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Convergence Structures in Topology and Analysis

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The international conference on "Convergence Structures in Topology and Analysis" was held from December 13 to December 19, 1987 in Oberwolfach, Germany. The organizing committee consisted of E. Binz (Mannheim), H. Herrlich (Bremen) and G. Preuß (Berlin). The meeting was opened by G. Preuß who welcomed 42 participants from 15 countries. Several conferences on convergence structures took place before in eastern countries as well as in western ones. But for the first time scientists working in so many different areas of convergence structures came together. Thus, e.g. pure topologists joined workers in the field of applications in analysis. These contacts were very fruitful.

The talks and discussions dealt mainly with the following:

I) Filter convergence (Adámek, Giuli, Kent, Lowen-Colebunders, Schwarz, Tholen)

II) Sequential convergence (Antosik, Frič, Novák, Tironi)

III) Convergence and generalized metric spaces (Kopperman, Lowen, Reichel)

IV) Convergence and nearness spaces (Bentley, Császár)

V) Convergence and bitopological spaces (Brümmer, Ivanov)

VI) Convergence and completeness problems (Brümmer, Császár, Doitchinov, Koutník)

VII) Convergence and order (Erné, Hoffmann, Vainio)

VIII) Convergence and algebraic structures (Frič, Hušek, Koutník, Porst)

IX) Applications of convergence structures in Analysis and Theoretical Physics (Beattie, Frölicher, Grimeisen, Kleisli, Kriegl, Michor, Nel, Schmid, Wegenkittl).

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It was demonstrated again, that for several kinds of problems convergence structures are more suitable than topological spaces.

Besides other ones the following results were very remarkable:

1° In the category SEQ of sequential spaces the question whether an intersection of full reflective subcategories of SEQ is always reflective is undecidable (being dependent of set theory). [Adámek]

2° There are convergence rings which cannot be completed. [Frič, Koutník]

3° Convergence spaces form a highly satisfactory setting for infinite dimensional Differential Calculus [Nel].

The chairman of the last session, P. Antosik, closed the meeting and expressed the gratitude of the participants for living together in such a nice and creative atmosphere. The success of this conference was also underlined by A. Császár, D. Kent, E. Lowen-Colebunders and A. Wegenkittl, who spoke as representatives of the eastern countries, the western countries, the female participants and the young scientists respectively. Finally, G. Preuß expressed the hope of all participants that such a conference should take place again in Oberwolfach in 1991.

ABSTRACTS

J. Adámek:

Reflective subcategories of convergence spaces

The older question of H. Herrlich whether an intersection of full, reflective subcategories of TOP is always reflective, can be considered for convergence instead of topology, where the situation is as follows:

I. Filter-convergence: in the category PSTOP of pseudotopological (Choquet) spaces the subcategory Comp of all compact Hausdorff spaces is not reflective (as observed by M. Hušek and, independently, by H.P. Butzmann and G. Kneis). However, $\mathrm{Comp} = \bigcap_{\alpha \in \mathrm{Card}} \mathrm{Comp}_\alpha$ where the category Comp_α (of those spaces in which each ultrafilter with a member of power $< \alpha$ has a unique limit) is reflective.

II. Sequential convergence: in the category SEQ of sequential spaces the above question is undecidable (being dependent of set theory). Namely, if no measurable cardinal exists, then every full subcategory of SEQ closed under the formation of limits is reflective. Conversely, if a huge cardinal exists (which implies that Weak Vopěnka Principle is true) then SEQ has full reflective subcategories whose intersection is not reflective. Both of those results are straightforward consequences of recent papers of V. Trnková, J. Rosický and J. Adámek.

P. Antosik:

On some diagonal property

Banach type theorems on uniform boundedness and equicontinuity, theorems of joint continuity, Nikodym type theorems for set functions make sense for mappings from sequential convergence structures to sequential convergence structures. To prove this type of theorems we need a new tool that could replace the Baire category type arguments.

The sort of this tool is provided by the following

LEMMA. Assume that X is an abelian group equiped with a convergence and assume that (x_{ij}) is a matrix (double sequence). Then $x_{ii} \to 0$ whenever the following conditions hold.

1^0 $x_{ij} \to 0$ as $j \to \infty$ for $i \in M$;

2^0 $x_{ij} \to 0$ as $i \to \infty$ for $j \in N$;

3^0 for every subsequence (m_i) of (i) there is a subsequence (n_i) of (m_i) such that $\sum_{j=1}^{\infty} x_{n_i n_j} \to 0$ and

4^0 the convergence is a FLUS-convergence and, additionally, it satisfies the following diagonal type property

Y. If (x_{ij}) is a matrix in X whose rows and columns converge to zero, then there is a subsequence (m_i) of (i) such that $\sum_{j \in A_i} x_{m_i m_j} \to 0$ whenever A_i are finite subsets of N and $i \notin A_i$ for $i \in N$.

PROBLEM. Can we drop condition Y with no harm for the lemma?

R. Beattie:

Continuous convergence and functional analysis

Continuous convergence can be a powerful tool for functional analysis even if the objects of interest are primarily locally convex topological vector spaces. There is, in fact, in the setting of convergence spaces a particularly attractive duality between locally convex spaces and their locally compact continuous duals.
Some well-known results of functional analysis, the Grothendieck completion, the closed graph theorem, the Banach-Steinhaus theorem and distribution theory results are examined in the context of convergence spaces and the role of continuous convergence is highlighted.

H.L. Bentley:

Zero sets and complete regularity for nearness spaces

Completely regular nearness spaces (cf. the work of H. Herrlich) are defined in a way that enables us to determine internally all completely regular extensions of topological spaces. A nearness space X is said to be completely regular provided that whenever $\mathcal{A}$ is a uniform cover of X then so is $\{B \subset X | B$ is completely within A for some $A \in \mathcal{A}\}$, where we say B is completely within A iff there exists a uniformly continuous $f: X \to [0,1]$ such that $f[B] \subset \{0\}$ and $f[X-A] \subset \{1\}$. We have "uniform $\Rightarrow$ normal $\Rightarrow$ completely regular $\Rightarrow$ regular" for nearness spaces and the arrows of implication are not reversible. A completely regular nearness space always has an underlying topological space which is completely regular; moreover, for topological nearness spaces the notion of complete regularity defined above coincides with the classical concept. We also study a closely related concept of zero spaces: Zero spaces are those nearness spaces having their structure determined by the collection of all zero sets in a natural way. z-clusters are maximal near collections of zero sets; these give rise to a new completion of the original space. Several open questions are posed.

G.C.L. Brümmer:

Completeness as firm saturation

One setting for such a theory is a topological category $\mathcal{A}$ which is universal in the sense of Marny (1979), and its subcategory $T_0\mathcal{A}$ of T_0-objects. An epireflection $\eta: I \to R$ in $\mathcal{A}$ is called firm iff whenever $f: X \to Y$ is a $T_0\mathcal{A}$-epi embedding with $Y \in \mathrm{Fix}\ R$, there is an isomorphism $h: RX \to Y$ with $h\eta_X = f$. Then $\mathrm{Fix}\ R$ is the class of $T_0\mathcal{A}$-saturated objects. In many examples such "complete" objects just don't exist, and in some categories of convergence spaces they are trivial. But when such a "completion" R exists, it preserves initial sources, and one can characterize when bireflections in $\mathcal{A}$ are stable under R. If $\mathcal{A}$ is simple, one can characterize firmness of $\mathcal{A}$. Consequence: We have "complete" objects in Bitop; they are not all the sober bispaces.

Ákos Császár:

Proximities and merotopies related with extensions of closures

If c is a closure on X (i.e. c: exp $X \to$ exp X with suitable properties), then there are a proximity δ_c and a merotopy M_c (in the sense of M. Katětov) associated with c:

$$A \, \delta_c \, B \Leftrightarrow c(A) \cap c(B) \neq \emptyset \, ,$$

M_c is generated by all c-neighbourhood filters.
The question dealt with is the following: given a proximity δ or a merotopy M on X, is there an extension (Y,c') of (X,c) such that δ or M is the restriction to X of $\delta_{c'}$ or $M_{c'}$, respectively?

D. Doitchinov:

Completeness in quasi-metric spaces and quasi-uniform spaces

The notion of completeness has to be founded on a concept of Cauchy sequence in a quasi-metric space X (or - of Cauchy filter in a quasi-uniform spaces). In the quasi-metric case it is required that:

(i) every convergent sequence is a Cauchy sequence;

(ii) when X is a metric space, then Cauchy sequences coincide with the usual ones. Further a completion X^* of the space X has to be constructed in a standard manner, such that:

(iii) when $X \subset Y$, then $X^* \subset Y^*$ (inclusion understood as quasi-metric embedding);

(iv) when X is a metric space, then X^* is the usual metric completion.

Some examples show that the last two requirements ((iii) and (iv)) contradict each other in the general (Hausdorff) case.
But it turns out that all this program can be fulfilled for a subclass $\underline{B}$ of the class of all quasi-metric spaces, which contains on the other hand the class of all metric spaces. The class $\underline{B}$ is defined by *imposing the following additional condition on the quasi-metric* $\underline{\underline{d}}$ *of* the space X:

(B) if $x', x'' \in X$, $\{x'_n\}$ and $\{x''_m\}$ are sequences in X and if $d(x', x'_n) \leq r'$ (for all n), $d(x''_m, x'') \leq r''$ (for all m) and $d(x''_m, x'_n) \to 0$ (which means: $\forall \varepsilon > 0\ \exists N_\varepsilon$ s.t. $d(x''_m, x'_n) < \varepsilon$ for $m,n > N_\varepsilon$), then $d(x', x'') \leq r' + r''$.

The quasi-uniform case is treated in a similar way.

M. Erné - R. Vaino:

Connectedness properties of lattices

Regard a convergence structure q on a lattice L. Given some natural conditions (T_1-ordered space, compact maximal chains and continuous lattice translations) connectedness components of (L,q) were characterized in [1] as maximal order dense, convex sublattices of L. In [2], so called i-spaces (q restricted to any maximal chain C of L equals the interval topology of C) were examplified and applied as a family of spaces having a neat connectivity theory.

Here these results will be extended and presented within a unifying theory covering a variety of different connectedness properties of lattices.

Connectivity concepts for (L,q) involving not only the convergence structure q, but also the order structure, will be introduced; such concepts are link-connectedness, l-connectedness, ordered path- and ordered arc-connectedness. A common background for these various types of connectivity, as well as for classical connectivity, is furnished by the concept of connectivity system. The component theorem for fairly general connectivity systems on a given lattice will be presented. As a corollary, we will also obtain an improved version of the component theorem of [1].

1. R. Vainio: Connectedness properties of lattices, Canad. Math. Bull. 29 (1986), 314-320.
2. R. Vainio: A maximal chain approach to topology and order, to appear in Internat. J. Math. and Math. Sci.
3. M. Erné and R. Vainio: Connectedness properties of lattices II, manuscript (1987).

R. Frič:

Coarse sequential convergence in groups, rings, etc.

Let G be a group. By a group convergence on G we understand a sequential convergence with unique limits (the Urysohn axiom is not assumed) compatible with the group structure of G . A group convergence on G is said to be coarse if there exists no strictly larger group convergence on G . Coarse convergences on semigroups, rings, fields and vector spaces are defined analogously. We characterize coarse convergences and give some applications, e.g. we construct a convergence field having no ring completion.

A. Frölicher:

Mackey convergence and calculus

Some basic ideas of the forthcoming book "Linear Spaces and Differentiation Theory" by A. Kriegl and the speaker are presented. Classical Banach space calculus is extended to a maximal class of linear spaces in a way which is absolutely natural if one wants to uphold some basic features (known since a few years) of Banach space calculus. The main purpose is achieved: closedness under formation of very general function spaces holds.

The linear spaces of that maximal class admit many equivalent descriptions, using e.g. topologies or bornologies or convergence structures. Bornologies are most useful for characterizing differentiability properties, convergence structures for describing derivatives in terms of limits. The link is given by Mackey convergence. It constitutes the natural convergence in bornological vector spaces and yields an embedding of their category into that of convergence vector spaces. There are several open questions.

E. Giuli:

Epis in categories of convergence spaces

For every closure operator $C = \{c_X : P(X) \to P(X)\}_{X \in \underline{X}}$ in a topological category $\underline{X}$ the following subcategories are introduced

$$\underline{X}_{0C} = \{X \in \mathrm{Ob}\underline{X} : y \in c_X x \text{ and } x \in c_X y \Rightarrow x = y\}$$
$$\underline{X}_{1C} = \{X \in \mathrm{Ob}\underline{X} : x = c_X x \text{ , for each } x \in X\}$$
$$\underline{X}_{2C} = \{X \in \mathrm{Ob}\underline{X} : \Delta_X = c_{X\times X}\Delta_X\} ,$$

and

$$\underline{X}_{0C^2} , \underline{X}_{0C^3} , \dots, \underline{X}_{0C^\alpha} , \dots$$

whenever C is not idempotent. All these subcategories are epireflective in $\underline{X}$. For the category $\underline{\mathrm{PrT}}$ of all pretopological spaces and for the induced Čech operator C in $\underline{\mathrm{PrT}}$ we obtain a chain

$$\underline{\mathrm{PrT}}_0 = \underline{\mathrm{PrT}}_{0C} \supsetneqq \underline{\mathrm{PrT}}_{0C^2} \supsetneqq \mathrm{PrT}_{0C^3} \supsetneqq \dots \supsetneqq \underline{\mathrm{PrT}}_{0C^\alpha} \supsetneqq \dots \supsetneqq \underline{\mathrm{PrT}}_{1C} \supsetneqq \underline{\mathrm{PrT}}_{2C} = \mathrm{PrT}_2 .$$

It is shown that: (a) $\underline{\mathrm{PrT}}_{iC}$-epis, $i = 0,1$, are onto; so the extremal monos in $\underline{\mathrm{PrT}}_{iC}$ are the $\underline{\mathrm{PrT}}$-embeddings; (b) there exist $\underline{\mathrm{PrT}}_{0C}$-epis which are not onto for each $\alpha \geq 2$; (c) the $\underline{\mathrm{PrT}}_2$-epis are precisely the $\hat{C}$-dense maps, where $\hat{C}$ is the idempotent hull of C , so the $\underline{\mathrm{PrT}}_2$-extremal monos are the C-closed embeddings; it follows directly from the fact that the category of Urysohn spaces is not cowellpowered, that $\underline{\mathrm{PrT}}_2$ is not cowellpowered; (d) $X \in \underline{\mathrm{PrT}}_2$ is absolutely $\underline{\mathrm{PrT}}_2$-closed iff X is compact. Moreover for these compact Hausdorff pretopological spaces the Kuratowski characterization holds.

G. Grimeisen:

<u>Contracting convergence of filtered set-families in the theory of integration for Banach spaces</u>

(W. ERBEN-G. GRIMEISEN) We introduce, for a Banach space E , a measure space (F,μ) , and a mapping $f\colon F \to \mathcal{E}(= \{X|X \subseteq E$ and $\operatorname{card} X \leq 1\})$ a Riemann-type integral ${}^{\mathcal{X}}\!\int f d\mu$, where $\mathcal{X}$ is a system of certain countable partitions of F . We indicate here only two essential properties of this integration: 1) If $\mathcal{X}'$ and $\mathcal{X}''$ are systems of the mentioned kind, then $\mathcal{X}' \subseteq \mathcal{X}''$ implies ${}^{\mathcal{X}'}\!\int f d\mu \subset {}^{\mathcal{X}''}\!\int f d\mu$. 2) ${}^{\mathcal{X}}\!\int \cdot d\mu$ is a socalled "pointwise integral" (for this notion, see G. Grimeisen, "Darstellung des iterierten Maßintegrals ...". J. f. reine und angewandte Math. <u>243</u> (1970), p. 94). In the special case that $\mathcal{X}$ consists of all admissable partitions of F , ${}^{X}\!\int f d\mu$ coincides with the G. BIRKHOFF integral (1938) and it lies between the BOCHNER and the PETTIS integral.

R.-E. Hoffmann:

The Hausdorff limit in continuous lattice theory

Starting out from an ordinary sequence of (extended) real numbers, we are led to introduce two - in general, different - concepts of limit superior and quasi-limit superior of a net $(x_j)_{j\in J}$ or filter $\mathcal{F}$ on a complete lattice L

$$\lim\sup \mathcal{F} = \inf\{\sup F \mid F \in \mathcal{F}\}$$

$$\text{q-}\lim\sup \mathcal{F} = \sup \bigcap \{\downarrow F \mid F \in \mathcal{F}\} = \sup\{\inf G \mid G \in \mathcal{G}(\mathcal{F})\}$$

where $\downarrow F = \{y \in L \mid y \leq x \text{ for some } x \in F\}$ and

$$\mathcal{G}(\mathcal{F}) = \{G \subseteq L \mid G \cap F \neq \emptyset \text{ for every } F \in \mathcal{F}\} .$$

In the lattice of closed subsets of a topological space, lim sup and q-lim inf are the Kuratowski-Hausdorff lim sup and lim inf , whereas q-lim sup and lim inf are the closures of the set-theoretic lim sup and lim inf , resp. . For a distributive complete lattice L , lim inf = q-lim inf and lim sup = q-lim sup if and only if L is completely distributive. For a filter $\mathcal{F}$ on a complete lattice L , let $\mathcal{F} \xrightarrow[\mathcal{H}_L]{} x$ iff $x = \lim\inf \mathcal{F} = \text{q-}\lim\sup \mathcal{F}$ (dual situation: O. Frink '42, G. Nöbeling '53; Hausdorff '14). G. Choquet ('47) showed that Hausdorff convergence $\mathcal{H}$ on $\underset{\sim}{O}(X)$, the lattice of open subsets of a topological space X , is pseudo-topological. An example is given to show that this does not extend to arbitrary complete lattices. Choquet's result that "(1) $\mathcal{H}$ is topological ⇔ (2) $\mathcal{H}$ is pretopological ⇔ (3) X is locally compact" for T_2-spaces is extended to the non-T_2-setting by replacing (3) by (3'): $\underset{\sim}{O}(X)$ is a continuous lattice. Moreover, a complete lattice L is a continuous lattice iff L is meet-continuous and $\mathcal{H}_L$ is induced by a topology. We conclude with a glimpse at a general framework: strongly sober convergence spaces and their induced "path structures" (seven different kinds, some of which coincide in special cases).

M. Hušek:

Convergence Groups, their completions and products

A unifying diagram concerning convergence and topological groups and their relations to uniform structures will be described. By means of natural structures, various kinds of completions of convergence groups may be introduced, none of them as satisfactory as in topological groups. Other completions may be provided using nice epireflective subcategories of convergence spaces (e.g. sequentially complete convergence groups, sequential modifications of compact groups). Their behaviour with respect to products will be discussed.

A.A. Ivanov:

Bitopological spaces: results and problems

There are more then 100 papers devoted to the theory of bitopological spaces. Initial understanding of bitopological space as a triplet (X,τ_1,τ_2) , where τ_1,τ_2 are topological structures on X , was generalized. Now it is a pair (X,β) , where β is a topological structure on $X \times X$. The speaker described the development of the theory beginning from the basic Kelly's paper (1963) up to nowdays and problems to research.

D. Kent:

P-regular convergence spaces

Let q and p be convergence structures on a set X ; then (X,q) is said to be p-regular if $F \xrightarrow{q} x$ implies $cl_p F \xrightarrow{q} x$. There is always a finest p-regular convergence structure $r_p q$ coarser than q , and if p is T_1 there is also a coarsest p-regular structure $r^p q$ finer than q ; these are called the lower and upper p-regular modifications of q . Various convergence properties can be described as special cases of p-regularity. For instance, if p is the finest completely regular topology coarser than q , then p-regularity is equivalent to ω-regularity (the ω-regular spaces are precisely the subspaces of compact, regular convergence spaces), and $r_p q$ is the

ω-regular modification of q . If q is T_1 and p the topology with closed base consisting of subsets of q-compact sets, then p-regularity is equivalent to local compactness, and $r^p q$ is the coarsest locally compact structure finer than q . p-regularity is well-behaved relative to projective limits, and has interesting ramifications in the theory of convergence space compactifications.

H. Kleisli:

A^*-closed category containing Ban_1^{op}

It is known that the category Ban_1^{op} is equivalent to the following category W_1 . The objects of W_1 are Waelbroeck-spaces X and the maps $f: X \to Y$ are Ban_1-maps such that the restriction $f|_{OX}$ to the unit-ball OX is continuous. Moreover, the category W_1 has a symmetric monoidal structure with tensorproduct the completion $X \hat{\otimes} Y$ of the projective tensorproduct in the sense of Grothendieck. Unfortunately the symmetric monoidal category W_1 is not closed.

PROBLEM. Is there a closed category $(V_1,\hat{\hat{\otimes}},\mathbb{C},[\,,\,])$ together with a natural equivalence $^*: V^{op} \to V_1$ and that 1) $(W_1,\hat{\otimes})$ and $(\mathrm{Ban}_1,\hat{\otimes})$ are monoidal subcategories of V_1 , 2) the restriction $^*|_{\mathrm{Ban}_1}$ is the equivalence $\mathrm{Ban}_1^{op} \to W_1^{op}$ mentioned above?

That problem has been solved by M. Barr (Springer LN 752 (1979)). I shall present another solution which puts the rôle of the tensorproduct into evidence and does not require completeness properties of the spaces involved.

Let A_1 be the category of MT-spaces in the sense of Semadeni: the objects of A_1 are Banach-spaces X with a loc. convex topology T_X , weaker than the norm-toplogy, and let us condider the following full subcategory B_1 of A_1 . The topology T_X of an object X in B_1 is given as follows: Choose a family W_X of pairs (W_α, i_α) where $W_\alpha \in \mathrm{ob}W_1$ and $i_\alpha: W_\alpha \to X \in \mathrm{Ban}_1$. Then T_X is the strongest loc. convex topology, weaker than the norm topology, such that all restrictions $i_\alpha|_{OW_\alpha}$ are continuous.

Tensorproduct. The tensorproduct in B_1 is the usual tensorproduct of Banach-spaces endowed with the topology given by the family $V_\alpha \hat{\otimes} W_\beta \xrightarrow{i_\alpha \otimes j_\beta} X \otimes Y$, where $(W_\alpha, i_\alpha) \in W_X$, $(W_\beta, j_\beta) \in W_Y$.

Duality. There is a functor $^*: B_1^{op} \to B_1$ with the following object-function: For each $X \in \mathrm{Ob}B_1$, let X^* be the Banach-space of all linear functionals φ , $\varphi|_{OX}$ continuous, endowed with the topology given by the family (B_σ^*, r_σ^*) , where σ is a continuous semi-norm on X and r_σ is the quotient-map $X \to \overline{X/\ker\sigma}$. Then the canonical map $\eta_X: X \to X^{**}$ has the following properties: (1) η_X is continuous, (2) $\eta(X^*) \subset X^{***}$ and $\eta(X^*\hat{\otimes}Y^*) \subset (X^*\hat{\otimes}Y^*)^{**}$ are dense, (3) X^{***} and $(X^*\hat{\otimes}Y^*)^{**}$ are reflexive, i.e. η is an isomorphism in B_1 .

Internal hom. Let now C_1 be the full subcategory of B_1 consisting of the reflexive objects in B_1 . Then the tensorproduct changes and is given by $X \times Y \xrightarrow{\pi} X\hat{\otimes}Y \xrightarrow{\eta} (X\hat{\otimes}Y)^{**}$, so that we set $X \hat{\hat{\otimes}} Y = (X\hat{\otimes}Y)^{**}$. Finally, there is a natural bijection $C_1(X\hat{\hat{\otimes}}Y,Z) \to C_1(X,(Y\otimes Z^*)^*)$, so that $[Y,Z] = (Y\hat{\otimes}Z^*)^*$ yield the internal hom.

R. Kopperman:

Continuity spaces

DEFINITION. A value semigroup A is an additive abelian semigroup with identity 0 and absorbing element $\infty \neq 0$, such that

(v1) if $a + x = b$ and $b + y = a$ then $a = b$ (thus $a \leq b \Leftrightarrow \exists x$, $a + x = b$, defines a partial order - the order used below)

(v2) there is a unique b (called $\frac{1}{2}$ a) such that $b + b = a$

(v3) each pair has an inf , $a \wedge b$

(v4) $a \wedge b + c = (a+c) \wedge (b+c)$

EXAMPLES. $[0,\infty]$, $\{0,\infty\}$, products.

DEFINITION. A set of positives is a $P \subset A$ such that

$\left.\begin{array}{l}\text{(p1) if } r,s \in P \text{ then } r \wedge s \in P \\ \text{(p2) if } r \in P,\ r \leq a \in A \text{ then } a \in P\end{array}\right\}$ filter axioms

(p3) if $r \in P$ then $\frac{1}{2} r \in P$

(p4) if $a \leq b + t$ for each $t \in P$, then $a \leq b$.

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DEFINITION. A continuity space is a quadruple $X = (X,d,A,P)$, A a value semigroup, P a set of positives on A , and $d\colon X\times X \to A$ such that:

(m1) $d(x,x) = 0$

(m2) $d(x,z) \leq d(x,y) + d(y,z)$

It is symmetric if (m3) $d(x,y) = d(y,x)$

It is separated if (m4) $d(x,y) = 0 \Rightarrow x = y$

A continuity multispace $X = (X,d,A,(P_i)_{i\in I})$ is such that X,d,A are as above, each P_i satisfies (p1) - (p3), and $\bigcup_{i\in I} P_i$ satisfies (p4).

DEFINITION. $N_r = \{(x,y) \mid d(x,y) \leq r\}$,
$\mathcal{U}_X = \{U \subset X\times X \mid \text{for some } r \in P , N_r \subset U\}$, $N_r(x) = \{y \mid d(x,y) \leq r\}$,
$\mathcal{T}_X = \{\mathcal{O} \subset X \mid \text{if } x \in \mathcal{O} \text{ then for some } r \in P , N_r(x) \subset \mathcal{O}\}$,
$\mathcal{C}_X = \{(\mathcal{J},x) \mid \mathcal{J}$ an ultrafilter on X , $x \in X$, and for some $i \in I$, if $r \in P_i$ then $N_r(x) \in \mathcal{J}\}$.

THEOREM. (A) $\begin{bmatrix}\mathcal{T}_X\\ \mathcal{U}_X\end{bmatrix}$ is a $\begin{bmatrix}\text{topology}\\ \text{quasi-uniformity}\end{bmatrix}$ and each such so arises from a continuity space.

(B) If X is symmetric, $\begin{bmatrix}\mathcal{T}_X\\ \mathcal{U}_X\end{bmatrix}$ is $\begin{bmatrix}\text{completely reg.}\\ \text{a uniformity}\end{bmatrix}$ and each such so arises from a continuity space.

(C) $\mathcal{C}_X$ is a convergence structure, and each such so arises from a continuity multispace.

Also discussed: dual, Boolean (and dimensionality), and applications to spaces of prime ideals.

V. Koutník:

Completion of convergence rings

The structure $(L,\mathcal{L},\lambda,+,\cdot)$ is said to be a convergence ring if $(L,+,\cdot)$ is a commutative ring with a unit and $(L,\mathcal{L},\lambda)$ is a maximal single-valued convergence space such that the algebraic operations are sequentially continuous. A sequence $\langle x_n\rangle$ is Cauchy in L if $x_n - x_{k_n} \to 0$ for every $\langle k_n\rangle$. L is complete if all Cauchy sequences converge in L . M is a completion of L if M

is complete and contains L as a subring and topologically dense subspace.

There are convergence rings which cannot be completed. A completion will be constructed for convergence rings satisfying certain conditions. As an example, the convergence ring of rational numbers will be shown to have a completion different from the usual real line.

A. Kriegl:

Smoothly real compact spaces

The starting point is "Milnor & Stasheff's exercise", which tells us that for smooth finite dimensional manifolds M every real-valued algebra homomorphism on the algebra $C^\infty(M,R)$ of smooth functions on M is just the evaluation at some point of M. This result is certainly one of the main tools for translating algebraic properties of the algebra $C^\infty(M,R)$ into properties of M itself, which is in particular important for synthetic differential geometry. The usual proof of it makes heavily use of the locally compactness of M. Nevertheless, joint work with P. Michor and W. Schachermayer resulted in an extension of this result to fairly general (infinite dimensional) smooth spaces, which we called for obvious reasons smoothly real compact spaces.

R. Lowen:

Convergence and metric spaces

We prove the existence of a topological universe (in the sense of L.D. Nel) in which convergence structured spaces and pseudo-quasi-metric spaces both with their natural maps i.e. continuous resp. non-expansive maps are nicely embedded. The advantages of doing this are: (1) unification of concepts; e.g. compactness, total boundedness, Kuratowski's measure of non-compactness all arise from a canonical compactness concept in the topological universe, also connectedness and Cantor's uniform connectedness arise from a categorical connectedness concept (in the sense of G. Preuss) in this topological universe. (2) the embedding of metric spaces is not reflective, in particular products in this topological universe differ from the metric product of metric spaces; the product in the topological universe however has as topological coreflection the topological product of the associated topological spaces which yields several pleasant permanence properties. (3) spaces of measures and spaces of (metric space valued-)random variables can be equipped with natural (proper) structures in the topological universe such that the topological coreflections give the usual weak topology resp. the topology of convergence in measure; the former structures however have far better relation with statistics than the topological structures.

E. Lowen-Colebunders:

On the non-simplicity of some convergence categories

An epireflective subcategory B of a topological category A is A-simple if there exists a single object E of B such that B is the epireflective hull in A of the class {E} .
We first study the case A = Conv.

Prtop and Top are known to be Conv-simple. We show that due to an old theorem of Herrlich, every epireflective subcategory of R Prtop or T_1 Prtop , containing all T_1 regular topological spaces is not Conv-simple.
With settheoretical arguments we show that every epireflective subcategory of Conv containing all c-embedded spaces or all compact Hausdorff pseudotopological spaces is not Conv-simple.
We extend the results to the cases A = Fil and A = Mer . We show that every epireflective subcategory of Fil (of Mer) containing all regular c-embedded filter nearness spaces is not Fil-simple (not Mer-simple).

P. Michor:

Gauge theory for diffeomorphism groups

For fibre bundles with structure group the diffeomorphism group of the typical fibre I will develop the theory of connections, curvature, holonomy group ...; I will consider the "nonlinear frame bundle" of

the bundle, a principal fibre bundle with structure group the diffeomorphism group of the typical fibre and show, that one can lift the theory there and that it gives the usual principal bundle geometry there: this is true for compact typical fibre. For non-compact one the Frölicher-Kriegl calculus might help.
Then I consider the classifying space for Diff(S), a new characteristic class, the gauge groups for a fibre bundle, it's action on connections and the moduli space, and self duality.

L.D. Nel:

Convergence spaces as setting for differential calculus

We call attention to four crucial properties of the real line as a convergence space: they encode all the general topology and real analysis needed for basic differential calculus.
Once these properties are established, the following can be obtained by purely algebraic manipulation of continuous maps (with no further call on things like filters or inequalities): the definition of continuously differentiable and smooth maps; smoothness of (multi-) linear maps; the chain rule; the usual properties of partial derivatives; the usual properties of integrals of vector valued curves; the Fundamental Theorem of calculus; the symmetry of higher order derivatives; the Taylor formula; the exponential laws for smooth maps. These results hold in infinite dimensional setting: for maps between arbitrary closed embedded linear convergence spaces, a class closed under all the usual constructions.
One cannot cast topological spaces in the role of convergence spaces without causing the above theory to fail miserably.
The above algebraification of differential calculus has far reaching consequences. One can put other categories in the role of convergence spaces, as long as they uphold the mentioned four crucial properties e.g. diffeological spaces, leading to the Frölicher-Kriegl calculus.

J. Novák:

Convergence of double sequences in Fréchet spaces

A double sequence (x_{mn}) of points of a Fréchet space (X,u) converges to a point $x \in X$ if $(*)$ holds

$(*)$ if $U(x)$ is a neighborhood of x then there is m_0 and $f: N \to N$ such that $x_{mn} \in U(x)$, $m \geq m_0$, $n \geq f(m)$. Notation $\operatorname{Lim} x_{mn} = x$.

The collection of all pairs $[(x_{mn}),x]$, where $\operatorname{Lim} x_{mn} = x$, is called a double convergence on X . It is possible to classify points of (X,u) by means of L''_u . Hence we have 4 kinds of points, viz $\rho\sigma-$, $\rho\bar{\sigma}-$, $\bar{\rho}\sigma-$, $\bar{\rho}\bar{\sigma}$-points. Some properties of this classification will be given.

H.-E. Porst:

Free algebras over convergence structures

If K is one of the categories Seq, Conv and A is a quasivariety, we investigate the adjoint G of the underlying functor $V: K\text{-}A \to K$ ($K\text{-}A$ being the category of A-objects in K). It is shown that due to cartesian closedness of K an explicit description of G (i.e. of the "tree topology") can be given.

This will be used to show that $VG\underline{X}$ is in K_H for $\underline{X} \in K_H$, where K_H is the subcategory of Hausdorff-objects (i.e. of objects $\underline{X}$ with closed diagonal $\Delta_{\underline{X}}$) .

This is contrasted with the far more unfavorable situation in classical topological algebra.

H.-Chr. Reichel:

Branche-spaces in topology (survey, new results and an old unsolved problem)

First, a characterization of linearly uniformizable spaces (resulting from joint work with P. Nyikos and continuating joint work with M. Hušek) is presented. Thm.: (X,τ) is κ-metrizable and non-metr. or metrizable and $\dim X = 0$ iff (1) X is a non-archimedean space, (2) for each non-isol. x, $\psi(x) = \psi(\Delta X)$, and (3) X has a κ-discrete dense subset. - From this thm. most of the older characterizations

follow more or less directly. Moreover, this thm. may provide new insights and methods for one of the oldest problems on branche-spaces: Is there a model of set theory where every perfectly normal n.-a. space is metrizable? (Nyikos and Reichel, 1971). The only counter-examples known up to now are derived from Suslin-trees!
Therefore, the second part of the talk deals with branche-spaces in general and some of their applications.
Concluding, two remarks show the relation of the whole field with (1) an old thm. of H. Herrlich on the orderability of totally disconn. metric spaces, and (2) with generalized metrics defining topologies (which connects my talk with the theme of the converence).

R. Schmid:

Fixed point theorem in Fréchet spaces and applications

Fixed point theorems in Banach spaces are widely used to study boundary value problems for ODEs and periodic solutions of deley-integral equations.
We prove a fixed point theorem for non linear operators in Fréchet spaces and apply it to integral evolution equations to obtain quasi periodic solutions. The convergence structure used in the differential calculus in Fréchet spaces is used to control the bifurcation of fixed points in a cone: i.e. nonzero fixed points are obtained (depending on a parameter) even though zero is known to be a fixed point.

F. Schwarz:

Extension-representable categories of convergence spaces

A monotopological category is called extension-representable (resp. strong extension-representable) iff partial morphisms (resp. strong partial morphisms) are representable. For topological categories the two notions coincide, since the strong monomorphisms are precisely the embeddings.
The familiar categories of convergence spaces are monotopological, many of them even topological. Recent results on extension - representability of topological categories - in particular, descrip-

tions of the one-point-extensions - are applied to convergence categories. None of the categories where the symmetry axiom R_0 is fulfilled is extension-representable.

Convergence spaces satisfying a separation axiom (T_0, T_1, T_2) generally form proper monotopological (i.e. monotopological, but not topological) categories. No proper monotopological category is extension-representable. The T_0-limit spaces (epis there being surjective) provide an example of a proper monotopological category that is not strong extension-representable. The question whether there are proper monotopological categories that are strong extension-representable is still unsolved; a negative answer would imply that no proper monotopological category forms a quasitopos.

W. Tholen:

Closure operators and cowellpoweredness

Several interrelationships between the following concepts are presented:

- factorization systems
- closure operators
- diagonal theorems
- the Pumplün-Röhrl correspondence
- the Salbany correspondence
- various reflective closures of subcategories
- cowellpoweredness of subcategories

(This is joint work with E. Giuli and D. Dikranjan, E. Giuli's talk gives applications)

G. Tironi:

$C_p(X)$ as chain-net space and products of chain-net spaces

Recently it was shown by Gerlits, Nagy and Szentmiklóssy that for spaces of the type $C_p(X)$ to be radial (or Fréchet chain-net) is equivalent to be Fréchet. However they also showed that there are spaces $C_p(X)$ that are chain-net but not Fréchet. This exactly happens when X is taken to be an ordinal number ξ, with the usual order topology, which is a regular cardinal and is ω-inaccessible (i.e. such that for any cardinal $\lambda < \xi$, $\lambda^{\omega} < \xi$).

The following problem was investigated in a joint paper with G. Dimov.

Is it true that if $C_p(X)$ is almost radial (this notion was introduced in a paper by Arhangel'skii, Isler and Tironi, presented at the Bechyně Conference in 1983) then $C_p(X)$ is Fréchet?

This can happen in some cases.

PROPOSITION. Let X be a Lindelöf space. Then $C_p(X)$ is chain-net if and only if it is Fréchet.

But, in general, $C_p(X)$ can be almost radial without being Fréchet. In fact

THEOREM. Let ξ be an ordinal number. Then $C_p(\xi)$ is chain-net if and only if it is almost radial.

Also the notions of u-chain-net and u-almost radial space were introduced and investigated.

Products of chain-net spaces are in general not chain-net. However some cases in which this happens for the product of two such spaces were again investigated by Gerlits et al. They left open the question for the product of two compact chain-net spaces and for the product of one compact Fréchet chain-net space and one compact chain-net space.

In a joint work with Z. Frolík, still in a preliminary form, the following result was obtained

THEOREM. Let X and Y be compact chain-net spaces. If one of them is Fréchet chain-net, then the product is chain-net.

A. Wegenkittl:

Manifolds of local diffeomorphisms defined by tensor-fields

Given manifolds M and N and covariant tensor fields ω and η on M and N respectively, then one can ask if the set $\{f \in C^{\infty}(M,N) : f^{*}\eta = \omega\}$ is a manifold (possibly infinite-dimensional). In general, this will be not true, but special cases are known, for example the set of all isometries of a Riemannian manifold, the set of all symplectomorphisms of a symplectic manifold and the set of all mappings preserving a contact-structure. Moreover, there is a theorem of E. Binz concerning isometric immersions into Euclidean space. A generalization of this theorem is given: the set of all smooth immersions $f: M \to \mathbb{R}^n$ (M a manifold) having the same pullback of a constant covariant tensor field on $\mathbb{R}^n$ is a manifold. (In fact, at this moment the speaker has only the proof for local diffeomorphisms, but it seems to him very likely that this theorem holds for immersions, too).

Berichterstatter: G. Preuß

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