MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 40/1987

Hysteresis-Phänomene: Modelle, Theorie und Verfahren

13.9. bis 19.9.1987

Die Tagung fand unter der Leitung von Herrn K.-H. Hoffmann (Augsburg) und Herrn J. Sprekels (Augsburg) statt.

Im Mittelpunkt des Interesses standen Fragen der mathematischen Behandlung einer Reihe physikalisch-technischer Problemstellungen, die mit dem Auftreten von Hysteresis einhergehen: Ferromagnetismus, Thermostaten und allgemeinere Schaltsysteme, das Wachstumsverhalten von Bakterien, elastisch-plastische Hysterese in gummiähnlichen Materialien und in Legierungen mit Gestalterinnerungsvermögen, optische Bistabilität in Lasern und die Kristallisation von Polymeren. Einen breiten Raum nahm auch die Untersuchung der Eigenschaften des PREISACH-Operators zur Beschreibung von Hysteresis-Phänomenen ein. Auch numerische Verfahren wurden vorgestellt, z. B. Algorithmen zur Identifizierung des definierenden Maßes beim PREISACH-Operator und Mixed Finite Elements zur Lösung der magnetostatischen Gleichungen.

Die Tagung wurde wesentlich mitgeprägt von einer ungewöhnlich intensiven Diskussion der Vorträge und angrenzender Problemstellungen; eine Reihe neuer Zielrichtungen für weitere Untersuchungen konnten aufgezeigt werden. Gefördert wurde dieser intensive Austausch von Problemen auch durch den bewußt klein gehaltenen Kreis der Tagungsteilnehmer.

Die angenehme Atmosphäre der Tagung, die nicht zuletzt der guten Betreuung durch die Mitarbeiter des Instituts zu verdanken ist, soll besonders erwähnt werden. Im Namen der Tagungsteilnehmer danken wir Herrn Prof. Dr. M. Barner und seinen Mitarbeitern herzlich dafür.

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Vortragsauszüge

H. W. ALT:

The Thermostat Problem

The thermostat problem for ODE is to solve

$$\dot{u} = f(u,s) \quad ,$$

where in the standard case (u,s) ranges in

$$\Sigma := \{(u,s) : (u \leq 1 \quad \text{and} \quad s = 0) \quad \text{or} \quad (u \geq 0 \quad \text{and} \quad s = 1)\} \quad .$$

Furthermore a jump condition is given, which in its weak formulation reads

$$\int_0^T (s - s_0)\dot{\eta} \leq 0 \quad , \quad (s_0 \text{ initial thermostat state})$$

for test functions η with $\eta \geq 0$ in $\{u = 1\}$ and $\eta \leq 0$ in $\{u = 0\}$. For the linear problem existence results of T. I. Seidman and B. Stoth about periodic solutions are reported. The corresponding PDE problem

$$\dot{u} - u'' + f(s) = 0$$

models diffusion with an "atomic" thermostat at each space point. Using an implicit discretization for diffusion and a suitable discretization of the jump condition, the existence of a solution is proved with $f(s)$ replaced by $f(\sigma)$, where $\sigma = s$ except that almost everywhere $f(\sigma) = 0$ and $\sigma \leq s \leq 1$ in $\{u = 1\}$, and $f(\sigma) = 0$ and $0 \leq s \leq \sigma$ in $\{u = 0\}$. Nonuniqueness questions are discussed.

E. BOHL:

Hysteresis in a Sewage Plant

In biological sewage plants degradation of matter via bacteria populations is used to convert toxic material into products which the environment can tolerate. The operation of such plants is sometimes delicate since they tend to suddenly break down for unknown reasons.

A mathematical model was presented whose analysis shows the possibility of hysteretic behaviour. It can offer explanations to some observations described in the literature on this subject.

A. BOSSAVIT:

"Mixed Elements" in Three-dimensional Field Computation

The magnetostatics equations

(1) $\operatorname{div} b = 0 \quad , \quad b = \beta(h) \quad , \quad \operatorname{curl} h = j$

can be presented in a very natural way (as Tonti first pointed out) in the context of cohomology (Fig. 1).

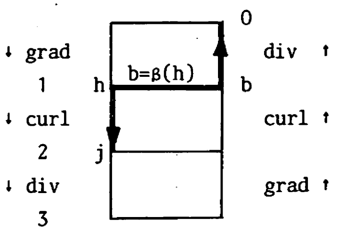


Fig. 1 de Rham's exact sequence (left), its dual (right) and how the magnetostatics equations fit within this structure.

Whitney's work contains a kind of "discrete" cohomology which appears to be home to the discretization of (1). We show that this leads to a new interpretation of the currently popular concept of "mixed elements" (unconventional finite elements with degrees of freedom associated with edges, facets, etc., of the mesh, instead of nodes). This approach allows one to cope with very general laws $b = \beta(h)$, in particular some which exhibit hysteretic behaviour.

M. BROKATE:

Some properties of the Preisach Operator

We define the Preisach hysteresis operator W as a composition of an input-state and a state-output map. Based on this, we characterize continuity, uniform continuity and Lipschitz continuity of W in C^o . We give sufficient conditions for W to operate in $W^{1,p}$ as well as in $C^{o,\lambda}$. Furthermore, we characterize the existence and continuity of the inverse of W .

A. DAMLAMIAN:

Multidimensional Extensions of the Preisach Model

The well-known Preisach model, obtained by superposition of elementary relays in a one-dimensional setting, is used in the representation of ferromagnetism. We give two vector-valued generalisations of this model in a purely phenomenological framework. Each is based on the definition of elementary relays whose behaviours are explained in detail. They are possible multidimensional extensions of the Preisach one-dimensional relays.

Some properties of the hysteresis functionals associated with the various ways of superposing these relays are investigated, namely rate independence, piecewise monotonicity, continuity of the output and vibrostability (in the sense of Krasnoselskii). Reference: A. Damlamian, A. Visintin: Une gênêralisation vectorielle du modele de Preisach pour l'hystêresis, Note CRAS Paris I (297) (1983), 437 - 440.

M. HILPERT:

Interacting Domain Models

In 1956 J. Enderby presented a generalization of the Preisach model, where interaction of the different domains is considered. Instead of having exact thresholds ρ for each domain there are regions $[\rho^N, \rho]$ where the domain may change from one state to another. If the input increases over ρ^N some part of the domain may change from state 0 to state 1, depending on the state of the neighbouring domains, if the input becomes greater than ρ the whole domain must be in state 1. For decreasing output there is a corresponding rule. The state of the domains is given by some number $\varphi(s)$ between 0 and 1. The influence of the neighbouring domains s' is formulated by the condition $\varphi(s) = \int p(s,s')\,\varphi(s')d_\mu s'$, where p is some nonnegative density with $\int p(s,s')\,d_\mu s' \leq 1$. The response of the whole system is the integral of $\varphi(\cdot)$ over all domains. Under suitable conditions we can show that this hysteresis operator W maps piecewise linear functions into $C[0,T]$. Also, W is rate independent and piecewise monotone.

K.-H. HOFFMANN

A Least-squares Technique for the Identification of Hysteresis Loops

A classical model representing several hysteresis phenomena (as they appear in ferromagnetism and in porous media filtration, e.g.) is considered here. It is demonstrated that the parameters of this model can be determined with any desired precision from suitable experiments. For that a special algorithm is developed which is based on least squares procedure and finite element technique. Numerical experiments as well as error estimates and order of convergence proof were presented. These results will be used in practical applications of electromagnetism in the future.

W. JÄGER:

Hysteresis in Growth of Bacteria

Growth of bacteria in spatial gradients of nutrients may show spatial patterns similar to precipitation patterns known as Liesegang phenomena. A report on the analysis and simulation of a model system, a diffusion reaction system with a hysteresis functional, is given and the results are compared with experiments. Recent experiments (by Winipenny and Jäger) relating the growth coefficient of the bacteria to the pH-value show that a new type of hysteresis functionals has to be introduced, taking adaption time into account. A new model is suggested. The equations lead to the well-known difficulties with discontinuous hysteresis functionals which can be overcome by averaging. The main new message is the fact that the hysteresis loops may depend on time and the hysteresis functionals may be no longer rate independent.

I. MÜLLER:

Simulation of a Pseudoelastic Hysteresis in a System of Rubber Balloons

Rubber balloons have a non-monotone pressure-radius characteristic. This fact leads to an unstable region of radii and to a hysteresis in an inflation-deflation experiment. The hysteresis becomes more and more like a pseudoelastic hysteresis when more and more interconnected balloons are inflated at the same time. The phenomenon simulates some aspects of a phase transition in that - during the inflation and deflation - all balloons but one are either small or big, only one is in transit. Thus the balloons can be said to separate into two phases.

M. NIEZGODKA:

A Thermomechanical Model for Shape-memory Effects

A phenomenological model of the dynamics of martensitic phase transitions in materials exhibiting shape memory is presented. The model refers to metallic alloys like Nitinol, especially. Constitutive relations are introduced within an extended Landau-Devonshire theory. The governing equations are those for momentum and energy balances, strongly coupled each to the other. For a relaxed version of the model, incorporating viscous terms, existence, uniqueness and stability properties are provided. As a verification of the model's relevance, results of a series of numerical experiments are shown.

J. OCKENDON:

Slowly Varying Nonlinear Oscillations with Hysteresis

A discussion will be given of hysteresis in nonlinear oscillators, both finite and infinite dimensional.
Particular attention will be paid to the dynamic response as the detuning parameter (the frequency of the drive mechanism) is slowly varied in the neighbourhood of critical values.

M. A. PINTO:

A Model for Conventional Ferromagnetism

The basic assumptions of the conventional theory of ferromagnetism are presented, in their static approximation (this excludes eddy currents but also more fundamental spin dynamics). The aim of this theory is to describe hysteresis, not to explain it. The Micromagnetic theory is powerful enough to give a qualitative explanation: this is presented in order to gain some physical insight into what is required. A short review of the art shows the necessity for a model that has the correct thermodynamical properties and that takes into account rotations of the magnetic field in a physical way. Such a model is presented and discussed with the help of geometrical tools that seem far more powerful than conventional hysteresis loops. This model has a fine geometrical interpretation, which we feel, is the direct generalization of Preisach diagrams.

W. SCHEMPP:

Nonlinear Laser Optics and Bistable Laser Resonators

Based on the classical Lorentz model and phased-array radar, the lecture describes how to calculate the phase matching condition of n-th harmonic generation in nonlinear crystals by the duality theory of semisimple rings. The underlying group symmetry is also exploited to study nonlinear phenomena that exhibit hysteresis; that is, the steady-state response to an input of the system depends upon the past history of the excitation. Because all memory elements involve hysteresis in one form or another, there is great interest in optical devices that exhibit hysteresis as potential optical memory elements.

T. I. SEIDMAN:

Switching Systems and Modelling of Thermostats

A "Switching System" evolves according to "modes" (semidynamical systems) with selection governed by "switching rules" of a certain form. A discussion is given of some general properties of such systems, including global existence and the possible existence of time-periodic solutions. Of particular significance to the theory is the acceptance of some possibility of non-unique evolution in order to preserve the principle that the limit of solutions is a solution. The relation of this notion to physical thermostats is related to occurence of such systems as reduced order models for (unmodelled) singular perturbations.

J. SPREKELS:

Thermodynamical Processes with Non-convex Free Energies of Ginzburg--Landau-type

We study the thermomechanical processes in a non-viscous one-dimensional heat-conducting solid of constant density $\rho \equiv 1$ which is subjected to heating and loading. We think of metallic solids (typically: Shape Memory Alloys) that respond to changes of the strain ε by an elastic stress $\sigma = \sigma(\varepsilon)$ and to changes of the curvature ε_x of their metallic lattices by a couple stress $\mu = \mu(\varepsilon_x)$. The corresponding free energy has the Ginzburg-Landau form $F = F(\varepsilon, \varepsilon_x, \theta)$, with the absolute temperature θ. If F is not convex in ε, temperature-dependent hysteresis loops may occur in the σ-ε-diagrams. For the case that F has the special form designed to explain the hysteresis loops in Shape Memory Alloys, an existence proof is given for the nonlinearly coupled system of partial differential equations which reflect the balance laws of linear momentum and energy.

A. VISINTIN:

Partial Differential Equations with Hysteresis

We deal with boundary and initial value problems for PDEs

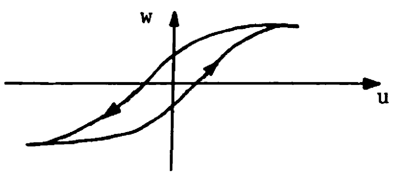
$$\frac{\partial}{\partial t}(u + w) - \Delta u = f \qquad \text{in} \quad (\Omega \times]0,T[)$$

$$\frac{\partial u}{\partial t} - \Delta u + w = f \qquad \text{in} \quad (\Omega \times]0,T[) \;,$$

each one coupled with a constitutive relationship of the form

$$w(x,t) = [F(u(x,\cdot), w^0(x))](t) \;, \; \forall \; t \in]0,T[\;, \text{ a.e. in } \Omega \;,$$

where $F : \mathrm{Dom}(F) \subset C^0([0,T] \times \mathbb{R}) \to C^0([0,T])$ is a continuous "hysteresis functional"



We present existence and uniqueness results for the corresponding weak formulations. Also discontinuous hysteresis functionals can be treated; in that case F is replaced by a multivalued functional, obtained as closure of F in suitable Banach spaces.

K. WILMANSKI:

Ersatzmodel for Crystallizing Polymers

The purpose of this work is to describe macroscopic mechanical properties of crystallizing polymers in terms of a simple model of a single polymeric chain. This ersatz model consists of two parts: amorphous and crystalline (folded and oriented) with two mechanisms, which are switched on and off by the external loading. On the basis of statistical mechanics, one derives the bulk Helmholtz free energy and, consequently, the stress-strain relation. The problem, considered in the work, is one-dimensional (macroscopically) and isothermal. Resulting stress-strain curves contain, apart from the classical non--linearities, connected with large deformations: i) the hysteresis loops, produced by so-called oriented crystallization and ii) residual deformations, depending on the extent of prior loading process (caused by melting of folded crystals). The model, in spite of its simplicity, agrees very well with experimental data and seems to form a proper basis for construction of phenomenological models, such as generalizations of Rivlin-Mooney model of rubber.

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