MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 35/1987

Workshop on theory and applications of quasigroups and loops

9.8. bis 15.8.1987

Organizer: Hala Pflugfelder, Philadelphia, PA, U.S.A.

The purpose of this workshop was to coordinate the final draft of a book on quasigroups and loops written jointly by 20 co-authors from 6 different countries. The book " Quasigroups and Loops: Theory and Applications", edited by O. Chein, H. Pflugfelder and J.D.H. Smith and to be published by Heldermann Verlag, Berlin, will contain up-to-date reports on research done in different branches of the theory of quasigroups, laying emphasis on cross-connections with such fields as Algebra, Combinatorics, Geometry, Topology, Differential Geometry and Differential Equations. The chapters of the book are:

I. Varieties of loops and quasigroups, by T. Evans, USA
II. Methods of construction and examples, by O. Chein, USA
III. Centrality, by J.D.H. Smith, USA
IV. Commutative Moufang loops and related groupoids, by L. Bénéteau, France
V. Cubic hypersurfaces, by L. Bénéteau, France
VI. Combinatorial structures arising from CMLs, by M.Deza, France and G. Sabidussi, Canada
VII. Systems with 2 binary operations, by E. Goodaire, Canada and M. Kallaher, USA

VIII. Geometry of quasigroups, by A. Barlotti, Italy

IX. Topological loops, by K. Hofmann, Germany and K. Strambach, Germany

X. Locally differentiable quasigroups and webs, by V.V. Goldberg, USA

XI. Locally compact double loops and ternary fields, by Th. Grundhoeffer, Germany and H. Salzmann, Germany

XII. Differential geometry and quasigroups, by P.O. Miheev, USSR and L.V. Sabinin, USSR

XIII. Loops arising from differential equations, by D. Gerber, USA

XIV. Ordered loops and ternary rings, by S. Priess-Crampe, Germany and F. Kahlhoff, Germany

Technical discussions with the publisher were followed by presentations dealing with topics of the book.

K. STRAMBACH

Es wurde vorgeschlagen, ein neues 14. Kapitel von den Autoren S. Priess- Crampe und F. Kahlhoff ueber "Ordered Loops and Ternary Rings" schreiben zu lassen. Es wurde ueber die ins Auge gefassten Saetze fuer dieses Kapitel diskutiert.

Vortragsauszüge

V.V. GOLDBERG

Classification of Grassmann (n+1)-webs.

Different classes of Grassmann (n+1)-webs were characterized. Among them are algebraic, (2n+2)-hedral, reducible, multiple and completely reducible, Bol and Moufang and parallelizable (n+1)-webs. The scheme of inclusions of those classes was presented. The Bol and Moufang (n+1)-webs which were recently introduced by S.A. Gerasimenko were additionally included in Chapter X of the book since they have a direct relationship with the theory of n-quasigroups.

T. EVANS

Varieties of loops and quasigroups

Two main topics were discussed. The first was the classification of varieties of loops in terms of their identities. This is done by using a standard form for loop identities and a description of the lattice of loop varieties as dually isomorphic to the lattice of fully invariant subloops of $F(\underline{\underline{L}})$, the free loop on a countably infinite set of generators. The second topic discussed was the use of varieties of loops and quasigoups in combinatorics A variety is described whose algebras may be regarded as pair of mutually orthogonal latin squares and it is shown that this variety contains algebras of order $\equiv 2 (\mathrm{mod} 4)$ giving infinitely many counterexamples to Euler's conjecture.
Finally, Steiner triplesystems with sets of involutions were discussed and a description given of loops associated with such designs.

J.D.H. SMITH

Representation theory of infinite groups and finite quasigroups

The representation theory of quasigroups builds on the concept of centrality discussed in the third chapter of the book. Quasigroup modules are defined as abelian groups in a comma category of quasigroups. They are equivalent to representations of universal multiplication groups of quasigroups. In this way they are classified by almost-periodic functions on these groups.

A.B. ROMANOVSKA

Quasigroups: geometry and convexity

Barycentric algebras over any field were discussed. One uses some constructions from barycentric algebras to obtain an invariant passage from affine to projective geometry. In the case of a field of odd characteristic, the construction reduces to a certain sum of idempotent commutative entropic quasigroups.

P.DEAN GERBER

<u>Loops and Quadratic Ordinary Differential Equations</u>

Connections between quadratic ordinary differential equations, local Lie loops and non-associative algebras were discussed. Let $q=q(x): R^n \to R^n$ be a quadratic map which determines a system of ordinary quadratic diff. eq. with initial condition $\dot{x}=q(x)$, $x(0)=\xi$. Then $q(x)=x*x$ for some commutative algebra $Q(*)$ called the <u>related algebra</u> of the quadratic system. Now let $D \subset R^n$ be an open set and let $F=F(u): D \to Gl(n)$ be analytic and satify $F(0)=I$. Define $L(F^{-1})$ to be the local loop with multiplication $x \circ y = F^{-1}(y)x+y$. We show that for every q there exists $Q=Q(u)$, analytic in u near 0 , such that $x(t,\xi)=-t^{-1}(t\xi)^{\lambda}$ in the loop $L(Q^{-1})$. This is the <u>related loop</u> of the quadratic system, and the related algebra of the loop is given by $pq=F_u(0)[q]p$. It follows that $x*y=\frac{1}{2}(xy+yx)$ so that the related algebra of the quadratic system arises from the related algebra of the loop. We say that the loop $L(F^{-1})$ is <u>q-admissable</u> if the solution of $\dot{x}=q(x), x(0)=\xi$ is given by $x(t,\xi)=-t^{-1}(\xi t)^{\lambda}$. We give conditions for L to be q-admissable.

Berichterstatter: H. Pflugfelder

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