MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t   32/1988

Fine Structure Theory

17.7.1988 to 23.7.1988

The organizer of the conference was Ronald Jensen (Oxford). The central topics of discussion were: fine structural models for large cardinals (Steel); core models for intermediate cardinals and their applications (Jensen, Koepke, Mitchell); application of coding (Friedman, Mack Stanley); applications of morasses (Donder, Lee Stanley, Velleman).

Abstracts of Talks

R. JENSEN (Oxford):

Core Model Theory for Measures of Order O

I developed the theory of mice, including a new method of defining the Σ^* preserving ultra power. I also discussed the problem of constructing morasses in core models and showed gap morasses exist at regular cardinals in this model.

J. STEEL (U.C.L.A.)

An Inner Model of a Woodin Cardinal and GCH

I outlined the basics of some work, done jointly with W.J. Mitchell, which we believe leads to the construction and fine structural analysis of an inner model with a Woodin cardinal. In particular, we are able to show that this model satisfies the GCH, a fact not known of those inner models of Woodin cardinals studied previously.

D. VELLEMAN (AMHERST):

Constructing w_1-Souslin Trees from a Cohen Real

I discussed the following conjecture:

Conj If there is an $(w_1,1)$ - morass, then adding a Cohen real to the universe adds an w_2-Souslin tree.

The conjecture is motivated by:

Thm1 (Shelah) Adding a Cohen seal to the universe adds an w_1-Souslin tree.

Thm2 (Shelah, Stanley). If there is an $(w_1,1)$ - morass and CH holds then these is an w_2-Souslin tree.

One proof of Thm1 involves constructing the Souslin tree generically along an (w,1) - morass. The intended proof of the conjecture involves constructing the Souslin tree along an (w,2) - morass, whose existence follows from the existence of $(w_1,1)$ - morass. It is consistent with ZFC that this construction yields an w_2-Souslin tree, but it is open whether every such construction does.

M. Stanley (San Jose State University)

A Non-Generic Real Over the Minimal Model of ZF

This talk outlined the proof of the following theorem: Suppose $\langle L;\in\rangle$ is the minimum standard model of ZFC (which is countable). There is an $x \subseteq \omega$ such that (1) $x \notin L$; (2) $\langle L[x]; \in\rangle$ satisfies ZF (in fact cardinals are preserved); (3) x is *not* weakly generic. A real x is weakly generic iff there is a $\mathbb{P}$ and a G which is $\mathbb{P}$-generic over $\Sigma_\omega(L[\mathbb{P}]; \mathbb{P})$ sit.(a) $\langle L[\mathbb{P}, G]; \mathbb{P}, G, \in\rangle$ satisfies ZF, (b) $x \in L[\mathbb{P},G]$, and (c) $x \notin L[\mathbb{P}]$. This real is "diagonally generic" over $\langle L;\in\rangle$: A sequence of $\langle L;\in\rangle$ class forcing properties $\mathbb{P}^n (n < w)$ is defined. For $n > m$ we have $\mathbb{P}^m \supseteq \mathbb{P}^n$. $\mathbb{P}^n$ has a logically more complex definition in $\langle L;\in\rangle$ than $\mathbb{P}^m$. It is arranged that x is "somewhat generic" over each of the $\mathbb{P}^n$ and that, consequently, on account of special properties of the $\mathbb{P}^n$'s, L[x] satisfies ZF_n for all n. Techniques involved include Jensen coding and the utilization of certain definability properties peculiar to the minimal model of ZF.

H.-D. DONDER (FU BERLIN):

<u>A Property of Ultrafilters in L</u>

We show that in the inner model L for every uniform ultrafilter U on a regular cardinal $\kappa > w$ there is $R \subset U$, $|R| = \kappa^+$, such that the intersection of $\bar{R}$ is not in U for any infinite $\bar{R} \subseteq R$. To prove this we introduce a two cardinal version of Prikry's principle. This, in turn, is proven with a gap one morass at κ together with a "higher gap" argument in L approximating κ^+ by countable structures.

S. FRIEDMAN (M.I.T.):

<u>A Π^1_2-singleton below O</u>

Using Jensen coding and an idea of Solovay's I outlined a proof of the existence of a Π^1_2-singleton R sit. $O < R < O$ in L. The recursion theorem is used to produce an index for the procedure that, when applied to the Silver indiscernables I, produces the P-generic real R. This index enters into the definition of P in the following way: Guesses $(i_1, \ldots, i_n)$ at an element of I^n are killed using backward Easton forcing when they produce via this procedure information contradicting the generic real.

P. KOEPKE (FREIBURG):

<u>On the Free Subset Property at Singular Cardinals</u>

Theorem. If κ is minimal such that every first order structure of size $\geq \kappa$ has an uncountable free subset, and if κ is singular, then there is an inner model with ω_1 many measurable cardinals.

The proof uses the Short Core Model and combines techniques of the embedding theorem for K with combinatorial ideas of Devlin and Paris.

W. MITCHELL (PENN. STATE):

<u>Absoluteness</u>

I outlined a proof of:

Theorem 1: Assume that there is no inner model in which a κ has order κ^{++}. If $a^\#$ exists for all reals a, and if M is any inner model containing those elements of K which are hereditarily countable, then M is correct for Σ^1_3 sentences.

Here K is the core model for sequences of measures. It is conjectured that when the core model for a Woodin cardinal has been constructed, the theorem will hold without the above assumption.

L. STANLEY (LEHIGH UNIVERSITY)

A Partition Relation

Miyamoto showed that the negative partition relation $w_3 w_1 \longrightarrow (w_3 w_1, 3)^2$ can be derived from an $(w_2, 1)$ - morass with some additional structure. Jointly with Velleman, I derived it from an $(w_1, 1)$ - morass together with the assumption $2^{\aleph_1} = \aleph_2$.

Report by: R. Jensen

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