MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Topology

31.8. bis 5.9.1987

The meeting was organized by M. Kreck (Mainz), A. Ranicki (Edinburgh) and L. Siebenmann (Orsay). About 45 participants from Northern and Western Europe, the United States and the Soviet Union attended the conference. The twenty talks dealt predominantly with geometric topology, in particular questions in low dimensions. Other topics were equivariant topology, links, manifolds, and combinatorial questions.

Vortragsauszüge

B.N. APANASOV

The topology of hyperbolic manifolds with boundary

The topology of geometrically finite hyperbolic n-manifolds with boundary is investigated by means of geometric methods of Kleinian groups in (n-1)-sphere. In particular, studied are problems close to S.P. Novikov's hypothesis on triviality of h-cobordism of the $K(\pi,1)$-type. In dimension 4 a new phenomenon has been discovered: the speaker and A.V. Tetenov constructed such 4-manifolds M, $bd(M) = N_0 \cup N_1$, such that the cobordism $(M;N_0;N_1)$ is homologically trivial: $H_*(M,N_i) = 0$, $i = 0,1$, but not an h-cobordism. An obstruction here is the fact that the limit set $L(G) \subset S^3 = bd(H^4)$ for the action of $G = \pi_1(M)$ in the hyperbolic space H^4 is a wildly embedded 2-sphere in S^3. Two important facts should be mentioned: 1.) $G \cong \pi_1(N)$ for a closed hyperbolic 3-manifold N and hence $Wh(G) = 0$ - see Farrell-Jones. 2.)The analogous cobordism in dimension 3 is a trivial product.

Ch. BONATTI

A common fixed point for commuting diffeomorphisms of the 2-sphere

Theorem: There exists a neighborhood U of the identity in $Diff_1(S^2)$ such that, if $f_1,\dots,f_n \in U$ are n commuting diffeomorphisms of S^2, then they have a common fixed point.

In 1964, E. Lima had proved that n commuting vector field of S^2 have a common zero. The proof of Lima's theorem used strongly Poincare-Bendixson's theorem. The proof of the theorem follows the proof of Lima's theorem. The main difficulty is to find an equivalent of Poincare-Bendixson's theorem. For this we need a good understanding of what is a diffeomorphsism C^1-close to identity.

The motivation of this work was to look at the deformations of the foliation given by the trivial fibration: $S^2 \times T^n \longrightarrow S^2$. As a trivial consequence of the theorem, we obtain the following result:

<u>Theorem</u>: Each foliation C^1-close to the foliation defined by the trivial fibration $S^2 \times T^2 \longrightarrow S^2$, has a compact leaf close to a fibre.

J. DAVIS (joint work with R.J. Milgram)

<u>Semicharacteristics, bordism, and free group actions</u>

Algebraic invariants of $\Omega_*(pt)$ are given by the signature and DeRham invariant. Characteristic class formulae were given by Hirzebruch and Lustig-Milnor-Peterson respectively. The obvious bordism invariants of $\Omega_*(BG)$, G a finite group, are given by the signature and semicharacteristic classes (invented by R. Lee). We completely determine these invariants and obtain characteristic class formulae. The motivations are (a) to obtain strong homological restrictions on $H_*(M;\mathbb{R})$ for manifolds with a free G-action. (E.g. $\mathbb{Z}/2 \times \mathbb{Z}/2$ cannot act freely on M^{4k+1}, $H_*(M;\mathbb{R}) = H_*(S^{4k+1};\mathbb{R})$.) (b) Calculations of a piece of the symmetric assembly map in L-theory.

We treat this as an analogue of the "oozing" problem. We use those techniques together with group cohomology computations to reduce to special cases.

S.C. FERRY

<u>On Novikov's conjecture</u>

<u>Theorem</u>: If M^n, $n \geq 5$, is a complete Riemannian manifold with nonpositive curvature and $f\colon N \longrightarrow M$ is a homotopy equivalence which is a homotopy equivalence near ∞, then f is (canonically and unstably) tangential relative

to the given identification of tangent bundles near ∞.

Using the above theorem and a surgery exact sequence, we deduce the Novikov conjecture for fundamental groups of complete Riemannian manifolds of nonpositive curvature. For $K(\pi,1)$ manifolds, the conjecture says that a certain map $H_n(\pi,\mathbb{L}(0)) \longrightarrow L_n(\pi)$ is a rational monomorphism.

The theorem, which is joint work with S. Weinberger, is proven by using a parameterized version of the α-approximation, β-domination theorems of Chapman and Ferry (Am.J.Math. 1979) to extend an argument of Farrell-Hsiang (Ann.Math. 1982) from the closed to the complete case. The Novikov conjecture in this generality was previously obtained by Kasparov using C^*-algebra techniques.

E. FLAPAN

Symmetries of embedded graphs

This paper considers the relationship between the automorphisms of a graph and the symmetries of that graph when it is embedded in S^3 or $\mathbb{R}^3$. More precisely, the following questions are addressed: Suppose Φ is a homeomorphism of a graph G, is there an embedding of G in S^3 or $\mathbb{R}^3$ such that Φ is induced by some diffeomorphism h of S^3 or $\mathbb{R}^3$? If h is also required to be orientation reversing is there still such an embedding? Finally if, in addition, Φ is of finite order will there be an h of the same finite order?

At the beginning of the paper an example is presented of a graph G with a homeomorphism Φ such that for any embedding of G in S^3 there is no diffeomorphism of S^3 which induces Φ on G. After this example, the above questions are answered in detail for a class of graphs which has arisen in chemistry, the Möbius ladders. A Möbius ladder M_n is a simple closed curve K together with n segments joining n pairs of antipodal points of K. In particular, it is shown that for n odd there is no embedding of M_n in S^3 which is invariant under an orientation reversing diffeomorphism of S^3, whereas for n even there is always such an embedding. The paper concludes with some observations about the symmetries of a slightly larger class of graphs which includes the Möbius ladders.

V. KHARLAMOV

<u>Estimates for Betti numbers in topology of real algebraic surfaces</u>

The treatment of topological properties of real algebraic surfaces has many direct relations with the treatment of real plane algebraic curves. One bridge is formed by branched coverings of the plane. It allows to transfer results, examples and conjectures from one field to another. Historically, the second one is richer of examples and conjectures. It is the main source of the following hypothetic bound (which is due to O. Viro): If X is a compact simply-connected real algebraic surface then

$$b_1(\mathbb{R}X) \le h^{1,1}(\mathbb{C}X), \qquad (1)$$

$$2b_0(\mathbb{R}X) \le h^{1,1}(\mathbb{C}X) + h^{0,0}(\mathbb{C}X) + h^{2,2}(\mathbb{C}X), \qquad (2)$$

where b_i denotes Betti numbers with respect to coefficients in $\mathbb{Z}/2$; $\mathbb{R}X$ - the set of real points, $\mathbb{C}X$ - the set of complex points, $h^{a,b}$ - a Hodge number. This conjecture is still open. The only bounds which are known follow from the relations

$$\Sigma b_i(\mathbb{R}X) \le \Sigma b_i(\mathbb{C}X), \qquad \text{(Thom)} \qquad (3)$$

$$|\chi(\mathbb{R}X) - 1| \le h^{1,1}(\mathbb{C}X) - 1 \qquad \text{(Comessatti)} \qquad (4)$$

$$\Sigma b_i(\mathbb{R}X) = \Sigma b_i(\mathbb{C}X) \Longrightarrow \chi(\mathbb{R}X) \equiv \sigma(\mathbb{C}X) \bmod 16 \qquad \text{(Rohlin)} \qquad (5)$$

$$\Sigma b_i(\mathbb{R}X) = \Sigma b_i(\mathbb{C}X) - 2 \Longrightarrow \chi(\mathbb{R}X) \equiv \sigma(\mathbb{C}X) \pm 2 \bmod 16 \qquad \text{(Kharlamov)} \qquad (6)$$

$$\left.\begin{array}{l}\mathbb{R}X \text{ has } k \text{ components} \\ \text{homeomorphic to} \\ \text{torus or sphere}\end{array}\right\} \Longrightarrow 2-\chi(\mathbb{R}X) \le h^{1,1}(\mathbb{C}X) - 2k \qquad \text{(Kharlamov)} \qquad (7)$$

In some special cases they imply (1) or (2). Thus one can obtain

(I) if $h^{0,2} < 6$ then (1) is true, if $h^{0,2} < 3$ then (2) is true,

(II) if all but one components of $\mathbb{R}X$ are tori and spheres then (1) is true.

Some trick (passage to Jacobian) allows to deduce from (II) the following result

(III) if X is a real algebraic elliptic surface then (1) is true.

In connection with (2) I should mention some results about surfaces of degree 5 in $\mathbb{RP}^3$. The maximal value of b_o for such surfaces must lie between 21 and 25. Due to (2) it does not exceed 23. According to (3), (4) only the following topological types can be realized by a surface of degree 5 with 25 components

$24S \amalg P(2)$, $23S \amalg S(1) \amalg P(1)$, $23S \amalg S(2) \amalg P$, $22S \amalg S(1) \amalg S(1) \amalg P$.

Viro proved that the forth type and I proved that the third one can not be realized. Probably the development of the method will allow to rule out the other two types.

U. KOSCHORKE

On the geometry of link maps

Given m and $p,q \geq 0$, let $LM^m_{p,q}$ denote the set (and often group) of link homotopy classes of link maps

$$f = f_1 \amalg f_2 : S^p \amalg S^q \longrightarrow S^m$$

i.e. $f_1(S^p) \cap f_2(S^q) = \phi$. Since the talk of Fenn on $LM^4_{2,2}$ here in Oberwolfach three years ago (showing $LM^4_{2,2} \neq 0$), tremendous progress has been made in the study of $LM^m_{p,q}$ so that now it is often possible

1.) to decide whether, within a link homotopy class, f_2 can be made into an embedding; or even

2.) to determine $LM^m_{p,q}$ althogether - in many cases it is not even finitely generated, as for example $LM^4_{2,2}$ (as was shown by Paul Kirk).

In my lecture I discussed the geometry of the relevant invariants, e.g. of the obstruction $\tilde{\beta}$ (with values in "graded, normal bordism with cohomotopy") to making f_2 into an embedding. I also indicated some relevant examples both of link maps and of link homotopies. There are interesting connections to homotopy theory, e.g. to the EHP-sequence of James and to the J-homomorphism.

M. KRECK

Exotic knottings of surfaces in the 4-sphere

In my talk I sketched the proof of the following

Theorem: There exists an infinite series S_1, S_2,... of smooth submanifolds of S^4 such that:

(1) for any i,j the pairs (S^4,S_i), (S^4,S_j) are homeomorphic via a map restricting to a diffeomorphism between appropriate neighborhoods of the surfaces.

(2) for any $i \neq j$ the pairs (S^4,S_i), (S^4,S_j) are not diffeomorphic:

(3) each S_n is homeomorphic to the connected sum $\#\mathbb{RP}^2$ of 10 copies of the real projective plane;

(4) $\pi_1\ (S^4,S_n) = \mathbb{Z}_2$

(5) the normal Euler number (with local coefficients) of S_n in S^4 is 16.

The proof uses recent results of Donaldson, Friedman and Morgan and independently Okonek and van de Ven about Dolgachev surfaces implying that for odd q and q', $q \neq q'$, $D_{2,q}$ is not diffeomorphic to $D_{2,q'}$. We construct antiholomorphic involutions such that the orbit space is S^4 and the fixed point set is $\#_{10}\ \mathbb{RP}^2$. These are knottings (S^4,S_q). On the other hand we prove that the number of homeomorphism types in the sense of (1) of these knottings is finite implying the theorem.

W. KÜHNEL

Combinatorial Manifolds with few Vertices

The minimal number $m(M^d)$ of vertices of a combinatorial triangulation of a given d-manifold M is known for d=2, and in the case d≥3 it is not known even for "well understood" standard spaces M. As main result it is shown that a combinatorial d-manifold with less than $3\lceil\frac{d}{2}\rceil+3$ vertices is PL homeomorphic to the sphere (where $\lceil x\rceil := \min\{k \in \mathbf{Z} | k \geq x\}$) and that a d-manifold M with $3\frac{d}{2}+3$ vertices is either a sphere or d=2,4,8 or 16 and M is a "manifold like a projective plane", i.e. a manifold of Morse number 3.

This result is sharp in the sense that for d=2,4,8 there exist such examples with $3\frac{d}{2} + 3$ vertices. In the case d=16 it is open. Furthermore it is shown that a d-manifold (d≥3) with less that 2d+3 vertices is simply connected, and that there are examples with 2d+3 vertices which are not simply connected. For lower bounds of the number of vertices for cohomology projective spaces compare the talk by A. Marin.

Lit.: U. Brehm and W. Kühnel, Combinatorial manifolds with few vertices, Topology 1987 (to appear)

J. Eells and N.H. Kuiper, Manifolds which are like projective planes, Publ. Math. IHES 14 (1962), 5-46.

R. LEE

Cohomology of mapping class groups

Let M_g denote the group $\pi_o(\mathrm{Diff}\ S_g)$ of diffeomorphisms of an orientable surface of genus g modulo those isotopic to the identity. In pure group theoretical language, this means the outer automorphism group $\mathrm{Out}(\pi)$ of the fundamental group $\pi = \pi_1(S_g)$ of S_g. If we let g be large, then the cohomology $H^q(M_g;\mathbf{Z})$ of M_g at a fixed degree q, ($q<\frac{g}{3}$) becomes the same. This is called the same cohomology of M_g. In the lecture, we described the proof of the following:

Theorem (with R. Charney). The stable cohomology $H^*(M_g;\mathbf{Z}[\frac{1}{2}])$ contains as a direct summand the cohomology $H^*(\mathrm{Im}J;\mathbf{Z}[\frac{1}{2}])$ of the space ImJ.

As an immediate consequence, we showed that the stable cohomology contains an element of odd order which is the same except for a power of 2 as the denominator of $B_{2k}/2k$ where B_{2k} is the 2k-th Bernoulli number.

In addition to these results, we discussed a conjecture of Mumford on the stable rational cohomology of these mapping class groups.

M. LUSTIG

On free group automorphisms, the rank of their fixed subgroup, and their realization by surface homeomorphisms.

The methods of R. Goldstein and E. Turner, introduced in their new proof that $\mathrm{Fix}\ \varphi = \{w \in F_n \mid \varphi(w) = w\}$ is finitely generated (for $\varphi: F_n \longrightarrow F_n$ an automorphism of the free group $F_n = F(a_1, \dots, a_n)$), are extended and interpreted in analogy with some elements of the Nielsen-Thursten theory for surface homeomorphisms. Our main results are:

1) Cor: $\mathrm{rank}(\mathrm{Fix}\ \varphi) \le \mathrm{card}\ \{$occurrences of a_i in $\varphi(a_i)$ as reduced word in a_i; $i = 1, \dots, n\}$; obtained from

   Thm: $\mathrm{rank}(\mathrm{Fix}\ \varphi) = \mathrm{card}\ \{\text{sources}\} - \mathrm{card}\ \{\text{sinks}\} - \mathrm{card}\ \{$attractive fixed points of $\bar{\varphi}$ at $\infty\}$, where a source (or a sink resp.) is a decomposition of the reduced word $\varphi(a_i) = u \cdot a_i \cdot v$ (or $\varphi(a_i) = u \cdot a_i^{-1} \cdot v$ resp.) with $u = x^{-1}\varphi(x)$ for some $x \in F_n$.

2) Examples of automorphisms $\varphi: F_n \longrightarrow F_n$ with trivial abelianization (and exponential or linear growth) are given s.t. none of their powers is induced by any homeomorphism of an orientable surface.

A. MARIN

Kühnel's complexes and minimal triangulations of projective spaces

The Veronese embedding $\mathbb{C}P^n \longrightarrow \mathbb{R}^{n^2+2n}$ is tight. For $n = 2$ it has a combinatorial analogue: the canonical imbedding in Δ^8 of $\mathbb{C}P^2_9$, Kühnel's nine vertex complex projective plane. We shall show that for $n > 2$ there is no triangulation of $\mathbb{C}P^n$ with $(n+1)^2$ vertices, in particular, Kühnel's $\mathbb{C}P^2_9$ is not a member of an infinite family of tight triangulations of $\mathbb{C}P^n$.

The first remark is that a finite complex K whose cohomology ring contains $\mathbb{Z}/_{2\mathbb{Z}}[X]/_{X^{n+1}}$ $(X \in H^w(K, \mathbb{Z}/_{2\mathbb{Z}}))$ has at least $\frac{(wn+2)(n+1)}{2}$ vertices. Let us call a K making this bound sharp a (w,n) Kühnel complex (if w=2, $\frac{(wn+2)(n+1)}{2} = (n+1)^2!$). Consideration of an explicit simplical counting cocycle establishes that for even w and any subset X of the vertex set K^o of a (w,2) Kühnel complex K either X or K^o-X is the vertex set of a simplex of K. That duality property enables us to get the non existence of $(n+1)^2$ vertex-triangulations of $\mathbb{C}P^n$ for n > 2 and a "human proof" of Kühnel-Laßmann's computer aided checking of the unicity of $\mathbb{C}P^2_g$.

We propose the following conjecture (which we proved for w=1 and w=2) (w,n) Kühnel-complexes with n > 1 exist only for n=2 and w=1,2,4,8, giving minimal triangulations of $\mathbb{R}P^2$, $\mathbb{C}P^2$, HP^2 and $\mathbb{C}aP^2$ respectively.

S. MAUMARY

The analytic and the de Rham torsion

It is proved that the torsion τ_{deRh} introduced by de Rham for manifolds with symmetry group Γ (respecting some riemannian metric), by means of open convex Γ-coverings $\mathfrak{S}$, coincides with the analytic torsion τ_{R-S} of Ray-Singer even when $\mathfrak{S}$ is not convex but has all its finite intersections equivariantly diffeomorphic to a disc with linear symmetries. This settles left over questions and brings insight in the non topological invariance of the R-torsion τ of symmetries, as exhibited by Cappell-Shaneson, by showing that a homeomorphism h preserves τ if $h^{-1}(\mathfrak{S})$ is de Rham when $\mathfrak{S}$ is, in the above sense. The proof requires on the one hand an abstract theory of analytic torsion for Hilbert chain complexes, and on the other hand, the variation of τ_{R-S} w.r.t. riemannian metrics on the disc with symmetries, taking care of piecewise boundary conditions along the boundary of the disc minus neighbourhood of strata.

W. METZLER

Homotopy versus Simple Homotopy Type of 2-complexes

Homotopy type and simple homotopy type coincide for 1-complexes and differ in dimensions ≥ 3. It is shown in this talk that they differ also in dimension 2. Let L be a finite 2-complex, $\tau \in Wh(\pi_1(L))$. Consider 2-complexes $L^* = L \vee (L_1 \vee \dots \vee L_n)$, where the L_i are copies of the standard complex of the $\mathbb{Z}_2 \times \mathbb{Z}_4$ presentation $\langle \alpha, \beta \mid \alpha^2 = [\alpha, \beta] = \beta^4 = 1 \rangle$. If n is big enough (for τ), there exists a 2-complex K^2_τ and a homotopy equivalence $f\colon K^2_\tau \longrightarrow L^*$ with $\tau(f) = \tau \in Wh(\pi_1(L^*)) = Wh(\pi_1(L))$. In order to restrict the possible torsion values of self equivalences of L^*, we refine the bias-invariant-method of Dyer - Metzler - Sieradski from h-type to simple h-type.
If L is the standard complex of the presentation $\langle a, b \mid a^m = 1,\ ab^l a^{-1} = b^l \rangle$; $l-k = \lambda k$; m, λ even; k prime ≥ 5, then there exist values τ with $K^2_\tau \not\!\nearrow\!\!\!\searrow L^*$.
The result was motivated by a joint paper of Cynthia Hog-Angeloni, Martin Lustig and the author on homotopy types of 2-complexes with π_1 a free product. A systematical treatment of these and "bias and S-h-type" is planned as further joint work.

H.J. MUNKHOLM

The bounded, algebraic Whitehead group and Chapman's controlled end theorem

Fix a (non compact) metric space Z. A boundedly controlled (bc) space over Z is a pair (X,p) where X is a space and $p\colon X \longrightarrow Z$ is a map. A bc map $f\colon (X,p) \longrightarrow (Y,q)$ is a map $f\colon X \longrightarrow Y$ s.t. $\mathrm{dist}(qf(x), p(x))$ is bounded. To any (X,p) we associate an abelian category $\mathbb{Z}\pi_1(X,p)$-mod (analogous to the category of $\mathbb{Z}\pi_1(X)$ modules in the uncontrolled case), and we define chain, homology and homotopy functors $C_*^c((X,p)^\sim)$, $H_*^c((X,p)^\sim)$, and $\pi_*^c(X,p)$ taking values in $\mathbb{Z}\Pi_1(X,p)$.

We establish Whitehead and Hurewicz theorems in this context and use these tools to prove a bc h-cobordism theorem with obstructions in a Whitehead group, $Wh(\mathbb{Z}\Pi_1(X,p))$, defined in the usual way from the category $\mathbb{Z}\Pi_1(X,p)$-mod.

We establish a strong connection between a variant of this group, $Wh(\mathbb{Z}\Pi_1(X,p)_\infty)$ and Chapman's obstruction groups $\varprojlim K_0^p(M)$ and $\varprojlim^1 Wh^p(M)$ appearing in the controlled end theorem.

We also compare our $Wh(\mathbb{Z}\Pi_1(X,p))$ to Siebenmann's proper simple homotopy group $\mathcal{S}(X)$ in case $X = N\times\mathbb{R}^k$ and p = proj.

We finally give an example of a compact PL manifold N parametrized over $\mathbb{R}$ where $\mathcal{S}(N) = 0$ but $Wh(\mathbb{Z}\Pi_1(N,p))$ is uncountable.

Ref.: Geometry and Topology (eds. McCrory and Shifrin) Marcel Dekker, 1987, pp. 13-42.

W. NEUMANN

Combinatorics of 3-cycles and hyperbolic geometry

An unusual chain complex associated to a compact oriented 3-cycle (= normal quasitriangulated pseudomanifold) was described. The computation of its homology refines earlier joint work with D. Zagier. It has applications to complete hyperbolic 3-manifolds of finite volume:

(i).A proof of Thurston's hyperbolic Dehn surgery theorem (this is already in the mentioned work with D. Zagier);

(ii). Using this description of Dehn surgery space plus the work of Culler and Shalen, Yoshida has described incompressible surfaces in the 3-manifolds;

(iii). A practical (and neat) formula for the volume and Chern-Simons invariant of the hyperbolic manifold.

M.M. POSTNIKOV

On the commutative comultiplication in the chain complex of a topological space

A commutative A_∞-coalgebra is a chain complex with chain maps $\nabla^n: K \longrightarrow K^{\otimes n}$ such that

1) $d\nabla^n = \sum_{p+q=n} \sum_i \pm (1\otimes\ldots\otimes \underset{(i)}{\nabla^p} \otimes\ldots\otimes 1)\circ\nabla^q$

where d is the Cartier differential $df = \partial_D \circ f + (-1)^{\deg f} f\circ\partial_C$, $f: C \longrightarrow D$,

2) $\sigma_{p,q}\circ\nabla^n = 0 \quad \forall_{p,q}$, $p+q=n$, where $\sigma_{p,q}$ is the sum of all (p,q) shuffles.

<u>Theorem</u>: On the chain complex C_*X of a topological space X there exists the structure of an A_∞-coalgebra.

Explicit formulas for ∇^n where found by Klimashev. They depend on elements $g_n \in k[\Sigma_n]$, k a field of characteristic 0, satisfying conditions

a) $I_n g_n = 0$

b) $g_n \equiv 1 \bmod I_n$

where I_n is the left ideal generated by all $\sigma_{p,q}$, $p+q = n$.

Klimashev's formulas for g_n are given by:

I) $g_n = \frac{1}{n}[[..[x_1,x_2],\ldots,x_n]$,

II) $g_n - 1 = \sum_{i=1}^{n-1} (-1)^i \rho_i \circ \sigma_{i,n-i}$, $\rho_i = \begin{pmatrix} 1 & \ldots & i & i+1 & \ldots & n \\ i & \ldots & 1 & i+1 & \ldots & n \end{pmatrix}$.

P. VOGEL

2 x 2 matrices and applications to link theory

Let $\mathcal{A}$ be the 2x2 matrix algebra over the commutative ring A. Denote by t and δ the trace and the determinant. We have the following:

i) t is linear

ii) δ is multiplicative

iii) δ is quadratic

iv) $\forall\, x,y \in M \; t(xy)-t(x) = -[\delta(x+y)-\delta(x)-\delta(y)]$

v) there is a unique involution $^{-}$ on $\mathcal{M}$ such that:

$\forall\, x \in \mathcal{M} : x + \bar{x} = t(x) \;,\; x\cdot\bar{x} = \delta(x).$

We will say that $\mathcal{M}$ is a quasi 2x2-matrix algebra if $\mathcal{M}$ is an A-algebra for some commutative ring A equipped with maps t and δ from $\mathcal{M}$ to A satisfying properties i) to v).

Proposition: For any ring Λ there exists a universal map $\varepsilon: \Lambda \longrightarrow \mathcal{M}$ where $\mathcal{M}$ is a quasi 2x2-matrix algebra over a ring A.

We set: $\mathcal{M} =: \mathcal{M}(\Lambda)$, $A =: C(\Lambda)$

Example: If Λ is $\mathbb{Z}[F(x,y)]$ where $F(x,y)$ is the free group generated by x and y, we have:

$C(\Lambda) = \mathbb{Z}[a,b,c,\alpha^{\pm},\beta^{\pm}]$ with: $a=t(x)$, $b=t(y)$, $c=t(xy)$, $\alpha=\delta(x)$, $\beta=\delta(y)$,

$\mathcal{M}(\Lambda) = C(\Lambda) \oplus C(\Lambda)x \oplus C(\Lambda)y \oplus C(\Lambda)xy$

Application

Let L be a link of 2 intervals imbedded in $\mathbb{R}^2 \times I$ in a trivial way on the boundary. Let G_L be the fundamental group of $\mathbb{R}^2\times I - L$ and x and y be the canonical elements of G coming from $\mathbb{R}^2\times 1$ and x' and y' those coming from $\mathbb{R}^2\times 0$.

Theorem. There exists a commutative ring A' containing $C(\Lambda) = \mathbb{Z}[a,b,c,\alpha^{\pm},\beta^{\pm}]$ such that the representation $\varepsilon: \mathbb{Z}[F(x,y)] \longrightarrow \mathcal{M}(\Lambda)$ extends for any link L to a map from $\mathbb{Z}[G_L]$ to $\mathcal{M}(\Lambda) \otimes_{C(\Lambda)} A'$.

Moreover, there exists for any link L an element $\omega_L \in \mathcal{M}(\Lambda) \otimes_{C(\Lambda)} A'$, unique up to a scalar, satisfying the following:

1) $\omega_L = u + vxy$, $u,v \in A'$

2) ω_L is invertible and $x' = \omega_L x \omega_L^{-1}$, $y' = \omega_L y \omega_L^{-1}$

3) ω_L depends only on the cobordism class of L

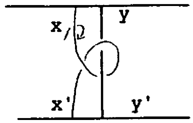
A' is defined as follows:

- $A = C(\Lambda)$ and $\Delta = c^2 - abc + \alpha b^2 + \beta a^2 - 4\alpha\beta$
- S is the set of polynomials $P(\frac{a^2}{\alpha},\frac{b^2}{\beta}) \in A$ such that $P(4,4) = 1$
- $A_o = S^{-1}A$

- A_1 is the completion of A_o with respect to ΔA_o
- A' is the ring of elements of A_1 that are algebraic over A.

Example

L is the following link:



Then $\omega_L = u+vxy$ and $\theta = \frac{u}{v}$ is the unique solution in A' of the following equation:

$$(\theta^2 + c\theta + \alpha\beta)(\theta + ab - c) + \Delta\theta = 0$$

congruent to c-ab modulo Δ.

S.H. WEINTRAUB

Rochlin invariants and theta functions.
(This is joint work with Ronnie Lee and Ed Miller.)

Let (M^{8k+2}, w) be a compact closed smooth spin manifold of dimension 8k+2 with torsion free middle integral homology $H_{4k+1}(M^{8k+2};\mathbb{Z})$. Moreover, for convenience, we assume that the quadratic map $q_w : H_{4k+1}(M^{8k+2};\mathbb{Z}) \longrightarrow \mathbb{Z}/2\mathbb{Z}$ associated to the spin structure w (via the work of E.H. Brown) has Arf invariant zero.

Let $F = (f,b)$ be a spin automorphism of (M^{8k+2}, w). Under favourable conditions, which are always satisfied if k equals 0 or 1, we may define an invariant of F, the Rochlin invariant $R(M,w,F)$, which is an integer mod 16. We announce a way of computing this invariant mod 8.

Further, for a suitable pair of spin automorphisms F_i of (M,w_i), $i = 1,2$, we express the complex number $\exp((2\pi i/8)[R(M,w_1,F_1) - R(M,w_2,F_2)])$ as a quotient of theta multipliers (the eighth roots of unity entering into the transformation law for theta functions). When M is a Riemann surface V, this number is the inverse of the holonomy of the flat determinant line bundle $(\det\partial_{w_1})\otimes(\det\partial_{w_2})^{-1}$. Similarly, the holonomy of the flat bundle $(\det\partial_w\otimes(\Delta^+)^h)\otimes(\det\partial_w)^{3h^2-1}$ (with Δ^+ the + chirality spin bundle) is completely determined. The motivation for this work has been twofold - to interprete theta multiplier in topological terms and to answer certain questions in physics. In the physics terminology we have calculated the "global anomaly" for certain coupled fields.

For more information see our announcement, to appear in the Bulletin of the A.M.S., October 1987.

S. WOLPERT

Degenerating Hyperbolic Surfaces

Let $\overline{M}_g$ be the stable curve compactification of the classical moduli space of Riemann surfaces. The compactification is constructed by the methods of algebraic geometry. It is also possible to define the compactification via 2-dimensional hyperbolic geometry. Our goal is to connect the two points of view.

Specifically let F be a degenerating family of Riemann surfaces over the unit disc $D = \{|t| < 1\}$, i.e. F is a 2-dimensional complex manifold and $F \xrightarrow{\pi} D$ is a holomorphic fibration, except for a node on the 0-fibre. The local model near the node of the 0-fibre is the (almost) fibration of

$$\begin{array}{c}\{zw = t\} \subset \mathbb{C}^3 \\ \downarrow \\ \{|t| < 1\}\end{array}$$

If the node of the 0-fibre is (temporarily) removed then each fibre has a complete hyperbolic metric. Thus $F \xrightarrow{\pi} D$ is also a family of hyperbolic metrics. Let (z,t) be local coordinates on F in a neighborhood of the general point of the 0-fibre, where t is the coordinate of the base. The problem is to write down the expansion in (z,t) of the family of hyperbolic metrics. We find an expansion of the following form:

s_t^2: hyperbolic metric on the t-fibre

$s_o^2 = \{$power series in $z, \bar{z}, t, \bar{t}, (\log \frac{1}{|t|})^{-2}\} + O(|t|^{1/2-\varepsilon})$.

In particular the first term has magnitude $(\log \frac{1}{|t|})^{-2}$ and is a nontrivial function of z.

Applications. The hyperbolic geometry of Riemann surfaces leads to a differential geometry on $\mathcal{M}_g$. The above expansion will provide for the extension of this geometry to $\overline{\mathcal{M}}_g$. A long range goal is to use the techniques of differential geometry, for instance harmonic tensors, to study the topology and geometry of $\overline{\mathcal{M}}_g$. A second project is to use the expansion to study the degeneration of the spectrum of the Laplace Beltrami operator of a compact surface.

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