MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 39/1987

Homotopietheorie

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This was the third homotopy theory conference at Oberwolfach organized by M. Mahowald (Evanston) and L. Smith (Göttingen). The meeting was attended by 40 participants from 10 countries.

Fourteen lectures of one hour and ten of 30 minutes were presented, each focusing on recent research developments. Topics receiving the most attention in these lectures were (unstable) homotopy groups of spheres, stable splittings of classifying spaces, and applications of Lannes' theory of unstable modules over the Steenrod algebra.

As always, participants benefited greatly from lively informal discussions in the evenings and between lectures.

Vortragsauszüge

Michèle Audin

Totally real embeddings

It is a question of interest in complex analysis and/or symplectic geometry to understand which closed n-manifolds admit totally real embeddings into $\mathbf{C}^n$.

(Recall an immersion $f : V^n \to \mathbf{C}^n$ is called "totally real" if the image $T_x f(T_x V)$ of each tangent space contains no non-trivial complex subspace).

After using the Gromov's h-principle, the question reduces to the following: "Let V be a closed connected n-manifold, and assume $TV \otimes \mathbf{C}$ is trivializable (necessary and sufficient condition for V to admit totally real immersions). Is there a regular homotopy class of immersions $V \to \mathbf{C}^n$ which contains both a totally real immersion and an ordinary embedding?"– which problem can be attacked by homotopy-theory methods.

It is an easy exercise to show that the answer is "yes, if and only if the Euler characteristic of V is zero" when V is orientable and n *even* (the case of non-orientable *even* dimensional V's is not difficult either). The case of odd n is more interesting. I show "Suppose n is an odd number ($n \geq 5$). Let V be a closed connected n-manifold such that $TV \otimes \mathbf{C}$ is trivializable. Suppose, either that V is framable, or that $n \equiv 1 \pmod 4$ and V is orientable. Then V admits a totally real embedding into $\mathbf{C}^n$ if and only if its Kervaire mod 2 semi-characteristic vanishes."

C. F. Bodigheimer

An unstable splitting of $\Lambda\Sigma X$

A well-known theorem of Milnor asserts that the loop space $\Omega\Sigma X$ of a suspension of a connected space X splits after a single suspension,

$$\Sigma\Omega\Sigma X \simeq \bigvee_{k\geq 1} \Sigma X^{(k)}.$$

For the free loop space $\Lambda\Sigma X$, it was proved by Th. Goodwillie, R. Cohen, and the author that there is a stable splitting,

$$\Sigma^\infty \Lambda\Sigma X \simeq \bigvee_{k\geq 1} \Sigma^\infty (S^1_+ \underset{\mathbf{Z}_k}{\wedge} X^{(k)}).$$

In the talk we reported on an unstable splitting.

THEOREM. $\Sigma^2\Lambda\Sigma X \simeq \bigvee_{k\geq 1} \Sigma^2(S^1_+ \underset{\mathbf{Z}_k}{\wedge} X^{(k)}).$

To prove this we use the usual configuration space model $C = C(S^1; X) \simeq \Lambda\Sigma X$. C is filtered, and the associated graded is

$$D = \bigvee_{k\geq 1} S^1_+ \underset{\mathbf{Z}_k}{\wedge} X^{(k)}.$$

We then study a "flag covering "$\Phi : F \to C$, where $\Phi^{-1}(\xi)$ is the set of all subconfigurations of ξ. We show that F embeds over C into the trivial plane bundle $C \times \mathbf{R}^2$. By "integration along the fibres of Φ " we obtain a map $\tau : C \to C(\mathbf{R}^2; D) \simeq \Omega^2\Sigma^2 D$, whose adjoint is an equivalence.

F. R. Cohen

Exponents for mod-2^r Moore spaces

Let $P^n(2^r)$ denote the cofibre of the degree 2^r map on S^{n-1}.

THEOREM. *If $n \geq 4$ and $r \geq 2$, then $2^{2r+3}\pi_* P^n(2^r) = 0$.*

The proof is obtained by finding a product decomposition for $\Omega P^n(2^r)$ and then fibering the resulting factors.

D. M. Davis

The complex bordism of groups with periodic cohomology

This is joint work with A. Bahri, M. Bendersky, and P. Gilkey. It is well-known that the following conditions are equivalent for finite groups G:

a) Every Sylow subgroup of G is cyclic or generalized quaternionic;

b) $H^*(BG)$ is periodic (G has periodic cohomology);

c) hom $\dim_{MU_*} MU_*(BG) \leq 1$;

d) G acts freely on a finite simplicial homotopy sphere.

The spherical space-form groups are those with a free orthogonal action on a standard sphere, and form a well-known subcollection.

Recall that $\pi_*(MU) \approx \mathbf{Z}[x_{2i} = i \geq 1]$ and $\pi_*(bu) \approx \mathbf{Z}[x_2]$. Our main result is

THEOREM. *If G is a group with periodic cohomology, then there is an isomorphism of graded abelian groups*

$$MU_*(BG) \approx bu_*(BG) \otimes \mathbf{Z}[x_{2i} : i \geq 2].$$

We can also write the explicit abelian group structure of $MU_*(BG)$, but, as it is quite complicated, it was omitted from the lecture.

We show by example that such a result is false for $B(\mathbf{Z}/2 \times \mathbf{Z}/2)$ and for a certain finite complex X satisfying hom $\dim_{MU_*} MU_*(X) = 1$.

The idea of the proof is to prove it for a cyclic or a generalized quaternionic Sylow subgroup H of G, and then deduce it for G from the p-local splitting of BG as a summand of BH.

E. Devinatz

K-theory and the generating hypothesis

In the mid '60's, Freyd conjectured that if X and Y are finite spectra and $f : X \to Y$ with $\pi_* f$ the zero homomorphism, then f is trivial. He called this conjecture the generating hypothesis.

One may attack this problem in the following way. Fix a prime number p, and let L_n be the stable localization functor with respect to $E(n)$. ($E(n)$ is a periodic homology theory related to Brown-Peterson homology; its coefficient ring is given by $E(n)_* = \mathbf{Z}_{(p)}[v_1, \ldots, v_n, v_n^{-1}]$ if $n > 0$ and $E(0)_* = \mathbf{Q}$.) In particular, L_0 is rational localization, and L_1 is localization with respect to complex K-theory at the prime p. Roughly speaking, L_n detects periodicity of order less than $n+1$ in a stable homotopy. Consider the following conjecture: If X, Y, and f are as above, then $L_n f = 0$. It follows immediately from a result of M. Hopkins and D. Ravenel that if this conjecture is true for each n, so is the p-local generating hypothesis.

If $n = 0$, the conjecture is essentially trivial; if $n = 1$, we have the following result.

THEOREM. *Let p be an odd prime and X a finite spectrum. If $f : X \to S^0$ with $\pi_* f = 0$, then $L_1 f = 0$.*

The proof of this theorem, though not long, uses much of the theory of periodicity in stable homotopy.

M. Feshbach

Varieties and the transfer

This talk is based on joint work with Leonard Evens. J.Carlson has shown for finite groups G that the variety in spec $H^{ev}(BG, \mathbf{F}_p)$ defined by the ideal generated by the images of transfers from all subgroups of index divisible by p is the same as the variety defined by the kernel of the restriction to the center of a p-Sylow subgroup. We prove a generalization of this theorem to compact Lie groups. If G is a compact Lie group with maximal torus T and normalizer N, we call $P < N$ a Sylow p-normalizer of T if it is the inverse image of a Sylow p-subgroup of the Weyl group N/T. We let $J_G = \sum \operatorname{im} t(H, G)$ where H is a closed subgroup of G such that $p | \chi(G/H)$.

We let $J'_G = \sum \operatorname{im} t(H, G)$, where $H = C_P(x)$, for $x \in P$, a *noncentral* element of order p. The center of P is denoted by Z.

THEOREM. *The varieties defined by the ideals J_G, J'_G, and $\ker \rho(Z, G)$ are equal. Equivalently, these ideals all have the same radical.*

The proof reduces first to the case $P = G$.

J. R. Harper

Cogroups and p-divisibility of double suspensions

Let $f : \Sigma^2 X \to S^{2n+1}$ where spaces are localized at an odd prime p. It is well known that f is a co-H-map if and only if $\Omega H \circ f^{**}$ is null, where ΩH is the pth James-Hopf invariant and f^{**} is the double adjoint of f. Given a co-H-map f and a primitive homotopy F, there is a function $A(f,F)$, called the co-A-deviation, from $\Sigma^3 X$ to 3-fold bouquet of S^{2n+1}, which measures the deviation from homotopy coassociativity of the map f. We calculate $A(f,F)$ in terms of the standard invariants of homotopy theory, and in particular characterize the co-A-maps (those where $A(f,F) \sim *$ for some F) as follows.

THEOREM. *f is a co-A-map if and only if $\Omega^2\partial \circ \Omega E^2 \circ T \circ \bar{f}$ is null, where $\bar{f} : X \to \Omega J_{p-1} S^{2n}$ lifts f^{**}, and the Toda-Hopf invariant T is chosen to be an H-map. The other maps are displayed,*

$$\Omega J_{p-1} S^{2n} \xrightarrow{T} \Omega S^{2np-1} \xrightarrow{\Omega E^2} \Omega^3 S^{2np+1} \xrightarrow{\Omega^2 \partial} \Omega^2 S^{2np+1}\{p\}.$$

Where one specializes to S^3, there is a further reduction.

THEOREM. *$f : \Sigma^2 X \to S^3$ is a co-A-map if an only if $f \simeq \Sigma^2 g$ where $g : X \to S^1$.*

An example of a co-A-map which is not a suspension is $\alpha_1\beta_2(5) : S^{4p^2-2} \to S^5$. The resulting 2-cell complex is a cogroup not homotopy equivalent to any suspension.

M. Hartl

On the classification of highly connected Poincaré-complexes

A $(n-1)$-connected $(2n+1)$-dimensional Poincaré-complex X has a decomposition

$$X \simeq M(H_n(X), n) \vee M(H_{n+1}(X), n+1) \cup e_{2n+1},$$

where $M(A,n)$ denotes the Moore space with homology group A in dimension n. A homotopy classification of arbitrary CW-complexes of this cell type with finitely generated homology and H_{n+1} free is obtained by describing a functorial algebraic model for the homotopy group $\pi_{2n} M(A,n)$, $n > 3$ and $n \neq 7$, which does not use a decomposition of A into cyclic factors. It transforms data from the homotopy of spheres into the corresponding data for Moore spaces.

The double suspension $E^2 : \pi_{2n}M(A,n) \to \pi_n^s MA$ has a natural splitting if $n > 7$, $n \not\equiv 6,7 \mod 8$. The subgroup $\ker E^2$ has period 4 and is determined explicitly. For example, one gets $\ker E^2 \cong \mathbf{Z}/8$ if $A = \mathbf{Z}/2$ and $n \equiv 1 \mod 8$. Thus an element is produced which has the maximal order that Barratt's exponent conjecture admits in this case.

H. W. Henn

Algebras over the Steenrod algebra of finite transcendence degree and injective modules over the Steenrod algebra

This is joint work with J. Lannes and L. Schwartz. Let V be a finite dimensional vector space over $\mathbf{F}_p$, and denote by $\mathcal{K}_V$ the category of unstable algebras K over the mod p Steenrod algebra A_p with transcendence degree $d(K) \le \dim V$, and by $\mathcal{S}_V$ the category of profinite $\mathrm{End}(V)$-sets. A morphism $f : K \to L$ in $\mathcal{K}_V$ is called an F-isomorphism iff $\ker f$ and $\operatorname{coker} f$ are nilpotent modules over A_p. Let $\mathcal{K}_V/\mathrm{Nil}$ be the category obtained from $\mathcal{K}_V$ by inverting all F-isomorphisms.

THEOREM. *The functor*

$$\begin{aligned} \alpha : \mathcal{K}_V &\to \mathcal{S}_V \\ K &\mapsto \hom_{\mathcal{K}}(K, H^*V) \end{aligned}$$

induces an equivalence $\bar{\alpha} : \mathcal{K}_V/\mathit{Nil} \to \mathcal{S}_V$.

Immediate consequences are

(1) The existence of the Adams-Wilkerson embedding of an integral domain $K \in \mathcal{K}_V$ into H^*V.

(2) Extensions of results of Rector and S. P. Lam on Noetherian algebras (resp. algebras) with a finite number of minimal prime ideals invariant under A_p.

Other topics discussed were injective algebras K without nilpotents and their associated $\mathrm{End}(V)$-sets $\alpha(K)$ as well as the realization of injective algebras without nilpotents. Finally we presented a complete list of indecomposable (as modules) injective algebras without nilpotents. It turns out that there are very few examples, all of the form $H^*(BV)^G$ for some P'-subgroup $G \subset \mathrm{GL}(V)$. Our proof of this fact depends on the classification of finite simple groups.

S. Jackowski

Homotopy approximations for classifying spaces of compact Lie groups

This was a report on a joint work with James McClure. By a homotopy approximation for a space BG at the prime p we mean a mod p-cohomology equivalence

$$\mathrm{hocolim}_C\, EG \times_G I \to BG$$

where C is a finite subcategory of the orbit category $O(G)$ and

$$(EG \times_G I)(G/H) := EG \times_G G/H \simeq BH.$$

We found such approximations for

$$G = SO(3),\ SU(2),\ SU(3),\ U(2).$$

Here are the diagrams for $BSU(3)$, with T denoting a maximal torus in $SU(3)$:

$$\begin{array}{ll} p \neq 2,3 & BSU(3) \underset{p}{\sim} \mathrm{hocolim}\left(BT \circlearrowright \Sigma_3\right) \\ p = 2 & BSU(3) \underset{2}{\sim} \mathrm{hocolim}\left(\Sigma_3 \circlearrowright BT \rightrightarrows BU(2)\right) \\ p = 3 & BSU(3) \underset{3}{\sim} \mathrm{hocolim}\left(SL_2\mathbf{Z}_3 \circlearrowright B\Gamma \rightrightarrows BN_3\right) \\ & \text{where } T \hookrightarrow N_3 \twoheadrightarrow \mathbf{Z}_3, \text{ and } |\Gamma| = 27. \end{array}$$

Such approximations can be applied to the classification problems for maps defined on classifying spaces. In particular we obtained a classification of the self-maps of $BSU(3)$. Every self-map is homotopic to one of the Adams maps

$$\psi_k : BSU(3) \to BSU(3), \qquad \text{where } (k,6) = 1.$$

The Adams map ψ_k is the unique up to homotopy extension of the map $BT \to BSU(3)$ induced by the kth power homomorphism $T \ni t \mapsto t^k \in T \subset SU(3)$.

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Ulrich Koschorke

A filtration of the stable homotopy groups of spheres

Given integers p, q and $n \geq 0$, define

$$\pi_n^{p,q} := \{\alpha(f) \mid f : S^p \cup S^q \to S^{p+q+1-n} \text{ link map, i.e. } f(S^p) \cap f(S^q) = \emptyset\}$$

which, apart from a few trivial cases, forms a subgroup of the stable n-stem π_n^s. Here the α-invariant is a natural generalization of the classical linking number (defined for $n = 0$); often it classifies link maps up to link homotopy so that in this case its value set becomes particularly relevant to the classification problem.

The family $\{\pi_n^{p,q}\}_{p,q\geq 0}$ forms a symmetric double filtration of π_n^S, (i.e. $\pi_n^{q,p} = \pi_n^{p,q} \subseteq \pi_n^{p+1,q} \cap \pi_n^{p,q+1}$) with good compatibilities with respect to the multiplicative structure on π_*^s (this is joint work with D. Rolfsen). Apart from some vanishing results following from Papakyriakopoulos' acyclicity of knots (noted by U. Kaiser) and the relation

$$E^\infty(\pi_p(S^{p-n})) \subseteq \pi_n^{p,q} \subseteq E^\infty(\pi_{p+q}(S^{p+q-n})),$$

where the first is "=" if $p - q \geq n$, we also have the important observation that

$$E^\infty \circ J(\pi_n(SO(p+q-2n))) \subseteq \pi_n^{p,q} \quad \text{for } p, q > n.$$

So the excess of $\pi_n^{p,q}$ over $E^\infty(\pi_p(S^{p-n}))$ can be considerable. It is measured by the β-invariant whose refinements impose further restrictions on $\pi_n^{p,q}$.

In many cases a geometric exact sequence involving the set of link homotopy classes of link maps is closely related, via α and β, to the EHP-sequence of James.

J. Lannes

Comparison between stable and unstable homotopy fixed points

Let X be a pointed space equipped with an action of a finite group σ (the base point is fixed under σ). One considers:

(1) the homotopy fixed point space : $X^{h\sigma} = \hom_\sigma(E\sigma, X)$

(2) the homotopy quotient space: $X_{h\sigma} = E_+\sigma \underset{\sigma}{\wedge} X$

(3) the Ω-spectrum $\{(Q\Sigma^n X)^{h\sigma}\}_{n\in N}$ which is denoted $(\Sigma^\infty X)^{h\sigma}$ (note that σ "does not act on $Q\Sigma^n$").

First let us assume that σ is a cyclic group of order a prime p. Using the wonderful properties of $H^*(B\sigma; \mathbf{F}_p)$ seen as an unstable algebra over the Steenrod algebra, we prove that the natural map:

$$\Sigma^\infty X^{h\sigma} \vee \Sigma^\infty X_{h\sigma} \to (\Sigma^\infty X)^{h\sigma}$$

is a homotopy equivalence after p-completion for a large class of spaces X.

Now, more generally, let us consider a finite p-group π and an exact sequence

$$1 \to \kappa \to \pi \to \sigma \to 1$$

with $\sigma \approx \mathbf{Z}/p$. The formula $(\ \ )^{h\pi} = ((\ \ )^{h\kappa})^{h\sigma}$ leads to an induction method à la Dwyer for computing $(\Sigma^\infty X)^{h\pi}$. This works in particular for $(\Sigma^\infty S^0)^{h\pi}$ and gives a proof by induction of the Segal conjecture for finite p-groups quite different in its spirit from the one by G. Carlsson.

M. Mahowald

bo-cover of S^3

The usual way of understanding the "Greek letter" family of classes in $\pi_* S^0$ is to consider maps of finite complexes $V(n) \to S^0$, where $V(n)$ has a selfmap v such that $MU_*(v) \neq 0$. Such maps do not desuspend to the 3-sphere since $\Omega^{\infty-3} MU = SU \times X$ for a suitable X. Thus in order to understand a version of the "Greek letter" family on S^3 one needs to "remove" the part of S^3 represented by SU. The result is called the bo-cover of S^3. Let

$$BSp(1) \xrightarrow{i} BSp \xrightarrow{f} BO[8]$$

where f is defined by $H_*(f)$ is epi and $f \circ i \sim 0$. Then i lifts to $\bar{\imath} : BSp(1) \to F(f)$ where $F(f)$ is the fiber of f. The bo-cover of S^3 is the fiber $F(\bar{\imath})$ of $\bar{\imath}$

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THEOREM 1. $H^*(F(\bar{\imath}); \mathbf{Z}/2) = \mathbf{Z}/2[x_{8k+4}] \otimes E[y_{8k+5}] \quad k \geq 1.$

THEOREM 2. *There is a map $g : F(\bar{\imath}) \to BP\langle 2\rangle \wedge M_{2^k}$ with $H_{12}(g)$ an isomorphism.*

From these a beginning of v_2-phenomena on S^3 can be understood.

S. Mitchell

A double coset formula and splitting BGL_n

This is joint work with Stewart Priddy. If K is a ring, we consider the natural filtration

$$BGL_1K \subseteq BGL_2K \subseteq \cdots \subseteq BGL_nK.$$

As a useful convention here we regard the symmetric group Σ_n as the general linear group of the "zero field".

THEOREM. *Let $K = \mathbf{R}, \mathbf{C}, \mathbf{H}, \mathbf{F}_q (q = p^m)$, or zero. If $K = \mathbf{F}_q$, assume p has been inverted. Then the filtration stably splits:*

$$BGL_nK \cong \bigvee_{i=1}^{n} BGL_iK/BGL_{i-1}K.$$

The theorem is new when $K = \mathbf{R}$ or $\mathbf{F}_q$. The case $K = 0$ was done by Kahn and Priddy, and the cases $K = \mathbf{C}$ or $\mathbf{H}$ by Snaith, who obtained similar but much coarser splittings for $K = \mathbf{R}$ or $\mathbf{F}_q$. We provide a uniform proof of all cases. The splitting maps are defined as the following composites:

$$\underset{(i+j=n)}{BGL_nK \xrightarrow{\mathrm{tr}} BGL_iK \times BGL_jK} \to BGL_jK \to BGL_jK/BGL_{j-1}K.$$

Here tr is the transfer associated to the inclusion $GL_i \times GL_j \to GL_{i+j}$. When $K = 0$, the double coset formula (for $\Sigma_i \times \Sigma_j$ in Σ_n) leads quickly to a proof of the splitting. An essentially identical double coset formula holds in the general case if $GL_i \times GL_j$ is first replaced by the corresponding parabolic subgroup P_{ij}. But $BP_{ij} \to B(GL_i \times GL_j)$ is an equivalence—provided p is inverted when $K = \mathbf{F}_q$. This idea leads to a general double coset formula for Levi factors of parabolic subgroups of (*a*) real reductive Lie groups and (*b*) finite Chevalley groups. In the former case we use a generalization of Feshbach's double coset formula, in which G-CW-decompositions are replaced by "equivariant bundle decompositions".

N. Oda

Equivariant phantom maps

This is joint work with Y. Shitanda. We shall consider the category of spaces with base point $*$ which is fixed under G-action. Let Y be a G-CW complex and $i^n : Y^n \to Y$ be the inclusion map from the G-n-skeleton of Y. Then a G-map $f : Y \to X$ is called an equivariant phantom map iff $f \circ i^n \simeq *$ for all n.

Let $\tau : Y_\tau \to Y$ and $\rho : X_\rho \to X$ be the homotopy fibres of the equivariant localization $\ell : Y \to Y_Q$ and the equivariant completion $\hat{e} : X \to X^\wedge = \prod_p X_p^\wedge$ respectively. Let X and Y be G-simple G-CW complexes of finite type.

PROPOSITION.

(1) $[S^n Y_Q, X^\wedge] = *$ *for all* $n \geq 0$.

(2) $[S^n Y_\tau, X_\rho] = *$ *for all* $n \geq 0$, *if* $\pi_1(X^H)$ *and* $\pi_1(Y^H)$ *are finite abelian groups for all closed subgroups* H *of* G.

THEOREM A. *Assume* Y *has finite* G*-cells or* X *has finite Postnikov system, then*

(1) $\tau^* : [Y, X] \to [Y_\tau, X]$ *is a monomorphism if* $\pi_1(Y^H)$ *is a finite group for all closed subgroups* H *of* G.

(2) $\ell^* : [Y_Q, X] \to [Y, X]$ *is trivial under the same condition as in (1).*

(3) $\hat{e}_* : [Y, X] \to [Y, X^\wedge]$ *is a monomorphism.*

LEMMA. $f : Y \to X^\wedge$ *is a phantom map if and only if* $f \simeq *$.

THEOREM B. *Assume* $\pi_1(X^H)$ *and* $\pi_1(Y^H)$ *are finite groups for all closed subgroups* H *of* G. *Then the following conditions on* $f : Y \to X$ *are equivalent.*

(1) f *is an equivariant phantom map*

(2) $f \circ \tau \simeq *$

(3) $\hat{e} \circ f \simeq *$

This theorem is an equivariant version of a result of A. Zabrodsky.

F. P. Peterson

A-generators of $H^*(RP^\infty \wedge \cdots \wedge RP^\infty; \mathbf{Z}_2)$

Let $P = H^*(RP^\infty; \mathbf{Z}_2)$, $\bar{P} = \bar{H}^*(RP^\infty; \mathbf{Z}_2)$. We want to find a minimal set of a A-generators for $\bar{P}^{\otimes n}$. As a first approximation, we suggest a conjecture.

CONJECTURE. *There is a set of A-generators for $\bar{P}^{\otimes n}$ whose dimensions d satisfy*

$$\alpha(d+n) \leq n.$$

Here are 2 corollaries of this conjecture.

COROLLARY 1. *There is a set of A-generators for $H^*(BO)/(\bar{H}^*(BO))^{n+1}$ whose dimensions d satisfy $\alpha(d) \leq n$.*

COROLLARY 2. *Let M^N be a C^∞, closed N-manifold such that the product of $n+1$ Stiefel-Whitney classes of M vanish. (e.g., M has Lusternik-Schnirelmann category $\leq n$). Then $\alpha(N) \leq n$ or M is a boundary.*

We have the following evidence.

THEOREM 1. *(Joint with E. Campbell and P. Selick.) The conjecture is true for $n \leq 5$.*

THEOREM 2. *A minimal set of A-generators for $\bar{P} \otimes \bar{P}$ is:*

$$x^{2^r-1} \otimes x^{2^s-1}, \quad r, s > 0 \qquad \text{and}$$

$$x^{2^r-1+2^s} \otimes x^{2^s-1}, \; 0 \leq r < s.$$

There is a relation between these results and those of W. Singer. He constructs a map $\theta : \mathrm{Tor}_s^A(\mathbf{Z}_2, \mathbf{Z}_2) \to (\mathbf{Z}_2 \otimes_A P^{\otimes s})^{GL(s,\mathbf{Z}_2)}$ and proves that θ is quite non-trivial. A solution to our problem could help understand $\mathrm{Tor}_s^A(\mathbf{Z}_2, \mathbf{Z}_2)$.

S. Priddy

Characterizations of summands in the classifying space of a finite group

For a finite p-group P, it is known that each summand of a stable decomposition

$$BP = \bigvee_{i=1}^{N} X_i$$

corresponds to a unique (up to isomorphism) minimal subgroup of P. We prove various theorems about the existence of summands derived from subgroups. The proofs use the Segal conjecture. For the summand

$$L(n) = \Sigma^{-n} Sp^{p^n}(S^0)/Sp^{p^{n-1}}(S^0)$$

of $B(\mathbf{Z}/p)^n$ we have

THEOREM. *Let n be the p-rank of P. Then $L(n)$ is a summand of BP if and only if P contains a self-centralizing p-torus E of rank n.*

By "self-centralizing" we mean $E = C_P(E)$, its centralizer. An easier result concerns $\mathbf{Z}/p$ itself; namely, $B\mathbf{Z}/p$ is a summand in BP if and only if $\mathbf{Z}/p$ is a retract of P. Concerning the center $Z \overset{i}{\hookrightarrow} P$, let $K \subset \{BP, BP\}$ be the ideal generated by maps $BP \to BZ \overset{Bi}{\hookrightarrow} BP$. Then

PROPOSITION. *Assume no central subgroup is a direct factor in P. Then $K \otimes \mathbf{F}_p$ is nilpotent and thus no summands come from the center.*

M. Raußen

Smooth representation forms

A G-representation form is a locally linear G-manifold whose underlying homotopy type is a homotopy representation in the sense of tomDieck and Petrie. Question: Which homotopy representations can be realized as G-CAT-representation forms?

$$(CAT = TOP,\ PL,\ O)$$

Joint work with Ib Madsen gave an easy answer under the following restrictions: G a cyclic group of odd order, $CAT = TOP$ or PL, and X a homotopy representation satisfying the gap-hypothesis. Then, X is CAT-realizable iff, for all isotropy subgroups $H \leq G$, X is H-homotopy linear ($X \simeq_H SU$, U an $\mathbf{R}H$-module).

In the smooth category (G, X as above), a homotopy representation X with $X * SV \simeq_G SW$ is realizable iff there is a G-map $X \to \mathrm{Epi}(W', V' \oplus \mathbf{R})$ for some $\mathbf{R}G$-modules V', W' such that $[\underline{W}' - \underline{V}'] = [\underline{W} - \underline{V}] \in JO_G(X)$. For a smooth representation form M, such a map arises from a stable trivialization $\tau M \oplus \underline{\mathbf{R}} \oplus \underline{V}' \cong \underline{W}' \downarrow M$ of the tangent bundle. An orthogonal splitting of this equation over M^H, H an isotropy subgroup, implies desuspensions of stunted G-spheres. A study of obstructions to those desuspensions allows to single out dimension functions of PL-representation forms which cannot be realized smoothly.

D. C. Ravenel

The EHP sequence, v_1-periodicity, and a computer calculation of the homotopy groups of spheres

Let $\pi_{n,k}$ denote the 2-component of $\pi_{n+k}(S^n)$. The EHP sequence,

$$\cdots \longrightarrow \pi_{n,k} \xrightarrow{E} \pi_{n+1,k} \xrightarrow{H} \pi_{2n+1,k-n} \xrightarrow{P} \pi_{n,k-1} \longrightarrow \cdots.$$

is the long exact sequence of homotopy groups associated with a certain fibration. In principle, it enables one to compute the homotopy groups of spheres inductively, using our knowledge of $\pi_*(S^1)$ to start the induction. If the groups $\pi_{n,k}$ and $\pi_{2n+1,k-n}$ are known inductively, then we can find $\pi_{n+1,k}$ if we know the behavior of the map P. We have written a computer program which does the necessary bookkeeping and which computes P in easy cases; human intervention is required to determine the map P in the more difficult cases.

Thanks to the work of Mahowald, Gray and Thompson, we know how P behaves in almost every case when the element is in the image of the J-homomorphism. Such elements are said to be *v_1-periodic*. They are quite numerous and it is desirable to replace the EHP-sequence with a gadget having similar inductive properties from which the v_1-periodic elements have been excluded. *The main object of this talk is to describe a construction that does this.*

We define an *EHP-system* to be a collection of fibrations having certain properties which enable one to compute homotopy groups inductively as above. The standard example involves the spaces $\Omega^n S^n$. If we map this EHP-system to one involving the spaces $\Omega^\infty J \wedge \Sigma^\infty RP^{n-1}$, we capture all of the information about v_1-periodic elements. The fiber of this map is also an EHP-system, and it is the basis of our computer calculation. Details can be found in a preprint available from the author.

All of the above remarks can be generalized to odd primes. With our program we have computed up to $k = 24$ for $p = 2$, $k = 80$ for $p = 3$, and $k = 600$ for $p = 5$. We have found in each case that the total number of basis elements in dimensions $\leq k$ grows cubically with k, with the coefficient being roughly $p^{-6.5}$. The number of v_1-periodic elements grows quadratically, but with a much larger coefficient.

N. Ray

Combinational models for MU_*, BP_* and some of their modules

Let π be a partition of a finite set with n_1 parts of size 2, n_2 parts of size 3, ..., and call the monomial $\phi_1^{n_1}\phi_2^{n_2}\cdots$ its type. Let Φ_* be the polynomial algebra $\mathbf{Z}[\phi_1, \phi_2, \ldots]$.

There are two apparently distinct extensions

$$\Phi_* \subset U_* \subset \Phi_* \otimes \mathbf{Q} \quad \text{and} \quad \Phi_* \subset {}^L\Phi_* \subset \Phi_* \otimes \mathbf{Q},$$

both defined by combinatorial means. In fact it is possible to identify these, by using techniques of the Roman-Rota umbral calculus.

It then follows that both U_* and ${}^L\Phi_*$ agree with MU_*, by mapping ϕ_i to the manifold dual to the sum of the 2-dimensional cohomology classes in $\underbrace{S^2 \times \cdots \times S^2}_{i+1}$.

These constructions help explain the occurrence of Stirling numbers in MU theory, as well as providing alternative proofs of the theorems of Quillen and Hattori-Stong.

Choosing p-typical partitions to be those with parts only of size p, $p^2, \ldots$ yields the projection onto, and the corresponding description of, BP_*.

Our eventual hope is to describe spaces $\Omega^{2\infty}(MU(\infty)\wedge X)$ and $\Omega^{2\infty}(BP(\infty)\wedge X)$ explicitly and combinatorially!

R. Schwänzl

Nonconnective delooping of K-theory of A_∞ ring spaces

This is joint work with Z. Fiedorowicz, R. Steiner, and R. Vogt.

In the context of extending work of Burghelea and Fiedorowicz relating hermitian K-theory of simplicial rings rationally to the space of simple h-equivalences mod homeomorplusms, it seems necessary to work with A_∞ ring spaces. A technical point is to pass from the given A_∞ ring X to its suspension sX, which is the space $EmX \times_{mX} cX$, where cX = bounded operators on X and mX = compact operators, turned into a monoid.

THEOREM I. *If X is a ringlike A_∞ ring, then so is sX.*

THEOREM II. $\Omega((B\widehat{GL}(sX))^+) = K_0(\pi_0 X) \times (B\widehat{G\ell}X)^+$.

This gives the nonconnective delooping of the title.

One can check that if I is an ideal in a semiring R, then the two-sided Bar construction $B(R, I, *)$ has a simplical semiring structure also. To prove Theorem I, one used the parametrized lifting theorem. Naturality ensures that one gets the correct homotopy type. One has a fibration

$$B\widehat{G\ell}mX \to B\widehat{G\ell}cX \to B\hat{E}sX,$$

where $E = \{$ elementary matrices$\}$. Theorem II follows readily from

PROPOSITION.

i) *This fibration survives the "+" construction as a fibration;*
ii) $H_*(B\widehat{GL}cX) = 0$

L. Smith

Extending representations of cyclic groups to dihedral groups

This is a report on joint work with D. Notbohm. Let p be an odd prime, G a compact connected Lie group, and $\rho : \mathbf{Z}/p \hookrightarrow G$ a representation. There is the automorphism $\alpha(x) = -x$ of $\mathbf{Z}/p$, and we ask the question: when is α induced by an inner automorphism of G? If $G = S^1$ the answer is never. This suggests we assume G simply connected. We then observe: if the answer were yes, then at the classifying space level, $\operatorname{Im} B\rho^* \subset H^*(B\mathbf{Z}_p; \mathbf{F}_p)^\alpha$. Using this, one sees the representation of $\mathbf{Z}_p$ in $SU(3)$ given by the matrix

$$\begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda^{-2} \end{pmatrix}, \qquad \lambda = \exp\left(\frac{2\pi i}{p}\right)$$

does not have the desired property. However any representation $\mathbf{Z}/p \hookrightarrow S^3$ does have the desired property. So we ask: When does ρ factor through $S^3 \hookrightarrow G$? We begin by offering a new proof of a theorem of Hopf/Bott-Samelson assuring the existence of a "nice" S^3 in G.

THEOREM 1. *If G is a compact simply connected Lie group, then associated to each root β of G is an $S^3 \overset{\lambda\beta}{\hookrightarrow} G$. If β is dominant, $[\lambda\beta]$ is a generator of $\pi_3(G)$.*

Our main result is that under suitable technical conditions and an obvious necessary condition the answer is yes at the level of classifying spaces at p:

THEOREM 2. *Assume $\rho : \mathbf{Z}/p \hookrightarrow G$, G has no p-torsion, and*

$$\operatorname{Im} B\rho* = \mathbf{F}_p[z] < H^*(B\mathbf{Z}/p; \mathbf{F}_p), \quad |z| = 4.$$

Then there exists

$$\begin{tikzcd} & BS^{3\,\wedge}_p \arrow[d, "f"] \\ B\mathbf{Z}/p \arrow[ur, "B\theta"] \arrow[r] & BG^{\wedge}_p. \end{tikzcd}$$

E. Vallejo

Polynomial operations on stable cohomotopy and Burnside rings

Following Segal we study polynomial operations $\eta : \pi_s^0 \to \pi_s^0$ on the zeroth stable cohomotopy. The idea is to define them on covering maps, and then to extend them by means of the group completion theorem. In order to do that we observe that a group completion $g : X \to Y$ induces a natural transformation $g_* : [-,X] \to [-,Y]$ which is not only universal with respect to additive transformations $\eta : [-,X] \to [-,Z]$, Z being a group-like space; but also with respect to polynomial transformations. Let A denote the Burnside ring functor and $\hat{A}$ be the adic completion of A with respect to the augmentation ideals. The observation above together with the Segal conjecture yields the following, where Pol( ) denotes the ring of polynomial operations.

THEOREM 1. *There is a ring isomorphism* $\mathrm{Pol}(\pi_s^0, \pi_s^0) \cong \mathrm{Pol}(A, \hat{A})$.

We then proceed to study polynomial operations $\eta : A \to R$, where R satisfies some axioms. For example R can be A, $\hat{A}$, or the representation ring functor $R(-,E)$ over a field E of characteristic 0. Let $S(m)$ denote the symmetric group of degree m. There is a family of "generating" operations $F(a)$, $a \in RS(m)$, $m \geq 0$; and there is a filtration of $RS(m)$ by ideals

$$0 = J_m^0 \subseteq J_m^1 \subseteq \cdots \subseteq J_m^m = RS(m), \quad m \geq 1.$$

One has

THEOREM 2. *Each polynomial operation $\eta : A \to R$ of degree $\leq n$ can be written in a unique way as a sum $\eta = \sum_{m=0}^{\infty} F(b_m)$, where $b_m \in RS(m)$ and if $m > n$, then $b_m \in J_m^n$.*

S. Zarati

Cohomology of symmetric group and the Quillen map

This is joint work with J. H. Gunawardena and J. Lannes. Let G be a compact Lie group. We denote by $\mathcal{C}(G)$ the category that has

i) as objects the elementary abelian 2-groups V of G, and

ii) as morphisms the linear maps induced by conjugations in G.

We have the following map (called the Quillen map)

$$q_G : H^*(BG;\mathbf{F}_2) \longrightarrow \varprojlim_{C(G)} H^*(BV;\mathbf{F}_2).$$

Let S_n the symmetric group of degree n and $S_n \int G$ the wreath product of S_n and G. We prove:

THEOREM. *If q_G is an isomorphism then $q_{S_n \int G}$ is an isomorphism. In particular q_{S_n} is an isomorphism.*

Let A be the mod 2 Steenrod algebra. Our method to prove this theorem is to forget the algebra structure of $H^*(BG;\mathbf{F}_2)$ and to focus on its structure as unstable A-module. We study the concept of Nil-closed modules, and by using elementary homological algebra in the category of unstable A-modules we show that $H^*(BG;\mathbf{F}_2)$ being Nil-closed is necessary and sufficient for q_G to be an isomorphism. We show that if $H^*(BG;\mathbf{F}_2)$ is Nil-closed then $H^*(B(S_n \int G);\mathbf{F}_2)$ is Nil-closed. The result given by the theorem is not true (in general) when we replace $H^*(\ \ ;\mathbf{F}_2)$ by $H^*(\ \ ;\mathbf{F}_p)$, p an odd prime.

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