulfills at the same time the energy inequality for almost every $s>0$, for $s=0$ and for all $t\geq s$. As we show, this implies $\|u(t)\|_{L^2(\Omega)} \to 0$ as $t\to+\infty$ $(T=+\infty)$. The work reported here is a joint one with H. Sohr and M. Wiegner.

Y.-H. WAN: **Instability of vortex streets with small cores**

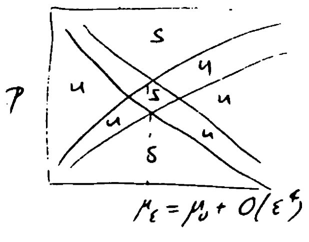
There exists one aspect ratio μ_ϵ close to $\mu_0 = .281$ for a desingularization of Karman street with area $O(\epsilon^2)$, in which the modified system is neutrally stable. This fact is verified by calculations while utilizing the Hamiltonian structure and symmetry of this modified system. The symmetry comes from the translation and reflection on the plane. Indeed, the modified system has the following bifurcation diagram:

s = stable region

u = unstable region

p = wave number

u = aspect ratio



H. ZHOU: Instability waves in the wall region of a turbulent boundary layer on a flat plate

Coherent structures play a very important role in determining the characteristics of turbulence in sheer flows. From the experimental observations it was found that the coherent structures in the wall region of a turbulent boundary layer bear many resemblances with the Görtler vortex and the instability waves in the transitional region of a laminar boundary layer. Using the mean flow profile as the basic flow, it was found all the Tollmien-Schlichting waves decay, thus linear stability theory can not explain the origin of the coherent structures. Resonant triad model does not work either. However for a pair of oblique waves, the threshold amplitude for instability is quite small for certain parameters of the waves. The parameters of the waves with the smallest threshold amplitude for instability seems to be very close to the parameters of the coherent structures observed experimentally.

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