MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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KOMBINATORIK : SYMMETRISCHE GRUPPEN, KLASSISCHE ALGEBRA UND SPEZIELLE FUNKTIONEN

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This time the main emphasis lay on the combinatorics of the symmetric group, on related series of symmetric polynomials, special functions, identities, codes, bijective proofs, combinatorial enumeration, q-analogues, etc. The abstracts show that also applications in sciences were discussed. The organizers of the conference, D. Foata (Strasbourg) and A. Kerber (Bayreuth) were glad to welcome forty-one participants from nine countries, who gave thirty lectures which showed the rapid progress in this field and stimulated intensive discussions.

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Abstracts

R. CANFIELD: The use of elliptic functions in solving quintic polynomials

We shall review the nineteenth century mathematics showing that irrationalities arising in the theory of elliptic modular functions are adequate for solving the general quintic. Two points of special interest are: a predominantly algebraic derivation of the modular equation of degree six; and Hermite's 1858 paper showing how this equation may be reduced in degree by 1.

M. CLAUSEN: Fast Fourier Transforms for Symmetric Groups

According to Wedderburn's theorem the group algebra $\mathbf{C}G$ of a finite group G of order N is isomorphic to a suitable algebra of block-diagonal matrices. Every such isomorphism $W: \mathbf{C}G \to \bigoplus_i \mathbf{C}^{d_i \times d_i}$ is called a Fourier transform for $\mathbf{C}G$. W.r.t. natural $\mathbf{C}$-bases W can be viewed as an N-square complex matrix. The linear complexity $L_s(W)$ of the matrix W is defined as the minimal number of $\mathbf{C}$-operations sufficient to compute $W \cdot x$, where $x = (x_i)$ is a column vector of indeterminates $x_1, \ldots, x_N$ over $\mathbf{C}$. The linear complexity $L_s(G)$ of the finite group G is defined by $L_s(G) := min\{L_s(W) \mid W$ is a Fourier transform for $\mathbf{C}G\}$. If $|G| > 1$ then $(|G| \leq) L_s(G) < 2 \cdot |G| + 2$. The classical FFT algorithms improve this trivial upper bound by showing that for cyclic groups G, $L_s(G) = O(|G| log |G|)$. Using Clifford theory, Beth (1984) has shown that for soluble groups $L_s(G) = O(|G| + 3/2)$. Using quite different methods, Beth's result can be extended to all finite groups (with a slightly bigger "constant".)

Motivated by real-time applications in digital signal filtering we are interested in extending the FFT results to other classes of finite groups.

Theorem 1. $L_s(S_n) = o(|S_n \mid \log^3 |S_n|)$.

Theorem 2. (jointly with U. Baum & T. Beth) *If G is a 2-group with an abelian normal subgroup of index ≤ 4, then $L_s(G) \leq 3/2|G| \log |G|$. In particular, this result applies to all groups of order* 64.

Theorem 3. *If G is a metabelian 2-group, i.e. $G + U = E$, then $L_s(G) \leq$ $2 \cdot |G| \log |G|$.*

The proofs "nearly automatically" translate into highly regular VLSI designs.

D. COHEN: Three Combinatorial Problems

I. The problem of Bulgarian Solitaire. Given the sequence

$$a_1 \geq a_2 \geq \cdots \geq a_n > 0$$

define β as the set $(a_1 - 1)\ (a_2 - 1) \ldots (a_n - 1)\ (n)$ deleting any zeroes.

β maps a partition of n into another partition of n and so induces on the set of all partitions a certain structure. It is proven that for $n = \binom{m}{2}$

i) the structure is a tree with root (fixed point) the set $m, m-1, \ldots, 1$

ii) the height of the tree is $m(m-1)$

iii) the top element is $(m-1), (m-1), (m-2), (m-3) \ldots 3, 2, 1, 1$

For n not triangular the transformation m induces a tree with a cycle attached to the root.

All of this follows immediately from looking at the right diagram.

β is important for its relationship to the conjugacy operator c. For all partitions p either $\beta c \beta c p = p$ *or* $c \beta c \beta p = p$.

II. Counting walks on the cubic lattice. By enumeration through generating functions physicists have shown that the number of paths on the cubic lattice from 0 to p of n steps is $\binom{n}{[n/2]}$ times the number of paths in the hexagonal planar (honeycomb) lattice of n steps from 0 to some point in it. This fact is shown not to be coincidence. The fact is that the cubic lattice itself can be decomposed in a natural way into tilted planes of honeycomb lattice. The result follows immediately.

III. A Petrie Net conjecture of Erdös. On the nodes of the n-cube place $2 + n$ tokens (arbitrarily many per node). A move is defined as follows: If node x is connected to node y and node x has ≥ 2 tokens it may then spend one (it is removed from the game) in order to send one other token to y.

Theorem. *It is always possible to obtain a token at the origin from any initial position.*

A. DÜR: On Reed-Solomon codes decoding and canonical forms for binary forms

The problem of computing the canonical form for binary forms in classical invariant theory and the problem of decoding generalized Reed-Solomon codes in algebraic coding theory are shown to be closely related. The covering radius of a generalized Reed-Solomon code over $GF(q)$ of length n and minimum distance d is proved to be either $d-2$ or $d-1$, and the exact value is determined unless $n \geq q+1$ and $q/2+3 < d < q$.

D. DUMONT: The parametrization of the curve $x+3+y+3=1$ with the help of Dixon's Elliptic functions

Functions cm and sm, introduced by A.C. Dixon in 1890, are defined by

$$\frac{d}{du}\mathrm{cm}\, u = -\mathrm{sm}^2 u\,; \quad \mathrm{cm}(0) = 1\,;$$
$$\frac{d}{du}\mathrm{sm}\, u = \mathrm{cm}^2 u\,; \quad \mathrm{sm}(0) = 0.$$

They satisfy $\mathrm{cm}^3 u + \mathrm{sm}^3 u = 1$, and have periods $B(\frac{1}{3},\frac{1}{3})$ and $\omega B(\frac{1}{3},\frac{1}{3})$. They are elliptic functions of order three (each has three simple poles and three zeroes in the parallelogram).

We give their relation with the Weierstrassian function $\wp$ and study their Taylor expansions around the origin, which has a combinatorial interpretation in terms of binary trees.

CH. F. DUNKL: Reflection groups and orthogonal polynomials

The weight functions are powers of products of linear functions whose zero-sets are the mirrors for the reflections in a Coxeter group, restricted to the surface of the sphere. A form of Selberg's integral, corresponding to the hyperoctahedral group, is an important special case. There is a commutative algebra of first-order differential-difference operators, which have an action in this structure analogous to that of the partial derivatives in the theory of spherical harmonics.

I. GESSEL: Coefficient Extraction for Symmetric Functions

Given a symmetric function $f(x_1, x_2, \ldots)$ we would like to compute the coefficient of $x_1^{n_1} x_2^{n_2} \cdots x_k^{n_k}$ in f. In many cases of interest there exists a sequence $p_0(u), p_1(u), \ldots$ of polynomials in u and a linear functional L on polynomials in u such that the desired coefficient is $L(p_{n_1}(u) p_{n_2}(u) \cdots p_{n_k}(u))$. For example, suppose that

$$f = \left(1 - \sum_{i=1}^{\infty} \frac{x_i}{1 + x_i}\right)^{-1}$$

and let L be the linear functional defined by $L(\psi) = n$, so $L(e + uz) = 1/(1-z)$. Then

$$f = L\left(\exp\left(u \sum_{i=1}^{\infty} \frac{x_i}{1+x_i}\right)\right) = L\left(\prod_{i=1}^{\infty} \exp \frac{ux_i}{(1+x_i)}\right).$$

Thus if the polynomials $p_n(u)$ are defined by

$$\exp \frac{uz}{(1+z)} = \sum_{n=0}^{\infty} p_n(u) z^n,$$

then the coefficient of $x_1^{n_1} x_2^{n_2} \cdots x_k^{n_k}$ in f is $L(p_{n_1}(u) p_{n_2}(u) \cdots p_{n_k}(u))$.

I. P. GOULDEN: Immanants of Combinatorial Matrices

The λ-th immanant of the $n \times n$ matrix A is given by

$$\mathrm{Imm}_\lambda(A) = \sum_{\sigma \in S_n} \chi^\lambda(\sigma) a_{1\sigma(1)} \cdots a_{n\sigma(n)}$$

where λ is a partition of n and χ^λ is the irreducible character of S_n corresponding to λ. If $\sigma \in S_n$ has i_j j-cycles in its disjoint cycle decomposition, then $c(\sigma)$ denotes the partition $1^{i_1} 2^{i_2} \ldots$, and $p_{c(\sigma)} = p_1^{i_1} p_2^{i_2} \ldots$ where p_j is the j-th power sum symmetric function, is the cycle indicator for σ. Then $\chi^\lambda(\sigma) = \langle s_\lambda, p_{c(\sigma)} \rangle$ where s_λ is the Schur symmetric function and $\langle \, , \rangle$ is defined by $\langle s_\lambda, s_\mu \rangle = \delta_{\lambda\mu}$.

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Let T_n be the $n \times n$ matrix whose (i,j)-entry is $\delta_{ij}(t_{i1} + \cdots + t_{i\,n+1}) - t_{ij}$. Then the Matrix Tree Theorem tells us, that $\det(T_n)$ is the generating function for trees on vertices $1, \ldots, n+1$, rooted at $n+1$, with all edges directed towards the root. We prove that each coefficient in the expansion of $\mathrm{Imm}_\lambda(T_n)$ (as a polynomial in the t_{ij}), is a non-negative integer, by identifying it as a sum over a subset of the standard tableaux of shape λ, of a combinatorially defined power of 2. This allows us to deduce that, for all partitions μ of $k \leq n$, with all parts ≥ 2,

$$(1) \qquad p_1^{n-|\mu|} \prod_{j \geq 1} (p_1^{\mu_j} + (-1)^{\mu_j} p_{\mu_j})$$

has a non-negative Schur function expansion.

Let $R_{\mu\nu}$ be the $m \times m$ matrix with (i,j)-entry $h_{\mu_i - \nu_j - i + j}$, where μ is a partition with $\leq m$ parts, and ν is contained in μ, and where h_k is a complete symmetric function. Then $\det(R_{\mu\nu})$ is the skew Schur function $s_{\mu\backslash\nu}$, by the Jacobi-Trudi identity, and thus has a non-negative Schur function expansion by the Littlewood-Richardson rule. If $\mu\backslash\nu$ is a border strip, we prove that each coefficient in the expansion of $\mathrm{Imm}_\lambda(R_{\mu\nu})$ is a non-negative integer, by identifying it as a sum over the standard tableaux of shape λ, of a non-negative integer weight, with an explicit combinatorial description. This allows us to deduce that

$$(2) \qquad (1 - p_1 - p_2 - \ldots)^{-1}$$

has a non-negative Schur function expansion. For arbitrary $\mu\backslash\nu$ we conjecture that each coefficient is non-negative. This would follow from the following, stronger

Conjecture. *The sum of the cycle indicators for the multiset*

$$S_{i_1} S_{i_2} \ldots S_{i_m},$$

$1 \leq i_1, \ldots, i_m \leq n-1$, *has a non-negative Schur function expansion, where* $S_i = id \cup (i, i+1)$.

L. HABSIEGER: Some irrationality results for the hypergeometric function

We give irrationality measures for ${}_2F_1\left({1,1/2 \atop (\alpha+3)/2}; x\right)$ under some conditions on the rational numbers α and x. When $\alpha = 0$, we find the classical results on $(1/\sqrt{x})\mathrm{Arctg}\sqrt{x}$ and $(1/\sqrt{x})\mathrm{Argth}\sqrt{x}$ (cf. Marc Huttner, Irrationalité de certaines intégrales hypergéométriques, *J. of Number Theory*, **26**, 166-178 (1987)). When $\alpha \neq 0$, we find the results of Huttner (Problème de Riemann et irrationnalité d'un quotient de deux fonctions hypergéométriques de Gauss, *C.R. Acad. Sc. Paris*, **302**, Série I, no. 17, 1986).

PH. HANLON: Idempotents of the descent algebra

Arnold computed the homology of the Braid group by utilizing its action on $C^n \backslash H_{ij}$ where H_{ij} is the complex hyperplane defined by the equation $x_i = x_j$. A key element in his analysis is the action of the symmetric group on the homology of $\mathbf{C}^n \backslash H_{ij}$. We consider the generalization to a finite group G wreathed over the Braid group.

D.M. JACKSON: Enumerative aspects of the group algebra of the symmetric group

The question of counting permutations, having a given number of cycles, which are a product of permutations in designated conjugacy classes is a common problem (embedding of graphs on surfaces, random walks, association schemes, for example). Such problems have a natural setting in the group algebra of the symmetric group. The character theory of the symmetric group and the algebra of symmetric functions are therefore available to us as devices for attacking these problems. In certain cases, the forms of the resulting generating functions indicate (potentially interesting) bijections, a knowledge of which would enhance our understanding of these problems. If a generating function has a convenient representation as a hypergeometric series, it is sometimes possible to develop a q-analogue for it algebraically, using the corresponding basic hypergeometric series. This poses questions about the combinatorial interpretation of the q-statistic, and the way it can be handled algebraically.

G. D. JAMES: The Permanental Dominance Conjecture

If $A = (a_{ij})$ is an $n \times n$ matrix over $\mathbf{C}$ which is Hermitian and positive semidefinite, then we write $A \geq 0$.

In 1918, Schur [4] proved that, for every subgroup G of S_n, every character χ of G and every $A \geq 0$, we have

$$\sum_{\pi \in G} \chi(\pi) a_{1,1\pi} a_{2,2\pi} \ldots a_{n,n\pi} \geq \chi(1) \det A.$$

The Permanental Dominance Conjecture [3] is that, under the same hypothesis, we have

$$\chi(1) \mathrm{per}\, A \geq \sum_{\pi \in G} \chi(\pi) a_{1,1\pi} a_{2,2\pi} \ldots a_{n,n\pi}.$$

There is little evidence for this conjecture (it is unresolved for $n = 4$, even), so we consider the special case where $G = S_n$. Let χ^λ denote the irreducible character of S_n, which corresponds to the partition λ of n. Then

$$d_\lambda(A) := \sum_{\pi \in S_n} \chi + \lambda(\pi) a_{1,1\pi} a_{2,2\pi} \ldots a_{n,n\pi}$$

is called an immanant of A. The Permanental Dominance Conjecture for this case states that $\mathrm{Per}\, A \geq d_\lambda(A)/\chi^\lambda(1)$ for all partitions λ of n. This has been proved [2] if $\lambda = (\lambda_1, \lambda_2, \lambda_3, \ldots)$ with $\lambda_3 = 0$ or 1. Recently, Peter Heyfron [1] has proved that $d_\lambda(A)/\chi^\lambda(1) \geq d_\mu(A)/\chi^\mu(1)$ if $\lambda = (r, 1{+}n - r)$ and $\mu = (s, 1{+}n - s)$ with $r \geq s$, and has verified that when $n \leq 8$, the only partitions which might provide a counterexample are $(2+3)$ and $(2+4)$.

1. P. Heyfron, Immanant dominance orderings for hook partitions, preprint, Imperial College, 1987.
2. G.D. James and M.W. Liebeck, Permanents and immanant of Hermitian matrices, *Proc. London Math. Soc.*, (3), **55** (1987), 243-265.
3. E.H. Lieb, Proofs of some conjectures on permanents, *J. Math. and Mech.*, **16** (1966), 127-134.
4. I. Schur, Über endliche Gruppen und Hermitische Formen, *Math. Z.*, **1** (1918), 184-207.

P. KIRSCHENHOFER: On some applications of series transformations due to Ramanujan in the analysis of algorithms

In Computer Science a data structure called "tries" is of great importance. The analysis of the higher moments of some characteristic parameters of this data structure yields complicated alternating sums, which are evaluated asymptotically via residue calculus. In order to get the correct order of asymptotic magnitude of the quantities one has to "compute" the mean of the square of some complicated periodic functions given by their Fourier series. It turns out that a set of series transformation formulae which occur in Ramanujan's Notebooks allow to establish the necessary identities.

R. LAUE: Expert systems for discrete structures

One important aspect of an expert system for discrete structures is to allow the usual constructions of objects. Here the most difficult problem is to avoid replication of isomorphic copies.

It is shown that many constructions, which are described by mappings, rely on the same principle, i.e. amalgamation. Here the isomorphism problem can be transformed into the double-coset problem in group theory. Thus, one tool can be used for many kinds of structures and many construction methods.

An expert system should be able to find isomorphisms of structures, automorphism groups and double coset representatives. In addition it has to be equipped with rules for choosing the appropriate operands for constructing the objects of a defined class and rules for stopping the generation process according to specified properties.

P. LEROUX: q-log-concavity of q-Stirling numbers

A sequence of real numbers $\{a_k\}_{0\leq k\leq n}$ is said to be *unimodal* if there exists $0 \leq m \leq n$ such that

(1) $a_0 \leq a_1 \leq \ldots \leq a_m \geq a_{m+1} \geq \cdots \geq a_n$.

The sequence is called *log-concave* if, for $1 \leq k \leq n-1$,

(2) $a_{k-1}a_{k+1} \leq a + 2_k$.

These concepts can be extended to *q-unimodality* and *q-log-concavity* for sequences of polynomials $a_k = a_k(q)$ in the variable q by interpreting the inequalities of polynomials in (1) and (2) as coefficient-wise inequalities. For positive numbers, log-concavity implies unimodality but in the case of polynomials, the two concepts are independant.

In a recent meeting of the Amer. Math. Soc., Lynne M. Butler proved, using injections, that the Gaussian polynomials $\left[{n \atop k}\right]_q$ are q-*log*-concave and conjectured that the same is true for the q-Stirling numbers of the second kind $S_q[n,k]$.

In this talk, we prove this fact by suitably extending Lynne Butler's proof. For this we introduce a combinatorial interpretation of p,q-Stirling numbers in terms of "0-1 tableaux" which is inspired from a reduced echelon matrix representation of restricted growth functions. This method can also be applied to prove the p,q-*log*-concavity of p,q-Stirling numbers of the first kind (in k) and of the second kind (in n).

Other proofs can be given to some of those results, for example using induction (Bruce Sagan) or the Jacobi-Trudi identity for Schur functions (Dennis Stanton).

J. D. LOUCK: Properties of generalized hypergeometric series

If I have the opportunity, I would like to speak about 40 minutes on a generalization of the Gauss hypergeometric function to an arbitrary number of variables by use of Schur functions. An integral representation of these hypergeometric functions that is closely related to Selberg's integral will also be given. A number of properties of the generalized hypergeometric coefficients that generalize the binomial, Saalschütz, and Bailey identities will also be discussed.

I. G. MACDONALD: A new class of symmetric functions

A survey of the properties of a class of symmetric functions $P_\lambda(q,t)$ depending on two parameters q,t and indexed by partitions λ. When $q=t, P_\lambda(q,t)$ is the Schur function s_λ; when $q=0$, it is the Hall-Littlewood symmetric function indexed by λ; when $t=1$, it is the monomial symmetric function indexed by λ; when $q=1$, it is the product of elementary symmetric functions corresponding to the parts of the partition conjugate to λ. Finally, when $q = t^\alpha$ and $(q,t) \to (1,1), P_\lambda(q,t)$ becomes in the limit the Jack symmetric function with parameter α for the partition λ.

ST. C. MILNE: Multiple basic hypergeometric series and their applications to combinatorial problems in the theory of partitions

We review Watson's transformation of a very well-poised ${}_8\Phi_7$ into a balanced ${}_4\Phi_3$ and his proof of the Rogers-Ramanujan-Schur identities. Utilizing our very well-poised/balanced multiple basic hypergeometric series in $U(n+1)$ we generalized the Bailey Transform and Bailey's Lemma to $U(n+1)$ in a canonical way that also works for the other classical "matrix groups". The second iteration of the $U(n+1)$ Bailey Lemma yields a new, more symmetrical generalization of Watson's ${}_8\Phi_7$ transformation, which includes as special cases a $U(n+1)$ generalization of: ${}_6\Phi_5$ summation theorem, q-Pfaff-Saalschütz, q-Gauss, q-Dougall summation, etc. A standard limiting case leads to a $U(n+1)$ generalization of the Rogers-Selberg identities, which under further spezialization, yields our first version of a $U(n+1)$ generalization of the Rogers-Ramanujan-Schur identities. The sum sides are the generating functions for the number of partitions whose parts differ by at least $(n+1)$, (the parts being $\geq n$, for the second identity).

A. O. MORRIS: q-polynomials

Hall-Littlewood polynomials in the special case $t = \zeta$, ζ a primitive l-th root of unity are considered. The $Q_\lambda(x;\zeta)$ are only defined when λ is a partition with no parts repeated more then $l-1$ times. Many special problems arise in this case.

(1) We give an explicit formula for $q_\nu^l = \sum_{\lambda > (\nu+l), l(\lambda) \leq l} f_\lambda \varepsilon_\lambda$

(2) We prove $\sum_{\mu \leq \lambda} \kappa_{\mu,\lambda}(\zeta)\chi_\rho^\mu = 0$, where $(\kappa_{\lambda,\mu}(\zeta))$ is the Kostka-Foulkes matrix, ρ an l-regular partition.
We *conjecture* that this matrix 'leads' to a complete set of $\mathbf{Z}$-linear relations between the ordinary characters of S_n on l-regular classes. A formula for $\kappa_{\lambda,\mu}(\zeta)$ is given in terms of the l-quotient diagram and l-core of λ.

(3) Let $X_\rho^\lambda(t)$ be the Green polynomials, put $\chi(l) = (\chi_\rho^\lambda(\zeta))$, ρ taken over the l-regular partitions, put $M(l) = (m_\rho^\lambda)$ be the table of irreducible Brauer characters of S_n, then

Conjecture. *There exists a triangular matrix* $A(l) = \begin{pmatrix} 1 & & 0 \\ & \ddots & \\ * & & 1 \end{pmatrix}$ *such that* $M(l) = A(l)\chi(l)$.

Problem. *Give a combinatorial description of* $A(l)$.

TH. MÜLLER: Combinatorial aspects of finitely generated virtually free groups

Let G be a finitely generated virtually free group and denote by m_G the l.c.m. of the orders of the finite subgroups in G. We investigate the function $b_G: \mathbf{N} \to \mathbf{N}$,

$$b_G(\lambda) = \text{number of free subgroups of index } \lambda m_G \text{ in } G.$$

Define the (free) rank $\mu(G)$ of G to be the rank of a free subgroup $\Gamma \leq G$ of index m_G. Our main result is

Theorem 1. *If $\mu(G) > 1$, then $b_G(\lambda+1)-b_G(\lambda) \geq \mu(G)$ for all $1 \leq \lambda < \infty$; in particular the function b_G is strictly increasing.*

Furthermore we introduce a notion of type $\tau(G)$ of G and prove

Theorem 2. $(m_G, b_G) = (m_H, b_H)$ *iff* $\tau(G) = \tau(H)$.

Theorem 2 implies in particular that there exist only finitely many non-isomorphic groups G having the same number of free subgroups for each finite index. The proofs involve Stallings' structure theorem and a group-theoretic generalization of binomial coefficients.

H. NIEDERHAUSEN: Umbral Calculus

"Classical" Umbral Calculus connects a certain type of polynomials, power series, and linear operators on polynomials. It is a rather limited, but important class of enumerative combinatorial problems where the results are values of polynomials. Such problems may be accessible by Umbral methods. If we have to work with generating functions, the classical calculus offers "only" the natural numbers for indexing. Multivariate generalizations, up to infinitely many variables, have been considered. Presently, I am working on an Umbral Calculus indexed by a bidirected semi-group.

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P. PRAGACZ: Algebro-Geometric Applications of Schur S-and Q-Polynomials

Being inspired by some problems in geometry we investigate algebro-combinatorial properties of Schur S- and Q-polynomials. We define these two families of symmetric polynomials in three ways (determinantal, á la H. Weyl, and, combinatorial via tableaux) We characterize S- and Q-polynomials in the rings of symmetric polynomials with the help of some cancellation properties. Using these polynomials we describe the ideal of all polynomials in the coefficients of two polynomials in one variable, which vanish if these equations have $r+1$ roots in common. We describe also the cohomology ring of the "isotropic" grassmannians $Sp(2n)/P_{\alpha_n}$ (and $SO(2n+1)/P_{\alpha_n}$) with the help of Q-polynomials, which describe the classes of the Schubert varieties in terms of the special ones. Finally, we apply the ideal of polynomials of universal cycles supported by degeneracy loci, their Chern numbers etc.

J. B. REMMEL: Calculating Kronecker Coefficients

The Kronecker product of two homogeneous symmetric polynomials P_1 and P_2 is defined by means of the Frobenius map F which takes the group algebra of the symmetric group into the space of homogeneous symmetric functions by the formula $P_1 \otimes P_2 = F(F^{-1}P_1)(F^{-1}P_2))$. When P_1 and P_2 are Schur functions S_I and S_J respectively, then the resulting product $S_I \otimes S_J$ is the Frobenius characteristic of the tensor product of the irreducible representations of the symmetric group corresponding to the diagrams I and J. Taking the scalar product of $S_I \otimes S_J$ with a third Schur function S_K gives the so called Kronecker coefficient $C_{IJK} = \langle S_1 \otimes S_J, S_K \rangle$ which gives the multiplicity of the representation corresponding to K in the tensor product. We shall discuss several results about the C_{IJK}'s for certain special class of shapes. For example, if I is a hook and J is either a hook or a two row shape, then we have explicit formulas for C_{IJK} which show that C_{IJK} is always ≤ 3. If both I and J are two row shapes, say $I = (m, n)$ and $J = (h, k)$ where $m \leq n$, $h \leq k$ and $m \leq k$, then we show that C_{IJK} is always bounded by $\binom{m+1}{2}$ and there are examples of J and K where C_{IJK} is of the order of $m+2$.

W. SCHEMPP: Symmetrische Gruppen und nichtlineare Optik

Since the invention of the laser, non-linear processes have attained increasing importance in optics and attracted physicists and engineers. In non-linear optics, the main object to be studied is the interaction between radiation and matter; depending on the particular physical situation, there arise phenomena such as optical phase conjugation by holographic grids in photorefractive materials, the generation of higher harmonics in uniaxial crystals, and the propagation of solitary pulses along optical fibers. It is the purpose of the present paper to apply the duality theory of semi-simple rings via the group algebra of the symmetric group to the generation of higher harmonics. The point is that the non-linear optical susceptibilities belong to the algebra of bisymmetric transformations due to their intrinsic permutation symmetries. Therefore, the hook products of the Young frames determine the multiplicities in the phase matching condition.

SHI HE: (and Guan Aiwen) q-Analogue of Li Shan-lan Identity

The expansions of the power of Gaussian polynomials

$$\begin{bmatrix} m+k \\ k \end{bmatrix}^n = \sum_i A_q(nk,i) \begin{bmatrix} m+nk-i \\ nk \end{bmatrix}$$

are studied. We obtain recurrence relations for the coefficients $A_q(nk,i)$, summation formulas, and explicit expressions. For $n = 2$, we prove the identity

$$\begin{bmatrix} m+k \\ k \end{bmatrix}^2 = \sum_i \begin{bmatrix} k \\ i \end{bmatrix}^2 q^{i+2} \begin{bmatrix} m+2k-i \\ 2k \end{bmatrix},$$

which is the q-analogue of the famous Li Shan-lan identity.

CH. SIEBENEICHER: (and A. W. M. Dress) On the number of solutions of certain linear diophantine equations

Let $P(A)$ denote the set of all periodic functions defined on the integers $\mathbf{Z}$ with values in the set A. Two such functions g and g' are called equivalent, if and only if there exists a positive integer m such that $g(i+m) = g'(i)$ for all $i \in \mathbf{Z}$. Consider the matrix Δ, where for $a \in A$ and $T \in \overline{P(A)}$, the set of equivalence classes of periodic functions, one defines $\Delta(a,T) = \#\{g \in T \mid g(0) = a\}$.

Theorem. *If A is a finite set and $s: A \to \mathbb{N}_0$ an arbitrary function, then the set*

$$U(s) := \{u \in \mathbb{N}_0 + \overline{P(A)} \mid \sum_{T \in \overline{P(A)}} \Delta(a,T) \cdot u(T) = s(a) \text{ for all } a \in A\}$$

of non negative integral solutions of the linear equation $\Delta \cdot u = s$ has cardinality $(\sum_{a \in A} s(a))/\prod_{a \in A} s(a)$.

Note that the matrix Δ is uniquely determined by the conditions of the theorem. The theorem turns up as a corollary in a "bijective" proof of the cyclotomic identity.

V. STREHL: Combinatorial Aspects of Brock's Identity

The following binomial identity, which originally arose in the study of a sorting problem, was posed as problem $60-2$ in the SIAM Review by P. Brock:

For nonnegative integers A, B let

$$H(A,B) = \sum_{i=0}^{A} \sum_{j=0}^{B} \binom{i+j}{j} \binom{A-i+j}{j} \binom{B+i-j}{B-j} \binom{A-i+B-j}{B-j}.$$

then the $H(A,B)$ satisfy the recursion

$$H(A,B) - H(A-1,B) - H(A,B-1) = \binom{A+B}{A}^2,$$

where $H(-1,B) = H(A,-1) = 0$.

Various proofs and extensions (by Baer and Brock, Slepian, Carlitz, Singhal, Andrews, Church jr.) are known. In general, the problem to be solved can be stated as follows:

Find a multivariate recursion with constant coefficients for the numbers

$$H^{(p,q)}(n_1, \ldots, n_p) = \sum \binom{i_1 + i_2}{i_1} \binom{i_2 + i_3}{i_2} \cdots \binom{i_{pq} + i_1}{i_{pq}},$$

where the summation runs (for fixed $(n_1, \ldots, n_p) \in \mathbb{N}^p$) over all

$(i_1, i_2, \ldots, i_{pq}) \in \mathbf{N}^{pq}$ such that $\sum_{\nu=0}^{q-1} i_{j+\nu p} = n_j$ $(1 \leq j \leq p)$.
It turns out that the (ordinary) generating function for these numbers $H^{(p,q)}$ can be nicely interpreted as the (exponential) g.f. for periodic, locally-injective endofunctions (by generalizing the combinatorial model for Jacobi-polynomials, as introduced by Foata and Leroux). This approach leads to a doubly infinite family of recursion relations involving matching polynomials related to the underlying structures. The method is different from the purely manipulative one used by earlier authors – apart from giving new recursion relations it also provides structural insight into this type of identities.

D. WHITE: Codes, transforms, and the spectrum of the symmetric group

Let G be the graph whose vertices are the permutations in the symmetric group S_n with two permutations being adjacent if they differ by an adjacent transposition. We consider what integers are in the spectrum of the adjacent matrix for G. These results have applications to the existence of perfect 1-codes in G and to the invertibility of certain discrete Radon transforms over S_n. Our techniques involve both representation theory of S_n and explicit production of certain eigenvectors.

D. ZEILBERGER: Bijective vs. Manipulative Proofs in Enumerative Combinatorics

A good bijective proof is great. A bad bijective proof is still better, in some sense, than a manipulative proof. This is the reverse snobism of combinatorics, in which less is more.

J. ZENG: Linearization of Products of Meixner, Krawtchouk and Charlier Polynomials

Let $(p_n(x))$ $(n \geq 0)$ be a sequence of orthogonal polynomial with respect to a functional $\mathcal{L}$. We propose a calculation of the functional $\mathcal{L}(\prod_{i=1}^{m} p_{n_i}(x))$ for the Meixner, Krawtchouk,. and Charlier polynomials, with the help of combinatorial techniques. The same problem for the polynomials of Hermite and Laguerre has been solved by Azor-Gillis-Victor and Foata-Zeilberger respectively. This combinatorial approach permits to refine several classical analytical results.

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