MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 5/1988

Methoden und Anwendungen der Approximationstheorie

31.1. bis 6.2.1988

The meeting was held under the chairmanship of D. Braess (Bochum), W. Dahmen (Berlin) und C.A. Micchelli (Yorktown Heights, USA). The research topics represented by the 49 participants from 12 countries highlighted in particular currently increasing connections and interaction between activities in approximation theory and other areas of mathematics which, in fact has been a central objective of the meeting. The respective contributions covered purely theoretical problems as well as practical applications. To this end, one may mention the following examples.

1. A central topic of the meeting was the theory and applications of spline functions. The connection with finite elements is to be mentioned. At first glance there seems to be a hard competition, but it turns out that in both theories different properties are considered as essential. This leads to a different weighting of the features as local flexibility, global smoothness, uniformity of the grid and simplicity of the ansatz, when one is looking for suitable spaces and representations of the functions. In this context as well as in connection with box splines and vertex splines, the construction of dual bases was discussed. The relevance of Fourier analysis was highlighted. Guided by regularity results from the theory of linear partial differential equations growth estimates for fundamental solutions of multivariate difference equations were applied to cardinal interpolation. The role of differential geometric aspects and topological arguments was discussed in connection with parametric representation of closed surfaces.

2. During the past few years a different type of functions namely so called radial functions have become a popular tool for treating surface fitting and interpolation problems. These funtions have

the form $\sum_i c_i \ \psi(\| x - x_i \|)$ where $\| \cdot \|$ denotes the Euclidean norm. A central problem concerns the denseness of such classes of functions in spaces of continuous functions on compact sets, say. Recent results were reported according to which Fourier analysis methods were used in a similar fashion as for splines on regular grids to derive conditions on the generating function ψ to ensure denseness when the data points x_i are located on a regular lattice. A somewhat unexpected dependence of the degree of approximation on the spatial dimension was pointed out.

3. Statistical methods come into play when estimating or recovering densities of functions from noisy data, again in the context of using spline methods. In particular, choosing the "balance between interpolation and smoothing" by means of cross-valisation techniques was discussed.

4. The approximation in the complex plane has recently given new insight into the difference between polynomial and rational approximation. The behaviour of best polynomials at the boundary seems to be comparable with bad cases of Taylor's series. On the other hand rational approximations do not often show such a bad behaviour. A nice theory which has attracted much attention in the past few years, goes back to results of Carathêodory and Fejer. Rational approximation is investigated via Hankel operators. This leads to a connection with system theory in electrical engineering and the design of optimal filters.

5. Approximation theory is important for the solution of elliptic differential equations; the finite elements have been already mentioned. Now for hyperbolic differential equations a different approach has been initiated. To this end, regularity results in terms of approximation rates were studied in the framework of Besov spaces.

Dietrich Braess
Wolfgang Dahmen
Charles Micchelli

Vortragsauszüge

Thomas Bloom

Convergence of Multivariable Lagrange Interpolants

Let X be a compact set in $\mathbb{C}^n$ and $\{A_{\alpha\nu}\}$ a triangular array in X. Let f be analytic in a neighborhood of X and let L_d denote the Lagrange interpolante to f at stage d of the triangular array. Is $\lim_{d\to\infty} \|f - L_d\| = 0$? For $n = 1$ (i.e. the one-variable case) the results are classical. For $n > 1$ the answer is positive if f is analytic on a "sufficiently large" open set containing X. For X locally regular we give such sufficiently large sets in terms of an extremal plurisubharmonic function.

Borislav Bojanov

Comparison of Approximation Methods

Let $R(E)$ denote the error of a certain approximation method which uses as information data $T(f) := \{f^{(j)}(x_i)\}$ of type, described by the incidence matrix E, where the nodes $\{x_i\}$ and the scheme of using $T(f)$ are determined in a certain best way. We give examples of comparison theorems of the form: $\mu(E_1) \prec \mu(E_2)$ implies $R(E_1) < R(E_2)$, where $\mu(.)$ is some characteristic of the structure of the matrix.

A. Bultheel

Matrix Euclidean Algorithm and Matrix Padé Approximation

A possible extension of the scalar Euclidean algorithm to matrix polynomials is given. It is also shown how it can be used to compute Padé approximations for rectangular matrix series. This definition of matrix Padé approximation is a natural generalization of the scalar definition which uses detailed information about the row degrees of the numerator and column degrees of the denominator.

Chang Gengzhe

A Converse Theorem of Convexity for Bernstein Triangular Polynomials

Assume that $f(x)$ is a continuous function defined on $[0,1]$. It is well-known that if $f(x)$ is convex in $[0,1]$ then the sequence of its Bernstein polynomials is decreasing as the degree of Bernstein polynomials goes to infinity. Kosmak, and then Ziegler showed that the converse is still valid in the univariate case. For Bernstein polynomials over triangles, Chang and Davis showed in 1984 that the convexity of a continuous function implies the decreasing of its Bernstein polynomials sequence, however the converse is not generally true. In this paper we show that if the sequence of Bernstein triangular polynomials is decreasing with respect to the degree, then the approximated function does not attain nontrival local maximum inside the domain triangle. This generalizes the theorem of Kosmak and Ziegler in univariate case. Our theorem can be conversed. This is a joint work with Professor Zhang Jinzhong, at Chengdu Branch, Academia Sinica.

Charles K. Chui

On Bivariate Super Vertex Splines

This is a continuation of my joint work with M.J. Lai. Three years ago in Oberwolfach, I introduced the notion of vertex splines on an arbitrary triangulation in $\mathbb{R}^2$ and discussed our results in constructing vertex splines and quasi-interpolation formulae. In particular, the spaces S_4^1 and S_5^1 were studied carefully. The S_5^1-result can of course be generalized to S_d^r for $d \geq 4r+1$. These results were again studied by us in great generality later, including mixed partitions with simplices and parallelpipeds, and any dimension $\mathbb{R}^s$. Recently, motivated by an important result of de Boor and Höllig who proved that $d \geq 3r+2$ is the correct degree in $\mathbb{R}^2$, we constructed a basis of vertex splines of a super spline subspace of S_d^r that gives the full approximation order of $d+1$ for $d \geq 3r+2$. Here, a super spline is one which has higher order derivatives than r at the vertices.

Z. Ciesielski

Nonparametric Density Spline Estimation

Bernstein-Schoenberg type spline operators together with approximation theory techniques are used to estimate the density and its distribution. In a wide class of densities (i.e. with derivatives of bounded variation) the mean L^1 error is optimal, namely it is $\mathcal{O}(h^{2/5})$, where h is the window parameter. For a fixed sample a new computationally feasible method for the choice of the window parameter is presented. To this end, the second L^1-modulus of smootheness of the density estimator is minimized in h.

Ronald A. DeVore

Non-linear Approximation and Applications

The application we have in mind is to regularity estimates for conservation laws: (*) $u_t + f(u)_x = 0$, $u(x,0) = u_0(x)$. We use Lax's method for describing a solution to (*). This method shows that $u(x,t)$ for $t > 0$ is equal to $u_0(y)$ where y is one of the values for which $\frac{x-y}{t} = u_0(y)$. From this, it is easy to show that if u_0 can be approximated to an error ϵ_n by piecewise constants with n (free) break points in the uniform norm. Then so can $u(x,t)$ for any $t > 0$. This is a regularity theorem if we take for ϵ_n a sequence which tends to 0. For example if $\epsilon_n = \mathcal{O}(1/n)$, this says that $u_0 \in BV$ implies $u \in BV$. Lucier has given similar results for p.w. linear approximations with n free knots in the L_1-norm, which can also be easily described using Lax's method. A typical result in this case is that if u_0 is in $B^\alpha_\sigma(L_\sigma)$ (the Besov space), $\sigma = \frac{1}{\alpha+1}$, $0 < \alpha < 2$, then u is also in $B^\alpha_\sigma(L_\sigma)$ for all $t > 0$.

Paul Dierckx

FITPACK: a Software Package for Curve and Surface Fitting Using Splines

FITPACK is a collection of FORTAN programs for curve and surface fitting with splines. Features included are automatic knot selection, error smoothing and data reduction. Beside the set of data points, the user merely has to provide a single parameter by which he can control the trade-off between closeness of fit and smoothness of fit. In this talk we will discuss the basic properties of the FITPACK routines and explain the knot placing strategy. We will report on some practical applications and mention some future developments.

Nira Dyn

4-Term Recurrence Relations for Tchebycheffian B-splines

Four-term recurrence relations with constant coefficients are derived for a wide class of Tchebycheffian B-splines, LB-splines and complex B-splines. Such a relation exists whenever the differential operator defining the underlying "polynomial" space can be factored in two essentially different ways. The four lower order B-splines in the recurrence relation appear in two pairs, each pair corresponding to one of these factorizations.

It is shown that the two term recurrence relations for polynomial, trigonometric and hyperbolic B-splines as well as other known two-term recurrence relations, are obtained directly from the four-term recurrence relations in a unified and systematic way.

The above derivation also yields two different two-term recurrence relations for the Green's functions of these "polynomial" spaces. In this context the special examples of exponential functions and rational functions are analyzed in detail.

Stephen D. Fisher

Analytic Functions with Bounded r^{th} Derivative

Fix an integer $r \geq 0$ and let $A = A_r$ be those analytic functions f on the open unit disc $\Delta = \{z : |z| < 1\}$ which satisfy (a) $|f^{(r)}(z)| < 1, z \in \Delta$ and $(b) f$ is real on the interval $(-1,1)$. Let $n \geq 0$ be an integer and let $x_0, \ldots, x_{n+r}$ be $n+r+1$ points in (-1,1), with not more than r consecutive coincidences. Set

$$\Lambda = \{(f(x_0), \ldots, f(x_{n+r})) : f \in A\}.$$

My talk will be concerned with the nature of the set Λ and several implications that can be drawn from it. Specifically, I will give

(1) A description of the boundary of Λ.

(2) An "envelope" theorem: $\max\{|f(x_0)| : f \in A, f(x_j) = 0, 1 \leq j \leq n+r\}$.

(3) The Kolmogorov $n+r$ width of A in $C_{\mathbf{R}}(E)$, where E is a subset of $(-1,1)$.

(4) A "Landau" theorem: $\max\{|f^{(k)}(x_0)| : f \in A, |f| \leq \sigma \text{on } E\}$, $1 < k < r$.

T.N.T. Goodman

Homogeneous Polynomial Splines

We construct functions which are piecewise homogeneous polynomials in three dimensions. These give a rich and elegant theory which combines properties of polynomial box splines with the explicit representation of simple exponential box splines, while enjoying complete symmetry in the three variables. By considering the restriction of these functions to suitable planes one obtains piecewise polynomial functions of two variables on a mesh formed by three pencils of lines. The vertices of these pencils may be finite or one or two may be infinite, i.e. the corresponding pencils may comprise parallel lines. As a limiting case all three vertices become infinite and one recovers polynomial box splines on a three direction mesh.

John A. Gregory

Piecewise Defined Parametric C^k Surfaces

An approximation theorist might view the construction of a piecewise defined C^k-parametric surface as a C^k-map from a partition of a domain in $\mathbb{R}^2$ to $\mathbb{R}^3$. A differential geometer would immediately dismiss this notion, for example, it is trivial that the plane is not topologically equivalent to a spherical surface. Differential geometry solves this by building up a surface as a sequence of regular C^k-mappings from open regions in $\mathbb{R}^2$ ("coordinate charts"). The problem in spline approximation is that surfaces are built as a sequence of non-overlapping mappings from closed domains in $\mathbb{R}^2$ and this leads to the concept of *geometric continuity* GC^k between such closed surface elements. This talk, which is based on joint work with J. Hahn, will present conditions which ensure geometric continuity between surface elements and will present a construction for developing a GC^k surface over an arbitrary mesh network in $\mathbb{R}^3$.

Rainer Hettich

On the Superlinear Convergence of an Algorithm for General Rational Tchebycheff Approximation

This talk is concerned with the approximation of a real-valued $f \in C(B)$, B a compact metric space, by rationals $r(p,x) = v(p,x)/w(p,x)$, v, w linear in p, and w restricted to $1 \leq w(p,x) \leq \gamma$ on B. Additional linear constraints (e.g. "restricted range") are admitted. Around 1980, Speide, Leucke and Hettich have shown that the problem is equivalent to finding the zero of the value-function of a parametric linear programming problem, and that this zero can be determined by a (globally converging) "Newton-like" method. The amount of work per step is the same as for the differential correction algorithm. Using recent results (obtained by Leucke and Hettich) from parametric programming it will be shown that the assumption of a Slater condition is sufficient for superlinear convergence, in contrast to the differential correction method, where this could be proved only under uniqueness assumptions.

Klaus Höllig

Multivariate Difference Equations and Cardinal Interpolation

Let $B : \mathbb{Z}^d \longrightarrow \mathbb{R}$ be a mesh function with compact support. It is shown that the difference equation

$$\sum_{j \in \mathbb{Z}^d} B(k-j)\, a(j) = f(k), \; k \in \mathbb{Z}^d,$$

has a fundamental solution Λ with [at most] polynomial growth, i.e.

$$a(j) = \sum_k \Lambda(j-k) f(k)$$

and $|\Lambda(j)| = \mathcal{O}\big((1+|j|)^m\big)$ with m depending on B. This result is applied to prove optimal convergence rates for interpolation with box-splines.

Ian R.H. Jackson

Approximation Using Radial Basis Functions

Quasi-interpolation on a regular grid in $\mathbb{R}^n$ of spacing h to a function f is considered, so that

$$a_n(\chi) = \sum_{z \in (h\mathbb{Z})^n} f(z)\psi(h^{-1}(\chi - z)), \ \chi \in \mathbb{R}^n$$

where ψ is a function decaying quickly for large argument.

Consider the case of radial basis functions

$$\psi(\chi) = \sum_{j=1}^{m} \mu_j \varphi(\|\chi - \chi_j\|_2), \quad \chi \in \mathbb{R}^n, \chi_j \in \mathbb{Z}^n,$$

where $\varphi : \mathbb{R}^t \to \mathbb{R}$ is known as a radial basis function. It is shown that for sufficiently smooth f, one can bound $\|a_h - f\|_\infty$ by a constant multiple of $h^{t+1}|\log h|$, if the formula reproduces polynomials of degree t and $|\psi(\chi)| \le A(1+\|\chi\|^{n+1+t})^{-1}$. Conditions are then considered for polynomial reproduction and under the same conditions on ψ, a characterization is given in terms of the Fourier transform of ψ. Various choices of φ are considered and, among other things, it is shown that, if $\varphi(r) = r$, then the order of convergence h^{n+1} can be achieved in $\mathbb{R}^n$ for n odd, a generalisation of the result for linear interpolation in $\mathbb{R}$.

Kurt Jetter

Birkhoff Quadrature of Double Precision

Im Vortrag behandeln wir – exemplarisch statt allgemeinerer und technisch aufwendigerer Fälle – Identitäten vom Typ

$$\int_0^1 f(x)dx = \sum_{i=1}^{n} a_i f^{(k_i)}(x_i) \quad \text{für } f \in P_{2n-1},$$

wobei die Knoten x_i angeordnet seien und k_i vorgegebene natürliche Zahlen sind. Im Fall $k_i = 0, i = 1, \ldots, n$, existiert eine solche Identität bekanntlich genau für die Knoten der n-Punkte-Gauß formel.

Fragen der Existenz und der Eindeutigkeit solcher Identitäten in Abhängigkeit des Ableitungsvektors $K = (k_1, \ldots, k_n)$ stehen im Vordergrund unserer Überlegungen. Für "pyramidales" K beweisen wir die Existenz unter Anwendung des Borsukschen Antipodenlemmas; dieser Beweis wurde in Zusammenarbeit mit N. Dyn (Tel-Aviv) erbracht.

Rong-Quing Jia

Fourier Analysis Methods in Multivariate Approximation

Fourier analysis methods were used by Schönberg to study approximation of equidistant data and cardinal spline interpolation. Following the lead of Schönberg, Strang and Fix developed this approach in the setting of finite element methods. They introduced the concept of controlled approximation and tried to characterize the controlled approximation order. However, their main characterization theorem was shown by Jia to be incorrect. Furthermore de Boor and Jia proposed to employ local approximation instead of controlled approximation, and in addition, gave a complete characterization for the local approximation order. In the meantime, Dahmen und Micchelli used Fourier analysis methods in their study of the algebraic properties of box splines.

In this talk we want to demonstrate that Fourier analysis methods still play an important role in several problems in multivariate approximation. Among them are partition of unity and approximation, subspaces invariant under translation and dual bases for box splines, approximation by box splines, multivariate cardinal interpolation by using B-nets. Also, we want to take this opportunity to announce several results obtained recently.

W. Krabs

On an Approximation Problem in Signal Theory

In the process of determining the probability distribution of the sampling error in a communication system in the worst case, one is led to the following approximation problem: Let, for some $L, N \in \mathbb{N}$,

$$V_L = \{\sum_{\ell=1}^{L} \chi_\ell \sum_{k=0}^{N-1} z_\ell(\omega + k\omega_0)e^{ik\omega_0 t} \mid t \in [-\tau, \tau],$$

$$\omega \in \Omega_N^0 = [-N\frac{\omega_0}{2}, (-N+2)\frac{\omega_0}{2}],\ \chi_\ell \in \mathbb{C} \quad \text{for } \ell = 1, \ldots, L\}$$

where $\omega_0 = \frac{2\pi}{\tau}, 0 \leq \tau \leq \frac{T}{2}$ $(T > 0)$ and $z_1, \ldots, z_L \in L^2[-N\frac{\omega_0}{2}, N\frac{\omega_0}{2}]$ are given and let $f(t,\omega) = e^{i\omega t}$ $(i = \sqrt{-1})$, $t \in [-\tau, \tau], \omega \in \Omega_N^0$. Find $\hat{v}_L \in V_L$ such that

$$\|f - \hat{v}_L\| \leq \|f - v_L\| \quad \text{for all } v_L \in V_L$$

where $\|g\| = \max_{t\in[-\tau,\tau]} \|g(t,\cdot)\|_{L^2(\Omega_N^0)}$ for every $g \in C([-\tau,\tau], L^2(\Omega_N^0))$.
Based on a characterization of best approximants $\hat{v}_L$ which involves the probability distribution of the worst sampling error, this problem can be solved explicitly for the special case $L = N \in \mathbb{N}$ and

$$z_\ell(\omega) = \begin{cases} 1 & \text{for } \omega \in [-\ell\frac{\omega_0}{2}, \ell\frac{\omega_0}{2}], \\ 0 & \text{for } \omega \in [-N\frac{\omega_0}{2}, N\frac{\omega_0}{2}] \backslash [-\ell\frac{\omega_0}{2}, \ell\frac{\omega_0}{2}]. \end{cases}$$

M. Lautsch

A Spline Inversion Formula for the Radon Transform

We give an approximate inversion formula for the Radon transform of a function f in an arbitrary number of dimensions. The basic idea is to consider the least-squares approximation of f in a finite element space and to transform the normal equations into equations involving Radon data. We express the Radon transform of finite elements in terms of multivariate B-splines and deduce error estimates in an L^2-setting.

Tom Lyche

Condition Numbers for B-splines

To a given B-spline expansion, $f = \sum_j c_j \; B_{j,k,\vec{\tau}}$ of order k on a knot sequence $\vec{\tau}$ and a $\vec{t} \supset \vec{\tau}$ we define for $1 \leq p \leq \infty$

$$\delta_{k,p,\vec{\tau},\vec{t}} = \sup_{\|Ac\|_{\ell^p,\vec{t}}=1} \|c\|_{\ell^p,\vec{\tau}}.$$

Here Ac are the B-spline coefficients of f on $\vec{t}$ and

$$\| c \|_{\ell^p,\vec{t}} = (\sum_j |c_j|^p(\tau_{j+k} - \tau_j)/k)^{1/p}$$

are weighted ℓ^p norms. We have $\delta_{k,p,\vec{\tau},\vec{t}} \leq D_{k,p}$ where $D_{k,p}$ is independent of $\vec{\tau}$ and $\vec{t}$. We find the smallest value for δ in the case of adding one knot: $\vec{t} = \vec{\tau} \cup \{t\}$. Indeed, if $\delta_k^{(1)} = \sup_{\vec{\tau}} \sup_{\vec{t}} \delta_{k,\infty,\vec{\tau},\vec{t}}$ then we show that $\delta_k^{(1)} = k$ if k is odd and $k - \frac{2}{k} \leq \delta_k^{(1)} < k$ if k is even.

Jean Meinguet

On the Optimal Hankel-norm Approximation Problem

The main purpose of this talk is to contribute to a more thorough understanding of the fine constructive mathematics involved in significant applications of the so called Adamjan-Arov-Krein theory (which is essentially concerned with infinite Hankel matrices bounded in ℓ^2 and best approximation or extension problems associated with them):

– by providing a tutorial presention of the background material (rationale for the Hankel-matrix approach and singular value decomposition of ℓ^2 bounded infinite Hankel matrices of finite rank) for system-theoretic applications.

– by emphasizing the main AAK results concerning the solution of the problem of optimal Hankel-norm approximations (model reduction problems) for discrete-time systems of finite degree.

– by outlining a pure matrix proof of the basic (and mysterious!) "zero-count" property, viz: the z-transform of the right singular vector belonging to the j'th singular value (supposed to be simple) of an ℓ^2 bounded infinite Hankel-matrix of finite rank has precisely j-1 zeros inside the unit circle.

Günter Nürnberger

Kolmogorov Criteria and Spline Approximation

The famous criterion of Kolmogorov characterizes best uniform approximation from finite-dimensional subspaces of $C(T)$, T compact. We investigate Kolmogorov type criteria in connection with the uniqueness of best approximation. The results are applied to obtain alternation characterizations for spaces of splines with fixed knots. Moreover, it is shown that the set-valued spline metric projection is continuous on an open and dense subset of $C[a,b]$.

In general, alternation characterizations do not hold for splines with free knots. We show that in this case various strong unicity properties can be completely described by alternation conditions for best approximations with simple knots.

The results (except on splines with free knots) were obtained in cooperation with H. Berens.

Vasil Popov

Approximation of Functions by Means of Rational Functions

A pair of Besov spaces is presented such that an approximation property and the associated inverse property holds, when the approximation by rational functions is considered.

M.J.D. Powell

Radial Basis Function Interpolation on $\mathbb{Z}$

More of the given results are due to my research student, M.D. Buhmann. The problem is to interpolate a bounded function at the integers by a linear combination of $\{\phi(r) : r \in \mathbb{R}\}$ and its integer translates, where $\phi(\cdot)$ is a radial basis function, e.g. $\phi(r) = \sqrt{r^2 + c^2}$ which is Hardy's multiquadric. The theory is relatively straightforward when ϕ is absolutely integrable. This case is considered first, including identification of the Fourier transform of the cardinal interpolating function. This work is extended to the case when the space of interpolating functions includes absolutely integrable functions whose fop symbols do not vanish, which takes care of the multiquadric radial function. Finally we consider a more general extension, in order to address the question whether bounded interpolation on the integers is possible for the inverse multiquadric radial function $\phi(r) = 1/\sqrt{r^2 + c^2}$. This analysis is also relevant to understanding the properties of linear radial function approximation, $\phi(r) = r$, to functions taking an even number of variables.

T.J. Rivlin

Generalized Taylor Series

Let β denote an infinite lower triangular array of complex numbers whose k^{th} row is $\beta^{(k)} = (\beta_1^{(k)}, \ldots, \beta_{k+1}^{(k)})$, $k = 0, 1, 2, \ldots$. Let f be a function from β into $\mathbb{C}$, and $I_j f$ be the divided difference functional, $f \longrightarrow f(\beta_1^{(j)}, \ldots, \beta_{j+1}^{(j)})$, $j = 0, 1, 2, \ldots$. It is not difficult to see that there exists a unique sequence of monic polynomials, $P_k \in \mathcal{P}_k$, $k = 0, 1, 2, \ldots$ which are biorthogonal to I_j, i.e. $I_j P_k = \delta_{j,k}$, $j, k = 0, 1, 2, \ldots$. We may associate to each f the series $\sum_0^\infty (I_j f) P_j(z)$, as biorthogonal expansion, which is called a Generalized Taylor Series. For example:

(1) $\beta_i^{(k)} = 0$, all i, k. The G.T.S. is the usual Taylor series.

(2) $\beta^{(k)}$ is independent of k, $P_k(z) = (z - \beta_1) \ldots (z - \beta_k)$, $P_0 = 1$, the familar Newton series.

(3) The entries in each row are all equal. P_k are the Gontcharov polynomials.

(4) $\beta_j^{(k)} = \omega_k^j$, $j = 1, \ldots, k+1$ where ω_k is a primitive $k+1^{st}$ root of unity. $P_k(z) = \sum_{j/n} \mu(\frac{n}{j}) z^j$, $k = 1, 2, \ldots$ (Ching and Chui, J. Approx. Theory *10* (1974), 324–336). μ is the Möbius function of number theory frame.
(5) $\beta_1^{(0)} = 1$, $\beta_j^{(k)} = \cos\frac{(j-1)\pi}{k}$, $j = 1, \ldots, k+1$ (extrema of the Chebychev polynomial). We show that: $P_0 = 1$; $2^{n-1} P_n = T_n - T_0$, $n = 2^q, q = 0, 1, \ldots$, and $2^{n-1} P_n = \sum_{d/m} \mu(\frac{m}{d}) T_{2^k d}$, $n = 2^k m, k \geq 0, m = 2q = 1, (q \geq 1)$. Bounds on the basic polynomials can be used to show that, for certain classes of f, if $I_j f = 0, j = 0, 1, \ldots$, then $f = 0$.

Robert Schaback

Adaptive Rational Splines in one and two Dimensions

Adaptive rational splines generalize the notion of C^2 rational interpolating splines with quadratic numerator and linear denominator by allowing cubic sections where the data make these necessary. This allows inflection points to be handled, while regions of strict convexity or concavity are preserved. An existence theorem based on Brouwer's fixed point theorem is given. Furthermore, the relation to Bézier rational functions is described. Finally, a series of numerical examples in one and two dimensions is presented.

Walter Schempp

Holographic Identities

The classical sampling theorem ist based on one-dimensional lattices and is therefore suitable for linear sequential (digital) signal processing. Holography, although also a sampling procedure, is radically different in concept. Indeed, second generation holography gives rise to planar and spatial lattices by off-axis techniques and is therefore applicable to parallel signal processing. – In this lecture we determine the wave functions having radial holographic transforms in terms of Hermite functions. Moreover, we compute the discrete intensity-interference patterns of elementary holograms in terms of Laguerre and Charlier-Poisson polynomials and the associated holographic identities. Various applications to information processing are briefly indicated.

Karl Scherer

Best Parametric Interpolation by Curves

The classical spline problem has for given data $\{y_i\}_{i=1}^n \in \mathbb{R}^d$ the following natural generalization

$$\inf_{0=t_1<t_2<\ldots<t_n=1} \inf\{\int_0^1 \|f^{(k)}\|^2 : f(t_i)=y_i,\ 1\le i\le n,\ f\in \mathbf{L}_2^k(0,1)\}$$

where $\mathbf{L}_2^k(0,1)$ is the space of $\mathbb{R}^d$ valued functions with components in $L_2^k(0,1)$.

The significance of this problem for applications is discussed (initiated by Toepfer, Marin). Further existence results are given (joint work with P.W. Smith) and a uniqueness result for the cubic case is announced.

Larry Schumaker

Penalized Least Squares

We discuss the method of penalized least squares for fitting data. The idea is to minimize a combination of smoothness and goodness of fit over a fixed given finite dimensional space (typically the span of B-splines). We also discuss a generalized cross validation method for choosing the smoothing parameter.

Philip W. Smith

The Spectrum of Spline Interpolation Matrices

For a fixed positive integer k, let $\mathbf{t}$ and $\mathbf{r}$ be two biinfinite increasing sequences satisfying $t_i < t_{i+k}$ and $\tau_i < \tau_{i+1}$. We errect the k-th order normalized B-splines $\{B_{j,k} := B_{j,k,\mathbf{t}}\}$ over $\mathbf{t}$ and consider the collocation problem:

$$\sum_j \alpha_j B_{j,k}(\tau_i) = f_i$$

for bounded $\mathbf{f}$. Or in matrix form, $A\alpha = \mathbf{f}$.

Suppose that A is boundedly invertible on ℓ^∞, then we ask whether the Neumann series, $\sum (I-A)^j$, converges to A^{-1}. We know this is so when A is Toeplitz and its central and main diagonal are equal. This leads us to the

CONJECTURE: If A is an invertible spline collocation matrix with central and main diagonals coinciding, then the spectrum of A is in $\{z : |z|\le 1 \text{ and } |z-1|\le 1\}$.

Florenzo I. Utreras

Tensor Product Spline Smoothing

We discuss the practical use of Tensor Product Smoothing Splines when choosing the smoothing parameter by Generalized Cross Validation. In particular we formulate a variational definition of smoothing splines that preserves the Tensor Product structure and leads to a practical algorithm. Numerical examples are given and some optimality results concerning GCV.

Grace Wahba

Problems in Estimating Functions of Many Variables from Scattered, Noisy Data

We describe partial and interaction splines for estimating functions of several variables given discrete, noisy experimental or survey data. A partial spline is a function of several variables which is a smoothing spline (univariate or multivariate) in some of the variables, and of specified parametric form in other of the variables. An interaction spline is a function of several variables which is a linear combination of spline functions of one variable, spline functions of two variables, etc. which satisfy some conditions guaranteeing uniqueness. Approaches to data based methods for selecting and fitting the model will be discussed, along with methods for detecting bad data points and for providing confidence intervals.

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