MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 26/1988

Graphentheorie

12.6. bis 18.6.1988

Die Tagung fand unter der Leitung von Herrn W. Mader (Hannover) und Herrn G. Ringel (Santa Cruz, Kalifornien) statt.
Die 40 Teilnehmer aus 14 Ländern (u.a. Neuseeland, Japan, Kanada, USA, UdSSR) berichteten einander über ihre Forschungsgebiete und über ihre neuesten Ergebnisse. Außerdem wurden in einer Sitzung unter Leitung von Herrn P.Erdös zahlreiche Probleme vorgestellt.

Vortragsauszüge

M. D. Plummer:
Some recent results on well-covered graphs

The maximum independent set problem for graphs is well-known to be NP-complete. But suppose one has a graph in which the greedy algorithm for an independent set always yields a maximum independent set. In other words, every maximal independent set is maximum. We call such graphs well covered (w-c).
The structure of well-covered graphs in general is far from completely understood. In this talk, we first consider the special case of cubic graphs (i.e., regular of degree 3). In work joint with Stephen Campbell, we present complete characterizations of 1- (but not 2-) and 2-

(but not 3-) connected cubic w-c graphs and 3-connected cubic planar w-c graphs. The first two families are infinite, but the third contains but four members.
We also discuss another approach taken by Denbow, Hartnell and Nowakowski who have recently characterized all w-c graphs of girth at least 5.

A. Hajnal:
On the number of distinct induced subgraphs

A graph $G = (V,E)$ is l-canonical if there is a partition $\cup_{i<l} A_i = V$ such that $x, x' \in A_i$, $y, y' \in A_j$ imply that $\{x,y\} \in E \Leftrightarrow \{x',y'\} \in E$. G is l,m-almost canonical if there is an l-canonical graph $G_0=(V,E_0)$ such that for the symmetric difference $G_1 = G \,\Delta\, G_0$, G_1 has only components of size at most m. Let i(G) denote the number of distinct induced subgraphs of G.

Theorem: $\forall\, \varepsilon > 0 \forall\, k \geq 1\, \exists\, \delta > 0\, \forall n\, \forall\, G$ with n vertices : $i(G) \leq \delta\, n^{k+1} \Rightarrow \exists\, W_{n,l,m}$:

($|W_n| < \varepsilon\, n$ and $l + m \leq k+1$ and $G[V\backslash W_n]$ is l,m-almost canonical).
This is a joint work with P. Erdös. The proof will appear in the Proceedings of Cambridge Combinatorial Conference held in March 1988.

Herbert Fleischner:
Transformationen Eulerscher Linien

Sei G ein zusammenhängender eulerscher Graph, M_E die Menge seiner eulerschen Linien. A. Kotzig führte den Begriff der κ-Transformation ein, um zu zeigen, daß je zwei Elemente aus M_E durch eine Folge von κ-Transformationen ineinander transformiert werden können. Beschränkt man sich auf spezielle Teilklassen $M_E' \subset M_E$, so ist diese Aussage nicht mehr gültig; Kotzig selbst zeigte dies anhand der nichtüberschneidenden eulerschen Linien 4-regulärer ebener Graphen. In diesem Fall bedarf es der Kombination von "κ-Abspaltungen" und "κ-Absorptionen", um eine analoge Aussage aufrechtzuerhalten. Ähnliches gilt, wenn M_E' die Menge der zu einem Durchgangssystem kompatiblen eulerschen Linien ist.

A. Schrijver:
Decomposition of graphs on surfaces

We discuss the following two theorems, which are motivated by "graph minors", VLSI-design, and multi-commodity flows.

Theorem 1. Let G be a graph embedded on a compact surface S, and let $C_1,\dots,C_k$ be closed curves embedded on S. Then G contains pairwise disjoint simple circuits $K_1,\dots,K_k$ so that K_i is freely homotopic to C_i (for $i=1,\dots,k$), if and only if:

(i) there exist pairwise disjoint simple closed curves $B_1,\dots,B_k$ on S so that B_i is freely homotopic to C_i (for $i=1,\dots k$);

(ii) $\mathrm{cr}(G,D) \geq \Sigma_{i=1}^{k} \mathrm{mincr}(C_i,D)$ for each closed curve D on S;

(iii) $\mathrm{cr}(G,D) > \Sigma_{i=1}^{k} \mathrm{mincr}(C_i,D)$ for each doubly-odd closed curve D on S.

Theorem 2. Let G be an eulerian graph embedded on the Klein bottle. Then the maximum number of pairwise edge-disjoint orientation reversing circuits in G is equal to the minimum number of edges intersecting all orientation-reversing circuits.

Gerhard Ringel:
Clean triangulation

In a triangulation T of a surface S each face is a triangle. If also each triangle is a face then T is called clean. If the number of triangles in T is minimal for a given S, T is called minimal. Denote by $\tau(S)$ the number of triangles in a minimal clean triangulation of S. Let S_g be the orientable surface of genus g. We can prove that $\tau(S_1) = 24$ and that $\lim_{g\to\infty} \tau(S_g)\, g^{-1}=4$. This was joint work with Nora Hartsfield.

K. Steffens:
f-factors of countable graphs

Let $G = (V,E)$ be a graph and $f:V\to N$ be a function such that $0 < f(x) \leq d_G(x)$ for each $x \in V$. A subgraph $F = (V^*,E^*)$ of G is said to be an f-factor if $d_F(x) \leq f(x)$ for all $x \in V^*$. An f-factor $F = (V^*,E^*)$ is called perfect if $V^* = V$ and $d_F(x) = f(x)$ for each $x \in V$. Let s be a fixed vertex and F be a fixed f-factor of G. A vertex v is called an <u>outer</u> vertex if there is an F-alternating trail from s to v starting with an edge of $E \setminus F$ and ending with an edge of F.

<u>Theorem.</u> A countable graph G has no perfect f-factor if and only if there exists an f-factor F of G, an unsaturated vertex s, a set O of outer vertices with $s \in O$ and a set L(v) of edges incident with v for each $v \in O$ such that

(i) $E(F) \cap L(v) = \emptyset$ for each $v \in O$

(ii) $|L(v)| = f(v) - d_F(v)$ for each $v \in O \setminus \{s\}$ and $|L(s)| = f(s) - d_F(s) - 1$,

(iii) there is no F-augmenting trail $(v_i : i < k \leq \omega)$ starting at s such that

$\{v_{2i}, v_{2i+1}\} \notin L(v_{2i})$ for each i with $2i+1 < k$.

Rainer Bodendiek:
<u>On almost planar graphs</u>

25 years ago Klaus Wagner found the concept of an almost planar graph! In this lecture, we are generalizing this concept for orientable or non-orientable surfaces or P_1 - where P_1 is the spindle-surface arising from a torus to a single point. Furthermore, two new methods of characterizing the set Δ of all almost planar graphs are given.

Tudor Zamfirescu:
<u>Intersections of longest circuits</u>

T. Gallai hat 1966 folgende Frage gestellt: Haben alle längsten Wege eines beliebigen (zusammenhängenden) Graphen einen gemeinsamen Punkt? Walther hat diese Frage negativ beantwortet. Sein Gegenbeispiel hatte 25 Punkte und hatte Zusammenhangszahl 1. Inzwischen sind auch 3-fach zusammenhängende Gegenbeispiele bekannt. Gibt es ein 4-fach zusammenhängendes Gegenbeispiel?

Sei A_k die Aussage: "Für je k Punkte eines zusammenhängenden (2-zusammenhängende) Graphen gibt es einen längsten Weg (Kreis), der all diese Punkte vermeidet". Wir wissen, daß A_1 und A_2 richtig sind. Ist auch A_3 richtig?

Sei B_k (C_k) die Aussage: "Je k längste Wege (Kreise) haben einen gemeinsamen Punkt". Seien $n_p = \sup\{k : B_k \text{ ist richtig}\}$, $n_c = \sup\{k : C_k \text{ ist richtig}\}$. Wir wissen, daß $2 \leq n_p$

≤ 6; $2 \leq n_c \leq 8$. Man bestimme n_p und n_c.

Wieviele gemeinsame Punkte haben zwei beliebige längste Kreise in einem k-zusammenhängenden Graphen. Scott vermutet, mindestens k. S.A. Burr und ich zeigen, daß es etwas mehr als $\sqrt{k}$ sein müssen.

H.-J. Bandelt:

Absolute retracts in graph theory

Both absolute retracts of reflexive graphs and absolute retracts of n-chromatic graphs (for $n \geq 2$) admit characterizations involving Helly type conditions, leading to polynomial time recognition procedures; for the bipartite case see Discrete Appl. Math. 16 (1987) 191 - 215, and for a survey see Mathematical Systems in Economics 110 (Athenäum Verlag, 1988). This is a joint work with E. Pesch, A. Dählmann & H. Schütte, resp.

Neil Robertson:

Algorithms related to the Hadwiger conjecture

This is joint work with Paul Seymour. We show that for $k \geq 4$, there is an algorithm, operating in V^5-time, with input a graph G and output either a k-coloring of G or a minor H of G which is not k-colorable. Thus if Hadwiger's conjecture is true, the algorithm would output a k-coloring or a K_{k+1}-minor. Our algorithm is based on a lemma which says that for fixed $k \geq 4$ there is a number f(k) such that for any graph G with K_{k+1} not a minor and with tree-width $\geq$ f(k) there is a reduction of the coloring problem for G to one for a simpler associated graph G*. This lemma is based on a general structure theory for graphs with K_{k+1} as an excluded minor, and a refinement of this needed to complete the proof. There will be some discussion of possible stronger results which would apply to the first open case, k = 5, of Hadwiger's conjecture.

D. Archdeacon:

Girth & face-width of embedded graphs

The face-width of an embedded graph measures how well a graph represents a surface. In this talk we present graphs which can represent two different surfaces very well, yielding counterexamples to a conjecture of Robertson and Vitray. We also present embeddings in which both the graph and its dual have large girth.

H.-J. Voss:

Groups and graphs

At the Symposium on Graph Theory and its Applications held at Smolenice in 1963 H. Sachs presented the following problem: Assign to each triple (x,y,z) of pairwise distinct elements x,y,z with xyz = e of a group (G,·) an oriented triangle with vertex set x,y,z, arc set (x,y), (y,z), (z,x) and one face. In the set Σ of so obtained triangles identify opposite directed arcs (x,y) and (y,x). Triangulations of orientable surfaces are obtained.

Thus to each group (G,·) a set of triangulations of orientable surfaces are assigned. Such triangulations have been intensively studied by W. Voss.

For a certain class C of triangulations of orientable surfaces St. Ulbricht, W. Voss and I could reverse this assignment, i.e. to each triangulation $T \in C$ an assigned group could be determined. This theorem could be extended to cellular decompositions of orientable surfaces, in which the bounding cycles of all cells (faces) have the some length q ($q \geq 4$).

Kathie Cameron:

Perfect graphs with additional min-max properties

A system L of linear inequalities in the variables x is called TDI (totally dual integral) if for every linear function cx such that c is all integers, the dual of the linear program: maximize {cx: x satiesfies L} has an integer-valued optimum solution or no optimum solution.

A system L is called box TDI if L together with any inequalities $l \leq x \leq u$ is TDI. It is a corollary of work of Fulkerson and Lovász that: where A is a 0-1 matrix with the 1-columns of any row not a proper subset of the 1-columns of any other row, the system $L(G) = \{Ax \leq \underline{1}, x \geq 0\}$ is TDI if and only if A is the matrix of maximal cliques (rows) versus nodes (columns) of a perfect graph. Here we will describe a class of graphs in a graph-theoretic way, and characterize them as the graphs G for which the system L(G) is box TDI. Thus we call these graphs box perfect. We also describe some classes of box perfect graphs.

W.R. Pulleyblank:

Cycle polyhedra

Let x^c be the incidence vector of a simple cycle C in an undirected graph G. Let C(G) and P(G) denote the cone and polytope respectively generated by all such vectors. Seymour (1979) gave a linear system sufficient to define C(G) for general graphs. Define, for each $\hat{x} \in C(G)$,

$$\lambda_{\hat{x}} = \min \{\Sigma\lambda_c : \hat{x} = \Sigma\lambda_c x^c, \lambda \geq 0\}$$

$$\mu_{\hat{x}} = \max \{\Sigma\lambda_c : \hat{x} = \Sigma\lambda_c x^c, \lambda \geq 0\}$$

where both sums are taken over all simply cyles. We let $L(G) = \{x \in C(G) : \lambda_x \leq 1\}$, $U(G) = \{x \in C(G) : \mu_x \geq 1\}$. Then $C(G) = L(G) \cup U(G)$; $P(G) = L(G) \cap U(G)$.

For the case of Halin graphs, we give linear minmax theorems characterizing $\lambda_{\hat{x}}$ and $\mu_{\hat{x}}$ for $\hat{x} \in C(G)$.

These enable us to give explicit formulations of L(G), U(G) and hence P(G).

This is a joint work with Collette Coullard.

Jack Edmonds:

Quantified logical networks

Any polynomial algorithm for a predicate P(x,y) implies polynomially growing Boolean networks $B_{n,m}(x^n,y^m)$ such that $\forall\, n,m \in N$, $B_{n,m}(x^n,y^m) = P(x^n,y^m)$.

A predicate Q(x) is called NP if it has the form $Q(x) = [\exists\, y : |y| = p(|x|)\ \&\ P(x,y) \text{ is true}]$ where $|x|$ is the length of x, p(n) is a polynomial, and P(x,y) has a polynomial algorithm.

Hence for any NP predicate Q(x) we have $Q(x^n) = [\exists\, y^{p(x)} : B_{n,p(n)}\, (y^{p(n)}, x^n)]$.

Hence it would be very good to have ways to eliminate quantified variables from Boolean networks.

Gerd Sabidussi:

Clumps, minimal asymmetric graphs, and involutions

We consider minimal asymmetric and minimal bilaterally asymmetric (= involution-free) graphs. A useful parameter for classifying such graphs is the induced length, i.e. the length of a longest induced path. Denote by A_n the class of all minimal asymmetric graphs of induced length n; similarly B_n for bilateral asymmetry. With J. Nesetril we conjecture that there are only finitely many minimal asymmetric graphs and that these are also the only minimal bilaterally assymmetric graphs. In fact, we believe there are only 18 such graphs (9 complementary pairs). We can prove that $A_n = B_n = \emptyset$ for n ≥ 6, $A_5 = B_5$ consists of two graphs, and $A_4 = B_4$ of seven. That $A_n = B_n = \emptyset$ for n = 1,2 is trivial. Thus the only open case is n = 3. These results follow from the following considerably stronger theorem dealing with the structure of graphs which contain no minimal asymmetric graph as an induced subgraph.

THEOREM: Let G be a finite graph of induced length n ≥ 4 , and suppose that G contains no minimal asymmetric graph as an induced subgraph (actually, none of a list of 13 minimal asymmetric graphs). Then G contains a non-trivial clump (homogeneous set) or G has an involution.

Hans Jürgen Prömel:

Graphs not containing certain subgraphs

For a finite graph K let Forb(K) denote the class of all finite graphs which do not contain K as a (weak) subgraph. In this talk we give a complete characterization of all those graphs K with chromatic number at least 3 which have the property that almost all graphs in Forb(K) are bipartite. This extends earlier results of Erdös, Kleitman and Rothschild (1976) showing that almost all triangle-free graphs are bipartite and of Lamken and Rothschild (1985) showing that almost all graphs in Forb(C_{2k+3}) are bipartite for every odd cycle C_{2k+3}.

Andrós Frank:

On sum of circuits of graphs

Let G = (V,E) be an undirected graph and $w : E \to \mathbf{Z}_+$ a non-negative integral vector on the edges such that $\Sigma(w(e) : e$ incident to $v)$ is even for every node $v \in V$. Assume furthermore that there are no 5 pairwise disjoint edges with weight bigger than 1.

THEOREM It is possible to assign non-negative integers z(c) to the circuits of G so that $w = \Sigma(z(c) \cdot \chi_c : c$ a circuit) if and only if $w(e) \leq 1/2\, w(B)$ holds for every cut B of G and edge $e \in B$.

The Petersen graph (along with a w that is 2 on a specific perfect matching and 1 on the other 10 edges) shows that the theorem does not hold if 5 is replaced by 6 in the assumptions.

László Lovász:
Entropy splitting and perfect graphs

The entropy of a probability distribution p on a set V with respect to a graph G = (V,E) can be defined as

$$H(G,p) = \min\,[-\Sigma_{i\in V}\, p_i \log a_i]$$

where $a = (a_i : i \in V)$ ranges over the vertex packing polytope of G. In this work with Csiszár, Körner, Marton and Simonyi it is shown that

$$H(G,p) + H(\overline{G},p) \geq H(p) = -\Sigma_{i\in V}\, p_i \log p_i$$

an equality holds if and only if G is perfect. The key in the proof is the following lemma: Let $A \subseteq \mathbf{R}_+{}^n$ be a convex corner and A^*, its antiblocker. Then the vectors

$$(a_i b_i : i = 1,\ldots,n) \quad (a \in A,\ b \in A^*)$$

fill the unit corner $\{x \in \mathbf{R}_+{}^n : \Sigma\, x_i \leq 1\}$. The results can be used to prove the existence of a cover of perfect graphs with cliques and independent sets with rather strong properties.

Michel Las Vergnas:
The Tree Game and the Arborescence Game

The Tree Game on a graph is a variant of the Shannon Switching Game, solved by A. Lehman in 1964 in terms of matroids. The Arborescence Game is a directed version of the Tree Game. In the Arborenscence Game, two players, black and white, play alternately edges of an undirected connected graph G with a distinguished vertex x_o. A move of black consists of deleting an unplayed edge. A move of white consists of directing an unplayed edge. White wins if he forms a spanning arborescence of G rooted at x_o, and loses otherwise.
We characterize winning positions for the Arborescence Game in the case when G is a union of two edge-disjoint spanning trees. A general strategy follows.
Joint work with Y.O. Hamidoune

Nora Hartsfield:
Hamilton surfaces

A Hamilton surface is a two-dimensional analog of a Hamilton cycle; that is, rather than a cycle which contains every vertex, a Hamilton surface is a collection of polygons which form a genus embedding of the graph after appropriate edge identifications have been made.

A decomposition of $K_{2n,\,2n}$ into Hamilton surfaces is presented. These results are from a joint paper by N. Hartsfield, B. Jackson, and G. Ringel.

Charles Little:
<u>Cubic combinatorial maps</u>

In recent years, there has been much interest in studying imbeddings of graphs in surfaces from a combinatorial point of view. For example, such imbeddings have been modelled by means of cubic combinatorial maps, i.e. cubic graphs with a proper edge colouring in three colours. We discuss the Jordan curve theorem in this context.

Roland Häggkvist:
<u>Decompositions of complete bipartite graphs into copies of a given k-regular bipartite graph</u>

I shall give some variations on the following theme:
For every natural number k there exists a constant c_k such that every k-regular bipartite graph on 2n vertices decomposes $K_{c_k n, c_k n}$.
In particular it shall be seen that every 3-regular 2n-order bipartite graph without any Heawood-graph as a component decomposes $K_{6n,6n}$.

Haruko Okamura:
<u>Paths and cycles in k-edge connected graphs</u>

For a graph $G = (V(G), E(G))$, $\lambda(G)$ denotes the edge-connectivity of G. We call a graph G weakly k-linked, if for every k pairs of vertices (s_i, t_i), there are edge-disjoint paths $P_1,\dots,P_k$ such that P_i joins s_i and t_i $(1 \le i \le k)$. Let

$$g(k) := \min \{n \mid \text{if } \lambda(G) \ge n, \text{ then G is weakly k-linked}\}.$$

It is known that $g(2) = g(3) = 3$, $g(4) = 5$ and $k \le g(k) \le 2k-3$ $(k \ge 5)$.
Our results are

THEOREM 1. If $\lambda(G) \ge 2k$ $(k \ge 2)$ and $f_1, f_2 \in E(G)$, then there is a cycle C such that

$$\{f_1,f_2\} \subset E(C) \text{ and } \lambda(G-E(C)) \ge 2k-2.$$

THEOREM 2. $g(5) \le 6$, $g(6) \le 8$, $g(7) \le 9$, $g(3k) \le 4k$ and $g(3k+1) \le g(3k+2) \le 4k+2$ $(k \ge 2)$.

Thomas Andreae:
<u>A binary search problem for graphs</u>

A search problem is considered which generalizes the group testing problems previously studied in papers of Chang/Hwang and Chang/Hwang/Lin. In its general form for arbitrary graphs this problem was proposed by M. Aigner. Let G be a finite simple graph with vertex set V(G) and edge set E(G), and let $e^* \in E(G)$ be an unknown edge. In order to find e^* we choose a sequence of test-sets $A \subseteq V(G)$ where after every test we are told whether or not e^* is an edge of the subgraph induced by A. Find the minimum number c(G) of tests required. G is called <u>optimal</u> if c(G) achieves the usual information theoretic lower bound, i.e. $c(G) = \lceil \log_2 |E(G)| \rceil$. We relax c(G) to the notion of k-degenerate graph. (A graph is <u>k-degenerate</u> if there exists a linear order $x_1, x_2, \ldots, x_n$ of the vertices of G such that x_i has at most k neighbors among $x_1, \ldots, x_{i-1}$ $(i = 1, \ldots, n)$.) Among other results we prove that all 2-degenerate graphs are optimal and we provide an upper bound for

$c(G) - \lceil \log_2 |E(G)| \rceil$ in terms of k (for k-degenerate G).

Robin Thomas:
<u>Well-quasi-ordering of infinite graphs</u>

Robertson and Seymour proved that given any infinite sequence $G_1, G_2, \ldots$ of finite graphs there are indices i and j such that $i < j$ and G_i is isomorphic to a minor of G_j. This has several interesting applications. We are interested in a generalization of their result to infinite graphs. It turns out that the infinite version is false in general, but holds if G_1 is finite and planar.

Gefördert durch DFG Deutsche Forschungsgemeinschaft

Bogdan Oporowski:
<u>On Seymour's self-minor conjecture</u>

Paul Seymour conjectured that every infinite graph is isomorphic to a proper minor of itself. A counter-example to this conjecture, presented in the talk, is based on the counter-example to the Wagner conjecture about well-quasi-ordering of infinite graphs due to Robin Thomas. The validity of the conjecture for graphs with an isolated planar end has been shown and the implications, if Seymour's conjecture in the still open countable case is true, have been discussed.

Reinhard Diestel:
<u>End-faithful spanning trees in infinite graphs</u>

Let G be an infinite connected graph. Two rays (one-way infinite paths) P, Q $\subset$ G are *end-equivalent* if there exists a ray R $\subset$ G which meets both P and Q infinitely often. E(G) denotes the set of the corresponding equivalence classes, the *ends* of G. For example, the 2-way infinite ladder has two ends, the infinite grid **Z** x **Z** has one end, and the dyadic tree has $2^{\aleph_0}$ ends.

If T is a spanning tree of G and P, Q are end-equivalent rays in T, then clearly P and Q are also equivalent in G. We therefore have a natural map $\eta : E(T) \to E(G)$ mapping each end of T to the end of G containing it. In general, η need be neither 1-1 nor onto; if it is both, then T is called end-faithful. The following question was raised by Halin in 1964:

Problem. *Does every infinite connected graph have an end-faithful spanning tree?*

Halin settled this question in the affirmative for countable graphs. We do the same for all graphs, irrespective of their cardinality, that do not contain any subdivided infinite complete graph as a subgraph. The general problem remains open.

Martin Grötschel:
<u>Applications of connectivity</u>

The unique capabilities of fiber optic technology have made it necessary to implement new communication networks. One of the most important practical problems in this area is the design of minimum-cost survivable networks. This problem leads to interesting new connectivity concepts in graph theory. We show in this talk how "survivability" can be phrased in terms of connectivity parameters, formulate integer programming models of the corresponding optimization problems and present optimum solutions of some real-world problems. This talk is based on joint work with Clyde Monma and Mechthild Stoer.

Walter Vogler:

Unboundedness of functions on graph languages generated by edge replacement systems

This reports work done in collaboration with A. Habel and H.-J. Kreowski, Bremen.
A graph grammar is a finite system for the generation of graphs, the generated set of graphs is called a language. We have studied a specific type of grammars, namely hyperedge replacement systems, which include edge replacement systems.
Given a graph function q into the natural numbers we have studied the following problem:
Decide whether for given graph grammar GG q is unbounded on the language generated by GG.
This decision problem can be solved for edge replacement systems, if q is compatible with the derivation process in a specific way, involving addition and maximumg taking only. Examples of such functions q are the number of edges, the maximum degree or the maximum path length.

W. Deuber:

Cai Ning's solution of the extremal problem for diameter 2 over the 3 element alphabet

Ning Cai proved in his dissertation (Bielefeld 1988) the following theorem:
In $\{-1, 0, 1\}^n$ a set of diameter at most 2r (taximetric: $d(x,y) = \Sigma|x_i - y_i|$) has not more elements than the unit ball around 0.

W. Mader:

Critically connected digraphs

A digraph is called critically connected, if it is connected, but the deletion of any vertex destroys the connectivity. We prove that every critically connected, finite digraph has at least two vertices of outdegree one.

Wilfried Imrich:

On transitive graphs of polynomial growth

Results of Gromov and Trofimov imply that transitive, connected, locally finite infinite graphs of polynomial growth are closely related to Cayley graphs of virtually nilpotent groups. This suggests that the automorphism groups of such graphs retain some of the properties of nilpotent groups.

A survey of results and open problems in this area is presented.

Tatsuro Ito:

Distance-regular digraphs with Q-polynomial property

Let $(X, \{R_i\}_{0 \le i \le d})$ be a commutative nonsymmetric association scheme. Let $A_0, A_1, \ldots, A_d$ and $E_0, E_1, \ldots, E_d$ be the adjacency matrices and the primitive idempotents of the Bose-Mesner algebra. Assume the association scheme is of (P and Q)-polynomial, i.e., there exist polynomials $v_i(x)$ and $v_i^*(x)$ of degree i ($i = 0,1,\ldots,d$) such that $A_i = v_i(A_1)$ w.r.t. the ordinary multiplication and $nE_i = v_i^*(nE_1)$ w.r.t. the Hadamard product, i.e., entrywise product, where $n = |X|$. Then it is shown that the association scheme is self-dual, i.e. $v_i(x) = v_i^*(x)$. This result is obtained by D. Leonard, independently. We expect that $v_i(x)$, $v_i^*(x)$ are a kind of Askey-Wilson polynomials with weight $w(x)$, $x \in \mathbb{C}$.

A.V. Kostochka:

Upper bounds of the chromatic number of graphs via clique number with restrictions to graphs' structure

It is a small review of results on coloring of graphs from family G_i (i=1,2,3,4) with given clique number or girth, where $G_1 = \{G \mid \Delta(G) \le k\}$, $G_2 = \{G \mid G \text{ is k-degenerate}\}$, $G_3 = \{G \mid G \notin G_2 \;\&\; \forall e \in E(G) : G \setminus e \in G_2\}$, $G_4 = \{G \mid G \notin G_2 \;\&\; \forall v \in V(G) : G \setminus v \in G_2\}$. The main result is a description of $\{G \in G_3 \mid \chi(G) > k\}$.

Y. Egawa (talk was given by K. Cameron):
Covering vertices of a graph with cycles

Our main result is: Let G be a 2-connected graph with n vertices, and k an integer such that $n > k \geq 1$. If the minimum degree of a vertex of G is greater than or equal to $n/(k+1)$, then there exist k cycles in G which cover all the vertices of G.
We conjecture: Let G be a graph and k a positive integer. If the maximum size of an independent set of vertices in G is less than or equal to k times the vertex connectivity of G, then there exist k cycles in G which cover all the vertices of G. Where $k = 1$, this is a theorem of Erdös and Chvátal.

Jaroslav Nesetril:
Complex graphs with large girth

We state the following results:

1) For every n there exists a graph G and two linear orderings $\subseteq_1, \subseteq_2$ of V(G) such that 1) $\chi(G) \geq n$, 2) there is no edge $\{x,y\}$ and z such that z lies between x and g in both orderings $\subseteq_1$ and $\subseteq_2$.
2) For every graph G there exists a graph H such that:
1) girth H = girth G
2) for every partition $E(H) = E_1 \cup E_2$ a copy of G is contained in either E_1 or E_2.

Several relevant results are considered.

Horst Sachs:
Matchings, monotone path systems and some selected applications

Eine Reihe graphentheoretischer Probleme hängen eng mit linearer Algebra zusammen: so können z.B. die Anzahl der Gerüste beliebiger Graphen und die Anzahl der Linearfaktoren gewisser (insbesondere: ebener) Graphen durch die Determinante einer Matrix ausgedrückt werden, die sich in einfacher Weise aus der Adjazenzmatrix des Graphen gewinnen läßt (Satz von KIRCHHOFF-TUTTE bzw. Sätze von KASTELEYN und LITTLE). Der Verfasser gibt eine graphentheoretische Methode zur Reduktion von Determinanten und linearen Gleichungssystemen an und benutzt diese zur Bestimmung der Anzahl der Linearfaktoren in Ausschnitten aus ebenen Gittergraphen.
Die entspringenden Algorithmen, besonders solche, die sich auf monotone Wege stützen,

erweisen sich als sehr effizient. - Die Resultate haben Anwendungen in der Chemie der aromatischen Kohlenwasserstoffe (Bindungsordnung) und in der Physik der Kristalloberflächen (Dimer-Problem).
(Teilweise gemeinsam mit K. Al-Khnaifes.)

Berichterstatter: W.Vogler

Gefördert durch
DFG Deutsche Forschungsgemeinschaft

Tagungsteilnehmer

Dr. T. Andreae
Institut für Mathematik II
der Freien Universität Berlin
Arnimallee 3

1000 Berlin 33

Prof. Dr. W. Deuber
Fakultät für Mathematik
der Universität Bielefeld
Postfach 8640

4800 Bielefeld 1

Prof. Dr. D. Archdeacon
Dept. of Mathematics
University of Vermont
16, Colchester Ave.

Burlington , VT 05405
USA

R. Diestel
Dept. of Pure Mathematics and
Mathematical Statistics
University of Cambridge
16, Mill Lane

GB- Cambridge , CB2 1SB

Dr. H. J. Bandelt
Fachbereich Mathematik
der Universität Oldenburg
Postfach 2503

2900 Oldenburg

Prof. Dr. J. Edmonds
Dept. of Management Sciences
University of Waterloo

Waterloo, Ontario N2L 3G1
CANADA

Prof. Dr. R. Bodendiek
Mathematisches Institut
Pädagogische Hochschule Kiel
Olshausenstr. 75

2300 Kiel

Prof. Dr. Y. Egawa
Dept. of Applied Mathematics
Faculty of Sciences
Science University of Tokyo
1-3 Kagurazaka, Shinjuku-Ku

Tokyo 162
JAPAN

Prof. Dr. K. Cameron
Dept. of Management Science
University of Waterloo

Waterloo, Ontario N2L 3G1
CANADA

Prof. Dr. P. Erdös
Mathematical Institute of the
Hungarian Academy of Sciences
Realtanoda u. 13 - 15, Pf. 127

H-1053 Budapest

Prof. Dr. H. Fleischner
Institut für
Informationsverarbeitung
Österreichische Akademie d.Wiss.
Sonnenfelsgasse 19/II. Stock

A-1010 Wien

Prof. Dr. R. Halin
Mathematisches Seminar
der Universität Hamburg
Bundesstr. 55

2000 Hamburg 13

Prof. Dr. A. Frank
Department of Computer Science
Eötvös University
ELTE TTK
Muzeum krt. 6 - 8

H-1088 Budapest VIII

Prof. Dr. N. A. Hartsfield
Dept. of Mathematics
University of California

Santa Cruz , CA 95064
USA

Prof. Dr. M. Grötschel
Institut für
Angewandte Mathematik II
der Universität
Memminger Str. 6

8900 Augsburg

Prof. Dr. R. Henn
Institut für Statistik und
Mathematische Wirtschaftstheorie
Universität Karlsruhe
Postfach 6380

7500 Karlsruhe 1

Prof. Dr. R. Häggkvist
Dept. of Mathematics
University of Stockholm
Box 6701

S-113 85 Stockholm

Prof. Dr. W. Imrich
Institut für Mathematik
und Angewandte Geometrie
Montanuniversität Leoben
Franz-Josef-Str. 18

A-8700 Leoben

Prof. Dr. A. Hajnal
Mathematical Institute of the
Hungarian Academy of Sciences
Realtanoda u. 13 - 15, Pf. 127

H-1053 Budapest

Prof. Dr. T. Ito
Dept. of Mathematics
Joetsu University of Education

Joetsu, Niigata 943
JAPAN

Gefördert durch
DFG Deutsche Forschungsgemeinschaft

Prof. Dr. A. V. Kostochka
Institute of Mathematics
Siberian Academy of Sciences of the USSR
Universitetskii pr. 4

Novosibirsk 630090
USSR

Prof. Dr. J. Nesetril
Dept. of Mathematics and Physics
Charles University
MFF UK
Malostranske nam. 25

118 00 Praha 1
CZECHOSLOVAKIA

Prof. Dr. M. Las Vergnas
Mathematiques
U.E.R. 48, Tour 45-46, 3eme etage
Universite Paris VI
4, Place Jussieu

F-75230 Paris Cedex 05

Dr. H. Okamura
Faculty of Engineering
Oaska City University
Sugimoto

Osaka 558
JAPAN

Prof. Dr. C. H. C. Little
Dept. of Mathematics and Statistics
Massey University
Private Bag

Palmerston North
NEW ZEALAND

B. Oporowski
Dept. of Mathematics
The Ohio State University
231 W. 18th Ave.

Columbus , OH 43210
USA

Prof. Dr. L. Lovasz
Department of Computer Science
Eötvös University
ELTE TTK
Muzeum krt. 6 - 8

H-1088 Budapest VIII

Prof. Dr. M. D. Plummer
Dept. of Mathematics
Vanderbilt University

Nashville , TN 37235
USA

Prof. Dr. W. Mader
Institut für Mathematik
der Universität Hannover
Welfengarten 1

3000 Hannover 1

Dr. H. J. Prömel
Institut für Ökonometrie und
Operations Research
der Universität Bonn
Nassestr. 2

5300 Bonn 1

Prof. Dr. W. R. Pulleyblank
Department of Combinatorics and Optimization
University of Waterloo

Waterloo, Ontario N2L 3G1
CANADA

Prof. Dr. A. Schrijver
Department of Econometrics
Tilburg University
P. O. Box 90153

NL-5000 LE Tilburg

Prof. Dr. G. Ringel
2256 Westcliff Drive

Santa Cruz , CA 95060
USA

Prof. Dr. K. Steffens
Institut für Mathematik
der Universität Hannover
Welfengarten 1

3000 Hannover 1

Prof. Dr. N. Robertson
Dept. of Mathematics
The Ohio State University
231 W. 18th Ave.

Columbus , OH 43210
USA

Prof. Dr. P. Terwilliger
Department of Mathematics
University of Wisconsin-Madison
Van Vleck Hall
480 Lincoln Drive

Madison WI, 53706
USA

Prof. Dr. G. Sabidussi
Dept. of Mathematics and Statistics
University of Montreal
C. P. 6128, Succ. A

Montreal , P. Q. H3C 3J7
CANADA

Prof. Dr. R. Thomas
Department of Mathematics
The Ohio State University
100 Mathematics Building
231 West 18th Avenue

Columbus , OH 43210-1174
USA

Prof. Dr. H. Sachs
Technische Hochschule Ilmenau
PSF 327

DDR-6300 Ilmenau

Dr. W. W. Vogler
Institut für Informatik
der TU München
Arcisstr. 21
Postfach 20 24 20

8000 München 2

Prof. Dr. H. J. Voss
Pädagogische Hochschule Dresden
Sektion Mathematik
Wigardstr. 17

DDR-8060 Dresden

Prof. Dr. T. Zamfirescu
Fachbereich Mathematik
der Universität Dortmund
Postfach 50 05 00

4600 Dortmund 50