MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Darstellungstheorie reduktiver Lie-Gruppen
und automorphe Darstellungen

26.7. bis 1.8.1987

Die Tagung fand unter der Leitung von Frau B. Speh (Ithaca), Herrn G. Harder (Bonn) und Herrn D.A. Vogan (Cambridge) statt. Obwohl das Tagungsthema weit gefaßt war, bildeten sich mehrere Schwerpunkte: In der Darstellungstheorie reduktiver Lie-Gruppen die Untersuchung von Kohomologiefunktoren für Moduln von Lie-Algebren sowie das Problem der L-Funktorialität von Darstellungen. Im Themenbereich "Automorphe Darstellungen" spielte die Theta-Korrespondenz für duale reduktive Paare ebenso eine dominierende Rolle wie das Studium der Kohomologie arithmetischer Gruppen.

Vortragsauszüge

M. Duflo:

Invariant differential operators and derived functors

Let G be a Lie group, H a closed subgroup, $\underline{g}$ and $\underline{h}$ their complexified Lie algebras. We put $s = \dim \underline{h}$, $\Delta(x) = \operatorname{tr} \operatorname{ad}(x)|\underline{h}$ ($x \in \underline{h}$), $\underline{h}^{-\Delta} = \{x-\Delta(x), x \in \underline{h}\} \subset U(h)$. The algebras $A := (U(\underline{g})/U(\underline{g})\underline{h})^{\underline{h}}$ and $A' := (U(\underline{g})/\underline{h}^{-\Delta}U(\underline{g}))^{\underline{h}}$ act respectively on $C^{\infty}(G/H)$ and $C_c^{\infty}(G/H)$ determining two isomorphisms of A and A' with the algebra D(G/H) of G-invariant differential operators on G/H. The resulting isomorphism $J: A \xrightarrow{\sim} A'$ will be described algebraically. If M is a $\underline{g}$-module we put $\gamma(M) = H^0(\underline{h},M)$: This is an A-module. The sequence of derived functors γ^i is isomorphic to $H^i(\underline{h},.)$. This gives $H^i(\underline{h},M)$ the structure of an A-module, which can be described explicitly in the standard complex. On the other side one considers the A'-module $\pi(M) = H_0(\underline{h},(\Lambda^s\underline{h})^* \otimes M)$ as an A-module via J. One proves the existence of an isomorphism $\pi_i \cong \gamma^{s-i}$ for the sequence of derived functors. All these questions are handled in a more general context which includes as a special case the functors of Zuckerman and Ducleux.

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David Wigner

Duality in the relative homology of Lie algebras (Zuckerman's conjecture)

Let $\mathfrak{g} \supset \mathfrak{k} \supset \mathfrak{h}$ be finite dimensional Lie algebras. Assign to each $\mathfrak{g}$-module M the submodule $Z^0(M)$ of $\mathfrak{k}$-finite vectors; this is a $\mathfrak{g}$-submodule, and the functor Z^0 from $\mathfrak{g}$-modules to $\mathfrak{g}$-modules is left exact. Zuckerman has introduced its right derived functors Z^i with respect to the class of $\mathfrak{h}$-split short exact sequences of $\mathfrak{g}$-modules, and conjectured the duality theorem $Z^i(M^*) \cong Z^0(Z^{m-i}(M)^*)$, where $m = \dim(\mathfrak{g}/\mathfrak{h})$. We study the Zuckerman functors and other related functors and give a new proof of Zuckerman's conjecture.

D.H. Collingwood:

Harish-Chandra modules with sparse cohomology

Let G be a connected semisimple real matrix group with Iwasawa decomposition $G = KAN$, $P = MAN$ a minimal parabolic subgroup, $\bar{P} = MA\bar{N}$ the opposite parabolic and V a fixed finite dimensional rep. of G. Let $\mathcal{H}\ell$ be the category of Harish-Chandra modules with infinitesimal character the same as V. To compute $H_\bullet(\bar{n},A)$ for $A \in \mathcal{H}\ell$ we introduce the Jacquet functor $J: \mathcal{H}\ell \to O'$, $J(M) = (M^*_{k\text{-finite}})^*_{p\text{-finite}}$, where O' is the Verma module category rel. p. $H_k(\bar{n},A) = H^k(n,J(A))$ for all irreducible $A \in \mathcal{H}\ell$. A spectral sequence is introduced which computes $H^*(n,J(A))$ modulo the determination of the E_1 differentials. Definition: $J(A)$ has sparse cohomology if

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$\sum_i \dim H^i(n,J(A))$ is minimal. Theorem: If (*) G has real rank one and A is irreducible or a principal series then $H^*(n,J(A))$ is sparse.

This leads to an explicit algorithm to compute $H_k(\bar{n},A)$, when A is as in the theorem. As a consequence $\dim \mathrm{Ext}^*_{g,k}(A,I^g_p(\tau))$ is determined whenever the assumptions (*) are fulfilled. Conjecture: Any self-dual indecomposable module in O' has sparse cohomology.

G.J. Zuckerman: Introduction to the Virasoro algebra

Let $\mathcal{L}$ be the unique one-dimensional central extension of the Witt Lie algebra of all derivations of $\mathbb{C}[t,t^{-1}]$ (discovered in the late 1960's by Gelfand-Fuks in mathematics and 1970 by Virasoro in physics). The most important cohomology functor for $\mathcal{L}$ is the semi-infinite cohomology $H^*_\infty(\mathcal{L},-)$ defined on an appropriate category of $\mathcal{L}$ modules (category "$\mathcal{O}$"). For Hermitian modules the speaker proved a Poincaré duality theorem (with I. Frenkel and H. Garland). The hermitian irreducible highest weight modules over $\mathcal{L}$ are classified by two real parameters $L(c,h)$. $L(c,h)$ is unitary if a) $1 \le c$, $0 \le h$ (continuous series) or b) $c = c_n = 1 - \frac{6}{n(n+1)}$, $h \in$ finite set depending on n.

Theorem: Let $L(c,h)$ be unitary. $H^i_\infty(\mathcal{L},L(c,h)) = 0$ unless $c = 26$, $h = 1$ and $i \in \{0,1\}$. $H^0_\infty(\mathcal{L},L(26,1)) \cong H^1_\infty(\mathcal{L},L(26,1)) \cong \mathbb{C}$.

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R. Rentschler:

Primitive ideals in the general case and the injectivity of the Duflo map (joint work with C. Moeglin)

Notations: $G/\mathbb{C}$ a conn. lin. algebraic group, $\mathfrak{g}$ = Lie G, $I \subset U(\mathfrak{g})$ a primitive ideal, $\mathfrak{k} \subset \mathfrak{g}$ an ideal, $P := I \cap U(\mathfrak{k})$, $U := U(\mathfrak{k})$, C = center of the total ring of quotients $Q(U/P)$ of the prime $\mathbb{C}$-algebra U/P. Consider the (rational) adjoint action of G on $U(\mathfrak{g})$, $U(\mathfrak{k})$ and U/P.

Theorem: (i) $\exists P_0 \subset U$ primitive (= rational, i.e. center of $Q(U/P_0) = \mathbb{C}$) ideal, unique up to G-conjugation, such that $P = \bigcap_{\gamma \in G} \gamma P_0$.

(ii) Let H be the stabilizer of P_0 in G. Then there is a canonical G-equivariant isomorphism between C and the function field of G/H.

(iii) $\exists\ J \subset U(\mathfrak{h})$, ($\mathfrak{h}$ = Lie H) primitive ideal, unique up to H-conjugation, such that $I = \mathrm{Ind}_{\mathfrak{h}}^{\mathfrak{g}} J$ and $J \cap U(\mathfrak{k}) = P_0$.

Consequence: The (surjective) Duflo classification map (Duflo, Acta Math., 1982) $(f,\xi) \to I_{\mathfrak{g}}(f,\xi) \in \mathrm{Prim}\, U(\mathfrak{g})$ (primitive spectrum) ($f \in \mathfrak{g}^*$ unipotent, ξ a primitive ideal of the stabilizer $\mathfrak{g}(f)$ of f such that $x - f(x) \in \xi$ for all x in the unipotent part of $\mathfrak{g}(f)$) is *injective* modulo G-conjugation.

W. Rossmann: Multiplicity polynomials

Suppose $H := (\mathbb{C}^*)^r$ acts on $\mathbb{C}^N$ by $h\cdot z = (h^{\alpha_1}\cdot z_1, \dots, h^{\alpha_N} z_N)$ with $\alpha_k \neq 0 \, \forall k$. H also acts on $R = \mathbb{C}[z_1, \dots, z_N]$ with character $ch_R(h) = \prod_k (1-h^{\alpha_k})^{-1}$. Similarly every R,H-module M (fin. gen. over R, locally semisimple over H, $h\cdot(r\cdot m) = (h\cdot r)\cdot(hm)$) admits a character ch_M. There exists a homogeneous polynomial $e_M \neq 0$ of degree N-n on $\mathfrak{h}$ = Lie H such that

$$ch_M(\exp x) = \frac{1}{\pi(x)} (e_M(x) + \mathcal{O}(|x|^{N-n})); \quad \pi(x) := \prod_k \alpha_k(x).$$

Suppose X is an N-dim. complex manifold with H-action, Z an H-stable n-dim. subvariety, $p \in Z$ a fixed point of H. Introduce analytic coordinates $z_1, \dots, z_N$ so that $p = (0, \dots, 0)$, $h\cdot z = (h^{\alpha_1} z_1, \dots, h^{\alpha_N} z_N)$ and assume $\alpha_k \neq 0 \, \forall k$. Apply the above construction with $R = \mathrm{gr}\, \mathcal{O}_{X,p} = \mathbb{C}[z_1, \dots, z_N]$, $M = \mathrm{gr}\, \mathcal{O}_{Z,p}$ to get a polynomial e_p on $\mathfrak{h}$, the "equivariant multiplicity" of p on Z in X. There exist equivalent analytic and geometric definitions for e_p. If Z is compact and all fixed points p on Z are of the above type, a localization formula can be proven.

Olivier Mathieu:

Canonical classes of Schubert varieties and affine Lie algebras

Let $\mathfrak{g}$ be a Kac-Moody Lie algebra, W ist Weyl group, $\mathfrak{b}$ its Borel subalgebra, B the associated group, P = {integral weights} , and h_i, α_i, s_i the simple coroots, roots and reflections.

To each $w \in W$ one can associate a Schubert variety S_w. Let $\mathcal{S}(w) := \{h_i \mid s_i \leq w\}$, $N(w) := \{\alpha \mid ws_\alpha \leq w$ and $\ell(ws_\alpha) = \ell(w)-1\}$, $P_w := \{\lambda \in P \mid \forall h \in \varphi(w): \lambda(h) = 0\}$.

For each B-module M , one constructs some sheaf $\alpha_w(M)$ over S_w. If $\dim \mathfrak{g} < \infty$, $\alpha_w(M)$ is the sheaf of sections of $\overline{BWB} \times^B M$. Let $D_w(M) = H^0(S_w, \alpha_w(M))$, and D_w^* = derived functors of D_w (on the category of B-modules).

<u>Theorem:</u> S_w is locally a unique factorization domain $\Leftrightarrow$

$$\Leftrightarrow \bigoplus_{\alpha \in N(w)} \mathbb{Z} h_\alpha \xrightarrow{\cong} \bigoplus_{i \in \mathcal{S}(w)} \mathbb{Z} h_i$$ by the natural homomorphism.

<u>Theorem:</u> S_w is Gorenstein $\Leftrightarrow$ D_w is "dualizable" $\Leftrightarrow$ $\exists \mu \in P \ \forall \alpha \in N(w): \mu(h_\alpha) = 1$.

The proof uses the natural maps $P_w \backslash P \to \mathrm{Pic}(S_w)$, $\lambda \mapsto \alpha_w(\lambda)$, and $D_w^* M \to H^*(S_w, \alpha_w(M))$ which are both shown to be isomorphisms.

<u>Application:</u> This can be used to show smoothness or non-smoothness of some Schubert varieties. For example: G_2 has 1,2,2,1,0,0,1 smooth Schubert varieties of dimensions 0,1,2,3,4,5,6 respectively. A Schubert variety of $A_1^{(1)}$ of dimension ≥ 3 is Gorenstein but not smooth.

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D. Vogan: $\mathcal{D}$-modules and Arthur's representations

Suppose X is a smooth projective algebraic variety over $\mathbb{C}$, stratified nicely by finitely many locally closed smooth subvarieties S_α. A regular holonomic $\mathcal{D}$-module $\mathcal{M}$ is said to be associated to $\{S_\alpha\}$ if $\mathrm{Char}(\mathcal{M}) \subset \bigcup_\alpha \overline{T^*_{S_\alpha} X}$. The irreducible $\mathcal{D}$-modules associated to $\{S_\alpha\}$ are parametrized as $\mathcal{M}(S_{\alpha_0}, L_0)$ with L_0 a local system on S_{α_0}, and

$$\overline{T^*_{S_{\alpha_0}} X} \subset \mathrm{Char}(\mathcal{M}(S_{\alpha_0},L_0)) \subset \bigcup_{S_\alpha \subset \overline{S_{\alpha_0}}} \overline{T^*_{S_\alpha} X} .$$

Problem: Fix S_{α_1} a closed stratum. For which (S_{α_0},L_0) is $T^*_{S_{\alpha_1}}(X) \subset \mathrm{Char}(\mathcal{M}(S_{\alpha_0},L_0))$?

Suppose now that G is a complex connected reductive algebraic group, $Q \subset G$ parabolic, θ an involutive automorphism of G, K its fixed points, and $X = G/Q$. Then all $(\mathcal{D}_X, K)$-modules of finite length are regular holonomic and associated to the K-orbit stratification of X. Fix a closed K-orbit X_0, and assume: (i) the moment map $\mu\colon T^*X \to \mathfrak{g}^*$ maps $T^*_{X_0}(X)$ onto the closure of one orbit $\mathcal{O}_\theta$ of K on $(\mathfrak{g}/\mathfrak{k})^*$, (ii) $\dim \mathcal{O}_\theta = \dim X$, (iii) $\mathcal{O}_\theta$ is contained in an even orbit of G on $\mathfrak{g}^*$, and Q is the parabolic attached to this even orbit. Then the set A of irreducible $(\mathcal{D}_X, K)$-modules $\mathcal{M}$ such that $\mathrm{char}(\mathcal{M}) \supset T^*_{X_0}X$ is in 1-1 correspondence with an Arthur ψ-packet consisting of representations of (real forms of) the dual group of G.

Conjecture: If $\mathfrak{M} \in A$ is associated to the K-orbit $X_1 \subset X$ (by the Riemann-Hilbert correspondence), then $\dim X_1 \equiv \dim X_0 \pmod 2$.

Jeffrey Adams: L-Functoriality for Dual Pairs

Let (G,G') be a reductive dual pair of subgroups of $Sp(2n,\mathbb{R})$ with inverse images $(\tilde{G},\tilde{G}')$ in the metaplectic group $Mp(2n,\mathbb{R})$. The restriction of the oscillator representation ω of $Mp(2n,\mathbb{R})$ yields a bijection $\pi \leftrightarrow \pi'$ between subsets $\Pi_\omega(\tilde{G}) \subset \Pi(\tilde{G})$ and $\Pi_\omega(\tilde{G}') \subset \Pi(\tilde{G})$ of the sets of admissible representations of $\tilde{G}$ and $\tilde{G}'$. We assume everything factors to G and G'. Langlands conjectured functoriality for this bijection in a sense similar to the conjecture below, with L-packets $\Pi(\varphi) \subseteq \Pi(G)$ of admissible homomorphisms $\varphi: W_{\mathbb{R}} \to {}^LG$ (and without SL_2 involved). Unfortunately, counter-examples arise when two discrete series in the same L-packet for G correspond to a tempered and a non-tempered representation of G', which cannot be in the same L-packet.

Arthur has conjectured that to an admissible map ψ: $W_{\mathbb{R}} \times SL_2(\mathbb{C}) \to {}^LG$, one can attach a packet $\Pi(\psi)$ of unitary representations with certain properties; a set $\Pi(\psi)$ with some of these properties is known. An Arthur packet $\Pi(\psi)$ may contain tempered and non-tempered representations.

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L-functoriality for Arthur packets is conjectured and holds in many cases:

Conjecture: There is a homomorphism $\gamma: {}^{L}G \to {}^{L}G'$ and a fixed homomorphism $T: SL_2(\mathbb{C}) \to {}^{L}G'$, such that $\pi \in \Pi_{\omega}(G)$, $\pi \in \Pi(\psi)$ implies $\pi' \in \Pi(\psi')$, where $\psi'(w,g) := \gamma(\psi(w,g))\cdot T(g)$ $(w \in W_{\mathbb{R}},\ g \in SL_2)$.

J. Johnson: Base change $\mathbb{C}/\mathbb{R}$ of certain derived functor modules

Let $G/\mathbb{R}$ be a reductive algebraic group, $\tilde{G}$ the Weil-restriction $R_{\mathbb{C}/\mathbb{R}}G_{\mathbb{C}}$, $A_{\tilde{\mathfrak{q}}}$ an irreducible unitary $\tilde{G}$-module affording non-zero $(\tilde{\mathfrak{g}}, \tilde{K})$ cohomology. Since $A_{\tilde{\mathfrak{q}}}$ is equivalent to $A_{\tilde{\mathfrak{q}}}^{\delta}$ (where $\mathrm{Gal}(\mathbb{C}/\mathbb{R}) = \{1,\delta\}$), there exists $A_{\delta} \in \mathrm{End}(A_{\tilde{\mathfrak{q}}})$ with $A_{\delta}\cdot g = g^{\delta}\cdot A_{\delta}$ $\forall g \in \tilde{G}(\mathbb{R}) = G(\mathbb{C})$. Let $\Theta^{\delta}_{A_{\tilde{\mathfrak{q}}}}$ be the locally L^{1} function representing the twisted-invariant distribution $C_c^{\infty}(\tilde{G}(\mathbb{R})) \ni f \mapsto \mathrm{tr}(A_{\delta}\circ A_{\tilde{\mathfrak{q}}}(f))$ (Clozel, Ann. E.N.S. 15). Let $A^{s}_{\mathfrak{q}}$ be the stable combination of $A_{\mathfrak{q}^{w}}$ defined by Adams-Johnson, Comp. Math. 86.

Theorem: $\Theta^{\delta}_{A_{\tilde{\mathfrak{q}}}}(g) = \pm\, \Theta_{A^{s}_{\mathfrak{q}}}(Ng)$ if $g \in \tilde{G}(\mathbb{R})$ is sufficiently regular, where N is the norm map on conjugacy classes.

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F. Shahidi: Plancherel measures for p-adic groups

Let G be a quasi-split connected reductive (linear) algebraic group over a p-adic field F of characteristic 0. Fix an endoscopic group H for G, and define tempered L-packets inductively. More precisely, assume Π is a tempered L-packet for $H(F)$. Assume that given $f \in C_c^\infty(G(F))$, there exists $f^H \in C_c^\infty(H(F))$ such that f and f^H have Δ-matching orbital integrals, where Δ is the transfer factor defined by Langlands and Shelstad. Let $\chi_\Pi = \sum_{\pi\in\Pi} \chi_\pi$, where for each $\pi \in \Pi$, χ_π is its character.

Then there exists a finite set Σ of tempered representations of $G(F)$ such that $\chi_\Pi(f^H) = \sum_{\sigma\in\Sigma} a_\sigma \chi_\sigma(f)$, where $a_\sigma \in \mathbb{C}^*$. We then prove that if the map $f \to f^H$ exists and is injective, then Π being generic, i.e. containing a generic representation, implies Σ is generic. Using Plancherel formula and a standard conjecture on stable distributions it can be shown that Plancherel measures are the same over L-packets, and therefore if the L-packet comes from a generic L-packet of $H(F)$, it would suffice to compute the Plancherel measure for representations induced from generic ones. Since for generic representations, the Plancherel measure is already proved to be arithmetic, as suggested by Langlands, this would then prove his conjecture in a large number of cases. The proof is based on certain results of Harish-Chandra and the work of Rodier and Moeglin-Waldspurger on characters of generic representations.

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F. Bien: Adelic representations of loop groups

Let Σ be a smooth Riemann surface over $\mathbb{C}$, F its function field and $\mathfrak{g} = \mathfrak{gl}_n(\mathbb{C})$. The speaker describes a construction of representations for the algebra $\mathrm{Map}(\Sigma, \mathfrak{g}) \cong F \otimes \mathfrak{g}$ of rational maps from Σ to $\mathfrak{g}$, which is motivated by an idea of E. Witten: Let $M = L(\lambda_0)$ be the basic module (a projective representation) for $\mathrm{Map}(S^1, \mathfrak{g}) = \mathbb{C}[t, t^{-1}] \otimes \mathfrak{g}$ in Kac-Moody notation. The completion $\bar{M}$ of M is a $\hat{\mathfrak{g}}$ module, where $0 \to \mathbb{C} \to \mathfrak{g} \to \mathbb{C}((t)) \otimes \mathfrak{g} \to 0$ is a central extension. For $p \in \Sigma$ let $\bar{F}_p \cong \mathbb{C}((t))$ be the completion of F at p, $\bar{R}_p \subset \bar{F}_p$ the local ring. Define $\mathfrak{g}_\Sigma$ to be the restricted direct product of the $\mathfrak{g}_p = \bar{F}_p \otimes \mathfrak{g}$ with respect to the $k_p = \bar{R}_p \otimes \mathfrak{g}$ and $\hat{g}_\Sigma$ to be the central extension of g_Σ by $\mathbb{C}$ using the sum of the local cocycles of the extensions $0 \to \mathbb{C} \to \hat{\mathfrak{g}}_p \to \mathfrak{g}_p \to 0$. A representation V_p of $\hat{\mathfrak{g}}_p$ is called unramified if it has a vector v_p^0 killed by k_p. An adelic representation $V_\Sigma = \bigotimes'_{p \in \Sigma} (V_p, v_p^0)$ of $\hat{g}_\Sigma$ is a restricted tensor product of representations V_p of $\hat{\mathfrak{g}}_p$ which are unramified for almost all p. $\mathrm{Map}(\Sigma, \mathfrak{g})$ injects to $\hat{g}_\Sigma$ by the residue theorem. V_Σ is called <u>automorphic</u> if it admits a functional fixed by $\mathrm{Map}(\Sigma, \mathfrak{g})$. Example: $M_\Sigma = \bigotimes'_{p \in \Sigma} M_p$, where M_p is $\bar{M}$ regarded as a representation of $\hat{\mathfrak{g}}_p$. Conjecturally, the automorphic representations of $\hat{g}_\Sigma$ should correspond to representations of the Galois group of F.

S. Osborne: Almost invariant Eisenstein series

Let G be a reductive Lie group, $\Gamma \subset G$ a lattice satisfying Langlands' axioms. Eisenstein series, c-functions and so on can be treated in a choice-free way (except for a maximal compact subgroup) by writing

$$E(\underline{H},\underline{\Phi},\underline{\Lambda},x) = \sum_{P\in\mathcal{C}_i} e^{<\underline{\Lambda}(P)+\rho_P, H_P(x)+\underline{H}(P)>} \underline{\Phi}_P(x),$$

where $\mathcal{C}_i$ is a G-conjugacy class of cuspidal parabolics, $\underline{\Lambda} \in \prod_{P\in\mathcal{C}_i} (\check{a}_P)_{\mathbb{C}}$ satisfies $\Lambda(xPx^{-1}) = Ad(x)\underline{\Lambda}(P)$, $\underline{H} \in \prod_{P\in\mathcal{C}_i} (a_P)_{\mathbb{C}}$ satisfies $\underline{H}(P) = Ad(\gamma^{-1})\underline{H}(\gamma P\gamma^{-1}) + H_P(\gamma)$, $\gamma \in \Gamma$ and $\underline{\Phi}$: $\mathcal{C}_i \times G \to \mathbb{C}$ satisfies $\underline{\Phi}_{\gamma P\gamma^{-1}}(x) = \underline{\Phi}_P(x\gamma)$ with $\underline{\Phi}_P$ appropriate for Eisenstein series on $\Gamma/\Gamma \cap P$. No choice is made of any representatives for the Γ-conjugacy classes in $\mathcal{C}_i$, but at the cost of introducing a parameter $\underline{H}$. There is no natural choice for $\underline{H}$ if the Weyl group is not represented in Γ as opposed to Arthur's T_0 in the Adelic situation.

Steve Rallis: Hypercusp forms for G_2

(joint work with G. Schiffmann)

The group G_2 of automorphisms of Cayley numbers has certain double transitivity properties in its 7 dimensional irreducible representation. This allows one to establish, over any field $k \supseteq \mathbb{Q}$, a correspondence between the representations of G_2 and those of $\overline{SL}_2$ (= 2 fold cover of SL_2) that occur in the action of $G_2 \times \overline{SL}_2$ on the oscillator representation of $O(7) \times SL_2$. In particular a θ-correspondence exists between the Hecke algebras of G_2 and $\overline{SL}_2$. Such a local correspondence in reflected by a global one. Namely in the case of the global oscillator representation of $\overline{Sp}_7$ restricted to $\overline{SL}_2 \times O(7)$, there is a θ-lifting of cusp forms on $\overline{SL}_2(\mathbb{A})$ to automorphic forms on $O(7)(\mathbb{A})$. A cusp representation π of $\overline{SL}_2(\mathbb{A})$ which lifts to a cusp form on $O(5)(\mathbb{A})$ (the Saito Kurokawa space) is lifted by θ to a hypercuspidal $\theta(\pi)$; this means $\theta(\pi)$ has no regular Whittaker model. Moreover, $\theta(\pi)$ is the "shadow of an Eisenstein series" on $O(7)(\mathbb{A})$, i.e. equivalent to an Eisenstein series on $O(7)(\mathbb{A})$ for almost all primes. We show that a Saito Kurokawa space for $G_2(\mathbb{A})$ can be constructed from the Saito Kurokawa space on $O(5)(\mathbb{A})$.

Stephen S. Kudla: The Siegel-Weil Formula
(joint work with S. Rallis)

Let $V,(\ ,\ )$ be a non-degenerate symmetric inner product space over a number field F. Then $H = O(V)$ and $G = Sp(n)/F$ form a dual reductive pair in $Sp(V^n+V^n)$. Assume that $m = \dim_F V$ is even and let α be the dimension of a maximal F-isotropic subspace of V. For a fixed nontrivial additive character $\psi: F\backslash\mathbb{A} \to \mathbb{C}^1$ let ω_ψ denote the smooth oscillator representation of $G(\mathbb{A}) \times H(\mathbb{A})$ on the

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Schwartz-Bruhat space $S(V(\mathbb{A})^n)$. The theta integral

$$I_\varphi(g) := \int_{H(F)\backslash H(\mathbb{A})} \sum_{x\in V(F)^n} \omega(g)\varphi(h^{-1}x))dh$$

$$(\mathrm{vol}(H(F)\backslash H(\mathbb{A})) = 1)$$

converges absolutely for $\alpha = 0$ or $m-\alpha > n+1$ (Weil).

The Eisenstein series

$$E(g,s,\varphi) := \sum_{\gamma\in P(F)\backslash G(F)} \omega(\gamma g)\varphi(0)|\alpha(\gamma g)|^{s-s_0}$$

$$(s_0 = \frac{m}{2} - \rho_n, \quad \rho_n = \frac{n+1}{2},$$

P standard parabolic)

converges absolutely for $\mathrm{Re}(s) > \rho_n$.

Theorem: If $\alpha = 0$ or $m-\alpha > n+1$ and $\varphi \in S(V(\mathbb{A})^n)$ is K-finite ($K \subset G(\mathbb{A})$ max. compact), then $E(g,s,\varphi)$ is holomorphic at $s = s_0$, and $E(g,s_0,\varphi) = kI_\varphi(g)$, $k = 1$ if $m > n+1$, $k = 2$ if $m \leq n+1$.

For $m > 2n+2$ this is the classical Siegel-Weil formula of Weil's Acta paper. For $\alpha > 0$ it may be proved by using the theory of singular modular forms.

M. Harris: Arithmetic of automorphic forms and representations with $\bar{\partial}$-cohomology

Let $S = S(G,X) = \varprojlim S_K$ be a Shimura variety, V the automorphic vector bundle over S associated to an irr. repr. δ of K_∞ with highest weight Λ. Over any smooth proj. toroidal compactification $j: S_K \hookrightarrow {}_\Sigma S_K$ V has the canonical extension V^{can} (Mumford) and the subcanonical extension $V^{sub} = V^{can} \otimes I_\Sigma$, I_Σ = ideal sheaf of ${}_\Sigma S_K - S_K$.

$$\tilde{H}^\bullet(V) = \varinjlim_{K,\Sigma} H^\bullet({}_\Sigma S_K, V^{can}) \quad \text{and} \quad \tilde{H}^\bullet(V(-\infty)) = \varinjlim_{K,\Sigma} H^\bullet({}_\Sigma S_K, V^{sub})$$

are admissible $G(\mathbb{A}_f)$-modules. Let $K^\bullet_{rd,\Lambda}$ resp. $K^\bullet_{si,\Lambda}$ denote the complexes on S of $\mathcal{C}^\infty$ rapidly decreasing resp. slowly increasing V-valued (0,q)-forms.

Theorem 1 (H., Phom) $\tilde{H}^\bullet(V) \cong H^\bullet(K^\bullet_{si,\Lambda})$, $\tilde{H}^\bullet(V(-\infty)) \cong H^\bullet(K^\bullet_{rd,\Lambda})$

Using techniques introduced by Borel, Langland's theory of Eisenstein series and a theorem of Wallach one can prove:

Theorem 2 a) The image $\bar{H}^\bullet(V)$ of $\tilde{H}^\bullet(V(-\infty))$ in $\tilde{H}^\bullet(V)$ is represented by automorphic forms in the discrete spectrum. b) For Λ far from the walls $\bar{H}^\bullet(V)$ is represented by cuspidal automorphic forms.

Theorem 3 (Blasius, Clozel, H., Ramakrishnan). Let F be a cuspidal Maass wave form on $GL_2(F)$, F a totally real number field, with $\lambda = \frac{1}{4}$ at each archimedean place.
a) The system of Hecke-eigenvalues $\{a_{\mathfrak{p}}\}$ generates a finite extension of $\mathbb{Q}$. b) For any $\delta \in \mathrm{Gal}(\bar{\mathbb{Q}}/\mathbb{Q})$, the collection $\{a^\delta_{\mathfrak{p}}\}$ is also the set of eigenvalues of a form f^δ as above. -

Furthermore, in joint work with Kudla, the speaker has established a criterion for rationality of forms in $\tilde{H}^i(V)$, where $G = GSp(2,\mathbb{Q})$, $i \in \{1,2\}$, Λ far from the walls. The criterion is based on restriction to imbedded products of modular curves.

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G. Harder: Some problems in representation theory occuring in the Eisenstein cohomology

For a reductive group $G/\mathbb{Q}$ and a rational representation $\rho: G\times_{\mathbb{Q}}\mathbb{C} \to GL(V)$ we study the cohomology group

$$H^\bullet(\tilde{S},V) = \varinjlim_{K_f} H^\bullet(S_K,V),$$

where $S_K = G(\mathbb{Q})\backslash G(\mathbb{A})/K_\infty K_f$, $K_f \subset G(\mathbb{A}_f)$ open compact. If $P/\mathbb{Q}$ is a rank-one parabolic subgroup with Levi quotient $M = P/U$ we want to understand the image of the restriction map

$$r: H^\bullet(\tilde{S},V) \to H^\bullet(\partial_P\tilde{S},V)) = H^\bullet(P(\mathbb{Q})\backslash G(\mathbb{A})/K_\infty,V)$$

to the part $\partial_P\tilde{S}$ of the Borel-Serre boundary. If π_M is a cuspidal automorphic representation on M contributing to cohomology one has to intertwine the induced representation $I_{\pi_M} = \mathrm{Ind}_{P(\mathbb{A})}^{G(\mathbb{A})}\pi_M$ with the space of automorphic forms $\mathcal{A}(G(\mathbb{Q})\backslash G(\mathbb{A}))$ using Eisenstein series. If they are holomorphic at $s = 0$ one gets an injection $I_{\pi_M} \hookrightarrow \mathcal{A}(G(\mathbb{Q})\backslash G(\mathbb{A}))$ and the contribution of I_{π_M} to $H^\bullet(\tilde{S},V)$ is easy to understand. If the Eisenstein series has a (simple) pole, the residue gives a map $I_{\pi_M} \twoheadrightarrow J_{\pi_M} \hookrightarrow \mathcal{A}(G(\mathbb{Q})\backslash G(A))$ and one has to describe the representation $J_{\pi_{M,f}}$ of $G(\mathbb{A}_f)$ and has to study the map

$$H^\bullet(\mathfrak{g},K_\infty,I_{\pi_{M,\infty}} \otimes V) \to H^\bullet(\mathfrak{g},K_\infty,J_{\pi_{M,\infty}} \otimes V).$$

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J.P. Labesse: Cusp cohomology for arithmetic groups

In the notation of Harder the problem discussed in the lecture is the description of the cuspidal part $H^{\bullet}_{cusp}(\tilde{S},V)$ of $H^{\bullet}(\tilde{S},V)$. If G_∞ has discrete series representations π_∞ one can compute Euler-Poincaré characteristics using either "geometric index theorems" or using the Selberg trace formula applied to a function $f_\infty \otimes f_{finite}$, where f_∞ is a pseudocoefficient or an "Euler-Poincaré function" (which is the stable version of it) for π_∞. If G_∞ has no discrete series Euler-Poincaré characteristics vanish. In some cases there exist involutions α preserving the whole situation and one can use Lefschetz numbers (Speh-Rohlfs) or a twisted trace formula (work in progress by Clozel and Delorme) to get non-vanishing results for $H^{\bullet}_{cusp}(\tilde{S},V)$ if V is regular enough and if at some finite place one can find an α-stable supercuspidal representation.

J. Rohlfs: Automorphic forms and Lefschetz numbers

Let G be a connected semisimple Lie group without compact factors, K a maximal compact subgroup and θ the corresponding Cartan involution. Let $\Gamma \subset G$ be a sufficiently small congruence subgroup with $\theta(\Gamma) = \Gamma$. Let V be a representation of $G \rtimes \{1,\theta\}$, which is irreducible as a G module and has a regular highest weight. Then

$$L(\theta,\Gamma,V) = \chi((K\backslash G/\Gamma)^{\theta})\cdot tr(\theta|V) > 0,$$

where $L(\theta,\Gamma,V) = \Sigma(-1)^{i}\cdot tr\ \theta|H^{i}(\Gamma,V)$ and $\chi((K\backslash G/\Gamma)^{\theta})$ is the Euler-Poincaré-char. of the fixpoint set of θ acting on $K\backslash G/\Gamma$.

Application: If G is a complex Lie group or if $G = SL_{2n+1}(\mathbb{R})$; $SL_{n}(\mathbb{H})$, ($n \geq 2$, $\mathbb{H}$ quaternions);

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$G = SO(n,1)(\mathbb{R})$ and if G/Γ is compact then

$$H^{\bullet}(K\setminus G/\Gamma,\tilde{V}) = H^{\bullet}(\mathfrak{g},K,H_{\pi}\otimes V)^{m(\pi,\Gamma)}, \quad m(\pi,\Gamma) > 0$$

where H_{π} is the unique representation with $H^{\bullet}(\mathfrak{g},K,H_{\pi}\otimes V) \neq \{0\}$ and where $m(\pi,\Gamma)$ is the multiplicity of H_{π} in $L^2(G/\Gamma)$.

Floyd Williams: An L^2-Riemann Roch problem

Let G be a connected semisimple Lie group, $H \subset G$ a compact Cartan subgroup, $\Gamma \subset G$ a discrete subgroup, $\mathrm{vol}(\Gamma\setminus G) < \infty$, λ a character of H, $\mathcal{L}_{\lambda} := G\times_{H}\lambda$ the induced holomorphic line bundle on G/H, $H_2^{*} = H_2^{*}(\Gamma\setminus G/H, \Gamma\setminus\mathcal{L}_{\lambda})$ the L^2-cohomology. By a theorem of Moscovici, H_2^{*} is finite dimensional, so the L^2-characteristic $\chi_H(\lambda;\Gamma) := \sum_j (-1)^j \dim H_2^j$ is defined. To compute it (at least for generic λ), we use the unitary representation theory of G.

Theorem: There are an integer $q(\lambda)$ and 1/2-spin bundles $\varepsilon_{\lambda}^{\pm} \to \Gamma\setminus G/K$ (both depending only on λ) where $K \supset H$ is a maximal compact subgroup of G, such that $\chi_H(\lambda;\Gamma) = (-1)^{q(\lambda)} I_2(\mathcal{D}^{+}_{\Gamma,\lambda})$, where I_2 is the L^2-index of the Dirac operator $\mathcal{D}^{+}_{\Gamma,\lambda}$. If G/K is of rank one then I_2 is completely known, by the work of Barbasch and Moscovici and (independently) by W. Müller.

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Henri Moscovici: "Infinite determinants, Selberg Zeta function and Eta invariants"

(joint work with Robert Stanton)

Let X^{2n-1} be a compact manifold, E a Clifford bundle over X, D its Dirac operator acting on L^2-sections of E. The family of infinite determinants with the zero eigenvalue removed, $\det' \frac{D+is}{D-is}$, is moromorphic for $s \in \mathbb{C}$ and holomorphic for $\mathrm{Re}(s) > 0$, and $\lim_{\mathbb{R}\ni s\to\infty} \det' \frac{D+is}{D-is} = e^{\pi i \eta_D(0)}$, where $\eta_D(0)$ is the eta invariant of D.

Theorem: Assume that X is locally symmetric and E is locally homogeneous. Then for $\mathrm{Re}(s) > 0$

$$e^{\pi i \eta_D(0)} \cdot \det' \frac{D-is}{D+is} = \sum_{\{\gamma\}\neq 1} \eta_{D,\{\gamma\}} e^{-s\ell(\gamma)}$$

where $\Gamma = \pi_1(X)$, $\ell(\gamma)$ is the common length of the periodic geodesics in the free homotopy class corresponding to the conjugacy class $\{\gamma\}$, and $\eta_{D,\{\gamma\}}$ is explicitly given involving the geometry of the geodesic flow at $\{\gamma\}$. In particular, $\eta_{D,\{\gamma\}} = 0$ unless the local Euclidean factor of the fixed point set of the geodesic flow corresponding to $\{\gamma\}$ is of dimension 1.

There are similar formulae for reduced eta invariants, which have topological significance.

Wilfried Schmid: Uniformization of algebraic varieties by hermitian spaces (thesis of Carlos Simpson)

A variation of Hodge structure (VHS) of weight $w \in \mathbb{Z}$ over a complex manifold X consists of a flat vector bundle $V \to X$ and C^∞ subbundles $V^{p,q}$ $(p+q = w)$ such that $V = \oplus V^{p,q}$, $\nabla V^{p,q} \subset A^{1,0}(V^{p-1,q+1}) \oplus A^{0,1}(V^{p+1,q-1}) \oplus A^{1,0}(V^{p,q}) \oplus A^{0,1}(V^{p,q})$, and a polarization is assumed to exist. The type of a VHS consists of w and the Hodge numbers $h^{p,q} = \mathrm{rk}\, V^{p,q}$. A system of Hodge bundles (SHB) $E = \{E^{p,q};\kappa\}$ consists of holomorphic vector bundles $E^{p,q} \to X$ and bundle maps $\kappa: E^{p,q} \to E^{p-1,q+1}$, with $\kappa^2 = 0$. The Chern classes and the rank of E are those of $\oplus E^{p,q}$, and the notions of slope, stability and polystability are defined in the category of SHB analogously to the case of vector bundles (note that stability of E is not equivalent to stability of each $E^{p,q}$).

Each VHS determines in a natural way a SHB.

Theorem: If X is smooth projective over $\mathbb{C}$, the functor $\{\mathrm{VHS}\} \to \{\mathrm{SHB}\}$ induces an equivalence $\{\mathrm{VHS}\} \to \{$polystable SHB with $c_1 = c_2 = 0\}$.

This generalizes a theorem of Narasimhan-Seshadri. The proof extends recent techniques of Donaldson and Uhlenbeck-Yau.

VHSs of a given type are induced by a "period map" $\Phi: \tilde{X} \to D$, where $\tilde{X}$ is the universal covering of X and

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D the classifying space for Hodge structures of given type. There is a semisimple G and a compact centralizer L of a torus such that $G/L = D$, and G/L has a G-invariant complex structure. From the first theorem, Simpson concludes:

Theorem: Let X be smooth projective over $\mathbb{C}$, G/K a non-compact hermitian space. Then $\tilde{X} \cong G/K$ if and only if the following conditions hold:

(i) there exists a holomorphic principal $K_{\mathbb{C}}$-bundle over X such that the holomorphic tangent bundle of X is associated to the adjoint action of $K_{\mathbb{C}}$ on $\mathfrak{p}_+$

(ii) the SHB associated by $(\mathfrak{p}_-, \mathfrak{k}, \mathfrak{p}_+)$ to the principal bundle, with κ induced by Lie bracket, is polystable (or stable, if G/K is irreducible)

(iii) the cohomology class $c_B \in H^4(X,\mathbb{C})$, represented by the contraction via the Killing form B of two copies of the curvature form of the principal bundle, vanishes.

This result leads to various known, as well as new, uniformization theorems. Note that (i), (ii), (iii) are algebraic conditions, by GAGA.

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