MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 28/1988

Probability in Banach Spaces

26.6. bis 2.7.1988

Die Tagung fand unter der Leitung von E. Eberlein (Freiburg), J. Kuelbs (Madison) und M. Marcus (New York) statt. In 38 Vorträgen wurde eine breites Spektrum von Themen aus der Stochastik, insbesondere auch der Statistik, angesprochen. Folgende Arbeitsrichtungen seien genannt: Eigenschaften stochastischer Prozesse unter minimalen Strukturannahmen, Grenzwertsätze und Invarianzprinzipien, Gesetze vom iterierten Logarithmus, Berry-Esseen-Resultate, nichtmeßbare Prozesse, Gaußprozesse und Diffusionen, Struktursätze für Wahrscheinlichkeitsverteilungen. Bei den Anwendungen auf statistische Fragestellungen sind vor allem die Vorträge über empirische Prozesse sowie über Bootstrapping zu erwähnen. Häufig ergaben sich im Anschluß an einen Vortrag anregende Diskussionen. Eine Reihe von Anwesenden nutzte den Aufenthalt in Oberwolfach um die Arbeit an gemeinsamen Forschungsvorhaben mit anderen Tagungsteilnehmern aufzunehmen bzw. weiter voranzutreiben.

Seit der ersten Tagung unter diesem Titel im Jahre 1975 – damals ebenfalls in Oberwolfach – hat sich inhaltlich eine starke Entwicklung vollzogen. Während in den siebziger Jahren Grenzwertsätze für Banachraum-wertige Zufallsvariablen im Zentrum des Interesses standen, hat sich heute der Schwerpunkt zur Diskussion stochastischer Prozesse im allgemeinen einschließlich verschiedener Anwendungen verschoben. Die nächste Tagung zu diesem Thema ist für Sommer 1991 an einem Ort in den USA geplant.

Vortragsauszüge

K. S. ALEXANDER:

Characterization of the cluster set of the LIL sequence in Banach space

Let $S_n = X_1 + \ldots + X_n$, where $X_1, X_2, \ldots$ are i.i.d. Banach-valued random variables with the weak mean 0 and weak second moments. Let K be the unit ball of the RKHS associated to the covariance of X_1. We show that the cluster set of $\{S_n/(2n \log\log n)^{\frac{1}{2}}\}$ is always either empty or of form αK, with $0 \leq \alpha \leq 1$. A series condition determines α, and all such α do occur.

R. C. BRADLEY:

Measures of dependence involving B–valued random variables

This talk concerned measures of dependence between pairs of σ–fields in a probability space, with special emphasis on certain measures of dependence involving B–valued random variables. One particular measure of dependence can be made equivalent to either that for "strong mixing" or that for "absolute regularity", depending on appropriate choices of a Banach space B, but it seems to be an open question whether these are the only two equivalence classes. A similar open question was also posed for another closely related measure of dependence.

H. DEHLING:

The empirical process of long–range dependent observations

Let X_i, $i \geq 1$, be a stationary Gaussian sequence with $EX_i = 0$, $\mathrm{Var}\, X_i = 1$ and $r(k) = EX_j X_{j+k} = k^{-D} L(K)$ for $0 < D < 1$ and a slowly varying function $L(k)$. Let $G: \mathbb{R} \to \mathbb{R}$ be measurable. We study the e.d.f. of $Y_i = G(X_i)$, i.e. $F_n(s) = \frac{1}{n} \Sigma_{j \leq n} 1\{Y_j \leq s\}$. Define $J_q(s) = \int (1_{\{x \leq s\}} - F(s))\, H_q(x)\, (2\pi)^{-\frac{1}{2}}\, e^{-x^2/2}\, dx$, where $H_q(x) = e^{x^2/2} \frac{d^q}{dx^q} e^{-x^2/2}$ is the q–th Hermite polynomial. Let $m = \inf\{q: J_q(s) \neq 0$ for some $s\}$.

Theorem: Assume $D < 1/m$. Then

$$\sup_{s \in \mathbb{R}} \sup_{0 \leq t \leq 1} (n^{2-mD} L^m(n))^{-\frac{1}{2}} \left| [nt](F_{[nt]}(s) - F(s)) - \frac{J_m(s)}{m!} \sum_{j=1}^{[nt]} H_m(X_j) \right| \to 0$$

in probability.

As a corollary of this and results of Taqqu, Dobrushin, and Major we obtain the weak convergence of the empirical process $(n^{2-mD} h(n))^{-\frac{1}{2}} [nt] (F_{[nt]}(s) - F(s))$ in $D(\mathbb{R} \times [0,1])$ to $\frac{J_m(s)}{m!} Z_m(t)$ where $Z_m(t)$ is an m–th order Hermite process. This limit process is deterministic in s, which differs markedly from the results for independent or weakly dependent observations.

This is joint work with M. Taqqu.

M. DENKER:

A law of the iterated logarithm

Let $T: X \to X$ be a pointwise dual ergodic, measure preserving transformation on the infinite (σ–finite) measure space $(X,\mathcal{F},m)$. Denote by $\hat{T}$ its dual operator on $L^1(m)$ and assume that $\frac{1}{n^\alpha h(n)} \sum_{k\leq n} \hat{T}^k f \to \int f dm$ $(\forall f \in L^1(m)^+)$, where $0 < \alpha < 1$ and h is slowly varying. Define recursively $\Lambda(0,t) \equiv 1$ $(t \geq 0)$ and

$$\Lambda(p+1,t) = \frac{\Gamma(1+\alpha(p+1))}{\Gamma(\alpha)\Gamma(1+\alpha p)} \int_0^1 u^{\alpha-1} (1-u)^{\alpha p} \frac{h(ut)}{h(t)} \left(\frac{h((1-u)t)}{h(t)} \right)^p \Lambda(p,(1-u)t)\, du,$$

and set $p^* = p^*(n) = [\frac{1}{1-\alpha} L_2 n]$ $(L_2 = \log\log)$, $\Lambda(n) = \Lambda(p^*,n)^{1/p^*}$. The following results are announced.

1) If $f \in L^1(m)^+$ then

(*) $$\limsup \frac{1}{n^\alpha h(n)(L_2 n)^{1-\alpha}\Lambda(n)} \sum_{k\leq n} f \circ T^k \leq \frac{\Gamma(1+\alpha)}{\alpha^\alpha (1-\alpha)^\alpha} \int f dm \quad \text{a.e.}$$

2) Assume that T admits a Darling–Kac set A, for which the return time process is uniformly mixing. Then for $f \in L^1(m)^+$ one has equality in (*).

3) Let $m(A) = 1$, $\beta' < 1 < \beta$. There exist constants M, $C(p,n)$, $C'(p,n) \sim \Lambda(p,n)$ such that for all $n, p \geq 1$

$$\int_A \left(\sum_{k=1}^n 1_A \circ T^k \right)^p dm \begin{Bmatrix} \leq C(p,n)\, \beta^p M \exp\left(M \frac{p^{\alpha+1}}{n^\alpha h(n)}\right) \\ \geq C'(p,n)\, \beta'^p \end{Bmatrix} \frac{p!\,\Gamma(1+\alpha)^p}{\Gamma(1+\alpha p)} h(n)^p n^{\alpha p}$$

where "$\geq$" holds if A is a Darling–Kac set. The results are obtained jointly with J. Aaronson.

E. DETTWEILER:

<u>Representation of Banach space valued martingales as stochastic integrals</u>

If $(M_t)_{t\geq 0}$ is a real–valued continuous local martingale whose quadratic variation is absolutely continuous relative to Lebesgue measure, then by a theorem of Doob, $(M_t)_{t\geq 0}$ is the stochastic integral of a certain function relative to a Brownian motion (on a possibly extended probability space). This theorem is also well known in the

$\mathbb{R}^d$–case. The classical method of proof does not work beyond the Hilbert space case. We give a completely different proof of Doob's theorem for continuous local martingales with values in a real separable Banach space. One application is a uniqueness theorem for the so–called martingale problem (in the sense of Strook and Varadhan) on Banach spaces.

V. DOBRIĆ:

The decomposition theorem for functions satisfying the law of large numbers

Suppose that B is a Banach space with the Radon–Nikodym property. Then $f \in LLN(\mu,B)$ if and only if there exist $f_1 \in L^1(\mu,B)$ and $f_2 \in L^1_p(\mu,B)$, $\|f_2\|_{GC} = 0$ ($\|\cdot\|_{GC}$ the Glivenko Cantelli norm) such that $f = f_1 + f_2$. An example of a function with the Pettis norm 0 but which does not satisfy $LLN(\mu,B)$ is given. Finally it is proved that the operator $T: L^\infty(\mu,\mathbb{R}) \to B$ defined by $Tg = \int gfd\mu$, $g \in L^\infty(\mu,\mathbb{R})$, is compact if $f \in LLN(\mu,B)$.

R. M. DUDLEY:

Nonlinear functionals of empirical measures and the bootstrap

Let $(X,\mathcal{A},P)$ be a probability space and $\mathcal{F}$ a class of functions on X such that the central limit theorem for empirical measures $\nu_n = n^{\frac{1}{2}}(P_n - P) \xrightarrow[\mathcal{L}]{} G_P$ holds in $\ell^\infty(\mathcal{F})$ for the sup norm $\|\cdot\|_{\mathcal{F}}$. Let T be a functional on a class $\mathcal{P}$ of laws on $(X,\mathcal{A})$ which is Fréchet differentiable for $\|\cdot\|_{\mathcal{F}}$, so that $T(Q) - T(P) = \int f_P d(Q-P) + o(\|Q-P\|_{\mathcal{F}})$, $P,Q \in \mathcal{P}$, where $f_P \in \mathcal{F}$ for all $P \in \mathcal{P}$. Then $n^{\frac{1}{2}}(T(P_n) - T(P))$ is

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asymptotically normal. This is extended to suitable equi–C^1 classes of functionals and to a bootstrap form.

E. EBERLEIN:

Strong approximation of continuous time stochastic processes

Let $(X^n(t))_{t\geq 0}$, $(Y^n(t))_{t\geq 0}$ be two sequences of stochastic processes. We study sufficient conditions under which an almost sure approximation

$$\|X^n(t) - Y^n(t)\|_{t_n} << \varepsilon_n$$

holds, where $(\varepsilon_n)_{n\geq 1}$ is a given approximation order, $(t_n)_{n\geq 1}$ a nondecreasing sequence of numbers and $\|\ .\ \|_t$ is the supremum norm on the interval $[0,t]$. Two different approaches are discussed. The first one is based on a Berkes-Philipp type theorem, the second one uses measurable selections.

U. EINMAHL:

Stability results and strong invariance principles for sums of Banach space valued random variables

We prove a strong approximation theorem for sums of i.i.d. d–dimensional r.v.'s with possibly infinite second moments. Using this result, one obtains strong invariance principles for Banach valued r.v.'s in the domain of attraction of a Gaussian law, generalizing the known strong invariance principle for sums of i.i.d. B–valued r.v.'s satisfying the central limit theorem. These new strong invariance principles imply compact as well as functional laws of the iterated logarithm. We also present a related stability result for sums of i.i.d. B–valued r.v.'s.

X. FERNIQUE:

Un modèle presque sûr pour la convergence en loi

Dans les années 50, Skorokhod prouvait que toute suite de mesures de probabilité sur un espace polonais convergeant étroitement est la suite des lois de certaine suite de variables aléatoires convergeant presque sûrement. L'exposé a été consacré à l'énoncé et à la démonstration du théorème suivant qui étend et précise le résultat de Skorokhod:
Soit E un espace polonais, il existe un espaces d'épreuves $(\Omega,\mathcal{A},P)$ et pour toute probabilité μ sur E, une variable aléatoire $X(\mu)$ sur Ω à valeurs dans E ainsi qu'une partie négligeable $N(\mu)$ de Ω telles que:

(1) $X(\mu)$ ait pour loi μ,

(2) pour tout filtre Φ sur l'ensemble $M(E)$ des probabilités sur E convergeant étroitement vers une probabilité μ et pour tout ω n'appartenant pas à l'ensemble $N(\mu)$, le filtre image $X(\Phi)(\omega)$ converge vers $X(\mu)(\omega)$.

E. GINÉ:

Necessary conditions for the bootstrap of the mean

If a very mild form of the bootstrap CLT holds for X (real valued) a.s. then $EX^2 < \infty$, and if it holds in probability then X is in the domain of attraction of a normal law. This is joint work with Joel Zinn.

F. GÖTZE:

Rates of convergence in the CLT via Stein's method

Let $X_1, \dots, X_n$ denote i.i.d. random vectors taking values in $\mathbb{R}^k$ with mean zero, identity covariance and finite absolute third moment, say β_3. Using solutions of the Ornstein-Uhlenbeck diffusion equation as a substitute for Stein's first order differential equation in one dimension the error in the CLT in $\mathbb{R}^k$ can be estimated by an inductive method. For a shift and scale invariant class of sets such that the Gaussian probability of the ε-boundary is uniformly bounded by $\Delta\varepsilon$ the error is less than $(5.4 + 23\Delta k^{\frac{1}{2}})\, \beta_3 n^{-\frac{1}{2}}$.

This method can be applied similar to the Bergström method of compositions to prove convergence rates for balls in Banach spaces. It may be modified to provide also estimates in the CLT for multivariate rank statistics and von Mises statistics under weak moment conditions.

M. G. HAHN:

The concentration of partial sums in small intervals: Improvements on Berry-Esseen

This is joint work with M. J. Klass. Let $X, X_1, X_2, \dots$ be i.i.d. mean 0 random variables with $P(-a \le X \le b) = 1$ for some $a, b > 0$. Let $S_n = \Sigma_1^n X_i$ and I be a closed interval. If $|I| \ge b + a$ and I is not too far into the tails of the distribution, then

$$P(S_n \in I) \propto |I|(\mathrm{Var}\, S_n)^{-\frac{1}{2}} \wedge 1.$$

The result is optimal in two senses: it can fail if either $|I| < b + a$ or if I is located too far into the tails. Explicit conditions specify how far is too far. The proof is derived from first principles modulo one application of the Berry-Esseen Theorem, which could in fact be circumvented.

E. HÄUSLER:

Laws of the iterated logarithm for trimmed sums

Let $X_1, X_2, \ldots$ be a sequence of non-negative independent and identically distributed random variables with common distribution function F in the domain of attraction of a non-normal stable law. We discuss the law of the iterated logarithm behavior of trimmed sums of the form $\Sigma_{i=1}^{n-k_n} X_{i,n}$, where $X_{1,n} \leq \ldots \leq X_{n,n}$ are the order statistics of $X_1, \ldots, X_n$ for $n \geq 1$ and where $(k_n)_{n\geq 1}$ is a sequence of integers with $k_n \to \infty$ and $k_n/n \to 0$ as $n \to \infty$.

B. HEINKEL:

Rearrangements of sequences of random variables and exponential inequalities

Exponential bounds are studied for $P(\|X_1+\ldots+X_n\| > t)$ where $(X_1,\ldots,X_n)$ denotes a sequence of independent random variables with values in a real separable Banach space $(B, \|\cdot\|)$. In our results the usual boundedness assumptions on $\|X_1\|, \ldots, \|X_n\|$ are replaced by hypotheses on the weak ℓ_p norm of the sequence $(\|X_1\|, \ldots, \|X_n\|)$.

J. HOFFMANN–JØRGENSEN:

Uniform convergence of martingales

Let $\{X_n(t), \mathcal{F}_n \mid n \geq 1\}$ be a martingale for each $t \in T$, and let $\{X(t) \mid t \in T\}$ be a Bochner measurable stochastic process (i. e. $X(\cdot,\omega)$ takes values in a $\|\cdot\|_T$–separable subset of $\mathbb{R}^T$), such that $X_n(t) \to X(t)$ a.s. $\forall t$. Now suppose that T is a separable topological space and

(i) $\sup_n E \sup_{t \in S} |X_n(t)| < \infty \quad \forall$ countable $S \subset T$,

(ii) $X(\cdot,\omega)$ is continuous for a.e. ω.

Let $\Delta_n(\omega) = \sup_{t \in t} \inf_{U \text{ nbh. of } t} \sup_{s \in U} |X_n(t) - X_n(s)|$, and suppose that $\Delta_n \to 0$ a.s., then $X_n(t) \to X(t)$ uniformly in t a.s. Moreover, if T is hereditarily separable, then this holds even if we drop condition (ii).

N. KÔNO:

An inequality on two dimensional Gaussian random variables

Let (X,Y) be a couple of real random variables with the binormal distribution such that $E[X] = E[Y] = 0$, $E[X^2] = E[Y^2] = 1$, $E[XY] = r$. Set $\varepsilon = (E[(X-Y)^2])^{\frac{1}{2}} = (2(1-r))^{\frac{1}{2}}$, $g(x) = \exp(-x^2/2)$, $\Phi(x) = \int_x^\infty g(u)du$.

Proposition 1. Assume that $r \geq 0$ ($\Leftrightarrow 0 \leq \varepsilon \leq \sqrt{2}$). Then, $\forall x>0, \forall y>0$

Type A: $P(X \geq x+\varepsilon y,\ Y \leq x) \leq 2g(y)\Phi(x)$,

Type B: $P(X \geq x+\varepsilon y,\ Y \leq x) \leq \varepsilon g(y)g(x)$.

P. KOTELENEZ:

Low and high density approximation of the reaction–diffusion equation

It is shown that a nonlinear reaction–diffusion equation can be approximated by stochastic space–time fields with local interaction (low density) and by fields with global interaction (high density).

J. KUELBS:

Self–normalized laws of the iterated logarithm

Using suitable self–normalizations for partial sums of i.i.d. random variables, a law of the iterated logarithm, which generalizes the classical LIL, is proved for all distributions in the Feller class. A special case of these results applies to any distribution in the domain of attraction of some stable law.

M. LEDOUX:

A remark on Gaussian isoperimetry and logarithmic Sobolev inequalities

We use the isoperimetric inequality for Gauss measure to show that a function on $\mathbb{R}^n$ whose gradient is integrable with respect to Gauss measure belongs to the Orlicz space $L^1(\mathrm{Log}\ L)^{\frac{1}{2}}$ of this measure. This complements the logarithmic Sobolev inequality of L. Gross.

G. LEHA:

Continuity properties of diffusion semigroups in Hilbert space

Let $(\xi_t)_{t\geq 0} = ((\xi_{x,t}))_{x\in H}$ be an Itô diffusion on a real separable Hilbert space H, i.e. solving the stochastic differential equation

$$d\xi_t = \sigma(\xi_t)d\beta_t + b(\xi_t)dt, \quad \xi_0 = x,$$

where (β_t) is Brownian motion on H with nuclear covariance, $\sigma: H \to \mathcal{L}(H,H)$ and $b: H \to H$ continuous, and let $P_t f(x) = E[f(\xi_{x,t})]$, $f: H \to \mathbb{R}$ bdd. meas. For two spaces V of continuous real functions on H we discuss the following continuity properties:

(a) $P_t f \in V$ if $f \in V$ (Feller property),

(b) $f \to P_t f$ continuous,

(c) $t \to P_t f$ continuous,

where V is endowed with a suitable topology. The conditions guaranteeing continuity are growth conditions on the diffusion coefficient σ and drift coefficient b of the stochastic differential equation above.

W. LINDE:

Gaussian measure of translated balls

Let μ be a centered Gaussian measure on a Banach space E. Then for $s > 0$ and $z \in E$ we define $F(s,z) := \mu\{x\in E: \|x-z\| < s\}$. This function has been thoroughly investigated as a function of $s > 0$ ($z\in E$ fixed). Our aim is to prove properties of F as function of z. The main result is that $z \to F(s,z)$ is Gateaux differentiable on the support of μ. Moreover, the derivative $d_{z,s} \in (\mathrm{supp}(\mu))'$ satisfies

$$\|d_{z,s}\| \leq \frac{d}{ds} F(s,z).$$

M. B. MARCUS:

Relationship between Gaussian processes and the local time of Markov processes

Dynkin's Isomorphism Theorem gives a relationship between Gaussian processes and the local time of a killed tied down right Markov process with symmetric transition probability density. The theorem shows that if the Gaussian process is continuous so is the local time. In fact if the Gaussian process is continuous the local time satisfies the central limit theorem in the space of continuous functions. This result is joint work with R. Adler and J. Zinn.

G. J. MORROW:

Large deviation result for a class of Markov chains

Let $\{X_n^{(N)}\}_{n>0}$ be an array of stationary Markov chains in $\mathbb{R}^d$. Suppose that with scaling $t = n\beta$, $\beta \to 0$ the chain (X_n) resembles a diffusion that solves a stochastic differential equation of Wentzell-Freidlin type. That is, the diffusion is a small random perturbation of a dynamical system. The time it takes the chain to escape a neighborhood of a stable fixed point of a dynamical system in discrete time is evaluated along an exponential scale as roughly the same time it takes the corresponding diffusion to leave this neighborhood. The Markov chains are motivated by models of population genetics.

R. OLSHEN:

Some aspects of the bootstrap

Let $Z, X_1, X_2, \ldots$ be i.i.d.; we study also $Z_1^*, X_{11}^*, Z_2^*, X_{21}^*, X_{22}^*, \ldots$ and assume that given $\{X_1,\ldots,X_n\}$, $\{X_{ij}^* \mid i,j \le n-1\}$ and $\{Z_i^* \mid i \le n-1\}$, the random variables Z_n^*, $X_{n1}^*, \ldots, X_{nn}^*$ are i.i.d. $P_n = \Sigma_1^n \delta_{X_i}$. Write $P^*\{\,.\,\}$ for $P\{\,.\mid X_1,\ldots,X_n\}$. For $0 < \alpha < 1$ define t_α, t_α^* by $P((\bar{X}_n - Z)/\sigma \le t_\alpha) = \alpha$, $P^*((\bar{X}^* - Z^*)/S_n \le t_\alpha^*) = \alpha + O_P(n^{-1})$, where $\sigma = (\mathrm{Var}\, Z)^{\frac{1}{2}}$, $S_n = (n^{-1} \Sigma_1^n (X_i - \bar{X})^2)^{\frac{1}{2}}$. Then under some regularity $t_\alpha - t_\alpha^* = O_P(n^{-\frac{1}{2}})$. A plausibility argument was given that notwithstanding, $P((\bar{X}_n - Z)/\sigma \le t_\alpha^*) - \alpha = O_P(n^{-1})$. At present, rigorous arguments prove only that the cited difference is $o_P(n^{-\alpha})$ $\forall \alpha < 1$. These results on prediction intervals are in contrast to those for confidence intervals insofar as $t_\alpha - t_\alpha^*$ is concerned. This work is joint with C. Bai and P. Bickel.

V. PAULAUSKAS:

Central limit theorem in Skorokhod space D[0,1]

In the talk the first estimate of the rate of convergence in the central limit theorem for i.i.d. summands with values in the separable metric space $D[0,1]$ is given. We consider the convergence on balls (with respect to sup norm) and under rather natural conditions we get a non-uniform (with respect to the radius of the balls) estimate of order $n^{-1/6}$ (here n is the number of summands). As a corollary we get the estimate of the rate of convergence in Kolmogorov-Smirnov criteria.

W. PHILIPP:

Embedding and approximating vector–valued martingales

Results of Morrow and Philipp (TAMS, 1982) suggest that the canonical process to embed $\mathbb{R}^d$–valued martingales is $\mathbb{R}^d$–valued Gaussian process $\{G(C), C \in \mathcal{C}\}$ (where $\mathcal{C}$ is the collection of all positive semidefinite $d \times d$ matrices) with the following properties:

(i) $G(0) = 0$,

(ii) $G(C) \sim N(0,C)$,

(iii) $G(C_1), G(C_1+C_2) - G(C_1), \dots, G(C_1+\dots+C_n) - G(C_1+\dots+C_{n-1})$ are independent for $C_1,\dots,C_n \in \mathcal{C}$, $n \geq 1$.

A simple argument shows that for $d > 1$ such processes do not exist.

Other possibilities to obtain strong approximation theorems for vector-valued martingales and counterexamples to some natural conjectures are also discussed.

J. ROSINSKI:

Series representation of i.d. random vectors with applications to 0–1–laws

A general form of LePage–type series representation for infinitely divisible (i.d.) random vectors without Gaussian component is given and some special cases are discussed. As an application of such representation it is shown that the zero–one laws for i.d. measures (Jansen 1984, LNM 1064) follow directly from basic zero–one laws (Hewitt-Savage, Borel-Cantelli lemma) and from a generalized version of a theorem of P. Lévy.

V. V. SAZONOV:

Asymptotically precise estimate of the accuracy of Gaussian approximation in Hilbert space

A new estimate of the speed of convergence in the central limit theorem in Hilbert space is presented. The estimate asymptotically coincides (up to an absolute constant) with the known lower estimate and thus is asymptotically precise. Some related results are also discussed.

M. TALAGRAND:

Sudakov's minorization for Rademacher processes

Consider $T \subset \mathbb{R}^n$. Set $r(T) = E \sup_{t \in T} |\Sigma_{i \leq n} \varepsilon_i t_i|$ where $(\varepsilon_i)_{i \leq n}$ is a Rademacher sequence. Denote by B_2 the Euclidean ball, and set $B_1 = \{t \in \mathbb{R}^n : \Sigma_{i \leq n} |t_i| \leq 1\}$. We prove the existence of a universal constant K such that if $D = Kr(T)B_1 + \eta B_2$, we have

$$\eta(\log N(T,D))^{\frac{1}{2}} \leq Kr(T),$$

where $N(T,D)$ denotes the minimum number of translates of D needed to cover T.

H. WALK:

Stochastic iterations for linear problems in a Banach space

For recursive estimates in linear filtering and prediction theory, problems of convergence and rate of convergence appear which can be reduced to corresponding problems with limit 0 for a sequence (X_n) of random elements in a real separable Banach space B (especially $C([0,1]^2)$ and Hilbert space). The sequence is iteratively defined by $X_{n+1} = X_n - a_n(A_n X_n - V_n)$ with $a_n \in [0,1)$, $a_n \to 0$, $\Sigma a_n = \infty$. Here A_n, V_n are L(B)– and B–valued random variables, resp., with a.s. convergence of weighted or arithmetic means of the A_n's to $A \in L(B)$ which satisfies a certain spectral condition. A.s. convergence of X_n (investigated jointly with L. Zsidó) and in the case $a_n = 1/n$ rates of convergence (functional central limit theorem and log log invariance principle) are obtained from corresponding assumptions on weighted and arithmetic means of the V_n's, resp., under weak additional assumptions.

M. WEBER:

The law of the iterated logarithm for subsequences

The law of the iterated logarithm for subsequences in Euclidean spaces is characterized, as well as the LIL behavior for subsequences in the Banach space setting is considered.

D. WEINER:

The asymptotic distribution of magnitude–Winsorized sums

This is joint work with Marjorie Hahn and Jim Kuelbs. Let $X, X_1, X_2, \ldots$ be i.i.d. $\sim F$, symmetric. Arrange $\{X_1,\ldots,X_n\}$ in decreasing order of magnitude, viz. $|X_1^{(n)}| \geq |X_2^{(n)}| \geq \ldots |X_n^{(n)}|$. Griffin and Pruitt (1987) have studied the asymptotic distribution of the *trimmed* sum

$$\sum_{i=r_n+1}^{n} X_i^{(n)} = \sum_{i=1}^{n} X_i \, I(|X_i| \leq |X_{n,r_n+1}|)$$

(if F is continuous), where $\{r_n\}$ are integers satisfying $r_n \to \infty$ and $r_n/n \to 0$. We study the related *Winsorized* sum

$$\sum_{i=r_n+1}^{n} X_i^{(n)} + |X_{r_n+1}^{(n)}| \sum_{i=1}^{r_n} \operatorname{sgn}(X_i^{(n)}) = \sum_{i=1}^{n} (|X_j| \wedge |X_{r_n+1}^{(n)}|) \operatorname{sgn}(X_j),$$

also popular in statistics. Our approach is based on a universal result (of independent interest) for the associated self-normalized (studentized) sums

$$\sum_{i=1}^{n} (|X_j| \wedge |X_{r_n+1}^{(n)}|) \operatorname{sgn}(X_j) \; / \; \{\sum_{i=1}^{n} (X_i^2 \wedge X_{r_n+1}^{(n)2})\}^{\frac{1}{2}}.$$

A. WERON:

Stochastic integral approach to the Prigogine theory of irreversible dynamical systems

A reformulation of the Prigogine theory of irreversible dynamical systems is given. In contrast to the previous works, a stochastic integral w.r.t. operator valued martingales is employed here. This new approach has an explicit random character and the reformulation of the theory in language of martingales might allow for future generalizations in both classical and quantum systems. The form of the Boltzmann kinetic equation for the irreversible dynamics is found.

W. A. WOYCZYNSKI:

Statistical mechanics on graphs

Random tree–type partitions for finite sets are used as a model of a chemical polymerization process when ring formation is forbidden. The study rigorously establishes theoretically the existence of three stages of polymerization and of a critical point dependent upon the ratio of association and dissociation rates. Distributions on Banach spaces arising in the study are also analyzed. This is joint work with Pittel and Mann.

J. E. YUKICH:

Rates for the CLT via ideal metrics

Let $(B, \|\cdot\|)$ be a separable Banach space and $\mathcal{X} := \mathcal{X}(B)$ the vector space of all random variables defined on a probability space and taking values in B. It is shown that new ideal metrics for $\mathcal{X}$ may be used to obtain refined rates of convergence of normalized sums to a stable limit law. The rates are expressed in terms of a variety of uniform metrics on $\mathcal{X}$. In the B–space setting, the rates hold w.r.t. the total variation metric and in the Euclidean space setting the rates hold w.r.t. uniform metrics between density and characteristic functions. The main result provides a sharp order estimate of rate of convergence in local limit theorems w.r.t. the uniform distance between densities.

The method is based on the theory of probability metrics, especially those of convolution type.

J. ZINN:

Bootstrapping general empirical measures

This is joint work with E. Giné. The a.s. central limit theorem for the bootstrap of empirical measures is characterized by the central limit theorem for the empirical measure and the finiteness of the second moment of the envelope function.

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