MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 44/1987

Funktionalanalysis und Operatorentheorie

4.10. bis 10.10.1987

Die diesjährige Tagung über Funktionalanalysis im Mathematischen Forschungsinstitut Oberwolfach fand in der Woche vom 04.10. bis 10.10.1987 statt. Sie stand - wie in den vergangenen Jahren - unter der Leitung der Herren Professoren K.-D. Bierstedt (Paderborn), Heinz König (Saarbrücken) und H.H. Schaefer (Tübingen).

Von insgesamt 45 Teilnehmern kamen hierbei 26 aus dem Ausland (Australien, Belgien, Brasilien, Dänemark, England, Frankreich, Italien, Österreich, Schweiz, Spanien und den USA).

In 32 Vorträgen wurden unter anderem folgende Themenkreise behandelt: Operatorentheorie, Spektraltheorie, Geometrie der Banachräume, Banachalgebren, Banachverbände, Kompaktheit in Funktionenräumen, Kompaktheit in lokalkonvexen Räumen, analytische Funktionen, Baire-1 Funktionen, automatische Stetigkeit, partielle Differentialgleichungen, Hyperräume und Multifunktionen, (DF)-Räume, topologische Tensorprodukte und Frécheträume.

Die anregende Atmosphäre und die Vielzahl der geknüpften Kontakte - die insbesondere für die jüngeren Teilnehmer von unschätzbarem Wert sind - ließen die Tagung, nicht zuletzt auch wegen der vorbildlichen Organisation, zu einem vollen Erfolg werden.

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Vortragsauszüge

W. ARENDT:

Tauberian theorems and asymptotic stability of C_o-semigroups

Let $\mathfrak{T}=(T(t))_{t\geq 0}$ be a one-parameter semigroup on a Banach space E with generator A. Stability of $\mathfrak{T}$ can be characterized by spectral properties:

Theorem (C.Batty & W.A.) Assume that

(i) $\mathfrak{T}$ is bounded

(ii) $P_\sigma(A')\cap i\mathbb{R}=\emptyset$

(iii) $\sigma(A)\cap i\mathbb{R}$ is countable.

Then $T(t)x\to 0$ $(t\to\infty)$ for all $x\in E$.

Here $P_\sigma(A')$ denotes the point spectrum of A'. Conditions (i), (ii) are necessary as well and condition (iii) is best possible in some sense. The proof is given by transfinite induction and based on a new Tauberian theorem for Laplace transforms.

J.BONET:

Weighted (DF)-spaces of continuous functions

We study the meaning of condition (D) of Bierstedt and Meise in the context of projective descriptions of weighted inductive limits of spaces of continuous functions $VC(X)=$ $=\text{ind}\, C(v_n)(X)$ in terms of their projective hull $C\overline{V}(X)$, where $V=$ $=(v_n)$ is a decreasing sequence of strictly positive continuous weights on the topological space X. It turns out that condition (D) on V characterizes the topological invariant "dual density condition" of $C\overline{V}(X)$. This invariant is a natural extension to (DF)-spaces of Heinrich's dual density condition of Fréchet spaces. Our study provides characterizations of several locally convex properties for the space of bounded sequences $l_\infty(E)$ with values in a (DF)-space E, and for the space $L_b(\lambda_1(A),E)$, where $\lambda_1(A)$ is a Köthe echelon space and E is bounded. (Joint work with K.-D.Bierstedt)

B.CASCALES:

Compactness in locally convex spaces

We introduce a class of locally convex spaces $\mathfrak{G}$ which has the following properties:

(1) If $E[\tau]\in\mathfrak{G}$, then $E[\tau]$ has metrizable precompact subsets and E is weakly angelic.

(2) Metrizable locally convex spaces and dual metric spaces belong to $\mathfrak{G}$ and $\mathfrak{G}$ is stable by countable products, countable locally direct sums, separated quotients, subspaces and completions.

(3) For inductive limits of increasing sequences of spaces of the class $\mathfrak{G}$ we prove the equivalence between sequential-retractivity, sequential compact-regularity, compact-regularity and precompact-retractivity.

(4) If $E[\tau]$ is a separable space of the class $\mathfrak{G}$, then E' is w*-analytic.

(5) The weakly compact subsets of spaces of the class $\mathfrak{G}$ are metrizable if and only if they are contained in a separable subspace. Moreover, we characterize the Talagrand-compact spaces as the weakly compact subsets of spaces of the class $\mathfrak{G}$.

(6) If X is a countably determined (or more general web-compact) space and $E[\tau]\in\mathfrak{G}$, then $C_p(X,E)$ is weakly angelic and the weakly compact subsets of this space are Gul'ko compact spaces.

(7) If E and F belong to $\mathfrak{G}$, then $E\varepsilon F$ and $E\hat{\otimes}_\varepsilon F$ are weakly angelic, have metrizable compact subsets and their weakly compact subsets are Talagrand-compact spaces.

V.CASELLES:

Remarks on a duality theorem for vector fields (the finite-dimensional case)

We consider the following problem:

Let $X:\mathbb{R}^n\to\mathbb{R}^n$ be a vector field such that the solutions of the differential equation

$$\left\{ \begin{array}{l} \frac{du}{dt} + X(u(t))=0 \\ u(o) = u_o \end{array} \right\}$$

form a contraction semigroup with respect to a norm N on $\mathbb{R}^n$: (i.e. $\| \delta^X(t)u_o - \delta^X(t)u_1 \|_N \leq \| u_o - u_1 \|_N$ (i.e. X is a N-accretive operator on $\mathbb{R}^n$)).

Suppose that there exists an adjoint vector field X^t (to precise the meaning of this later). Is X^t accretive with respect to the dual norm $\hat{N}$ of N? I.e., do the solutions of

$$\left\{ \begin{array}{l} \frac{du}{dt} + X^t(u(t))=0 \\ u(o) = u_o \end{array} \right\}$$

generate a contraction semigroup with respect to $\hat{N}$?

H.G.DALES:

Derivations from Banach algebras

We discuss some recent results concerning the automatic continuity and form of derivations from Banach algebras A into a Banach A-bimodule. The results include the following:

1. (Read) There is a Banach space X and a discontinuous derivation from $\mathfrak{B}(X)$.
2. (Thomas-resolving a conjecture of Singer and Wermer) Let A be a commutative Banach algebra with radical rad A, and let $D: A \to A$ be a (possibly discontinuous) derivation. Then $D(A) \subset \mathrm{rad}\ A$.

We also discuss the continuity of derivations from Banach algebras of power series (results of Bade-Dales), and weakly amenable Banach algebras (results of Bade-Curtis-Dales).

J.ESCHMEIER

Invariant subspaces and Bishop's property (β)

Recently S.Brown proved that each hyponormal operator on a Hilbert space with sufficiently rich spectrum has a nontrivial invariant subspace. S.Brown's paper is based on a result of M.Putinar which states that each hyponormal operator is a restriction of an operator with a C^∞-functional calculus. S.Brown's result is generalized in a preprint of Albrecht and Chevreau. First, hyponormal operators are replaced by subdecomposable operators, secondly Hilbert spaces are replaced by quotients of closed subspaces of ℓ^p $(1<p<\infty)$.

In the planned talk two results of this kind will be presented. First, it is shown that each subdecomposable operator with rich spectrum defined on a subspace of a space with unconditional basis has a non-trivial invariant subspace. Secondly, the same result is proved to hold true on an arbitrary Banach space for C^∞-restrictions with rich spectrum.

K.FLORET:

Cofinitely generated tensornorms

A tensornorm α on the class NORM of all normed spaces assigns to each $(E,F)\in \mathrm{NORM}^2$ a norm $\alpha(\cdot;E,F)$ on $E\otimes F$ such that

(1) $\varepsilon \leq \alpha \leq \pi$

(2) metric mapping property: If $T_i \in \mathcal{L}(E_i;F_i)$, then
$$\| T_1 \otimes T_2 : E_1 \otimes_\alpha E_2 \to F_1 \otimes_\alpha F_2 \| \leq \|T_1\| \, \|T_2\|$$

If α is a tensornorm on the class FIN of all finite-dimensional normed spaces (analogous definition) there are two ways to extend it to a tensornorm on NORM:

$$\overrightarrow{\alpha}(z;E,F) := \mathrm{INF}\{\alpha(z;M,N) \mid M \in \mathrm{FIN}(E),\ N \in \mathrm{FIN}(F),\ z \in M\otimes N\},$$

$$\overleftarrow{\alpha}(z;E,F) := \mathrm{SUP}\{\alpha(Q_K \otimes Q_L(z);\ E/_K, F/_L) \mid E/_K, F/_L \in \mathrm{FIN}\}.$$

A.Grothendieck in his "Resumé" (Sao Paulo 1956) considered only finitely generated tensornorms (i.e. $\alpha = \overrightarrow{\alpha}$) explicitly, but a consequent use of cofinitely generated tensornorms gives a better understanding of various phenomena concerning the duality of tensornorms and the bounded approximation property. Two applications are given:

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(1) Conditions under which the continuity of

$$T_1 \otimes T_2 : E_1 \otimes_\alpha E_2 \to F_1 \otimes_\beta F_2$$

implies the continuity of $T_1 \otimes T_2$".

(2) If α' is totally accessible (i.e. $\alpha' = \overleftarrow{\alpha}'$) then every Banach space the identity of which is α-integral has the bounded approximation property.

(Joint work with A.Defant)

M.GONZALES:

Sequence operator ideals and semi-Fredholm operators

Several classical operator ideals are defined in terms of the action of their operators on a class of bounded sequences: compact, weakly compact, l_1-singular and (weakly) completely continuous. Such an operator ideal U has associated a semigroup of operators SU definined by means of sequences as well.

We characterize semi-Fredholm operators with finite-dimensional Kernel SF_+ in SU in terms of their restrictions to infinite-dimensional subspaces with identity in U. Moreover there are not such subspaces if and only if SF_+ coincides with SU.

(Joint work with Victor M.Onieva)

G.GREINER:

One-parameter semigroups of positive operators and evolution equations

The linear Cauchy problem

$$\dot{u}(t) = Au(t)\ (t \geq 0);\ u(0) = u_0$$

is well-posed if and only if the closed linear operator A generates a strongly continuous semigroup $(T(t))_{t \geq 0}$. In many concrete situations the underlying space is a Banach lattice and A is a differential operator. Moreover, a maximum principle holds true thus ensuring that the semigroup consists of positive operators.

A survey on results concerning the asymptotic behavior for $t\to\infty$ of semigroups of positive operators is given. Depending on spectral properties of the generator A, the (rescaled) semigroup converges to an equilibrium state (in case there is a dominant spectral value) or it behaves like a rotation semigroup on the unit circle (in case the boundary spectrum is non-trivial). As an application we discuss an equation describing the growth of a cell population.

H.JARCHOW:

Remarks on weak cotype

The relations between cotype, equal norm cotype, and weak cotype were recently investigated by Mascioni and Matter, in particular by considering operators $T:X\to Y$ between Banach spaces such that $(\|Tx_n\|)\in l_{s,\infty}$ for every weak l_r-sequence (x_n) in $X(1\leq r\leq s\leq\infty)$. This yields, for example, another proof for the equivalence of equal norm cotype 2 and cotype 2 (Pisier), and also shows that, in contrast, equal norm cotype q is equivalent with weak cotype q if $q>2$. Further characterizations, e.g. in terms of Banach-Mazur distances, are available.

G.KÖTHE:

Duality of tensor products of convergence-free spaces

Let λ,μ be two convergence-free spaces equipped with the normal topology $T_n(\lambda^x)$. With $\lambda\tilde{\otimes}_n\mu$ we denote the completion of the tensor product $\lambda\otimes\mu$ equipped with the normal topology. With these notations the duality theorem has the form

$$(\lambda\tilde{\otimes}_n\mu)'_n \simeq \lambda^x\tilde{\otimes}_\varepsilon\mu^x \text{ and } (\lambda\tilde{\otimes}_\varepsilon\mu)'_n \simeq \lambda^x\tilde{\otimes}_n\mu^x.$$

The normal topology on the tensor product $\lambda\otimes\mu$ coincides with Grothendieck's inductive topology. With the exception of $\lambda=\mu=\varphi$ and $\lambda=\mu=\omega$ the topology T_n on $\lambda\otimes\mu$ is strictly finer than the topology $T_\varepsilon=T_\Pi$.

Ruckle proved that a continuous linear mapping $A=(a_{ik})$ of a perfect sequence λ into a perfect sequence space μ is continuous for the normal topologies iff $(|a_{ik}|)$ is continuous. In the case of $\lambda=\mu=l^2$ this leads to interesting problems first tackled by Hilbert, Schur and Toeplitz.

N.KUHN:

On Baire-1-functions with values in normed spaces

For topological spaces X and E define:

$B_1(X,E)$:=space of the pointwise limits of continuous functions $X\to E$

$C_1(X,E)$:=space of the functions $f:X\to E$ such that for every closed $\emptyset\neq A\subset X$ the function $f|A$ has a point of continuity.

The following theorems were discussed and applications were given:

(1) If X is paracompact and perfect(=closed subsets are G_δ) and E is a normed space, then $C_1(X,E)\subset B_1(X,E)$.

(2) X Polish or compact and perfect, E normed space. Then: $f\in B_1(X,E) \Leftrightarrow g\circ f\in B_1(X,\mathbb{R})\ \forall\, g\in C(E,\mathbb{R})$

More general results are true, too.

K.B.LAURSEN:

Spectral decompositions as a way of automating continuity

For a linear operator $T, \sigma_\delta(T):=\{\lambda\in\mathbb{C}\,|\,T-\lambda \text{ not onto}\}$.

For $x\in X$ (B-sp): $\rho_T(x):=\{\lambda\in\mathbb{C}\,|\,\exists N(\lambda)\ \&\ f \text{ holom.} \neq 0: N(\lambda)\to X: (T-\mu)f(\mu)\equiv x\}$

Also $S(T):=\rho_T(0)$ and $\sigma_T(x)=\mathbb{C}\setminus\rho_T(x)$.

For $A\subseteq\mathbb{C}$

$$\chi_T(A):=\{x\in X\,|\,\sigma_T(x)\subseteq A\} \text{ and } E_T(A):=\operatorname{span}\{Y\,|\,\sigma_\delta(T|Y)\subseteq A\}.$$

Then $\chi_T(A)\subseteq E_T(A)$ and $S(T)\subseteq\sigma(T|E_T(A))$. The case of closed $E_T(A)$ is discussed, as is the case when T is well decomposable.

A typical application to automatic continuity: $T\in B(X)$, $S\in B(Y)$, X,Y B-spaces; $F:=\sigma_\delta(S)\cap\sigma_\delta(T)$ countable and $E_S(F)$ closed. Then any $\theta:X\to Y$ (linear) is continuous if and only if (S,T) has no critical eigenvalues and either T is algebraic or $E_S(0)=\{0\}$.

S.LEVI:

Hyperspaces and Multifunctions: Some applications to functional analysis

1. (With Z.Słodkowski) Measurability properties of the spectrum in a topological algebra; here the spectrum is viewed as a set-valued mapping. We prove:
   a) Let X be a complex algebra which is a t.v.s over $\mathbb{C}$. Then the set of invertible elements of X is open iff the spectrum is upper semicontinuous (USC). In this case the spectrum of each element is compact.
   b) Let X be a polish algebra with continuous multiplication. Then the set of invertible elements is $F_{\sigma\delta}$ and the mapping $x\to x^{-1}$ is a Borel function of the second class.

2. (With A.Lechieki) We study the problem of extending semicontinuous multifunctions defined on a dense subset of a topological space. We obtain some necessary and sufficient conditions for the USC case and as an application we prove:
   c) Let X be a Namioka space, Y a compact Hausdorff space and $f:X\to C(Y)$. Then f is norm-continuous iff it is continuous for the topology of pointwise convergence and $f|(X-A)$ is norm-subcontinuous on X-A, where A is dense in X and $f|A$ is norm-continuous.

R.MEISE:

Convolution operators on non-quasianalytic Gevrey classes

For d>1 define the classical Gevrey class $\Gamma^{\{d\}}(\mathbb{R})$ by

$$\Gamma^{\{d\}}(\mathbb{R}) := \{f \in C^\infty(\mathbb{R}) \mid \forall K \subset\subset \mathbb{R}\ \exists h>0: \sup_{x \in K} \sup_{j \in \mathbb{N}_0} \frac{|f^{(j)}(x)|}{h^j (j!)^d} < \infty\}.$$

For $\mu \in \Gamma^{\{d\}}(\mathbb{R})'$ the convolution operator $T_\mu : \Gamma^{\{d\}}(\mathbb{R}) \to \Gamma^{\{d\}}(\mathbb{R})$ is defined by $T_\mu(f) : x \to \langle \mu_y, f(x-y) \rangle$. The proof of the following theorem is based on Vogt's work on the projective limit functor of Palamodov and on a sequence space representation for kernels of convolution operators given by Meise.

Theorem. For $\mu \in \Gamma^{\{d\}}(\mathbb{R})'$ the following are equivalent:

(1) $T_\mu : \Gamma^{\{d\}}(\mathbb{R}) \to \Gamma^{\{d\}}(\mathbb{R})$ is surjective.

(2) μ satisfies (i) and (ii).

(i) T_μ admits a fundamental solution.

(ii) The zero set $V(\hat{\mu})$ of the Fourier-Laplace transform $\hat{\mu}$ of μ can be decomposed as $V(\hat{\mu}) = V_0 \dot{\cup} V_1$ with

$$\lim_{\substack{|z| \to \infty \\ z \in V_0}} \frac{|\operatorname{Im} z|}{|z|^{\frac{1}{d}}} = 0 \text{ and } \liminf_{\substack{|z| \to \infty \\ z \in V_1}} \frac{|\operatorname{Im} z|}{|z|^{\frac{1}{d}}} > 0.$$

The analytic significance of the conditions in (2) was explained and it was indicated that the result is also valid in the class $\mathcal{E}_{\{\omega\}}(\mathbb{R})$ of ultradifferentiable functions.

(Joint work with R.W.Braun and D.Vogt)

V.MONTESINOS:

On drop property

The drop $D(x)$ defined by an element x, $\|x\| > 1$, in a Banach space $(X, \|\cdot\|)$, is the convex hull of $\{x\}$ and B_X, the closed unit ball of X. A Banach space $(X, \|\cdot\|)$ has the drop property (in short, DP) if given a non-empty closed subset of X such that $S \cap B_X = \emptyset$, there exists a point $x \in S$ with the property $D(x) \cap S = \{x\}$. This concept was introduced by S.Rolewicz (Studia Math.85,1(1986),27-35) and he proved that every uniformly convex Banach space has (DP) and that (DP) implies reflexivity. We prove that for a Banach space $(X, \|\cdot\|)$, the following conditions are equivalent: (i) X has (DP), (ii) X has property (α) (i.e., small slices of B_X have small Kuratowski index of non-compactness), (iii) X has the Kadec property (i.e., the w and $\|\cdot\|$ topologies coincide on the unit sphere S_X) and X is reflexive, and (iv) X has the Kadec-Klee property (i.e., the same sequences converge for w or $\|\cdot\|$ in S_X) and X is reflexive. Then, a Banach space is reflexive if and only if it can be renormed with an equivalent (DP)-norm. Some stability properties are derived.

J.MUJICA:

Holomorphic functions and the Michael problem

It is assumed that all topological algebras are complex, commutative, Hausdorff and have an identity element. The following theorem is proved.

Theorem. The following conditions are equivalent.

(a) For each complete, locally m-convex algebra A, every homomorphism $h:A\to\mathbb{C}$ is bounded.

(b) For each Fréchet algebra A, every homomorphism $h:A\to\mathbb{C}$ is continuous.

(c) For each Banach space E, every homomorphism $h:\mathcal{H}(E)\to\mathbb{C}$ is bounded.

(d) There exists an infinite-dimensional Banach space E such that every homomorphism $h:\mathcal{H}(E)\to\mathbb{C}$ is bounded.

(e) For each Banach space E, every homomorphism $h:\mathcal{H}_b(E)\to\mathbb{C}$ is continuous.

(f) There exists an infinite-dimensional Banach space E such that every homomorphism $h:\mathcal{H}_b(E)\to\mathbb{C}$ is continuous.

M.M.NEUMANN:

Decomposable operators and intertwining linear transformations

The lecture introduces the class of super-decomposable operators for which it is possible to give a very useful algebraic description of the spectral maximal spaces in the absense of non-trivial divisible subspaces. This class includes, for instance, all spectral operators in the sense of Dunford, all A-spectral operators in the sense of Colojoară-Foias, all multiplication operators on regular Banach algebras, and certain convolution operators on locally compact abelian groups. After developing the basic theory, the rôle of super-decomposable operators is examined in the context of automatic continuity theory. In particular, necessary and sufficient conditions on a pair of super-decomposable operators are given which ensure the continuity of every generalized intertwining linear transformation. Among the corollaries are the automatic continuity of all periodically-invariant linear systems on $L^p(\mathbb{R})$ for $1 \leq p < \infty$ as well as the automatic continuity of linear transformations intertwining certain convolution operators. This includes the solution of problems posed by B.E.Johnson, N.Jewell, and others. Further applications to module derivations and generalizations of the theory to well-decomposable operators are mentioned.

J.ORIHUELA:

Compactness in function spaces

We describe a class of topological spaces X such that $C_p(X)$, the space of continuous functions on X endowed with the topology of pointwise convergence, is an angelic space. This class contains the topological spaces with a dense and countably determined subspace; in particular the topological spaces which are K-analytic in the sense of G.Choquet. Our results include previous ones of A.Grothendieck, J.L.Kelley and I.Namioka, J.D.Pryce, R.Haydon, M.DeWilde, K.Floret and M.Talagrand. As a consequence we obtain an improvement of the Eberlein-Šmulian theorem in the theory of l.c.s. This result allows us to deduce, for instance, that (LF)-spaces and dual metric spaces, in particular (DF)-spaces of Grothendieck, are weakly angelic. This results answer a question posed by K.Floret. Moreover, for this class of topological spaces X, called web-compact spaces, it is proved that every compact subset of $C_p(X)$ is a Gulko compact space and so they are metrizable if and only if they are separable, even more they have a dense G_δ-set which is metrizable.

A.PELCZYNSKI:

Theorems of Hardy and Paley for vector-valued analytic functions and related classes of Banach spaces

The talk is based on a joint paper with O.Blasco of the same title. We investigate Banach spaces with the property that for the Fourier coefficients of analytic functions with values in these spaces analogues of the Hardy inequality and the Paley gap theorem hold. We show that the vector-valued Paley theorem is valid for a large class of Banach spaces (necessarily of cotype 2) which includes all Banach lattices of cotype 2, every Banach space whose dual is of type 2 or is a C*-algebra. For the trace class S_1 and the dual of the algebra of all bounded operators on a Hilbert space a stronger result holds, namely the vector-valued analogue of the Fefferman theorem on multipliers from H^1 into l^1; in particular for the latter spaces the vector-valued Hardy inequality holds. The vector-valued Hardy inequality is also true for every space of type > 1 (Bourgain).

W.RICKER:

Spectral properties of the Laplace operator in $L^p(\mathbb{R})$

This talk will discuss the operator $L = -d^2/dx^2$ in $L^p(\mathbb{R})$, $1<p<\infty$, $p\neq 2$, from the point of view of functional calculi. For $p=2$ it is the case that L is self-adjoint and, accordingly, L has a rich functional calculus. The case when $p\neq 2$ turns out to be quite different, although some positive results are still possible. These results will form the theme of this talk.

W.RUESS:

The ergodic theorem for semigroups of nonlinear contractions

Starting from Baillon's original nonlinear ergodic theorem for contraction semigroups in Hilbert space, and extensions to more general Banach spaces by Brézis-Browder, Bruck, Pazy, Reich and others, Kobayasi/Miyadera obtained the following most general version of this result so far:

Kobayasi/Miyadera (1982): Assume that $(S(t))_{t\geq 0}$ is a strongly continuous semigroup of contractions on a closed convex subset C of a uniformly convex Banach space X, and let $S(\cdot)x:\mathbb{R}^+\to C, x\in C$, be a bounded motion such that $\lim_{t\to\infty} \|S(t+h)x-S(t)x\|=\rho(h)$ exists uniformly over $h\in\mathbb{R}^+$. Then

$$\|\,\|-\lim_{T\to\infty} \frac{1}{T}\int_0^T S(t+h)x\,dt=z \text{ exists uniformly over } h\in\mathbb{R}^+.$$

We show that, under the assumptions of this result, much more precise information on the asymptotic behavior of the motion $S(\cdot)x$ can be obtained.

Theorem: Under the assumptions of the Kobayasi/Miyadera theorem, there exist (uniquely determined) elements $y\in C$ and $\varphi\in W_0(\mathbb{R}^+,X)$ such that

(i) $S(\cdot)x=S(\cdot)y+\varphi$ and (ii) $S(\cdot)y$ is almost periodic. (Here $W_0(\mathbb{R}^+,X)$ is the space of all bounded continuous functions φ from $\mathbb{R}^+$ into X for which the set $H(\varphi)$ of translates is weakly relatively compact in $(C_b(\mathbb{R}^+,X),\|\,\|_\infty)$ and for which $0\in w\text{-cl } H(\varphi)$.)

This theorem allows the known nonlinear strong ergodic limit theorems to be derived as a further special case of Eberlein's ergodic theorem from 1949. Moreover, one can reduce the existence of almost periodic solutions to the (nonlinear) abstract Cauchy problem without a priori norm-compactness assumptions.

(Joint work with W.H.Summers)

W.SCHACHERMEYER:

The Radon-Nikodym and related geometrical properties of Banach spaces

The Radon-Nikodym and the Krein-Milman-property of Banach spaces are defined. The former property implies the latter while the converse is an open question.

We show that for strongly regular Banach spaces the two properties are equivalent and derive some corollaries.

J.SCHMETS:

The space of vector-valued bounded continuous functions

Let X be a Hausdorff completely regular space and E be a locally convex space, and denote by CB(X;E) the vector space of the bounded continuous functions on X with values in E endowed with the uniform topology.

We essentially have the following results:

1) If X is paracompact, CB(X;E) is quasinormable (resp. (gDF),(DF)) if and only if E has the same property.

2) If X is pseudocompact or if $\hat{E}$ is semi-Montel, then CB(X;E) is (gDF) (resp.(DF); quasibarrelled) if and only if E is (gDF) (resp.(DF); quasibarrelled and E_b' has property (B)).

3) If E is a (DF)-space, then CB(X;E) is quasibarrelled if and only if we are in one of the following two settings:

a) either X is pseudocompact and E is quasibarreled,

b) or X is not pseudocompact and every bounded subset of E is metrizable.

(Joint research project with K.-D.Bierstedt and J.Bonet)

S.SIMONS:

The continuity of infsup, with applications

Abstract: Let X and Y be nonempty sets and $a,b:X\times Y\to\mathbb{R}$. We give conditions under which the map on $\mathbb{R}_+^2$ defined by

$$(\lambda,\mu)\to \inf_X \sup_Y(\lambda a+\mu b)$$

is continuous. Our results imply infinite-dimensional generalizations both of a result of Fan on the equilibrium value of a system of convex and concave functions and also of a result of Aubin on eigenvalues of a multifunction. Our results also have applications to matrix theory (the von

Neumann-Kemeney theorem and the Perron-Frobenius theorem). We use the Hahn-Banach theorem and do not use any fixed-point related concepts.

W.H.SUMMERS:

Positive limit sets consisting of a single periodic motion

For a dynamical system $\pi:\mathbb{R}\times X\rightarrow X$ on a complete metric space X, a longstanding problem has been that of finding topological criteria for the existence of periodic motions (cf. [G.R.Sell, J.Differential Equations 2(1966), 143-157]). This talk will summarize recent joint work with W.M.Ruess (Essen) in which we characterize those positively Lagrange stable motions of π which are either periodic or for which the corresponding positive ω-limit set consists of a single periodic motion.

M.VALDIVIA:

Some properties of Fréchet spaces

If A is a subset of a Fréchet space E, we set $\tilde{A}$ to denote the closure of A in $E''[\sigma(E'',E')]$. The following results are given:

a) Let E be a Fréchet space with strong bidual $E''[\beta(E'',E')]$ separable. Let G be a closed subspace of $E''[\beta(E'',E')]$ with $E\subset G$. Then there is a closed subspace F of E such that $E+\tilde{F}=G$

b) A Fréchet space is totally reflexive if and only if there is a sequence (X_n) of reflexive Banach spaces such that

   E is isomorphic to a closed subspace of $\prod_{n=1}^{\infty} X_n$.

c) As consequence of b), if E_1 and E_2 are totally reflexive Fréchet spaces then $E_1\times E_2$ is totally reflexive, which is the solution of a problem of Grothendieck (Summa Brasil. Math.1954).

D.VOGT:

Projective limits of (DF)-spaces

Let $(a_{j;k,n})$ be an infinite matrix with positive entries increasing in k, decreasing in m; $1 \leq p \leq \infty$. Put for $1 \leq p < \infty$

$$X_{k,m} = \{x=(x_1,x_2,\ldots):(x_j a_{j;k,m})_j \in l^p\} \quad (c_0 \text{ for } p=\infty);$$

$$X_k = \bigcup_m X_{k,m} \; ; \quad X = \bigcap_k X_k \; ;$$

$X^*_{k,m}$, X^*_k the respective duals; $X^* = \bigcup_k X^*_k$.

Theorem: The following are equivalent:

(1) $\forall \mu \, \exists n,k \; \forall m,K \, \exists N,S \; \forall j: \dfrac{1}{a_{j;k,m}} \leq S \max \left(\dfrac{1}{a_{j;K,N}}, \dfrac{1}{a_{j;\mu,n}}\right)$.

(2) $\mathrm{Proj}^1 (X_k)_k = 0$.

(3) X is bornological.

(4) X is barrelled.

(5) X'_σ is sequentially complete.

(6) Every bounded set in X'_σ is contained in some X^*_k and bounded there.

(7) X'_b is complete.

(8) X^* is regular.

(9) Every sequentially continuous linear form on X is continuous.

(10) Every sequentially continuous linear map into a locally convex space (Banach space) is continuous.

(11) $X' = \{y=(y_1,y_2,\ldots):\sum_j |x_j||y_j|<\infty \text{ for all } x\in X\}$.

This contains and extends results of Grothendieck, Palamodov and Retakk, Krone-Vogt. (1) was evaluated in a special example which has analytical applications (see lecture of R.Meise in this conference).

L.WAELBROECK:

Quotient bornological spaces and applications

In 1956, N.Bourbaki observed that one could generalize my

thesis, dealing with a quasi-complete algebra with a hypocontinuous multiplication. My answer is that one can generalize even more, to consider an algebra on which a "boundedness" is defined, including among others the condition that every bounded subset is contained in a completant bounded set B where B is completant if A_B is a Banach space if A_B is the space absorbed by B with the Minkowski functional of B.
In my Thèse d'Agrégation (Habilitationsschrift 1960), I used b-algebras and b-ideals of b-algebras. Using again the holomorphic functional calculus, I constructed $f(a_1,\ldots,a_n)$ where the elements a_i belong to a commutative b-algebra A. If one can construct both f(a) and f(a') then one is able to prove that f(a)-f(a') belongs to the ideal $idl(a_1-a'_1,\ldots$ $\ldots,a_n-a'_n;A)$. This is useful: observe that the difference belongs to the ideal, not to its closure.
I ended my talk by speaking of four quotient spaces considered at this period: the continuous germs, the singularities of distributions, the hyperfunctions, and the New Generalized Functions due to J.-F.Colombeau.

L.WEIS:

Banach lattices with the subsequence splitting property

A Banach lattice X has the subsequence splitting property if every bounded sequence $f_n \in X$ can be written as

$$f_{n_k} = g_k + h_k$$

where the g_k's have disjoint support and the h_k's form an equiintegrable set. This splitting has proven to be useful in studying the structure of Banach lattices and positive operators. We characterize this property in terms of the ultra-power of X, uniform order-continuity conditions and finite representability of l_∞^n's.

M.WOLFF:

<u>On the dilation of C_0-semigroups of positive contractions on an L^1-space</u>

Let $\mathfrak{T}=(T_t)_{t\geqq 0}$ be a C_0-semigroup of positive contractions on the space $E=L^1(\Omega,\mu)$ (where (Ω,μ) is without l.o.g. a probability space). Then there exists another space $\hat{E}=L^1(\hat{\Omega},\hat{\mu})$ containing E as a sublattice, there exists a strongly continuous group of isometric lattice isomorphisms and a positive projection Q from $\hat{E}$ onto E such that the following diagram commutes for all $t\geqq 0$

$$\begin{tikzcd} E \arrow[r, "T_t"] \arrow[d, hook] & E \\ \hat{E} \arrow[r, "U_t"'] & \hat{E} \arrow[u, "Q"'] \end{tikzcd}$$

The proof is based on an abstract approximation principle for dilations and uses heavily nonstandard analysis.

V.WROBEL:

<u>Joint spectra of linear operators</u>

After discussing algebraic concepts of joint spectra $Sp(a_1,\dots,a_n;A)$ for an n-tuple of pairwise commuting operators $a_i:X\to X$ on a complex Banach space X (A denoting a unital Banach subalgebra of L(X) containing $a_1,\dots,a_n$), J.L.Taylor's spatial joint spectrum $\sigma(a_1,\dots,a_n;X)$ was introduced. Among other things the following results were presented

(1) The convex hulls conv $\sigma(a_1,\dots,a_n;X)=$conv $Sp(a_1,\dots$ $\dots,a_n;A)$ are contained in the closure of the spatial joint numerical range $V(a_1,\dots,a_n;X)$.

(2) (Tensor stability) The following are equivalent

(i) $\sigma(a_1\otimes I_Y,\dots,a_n\otimes I_Y;X\hat{\otimes}_\pi Y)=\sigma(a_1,\dots,a_n;X)$ for all Banach spaces Y

(ii) $\sigma(a_1,\ldots,a_n;X)=\sigma_{split}(a_1',\ldots,a_n';X')$
($a_i':X'\to X'$ denoting the dual operators)

(3) If X is an $\mathcal{L}_p$-space ($p=1,2,\infty$) then
$\sigma(a_1\otimes I_Y,\ldots,a_n\otimes I_Y;X\hat{\otimes}_\alpha Y)=\sigma(a_1,\ldots a_n;X)$
for all Banach spaces Y and all uniform crossnorms α.

Berichterstatter: S. Kremp

Gefördert durch DFG Deutsche Forschungsgemeinschaft

Tagungsteilnehmer

Prof.Dr. J. M. Ansemil
Depto. de Analisis Matematicas
Universidad Complutense de Madrid

E-28040 Madrid

Prof.Dr. V. Caselles
Laboratoire de Mathematiques
Faculte de Science
Universite de Franche-Comte
route de Gray

F-25030 Besancon Cedex

Prof. Dr. W. Arendt
Laboratoire de Mathematiques
Faculte de Science
Universite de Franche-Comte
route de Gray

F-25030 Besancon Cedex

Prof.Dr. G. H. Dales
Dept. of Mathematics
School of Leeds

GB- Leeds , LS2 9JT

Prof.Dr. K.-D. Bierstedt
Fachbereich Mathematik/Informatik
der Universität Paderborn
Warburger Str. 100

4790 Paderborn

Dr. J. Eschmeier
Mathematisches Institut
der Universität Münster
Einsteinstr. 62

4400 Münster

Prof.Dr. J. Bonet
Departamento de Matematicas
E. T. S. I. Industriales
Universidad Politecnica
c. de Vera

E-46071 Valencia

Prof.Dr. K. Floret
Fachbereich Mathematik
der Universität Oldenburg
Postfach 2503

2900 Oldenburg

Prof.Dr. B. Cascales
Departamento de Matematicas
Facultad de Matematicas
Universidad de Murcia

E-30 001 Murcia

Prof. Dr. M. Gonzalez
Departamento de Matematicas
Facultad de ciencias
Universidad Santander
Avda. de los Castros, s/n

E- Santander 39005

Prof.Dr. B. Gramsch
Fachbereich Mathematik
der Universität Mainz
Saarstr. 21

6500 Mainz

Dr. G. Greiner
Mathematisches Institut
der Universität Tübingen
Auf der Morgenstelle 10

7400 Tübingen 1

Prof.Dr. H. Jarchow
Institut für Angewandte Mathematik
der Universität Zürich
Rämistr. 74

CH-8001 Zürich

Prof.Dr. H. König
Fachbereich 9 - Mathematik
Universität des Saarlandes
Bau 27

6600 Saarbrücken

Prof.Dr.Dr.h.c. G. Köthe
Parkstr. 14

6000 Frankfurt 1

S. Kremp
Fachbereich 9 - Mathematik
Universität des Saarlandes
Bau 27

6600 Saarbrücken

N. Kuhn
Fachbereich 9 - Mathematik
Universität des Saarlandes
Bau 27

6600 Saarbrücken

Prof.Dr. K. B. Laursen
Matematisk Institut
Kobenhavns Universitet
Universitetsparken 5

DK-2100 Kobenhavn

Prof.Dr. S. Levi
Scuola Normale Superiore
Piazza dei Cavalieri, 7

I-56100 Pisa

Prof.Dr. R. Meise
Mathematisches Institut
der Universität Düsseldorf
Universitätsstraße 1

4000 Düsseldorf

Prof.Dr. P. Meyer-Nieberg
Fachbereich Mathematik/Informatik
der Universität Osnabrück
PF 4469, Albrechtstr. 28

4500 Osnabrück

Prof.Dr. V. Montesinos
Alboraya 18, B-19

E-46010 Valencia

Prof.Dr. J. Mujica
Instituto de Matematica
Universidade Estadual de Campinas
Caixa Postal 6065

13081 Campinas S. P.
BRAZIL

Prof.Dr. P. Perez Carreras
Avda. de Baleares 3 -7a

E- Valencia 46023

Prof.Dr. L. Nachbin
Av. Gilberto Amado 223
Barra da Tijuca

22600 Rio de Janeiro
BRAZIL

Prof.Dr. W. Ricker
c/o Professor H.H. Schaefer
Mathematisches Institut der
Universität Tübingen
Auf der Morgenstelle 10

7400 Tübingen 1

Prof.Dr. M. M. Neumann
Dept. of Mathematics
Southwest Missouri State University
901 South National Avenue

Springfield , MO 65804
USA

Prof.Dr. W. M. Ruess
FB 6 - Mathematik
Universität-GH Essen
Universitätsstr. 3
Postfach 103 764

4300 Essen 1

Prof.Dr. V. M. Onieva
Facultad de Matematicas
Ciudad Universitaria

E-50009 Zaragoza

Prof.Dr. W. Schachermayer
Institut für Mathematik
Universität Linz
Altenbergerstr. 69

A-4040 Linz

Prof.Dr. J. Orihuela
Departamento de Matematicas
Facultad de Matematicas
Universidad de Murcia

E-30 001 Murcia

Prof.Dr. H.H. Schaefer
Mathematisches Institut
der Universität Tübingen
Auf der Morgenstelle 10

7400 Tübingen 1

Prof.Dr. A. Pelczynski
Institute of Mathematics of the
Polish Academy of Sciences
ul. Sniadeckich 8

00-950 Warszawa
POLAND

Prof.Dr. J. Schmets
Quai de Rome, 26/091

B-4000 Liege

Prof.Dr. S. Simons
Dept. of Mathematics
University of California

Santa Barbara , CA 93106
USA

Prof.Dr. J. Voigt
Fachbereich Mathematik
der Universität Oldenburg
Postfach 2503

2900 Oldenburg

Prof.Dr. W. H. Summers
Dept. of Mathematics
University of Arkansas

Fayetteville , AR 72701
USA

Prof.Dr. L. Waelbroeck
36, Avenue de Hannetons

B-1170 Bruxelles

Prof.Dr. H.G. Tillmann
Mathematisches Institut
der Universität Münster
Einsteinstr. 62

4400 Münster

Prof.Dr. L. Weis
Dept. of Mathematics
Louisiana State University

Baton Rouge , LA 70803
USA

Prof.Dr. M. Valdivia
Facultad de Matematicas
Universidad de Valencia
Doctor Moliner, 50, Burjasot

E-46071 Valencia

Prof.Dr. M. Wolff
Mathematisches Institut
der Universität Tübingen
Auf der Morgenstelle 10

7400 Tübingen 1

Prof.Dr. D. Vogt
Fachbereich 7: Mathematik
der Universität/Gesamthochschule
Wuppertal, Gaußstr. 20
Postfach 10 01 27

5600 Wuppertal 1

Prof.Dr. V. Wrobel
Mathematisches Seminar
der Universität Kiel
Olshausenstr. 40, Haus 12 a

2300 Kiel 1