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MATHEMATISCHES FORSCHUNGSIINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 33/1988

Hyperbolic Systems of Conservation Laws

24.7. bis 30.7.1988

The conference was organized by C. Dafermos (Brown University, Providence, R.I.) and W. von Wahl (Bayreuth). It was centered around the following subjects:

Compensated Compactness and Applications,

Unusual Riemann-Problems,

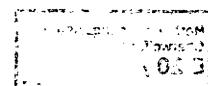
Rich Systems of Conservation Laws,

Gas-Dynamics,

Functional-Analytic Treatment of Nonlinear Wave Equations,

Periodic Solutions of Nonlinear Wave Equations,

Elasticity.



### Vortragsauszüge

H.-D. ALBER:

#### The asymptotic behavior of non-isentropic flow at large times

The asymptotic behavior of solutions to the system of conservation laws describing one-dimensional, compressible flow is discussed. Such results are known for systems consisting of not more than two equations. The system under consideration consists of three equations, and the methods used can be generalized to treat systems with  $m$  equations.

P. BRENNER:

#### Regularity of solutions of certain nonlinear hyperbolic equations

First a short review of some recent progress and the present state of art concerning the solutions to the hyperbolic problem

$$(*) \quad u_{tt} + P(x, D)u + f(u) = 0, \quad x \in \mathbb{R}^n, \quad t \in \mathbb{R}_+, \\ \text{data at } t=0 \text{ in } H^1 \times H^0$$

where

$P \sim \Delta$  for  $x$  large

$P$  2nd order smooth elliptic positive operator (hyperbolic problem)  
and  $f(u)$  is of power growth at  $\infty$ .

The discussion was mainly "limited" to the case of "large" initial data, and only touched on the works of Klainerman, Hörmander and John in the case of small initial data.

Finally a recent result by the author was presented: Strong solutions (Brenner, -88. To appear in Math. Z.). Let

$$(*) \quad u_{tt} + P(x, D)u + f(u) = 0, \quad x \in \mathbb{R}^n, \quad t \in \mathbb{R}_+$$

with

data at  $t=0$  in  $H^2 \times H^1$ .

We then have the following result:

THM. There exist a unique global solution  $u$  to  $(*)$  in  $C^2(\mathbb{R}_+) \times L_2(\mathbb{R}^n) \cap C(\mathbb{R}_+) \times H^2(\mathbb{R}^n)$ , provided  $|f(u)| \leq C|u|^p$ ,  
 $p < (n+2)/(n-2)$ ,  $n > 2$ .

H. ENGLER:

Singular kernels in nonlinear viscoelasticity of integral type

The description of finite shear deformations for viscoelastic liquids and solids leads to systems of two conservation laws, one with a memory term, in one or more space dimensions. If the memory term contains an integral with a singular kernel, then global weak solutions of this nonlinear system can be found that satisfy a physically meaningful entropy inequality. My work is concerned with consequences and variants of this entropy property and with results on the properties of the weak solutions.

H. FREISTÜHLER:

Rotational degeneracy of hyperbolic systems of conservation laws

Rotational symmetry of the flux function induces a loss of strict

hyperbolicity at each invariant point in the system's state space, which makes the pattern of elementary waves delicate. Nevertheless, near any such point, if certain genericity assumptions are satisfied, Riemann's initial value problem has a unique stable solution, depending continuously on the data.

The results can be applied to interesting cases from continuum mechanics, where the invariance property is a consequence of isotropy.

J. M. GREENBERG:

Continuum limits of discrete gases or surges in glacial flows - An example of conservation laws with limit cycle oscillations

We look at motions of discrete particles moving on the line. The particles are free streaming and interact elastically. Limit flows are characterized (as the mass of each particle and the initial particle spacing tend to zero) and we show that in certain cases the limit fields satisfy the usual gasdynamics equation with a  $p = S\rho^3$  law. The limit motions are characterized without solving the limiting pde's directly.

E. HORST:

Symmetric solutions of the Vlasov-Maxwell system revisited

The Vlasov-Maxwell system describes the evolution of a plasma in  $\mathbb{R}^3$  (consisting of ions and electrons) that moves under the influence of the electric field  $E$  and the magnetic field  $B$  that it generates itself.

If the initial data are symmetric in the sense that they are invariant under all rotations of  $\mathbb{R}^3$ , the global existence of unique classical solutions was shown by R. Glassen and J. Schaeffer in 1985. In this case B vanishes identically.

Recent work of Schaeffer suggests that one can expect global classical solutions also for data that are near symmetric data, provided the symmetric solution satisfies a decay estimate like  $\|E(t)\|_\infty \leq \text{const.} \cdot t^{-2}$ . We give a necessary and a sufficient condition for the validity of such an estimate.

C. KLINGENBERG:

On hyperbolic conservation laws in two space dimensions

After being able to explicitly construct the solution to Cauchy problems

$$u_t + f(u)_x + g(u)_y = 0, \quad u(x,y,t) \in \mathbb{R}$$

$$u(0,x,y) = u_0$$

which are invariant under the transformation

$$(x,y,t) \rightarrow (\alpha x, \alpha y, \alpha t), \quad \alpha > 0$$

for generic cases, we constructed a numerical algorithm which finds the solution to these two dimensional Riemann problems. These were used as part of numerical schemes which solve the Cauchy problem, namely a front tracking code and a two dimensional Glimm scheme.

D. KRÖNER:

Numerical schemes for the Euler equations in 2-D without dimensional splitting

We consider a numerical scheme for the nonlinear Euler equations of gasdynamics in 2-D. The algorithm doesn't use any dimensional splitting. It is a generalization of a scheme developed by Roe for the linearized Euler equations. In 1-D perturbation can propagate only in two directions but in 2-D there are infinitely many directions of propagation. Therefore the algorithm should be able to select the most important directions and to ensure that the scheme takes this fact into account. In this lecture we shall describe some details of this algorithm and we shall present some numerical results in 1-D and 2-D. Furthermore we shall compare our scheme with some Godunov-type schemes of first order.

N. A. LAR'KIN:

Hyperbolic equations in the theory of transonic flows

Initial-boundary value problems for nonlinear equations

$$u_{xt} + u_x u_{xx} - u_{yy} = 0$$

modelling nonsteady 3-D transonic gas flows near a slender body are considered. Local in time existence and uniqueness of classical solutions are proved.

M. MEIER:

Harmonic maps on Minkowski space and related nonlinear wave equations

We consider harmonic maps from  $(n+1)$ -dimensional Minkowski space-time with values in a Riemannian manifold  $M$ . Such maps arise, for instance, in  $SU(2)$ -gauge theory and in general relativity. We survey the conserved quantities of the system and some theorems concerning the local existence for the Cauchy problem and the global existence in case of small initial data. The latter result is compared to a perturbed nonlinear wave equation where blow-up of classical solutions always occurs in 2 and 3 space dimensions. I. Shatah has given an example of a self-similar rotationally symmetric harmonic map in 3 space dimensions with values in  $S^3$ , thus showing that harmonic maps with large Cauchy data may develop singularities in finite time. We present a number of sufficient conditions in terms of geometric properties of the target manifold  $M$  which exclude the existence of self-similar solutions. Nonexistence is proved, e.g., when  $n \leq 3$  and  $M$  has non-positive sectional curvature. For rotationally symmetric harmonic maps, a nonexistence result is obtained for  $n \leq 7$  and special target manifolds including hyperbolic space.

G. NAKAMURA:

An inverse problem for a stratified elastic medium

Let  $\Omega = \{x = (x_1, x_2, x_3); x_3 > 0\}$  be a stratified elastic medium with free boundary. Denote its density, Lamé parameter functions by  $p(x_3), \lambda(x_3), \mu(x_3) > 0$ . Now put an impulse (eg. a dynamite) at a boundary point  $x^0$  at a time  $t = 0$ . Then it generates an elastic wave for  $t > 0$ .

We have an affirmative answer to the following problem: Can you globally determine  $p(x_3)$ ,  $\lambda(x_3)$ ,  $\mu(x_3)$  by observing certain components of the  $o$ -th and first order velocity moments of the elastic wave.

H. PECHER:

Some remarks on the scattering theory for nonlinear wave equations

Consider as a model the Cauchy problem for semilinear wave equations with power type nonlinearities

$$u_{tt} - \Delta u = |u|^p, \quad u = u(x, t), \quad x \in \mathbb{R}^3, \quad t \in \mathbb{R}^+$$

It is well known that global classical solutions do not exist for any nontrivial smooth data with compact support in the range  $1 < p < 1 + \sqrt{2}$  (F. John 1979). So especially a scattering theory does not exist in these cases. On the other hand we are able to show that for  $p > 1 + \sqrt{2}$  the scattering operator which belongs to the pair of equations  $u_{tt} - \Delta u = |u|^p$  and  $u_{0,tt} - \Delta u_0 = 0$  exists in the sense of energy norms for smooth and small data if they decay sufficiently rapidly at  $\infty$ . The proof requires the use of space-time weighted  $L^\infty$ -norms and is completely elementary. The idea of using norms like these goes back to F. John who already proved a global existence result in the above case for small data in  $C_0^\infty(\mathbb{R}^3)$ .

B. PERTHAME:

Numerical approximation of gas dynamics using Boltzmann equations

Boltzmann schemes for solving gas dynamics equations have been used for a few years by several authors. They can be related rigorously to Boltzmann equations and they provide schemes which have remarkable properties: Unconditional stability, the entropy

condition is satisfied ... etc. ...

We generalize the class of Boltzmann schemes used in the literature, analyse the above properties and perform numerical computations - It turns out that the class of Boltzmann schemes contains many "classical" schemes: Steger & Warming, Osher & Engquist, Lax & Friedrichs ...

It gives also a way to build entropies for gas dynamics equations.

R. RACKE:

Global existence resp. blow-up phenomena for solutions to the equations of 3-d-thermoelasticity

In the first part we discuss the Cauchy problem for the hyperbolic-parabolic system of 3-d nonlinear thermoelasticity. The assumption that the nonlinear coefficients differ from those of the initial constant state by terms of at least order two for small values of the variables leads to cubic but also to some quadratic nonlinearities in the final setting. (The latter results from the parabolic part.) Global smooth solutions are shown to exist if the initial data are small in special Sobolev norms. We also point out that in the genuinely nonlinear case blow-up phenomena have to be expected.

In the second part we prove  $L^p-L^q$ -estimates for the solution to a special linear initial boundary value problem in an exterior domain, proposing a method using generalized eigenfunction expansions.

M. RASCLE:

Hyperbolic systems of conservation laws and compensated compactness

We study the extension of previous works by R. Di Perna, M. Rascle, D. Serre, as well as V. Roytburd and M. Slemrod, on the applications of the compensated compactness method to study the structure of oscillating solutions (or approximate solutions) to some nonlinear hyperbolic systems of conservation laws:

$$u_t^\varepsilon + f(u^\varepsilon)_x \approx 0$$

with initial data

$$u^\varepsilon(x, 0) = u_0(x).$$

B. SCARPELLINI:

Periodische und fastperiodische Lösungen in der Nähe von Gleichgewichtslösungen der nichtlinearen Wellengleichung  $u_{tt} = u_{xx} - g(u)$  auf der Halbgeraden

Wir betrachten die Wellengleichung (\*)  $u_{tt} = u_{xx} - g(u)$  auf  $\mathbb{R}_+ = \{x \geq c\}$ ,  $t \in \mathbb{IR}$ , wo  $g(u) = m^2 + \tilde{g}(u)$  ist und  $\tilde{g}(u) = Au^2 + \dots$  eine ganze Funktion. Sei  $u_0$  eine Gleichgewichtslösung von (\*), d.h.  $u_{0xx} = g(u)$ , für welche  $\lim_{x \rightarrow \infty} (|u_0(x)| + |u_{0x}(x)|) = 0$  gilt. Wir suchen Lösungen  $u(x, t)$  von (\*), die folgendes erfüllen:

- (1)  $u(x, t)$  ist periodisch oder fastperiodisch in  $t$ , ( $\forall x \geq c$ )
- (2)  $\Delta(x) := \sup_{t \in \mathbb{IR}} \{|u(x, t) - u_0(x, t)| + |u_x(x, t) - u_{0x}(x, t)|\}$

strebt in einem zu präzisierenden Sinne gegen Null,

(3)  $\sup_{x \geq 0} |\Delta(x)|$  ist klein.

Familien von Lösungen solcher Art werden konstruiert mit Hilfe einer Technik, die von A. Weinstein für den  $2\pi$ -periodischen Fall und  $u_0 = 0$  und die von Scarpellini-Vuillermot auf den fastperiodischen Fall und  $u_0 = 0$  ausgedehnt wurde.

M. SCHATZMAN:

Post-processing of dispersive numerical schemes

Dispersive approximations of the transport equation  $u_t + u_x = 0$ , such as the leap-frog scheme, or particle discretisation with non-characteristic speed can be summarized in the following model problem:

$$(1) \quad u_t^\varepsilon + \zeta^\varepsilon * u_x = 0$$

where  $\zeta^\varepsilon(x) = \varepsilon^{-1} \zeta(x/\varepsilon)$ ,  $\zeta \in S(\mathbb{R})$ ,  $\zeta(x) = \zeta(-x)$ ,  $\forall x \in \mathbb{R}$ ,  $\int \zeta(x)x^2 dx \neq 0$ . Eqn. (1) is stable in  $L^2(\mathbb{R})$ , but not in any other  $L^p$  space. If  $\rho$  is an arbitrary kernel,  $\rho \in S$ , and  $\rho_\alpha = \alpha^{-1} \rho(\cdot/\alpha)$ , the solution of (1) is postprocessed by convolution with  $\rho_\alpha$ , and the following holds: if  $\kappa = \varepsilon^{2/3}/\alpha$  is bounded from above by  $K$ , there exists a constant  $C(t, \rho, K)$  such that for all initial data  $u_0$ ,

$$\|u^\varepsilon(., t) * \rho_\alpha\|_1 \leq C(t, \rho, K) \|u_0\|_1.$$

Moreover, if  $\kappa \rightarrow 0$ ,  $u^\varepsilon(., t) * \rho_\alpha - u(., t) \rightarrow 0$  in  $L^1$ . The critical exponent  $\frac{2}{3}$  is optimal, if  $\zeta$  has more vanishing moments ( $\int x^p \zeta(x) dx = 0$ ,  $1 \leq p \leq 2n-1$ ), then the same result holds for critical exponent  $\frac{2n}{2n+1}$ .

D. SERRE:

Rich hyperbolic systems of first order

In order to understand the complexity of hyperbolic systems of the first order, one may start to characterize which ones are endowed with a rich structure: Diagonalization, a large set of entropies. We shall describe what they are, how to construct them and how to find the entropies. For such systems, the compensated compactness goes on as for the  $2 \times 2$  systems. Finally, we shall show an example of such a rich system. It is provided by the zero-dispersion limit of the Korteweg de Vries equation, as given by Lax and Levermore.

M. SHEARER:

Nonstrictly hyperbolic systems of conservation laws (with S. Schecter)

It is shown how to solve the Riemann problem for the  $2 \times 2$  system

$$(1) \quad U_t + dC(U)_x = 0,$$

where  $U = (u, v)$ ,  $C(U) = au^3/3 + bu^2v + uv^2$ , with  $a < 3b^2/4$ . The solution involves undercompressive shocks, which are characterized using a result of Chicone on quadratic gradient vector fields in the plane. Perturbing the flux in (1) by cubic and higher order terms leads to a bifurcation problem which is solved using Melnikov's method.

Y. SHIBATA:

The local and global existence theorem of solutions to Neumann problem for some quasilinear hyperbolic operators

It is considered the local and global existence theorems of classical solutions to the following mixed problem:

$$\partial_t^2 u - \partial_i (\partial_i u / \sqrt{1 + |\nabla u|^2}) = 0 \text{ in } (0, T) \times \Omega;$$

$$v_i (\partial_i u / \sqrt{1 + |\nabla u|^2}) + b(u, \partial_t u) = 0 \text{ on } (0, T) \times \Gamma;$$

$$u = u_0 \text{ and } \partial_t u = u_1 \text{ at } t = 0 \text{ in } \Omega.$$

Here, the summation convention is understood;  $\Omega$  is a domain in  $\mathbb{R}^n$ , its boundary  $\Gamma$  being a  $C^\infty$  and compact hypersurface;  $v = (v_1, \dots, v_n)$  is the unit outer normal to  $\Gamma$ ;  $\nabla u = (\partial_1 u, \dots, \partial_n u)$  ( $\partial_j = \partial/\partial x_j$ ).

Furthermore, the local existence theorem is extended to the case where the operators are some quasilinear hyperbolic system of 2nd order with fully nonlinear boundary condition. And, as an application, it is proved a local existence theorem of classical solutions to some nonlinear elastodynamics equations with some applied surface force which is not dead load.

T. C. SIDERIS:

Long time behavior of solutions to the three-dimensional compressible Euler equations

We show that the life space  $T(\epsilon)$  of classical solutions to the three-dimensional, compressible, isentropic Euler equations, with initial data that is a  $C_0^\infty$  perturbation of a constant state of amplitude  $\epsilon$ , satisfies the lower bound  $T(\epsilon) > \exp(c/\epsilon)$ .

M. SLEMROD:

Weak decay of a nonlinear wave equation

In this talk I will explain how some of the ideas expressed by Luc Tartar in the use of Young measures for proving existence of solutions in hyperbolic conservation laws may be used for describing asymptotic behavior as  $t \rightarrow \infty$  for a nonlinear wave equation.

In particular I show that for the equation  $u_{tt} - \Delta u = -a(x)g(u_t)$ ,  $x \in \Omega$  (bounded smooth domain in  $\mathbb{R}^N$ ),  $u=0$  on  $\partial\Omega$ ,  $a(x) \geq 0$ ,  $a \in C^\infty(\Omega)$ ,  $\text{meas}\{x; a(x) > 0\} > 0$ ,  $\xi q(\xi) \geq 0$ ,  $\ker q \subseteq [0, \infty)$  or  $\ker q(-\infty, 0]$ , we have weak  $\lim_{t \rightarrow \infty} (u, u_t) = (0, 0)$  in  $H_0^1(\Omega) \times L_2(\Omega)$ .

S. J. SPECTOR:

Nonuniqueness for a hyperbolic system: Cavitation in nonlinear elastodynamics

We let  $\Omega \subset \mathbb{R}^n$ ,  $n \geq 3$ , and consider the problem: Find  $u: \Omega \times [0, T] \rightarrow \mathbb{R}^n$  that satisfies

$$\operatorname{div} S(\nabla u) = u_{tt} \text{ in } \Omega \times [0, T],$$

$$u(x, t) = \lambda x \text{ für } x \in \partial\Omega \text{ and } t \in [0, T],$$

$$u(x, 0) = \lambda x \text{ for } x \in \Omega,$$

$$u_t(x, 0) = 0 \text{ for } x \in \Omega,$$

where

$$S(F) := F + h'(\det F) \operatorname{adj} F$$

with  $h'' > 0$ ,  $h''' < 0$ ,  $h'(v) \rightarrow -\infty$  as  $v \rightarrow 0^+$ , and  $h'(v) \rightarrow +\infty$  as  $v \rightarrow +\infty$ .

We prove that there are  $\lambda_i \rightarrow +\infty$  such that the above problem, with  $\lambda = \lambda_i$ , has multiple solutions. Moreover we show that each such solution is admissible according to the standard entropy criterion. The solutions that we construct are superpositions of special cavitation similarity solutions; that is, solutions of the form

$$u(x,t) := \frac{\varphi(s)}{s}(x-x_0), \quad s := |\dot{x}-x_0|/(t-t_0),$$

for  $t \geq t_0$ , where  $\varphi$  is continuous, piecewise  $C^2$  and satisfies

$$\varphi(0) > 0, \quad \varphi(s) = \lambda s \text{ for } s \geq \sigma,$$

for some  $\sigma > 0$ . Thus each such solution is discontinuous, opening a hole in the region  $\Omega$ .

L. TARTAR:

#### Compensated compactness

A new concept, generalizing the now classical theory of compensated compactness, will be presented. It involves the description of some measures (called H-measures) which are associated to any (sub)sequence of a sequence converging to 0 weakly in  $L^2$  and is indexed by  $(x, \xi)$  with  $x \in \Omega$  and  $\xi \in S^{N-1}$ . It permits to solve exactly some questions of homogenization of lower order or small amplitude; it sees the complete characteristic set given by the balance equations and allows some description of propagation of oscillations and concentration effects. The case of a scalar linear equation with variable coefficients and of the wave equation shows its ability to describe the propagation of energy along bicharacteristic rays; nothing on the phase can be seen by these measures. The basic formula involves a limit of  $\int_{\mathbb{R}^N} F(\varphi_1 U_i^\varepsilon) F(\varphi_2 U_j^\varepsilon) \psi(\xi/|\xi|) d\xi$ .

T. C. T. TING:

The Riemann problem and the generalized Riemann problem for elastic-plastic waves

For wave propagation in solids, the Riemann problem prescribes the initial condition  $U(x,0)$ ,  $x > 0$  and the boundary condition  $\sigma(0,t)$ ,  $t > 0$ ; both are constant.  $U$  contains the velocity vector  $v$  and the stress vector  $\sigma$ . The solution to the Riemann problem, when it exists, is valid for the entire region  $x > 0$ ,  $t > 0$ . The generalized Riemann problem prescribes the initial condition  $U(x,0)$ ,  $x > 0$  which is a constant but the boundary condition is given by  $\sigma(0,T) = \sigma(0,0^+) + \dot{\sigma}(0,0^+)t$ ,  $t > 0$ . If  $\dot{\sigma}(0,0^+) = 0$  we have the Riemann problem. For  $\dot{\sigma}(0,0^+) \neq 0$ , we are interested in the solution near the origin  $x = 0$ ,  $t = 0$ . We want to know how much the solution differs from the solution to the Riemann problem. The tension-torsion of a thin-walled tube of elastic-plastic materials is used as an example to illustrate how one can find the solution to the generalized Riemann problem. The differential equations are a  $4 \times 4$  non-strictly hyperbolic system with one umbilic point.

A. E. TZAVARAS:

Weak solutions for a nonlinear system in viscoelasticity

We consider a one-dimensional model problem for the motion of a viscoelastic material with fading memory governed by a quasilinear hyperbolic system of integrodifferential equations of Volterra type. For given Cauchy data in  $L^\infty \cap L^2$ , we use the method of vanishing viscosity and techniques of compensated compactness to obtain the existence of weak solutions (in the class of bounded measurable functions) in a special case.

P. A. VUILLERMOT:

Smooth manifolds for certain dynamical systems on tori:

Quasiperiodic soliton solutions to nonlinear Klein-Gordon  
equations on  $\mathbb{R}^2$

Invoking the theory of stable and unstable manifolds for infinite-dimensional dynamical systems, we present new theorems concerning the existence and the regularity of certain real solutions to a class of semilinear wave equations in  $\mathbb{R}^2$ . Those solutions are Bohr almost periodic in time and converge exponentially rapidly to a constant equilibrium solution as the spatial variable goes to plus or minus infinity.

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