MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 34/1988

Theory of Large Deviations

31.7. bis 6.8.1988

Die Tagung stand unter der Leitung von E. Bolthausen (Berlin) und S. R. S. Varadhan (New York). Unter einem verbindenden Thema trafen sich Wissenschaftler mit einem sehr breiten Spektrum von Interessen: von der Mathematischen Statistik bis zur Quantenfeldtheorie.

Die Theorie der Wahrscheinlichkeiten großer Abweichungen war einer der Schwerpunkte der wahrscheinlichkeitstheoretischen Forschung der letzten 10 bis 15 Jahre. Es handelt sich um Präzisierungen von Gesetzen großer Zahlen: man untersucht die kleinen Wahrscheinlichkeiten, mit denen ein untypisches Verhalten eines stochastischen Prozesses auftritt. Die Untersuchung dieser kleinen Wahrscheinlichkeiten ist für viele Fragen interessant, u. a., da die untypischen Pfade eines einfachen Modells oft den Hauptbeitrag zu wichtigen Größen eines komplexeren liefern.

Wichtige Themen der Tagung waren:
- Große Abweichungen für angewandte stochastische Prozesse (Risikotheorie, Verzweigungsprozesse, simulated annealing)
- Effizienz statistischer Tests
- Große Abweichungen für Markoff-Prozesse
- Malliavin-Kalkül und Asymptotik des Wärmeleitungskerns
- Dynamische Systeme
- Wechselwirkende Teilchensysteme
- Statistische Mechanik, Gibbs-Maße, Hydrodynamik
- Quantenmechanik und Quantenfeldtheorie.

Die Tagung hatte 41 Teilnehmer. Es wurden 34 Vorträge gehalten.

Gefördert durch
DFG Deutsche Forschungsgemeinschaft

Abstracts

A. de Acosta

Large deviations for empirical measures of Markov chains in the τ-topology

Let S be a Polish space, π a Markov kernel on S, and $\{X_j : j \geq 0\}$ a Markov chain with state space S and transition probability π. The τ-topology on the space of probability measures $M^+{}_1(S)$ is the weakest topology making all maps $\mu \to \int f\, d\mu$ continuous, where f is a bounded measurable function on S. The following result improves existing results in two ways: It relaxes the assumptions on π and it gives large deviation lower bound rates based on the τ-topology, which are finer than those based on the usual weak topology on $M^+{}_1(S)$.

Lower bound result:
Assume that π is ψ-irreducible and that for all measurable $A \subset S$ with $\psi(A) = 0$ and for all $x \in S$ there exists $n \in N$ with $\pi^n(x,A) = 0$ (a slightly weaker assumption is possible). Then for all $x \in S$ and all measurable $B \subset M^+{}_1(S)$

$$\underline{\lim}_{n\to\infty} \frac{1}{n} \log P_x\left(\frac{1}{n}\sum_{j=0}^{n-1} \delta_{X_j} \in B\right) \geq -\inf\{I(\mu) : \mu \in \tau\text{-interior of } B\}$$

where I is the Donsker-Varadhan functional.

The following result improves existing upper bound results in the τ-topology.

Upper bound result:
Assume that there exists $m \in N$ such that $\{\pi^m(x,\cdot) : x \in S\}$ is uniformly absolutely continuous w. r. t. a p. m. ψ. Then for all $x \in S$ and all measurable $B \subset M^+{}_1(S)$

$$\overline{\lim}_{n\to\infty} \frac{1}{n} \log P_x\left(\frac{1}{n}\sum_{j=0}^{n-1} \delta_{X_j} \in B\right) \geq -\inf\{I(\mu) : \mu \in \tau\text{-closure of } B\}.$$

We also give extension to empirical measures of the multistate chain $\{(X_j, \ldots, X_{j+n-1}) : j \geq 0\}$, where $n \in N$ is fixed.

O. E. Barndorff-Nielsen

Large deviations and statistical inference

The talk concentrated on properties, and in particular the accuracy, of the formula

$$p(\hat\omega; \omega | a) \doteq p^*(\hat\omega; \omega | a) = c(\omega, a)\, |\hat{j}|^{1/2} \exp(l - \hat{l}),$$

where the expression p* defined on the right side of the approximation sign is a probability density function which in general constitutes a highly accurate approximation to the conditional distribution of the maximum likelihood estimator $\hat{\omega}$ given an (exact or approximate) ancillary statistic a, under a parametric statistical model $p(x; \omega)$. Actually, p^* is not only approximate but exact in a rather wide range of interesting cases, including virtually all transformation models. The problem of characterizing all exactness cases was raised. This problem is of interest both intrinsically and because the known exactness cases all exhibit various further properties of statistical use. Outside the exactness cases, and under ordinary repeated sampling, the formula is typically accurate to a relative error of order $O(n^{-3/2})$, often over the entire sample space. (For details see forthcoming volume in the Springer "Lecture Notes in Statistics Series", entitled "Parametric Statistical Models and Likelihood".)

G. Benarous

Large deviations and Malliavin calculus

If $L = \frac{1}{2} \Sigma_1{}^m X_i{}^2 + X_0$ is a Hörmander type second order operator on $\mathbb{R}^d$ and the X_i are smooth vector fields such that the restricted Hörmander hypoellipticity hypothesis is satisfied ($\mathrm{Lie}(X_1, \ldots, X_m)(x) = T_x\mathbb{R}^d$ for all $x \in \mathbb{R}^d$) the heat kernel $p_t(x, y)$ associated to L can be studied in small time for "good points" (x, y). The result is that if there is a unique subriemannian geodesic joining x to y, if this geodesic is the projection of a (unique) bicharacteristic curve, and if (x, y) are non-conjugate along this geodesic, then we have the asymptotic expansions

$$p_t(x,y) = t^{-1/d} \exp\left(-\frac{d^2(x,y)}{2t}\right)\left(\sum_{k=0}^{N} c_k(x,y)\,t^k + O(t^{N+1})\right)$$

for all N, where d(x, y) is the subriemannian distance associated to L, c_k are smooth functions (locally), $c_0(x, y) > 0$, and the expansion is uniform on compact sets of good points.

This result can be obtained by using large deviation techniques (Laplace method) to study the behaviour of the Fourier transform of $p_t(x, y)$ for small t, and by using Malliavin calculus to integrate this expansion to obtain the expansions of $p_t(x, y)$ by Fourier inversion formula. The interesting point here might be that these estimates can't be obtained for close points and then propagated by the semigroup property to distant points (as can be done in the elliptic case). This is so because on one given minimizing geodesic two points might be non-conjugate while intermediate points could be conjugate. It is then clear that a direct *global* approach is needed. That is what the large deviation principle (+ Laplace method) gives.

(This work is in part independent and in part joint work with R. Léandre)

T.-S. Chiang

Convergence rate for simulated annealing processes

An inhomogeneous Markov process X_t with transition rates $Q_{ij}(t) = p_{ij}\exp(-(U(j)-U(i))^+/T(t))$ where $T(t)\downarrow 0$ as $t\to\infty$, U is a function on the state space $S=\{1, 2, ..., n\}$ assumed to be integer-valued and non-negative, is called a simulated annealing process. Let $\lambda(t)=\exp(-1/T(t))$ and denote by d_H the maximum depth of the local minimum (minima) of U. Under the assumption that p_{ij} is irreducible and reversible, it is proved that

$$\lim_{t\to\infty} P(X_t = i)\exp\frac{U(i)}{T(t)} = \beta_i \in (0,\infty)$$

for each $i\in S$ if $\lambda'(t)/\lambda(t)=o(\lambda^{d_H})$ and $\int_0^\infty \exp(-d_H/T(t))\,dt = \infty$. The typical case for $T(t)$ is $c/\log t$ for some constant c. An optimal c exists if and only if $d_H = d_V$ where d_V is the convergence rate of the 2nd eigenvalue of $(Q_{ij}(t))$.

F. Comets

Large deviations for Gibbs measures with random interaction

Let (X_i,Y_i), $i\in Z^d$, be i. i. d. random vectors with arbitrary distribution. We show that, for a. e. $(Y_i)_i$, the conditional law of the empirical field given $(Y_i)_i$ has large deviations property with a deterministic rate function.

We use this to study Gibbs measures with random interaction (i. e. the distribution of dependent variables $(X'_i)_i$ with coupling dependent of $(Y_i)_i$). We obtain:
- a (deterministic) variational formula for the free energy
- large deviations property for the Gibbs measure, and by localization
- we relate some of the measures to conditional versions of the probabilities which are solution of the variational formula (this is a variational principle).

Jean-Dominique Deuschel

Critical Ornstein-Uhlenbeck process

Consider the symmetric diffusion process on $E=M^{Z^d}$ generated by

$$Lf = \frac{1}{2}\sum_{k\in Z^d} \exp(H_k)\,\mathrm{div}_k(\exp(-H_k)\,\mathrm{grad}_k f)$$

where $H_k = \Sigma_{F\ni k} J_F$. M is a Riemannian manifold. If the interaction is small enough, one can check the hypercontractivity of the process. For the critical Ornstein-Uhlenbeck process where $M=\mathbf{R}$ and

$$J_F(x) = \begin{cases} +(x(i))^2/2 & F=\{i\} \\ -\gamma(i-k)\,x(i)\,x(k) & F=\{i,k\} \\ 0 & |F|\geq 3 \end{cases}$$

with $\Sigma_i\ \gamma(i)=1$, one sees that the Gibbsian field admits a phase transition when $d\geq 3$. In this case the process is not hypercontractive and the following large deviation results for $Z_t(0)=1/t\int_0^t x_s(0)ds$ holds

$$\lim_{t\to\infty} \frac{1}{r_t(d)} P_\mu\left(Z_t(0)>\beta\right) = -\frac{1}{t}\, s_\mu^2(d)\,\beta^2$$

where

$$r_t(d) = \begin{cases} t^{1/2} & d=3 \\ t/\log t & d=4 \\ t & d\geq 5 \end{cases}$$

and $s_\mu^{\,2}(d)\in\mathbf{R}_+$ is the independent of the initial distribution μ for $d\geq 4$ and dependent of μ for $d=3$.

Hermann Dinges

Unified treatment of small and large deviations

A family of probability densities $\{f_\varepsilon(x):\varepsilon\to 0\}$ on $U\subseteq\mathbf{R}^d$ is called a (smooth) Wiener germ of order m if there exist functions (satisfying smoothness conditions) $K(x)$, $S_0(x), \ldots, S_{m-1}(x)$ on U such that uniformly on compact subsets of U

$$f_\varepsilon(x) = (2\pi\varepsilon)^{-d/2}\exp\left(-\varepsilon^{-1}K(x)+S_0(x)+\varepsilon S_1(x)+\ldots+\varepsilon^{m-1}S_{m-1}(x)+O(\varepsilon^m)\right).$$

Wiener germs are called equivalent of order j if $K(x)$, $S_0(x)$, ..., $S_{j-1}(x)$ agree. Transform Wiener germs by mappings of the type $T(\varepsilon,x)=T_0(x)+\varepsilon T_1(x)+\ldots+\varepsilon^m T_m(x)+o(\varepsilon^m)$ (uniformly) where $T_0(x)$ is a diffeomorphism and the $T_j(x)$ satisfy suitable smoothness conditions; then you get Wiener germs. Under certain conditions the marginals of Wiener germs form a Wiener germ. Similarly conditional densities derived from Wiener germs are Wiener germs.

Theorem: Let $\{X_\varepsilon:\varepsilon\to 0\}$ be a family of random vectors such that the distributions $L(X_\varepsilon)$ form a Wiener germ. Then there exists a transformation $T(\varepsilon,x)=T_0(x)+\ldots+\varepsilon^m T_m(x)$ such that $\{L(X_\varepsilon):\varepsilon\to 0\}$ is equivalent of order m to $\{L(T(\varepsilon,\varepsilon^{1/2}Z)):\varepsilon\to 0\}$ where Z is standard normally distributed.

The following lesson might be learned from the theory. In cases where the "Large Deviation Principle" holds and everything is very smooth, it may be worthwhile not to study the refined limiting behaviour of $-n^{-1}\ln P(X^{(n)}\in A)$; one might rather hope to find an asymptotic expansion of the form

$$n^{-1/2}\Phi^{-1}\left(P\left(X^{(m)}\in A\right)\right)=a_0(A)+\frac{1}{n}a_1(A)+\ldots+\left(\frac{1}{n}\right)^m a_m(A)+o(n^{-m}).$$

Such an asymptotic formula may even hold uniformly for $A_c=\{x: f(x)\leq c\}$ if f is a nice function - and this comprises asymptotic normality in the range of small deviations.

Richard S. Ellis

A unified approach to large deviations for Markov chains

Let S be a Polish space, $X_0=x$, X_1, X_2, ... a Markov chain taking values in S with transition probability function $\pi(x, dy)$. We prove uniform large deviation properties for the following classes of random measures:

$$M_{n,1}(\omega,.)=\frac{1}{n}\sum_{j=0}^{n-1}\delta_{X_j(\omega)}(.),\quad M_{n,a}(\omega,.)=\frac{1}{n}\sum_{j=0}^{n-1}\delta_{Y_j^{(a)}(\omega)}(.),\quad R_n(\omega,.)=\frac{1}{n}\sum_{j=0}^{n-1}\delta_{T^jX(n,\omega)}(.),$$

where $a\in\{2, 3, \ldots\}$, $Y_j^{(a)}(\omega)=(X_j(\omega), X_{j+1}(\omega), \ldots, X_{j+a-1}(\omega))\in S^a$, $X(n, \omega)$ is the periodic point in S^Z satisfying $(X(n, \omega))_{kn+i}=X_i(\omega)$, for $i=0, 1, \ldots, n-1$, $k\in Z$, and T is the shift mapping on S^Z.

Theorem I: Assume that $\pi(x, dy)$ satisfies some hypothesis (H) on S such that

a) $\{M_{n,1}\}$ has a uniform LDP,

b) π_a, the transition probability function of the Markov chain $\{Y_j^{(a)}: j=0, 1, \ldots\}$ on S^a, satisfies (H) on S^a for each $a\in\{2, 3, \ldots\}$.

Then c) $\{M_{n,a}\}$, $a\in\{2, 3, \ldots\}$, has a uniform LDP,

d) $\{R_n\}$ has a uniform LDP.

Theorem II: Here is an example of a hypothesis (H) such that a) and b) in Theorem I hold (and thus c) and d)): There exist $\beta\in N$ and $M\in[1,\infty)$ such that for all $x, x'\in S$ and all measurable $A\subset S$ $\pi^\beta(x, A)\leq M\,\pi^\beta(x', A)$.

The entropy functions in a), c), and d) of Theorem I are all closely related.

H. Föllmer

Surface entropy

For an ergodic Gibbs measure P on the space Ω of lattice configurations $\omega: Z^d\to S$, large deviations of the empirical field

$$R_n(\omega) = \frac{1}{|V_n|}\sum_{t\in V_n}\delta_{\theta_t\omega}$$

along the sequence of d-dimensional cubes V_n are of the form

$$\frac{1}{|V_n|} \log P(R_n \in A) \sim -\inf_{Q \in A \cap M_s} h(Q|P),$$

where M_s is the class of stationary probability measures on Ω, and where $h(Q|P)$ denotes the relative entropy of Q with respect to P (Comets, Olla, Föllmer-Orey). The lower bound follows from a Shannon-McMillan theorem for $h(Q|P)$. But in the case of a phase transition, the right side may be 0 even if P does not belong to the closure of A. This leads to a refined description of large deviations where the volume $|V_n|$ is replaced by surface area $|\partial V_n|$ (cf. R. Schonmann), and where $h(Q|P)$ is replaced by a surface entropy $s(Q|P)$ (Föllmer-Ort). In this talk, we prove the Shannon-McMillan theorem for $s(Q|P)$, defined in terms of cubes. We also sketch its extension to surface entropy around C^1-shapes; this is analogous to a recent construction of Dobrushin-Kotecki-Schlossmann.

Mark Freidlin

Weakly coupled dynamical systems

Let $\dot{u} = f(x, u)$ be a dynamical system in $\mathbb{R}$ depending on a parameter $x \in X$, and let A be the generator of some Markov process in the space X. Consider the dynamical system or semi-flow $u(t,.)$ in the space of functions on X:

$$(*) \qquad \frac{\partial u(t,x)}{\partial t} = f(x,u) + \lambda A u, \; u(0,x) = g(x), \; x \in X, \; t > 0.$$

The dynamical system $u(t,.)$ is said to be coupled by the generator λA. If λ in (*) is large we speak about strong coupling; if λ is small - about weakly coupled systems. We consider also the case when the initial dynamical system is replaced by a semi-flow of general type. Using large deviations principle for the family of processes, corresponding to the generator λA, we study wave front propagation in the system (*) and other problems concerning the long time behaviour of solutions of the system (*).

József Fritz

An extension of the Guo-Papanicolau-Varadhan method to the infinite line

We consider a stochastic system

$$d\sigma_k = \frac{1}{2}\left[V'(\sigma_{k+1}) - 2V'(\sigma_k) + V'(\sigma_{k-1})\right] dt + dw_k - dw_{k-1}$$

where $\sigma_k \in \mathbb{R}$, $k \in \mathbb{Z}$, $\sup |V''| < \infty$, $\underline{\lim}_{x\to\infty} V''(x) > 0$, and w_k is a family of independent Wiener processes. The Holley-Stroock (76) estimate on the local rate of entropy production extends also to this case, and it has two consequences. Under some moment condition we show that every stationary measure is a canonical Gibbs state with energy $H=\Sigma V(\sigma_k)-z_k\sigma_k$, where $2z_k=z_{k+1}+z_{k-1}$. For the hydrodynamic (diffusive) scaling we show that the entropy remains proportional to the macroscopic volume even for macroscopic times, while the rate of entropy production is an order of (d-2)-th power of the volume (d is the dimension of the space). These two bounds allow us to extend the Guo-Papanicolau-Varadhan theorem to infinite systems.

Jürgen Gärtner

Large deviations for weakly interacting diffusions and hierarchical models

There are essential differences in the qualitative behaviour of mean-field models and lattice models with short-range interaction. We consider (two-level) hierarchical models which, on the one hand, are similar to mean-field models but, on the other hand, reflect the behaviour of short-range lattice models (e. g. the dimension dependence of critical fluctuations) in a more proper way. The hierarchical analog of the two-dimensional stochastic Ising model with continuous spin may be described by the following system of weakly interacting diffusions:

$$dx_{ij} = -\left\{V_h'(x_{ij}) - J_1\left[\frac{1}{N}\sum_{l=1}^{N} x_{il} - x_{ij}\right]\right\} dt + \sigma\, dw_{ij}$$

$$+\frac{J_2}{N}\left\{\frac{1}{MN}\sum_{k=1}^{M}\sum_{l=1}^{N} x_{kl} - \frac{1}{N}\sum_{l=1}^{N} x_{il}\right\} dt, \qquad 1\le i\le M,\ 1\le j\le N.$$

Here V_h is a potential of the form $x^4/4-x^2/2+hx$ and w_{ij} are independent Brownian motions. J_1, J_2, h, and σ denote positive constants. We define the level II empirical process $X^{MN}(\,.\,)$ by

$$X^{MN}(t) := \frac{1}{M}\sum_{i=1}^{M}\delta_{X_i^N(t)}, \text{ where } X_i^N(t) = \frac{1}{N}\sum_{j=1}^{N}\delta_{x_{ij}(t)}.$$

Let Π^{MN} be the probability law of the stationary distributrion of the process $X^{MN}(\,.\,)$. We prove large deviation theorems for Π^{MN} as $M, N\to\infty$ both in the $M{\cdot}N$-scale and the M-scale and discuss the structure of local minima of the associated rate functions (free energy functionals). In the case $0<\sigma^2<\sigma_{cr}^2$, $h=h_N=h^*/N$ ($h^*\in\mathbf{R}$), these results can be interpreted (in a certain sense) as a nucleation effect. (Joint work with D. A. Dawson)

Andreas Greven

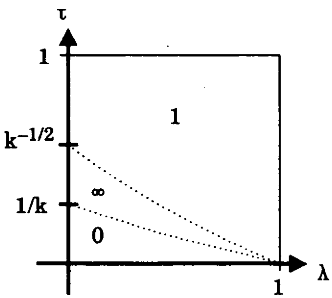
Branching random walks in random environment

We consider a branching random walk on Z^d with birth and death rates assigned to the sites according to some random mechanism. This system exhibits a phase transition with respect to the drift of the underlying random walk. With that we mean that for small drift the number of particles per site grows exponentially fast for almost all environments, while for large dirft the system becomes (locally) extinct. Various refinements and related questions are considered. For example: Look at the total particle density $\lim_{n\to\infty} |V_n|^{-1} \Sigma_{x\in V_n} \eta_t(x)$ and call the limit D_t. Then this quantity is growing for certain values of the drift exponentially fast even though for these values of the drift the system becomes already locally extinct.

Geoffrey Grimmett

Percolation in $\infty + 1$ dimensions

For percolation on a finite-dimensional lattice the number of infinite clusters is either 0 or 1, whereas for a tree it is either 0 or ∞. What is going on? Certainly the infinite cluster is a. s. unique (when it exists) for the Cayley graph of any infinite finitely-generated group with sub-exponential growth function. An interesting example is the graph $L = T \times Z$, the direct product of a regular tree T and the line Z. If each edge in a tree is open with probability τ and each edge in a line with probability λ, then the "phase diagram" may be represented by the opposite illustration, where $k+1$ is the common degree of vertices of T and 0, 1, ∞ indicate the a. s. number of infinite clusters. There are corresponding results for $T \times Z^d$, and consequences for the phase diagram of the corresponding Ising system. (Joint work with Chuck Newman.)



N. C. Jain

Large deviations for additive functionals of irreducible Markov processes

Let X_t, $t \geq 0$, be a Markov process with transition kernel $\pi(t, x, .)$. Let $\psi(x, A) = \int_0^\infty e^{-t} \pi(t, x, A)dt$. The process is said to be irreducible if there is a measure α on the state space E (assumed Polish) such that $\alpha(A)>0$ implies $\psi(x,A)>0$ for every $x\in E$. This condition does not suffice for the lower bounds at level 2 in terms of the Donsker-Varadhan rate function. If one also assumes that $\alpha(A)=0$ implies $\int_t^\infty e^{-s}\pi(s,x,A)ds=0$ for some t, then one has $\underline{\lim}_{t\to\infty} 1/t \log P_x[L_t(\omega,.)\in G]\geq -I(\mu)$, for all $x\in E$. Here $I(\mu)=-\inf_{u\in U}\int(Lu)/u\,d\mu$ and G is a weak neighborhood of μ. Lower bounds for more complicated sets also follow. The method applies to bounded measurable additive functionals taking values in a separable Banach space. Everything works as well in the discrete parameter case. We also define $\bar{I}(\mu)$ which gives lower bounds under the irreducibility condition above. $\bar{I}=I$ when the second condition holds.

G. Jona-Lasinio

Large deviations for a stochastic PDE with renormalization

We study in the small noise limit the process obtained as a weak solution of the stochastic PDE

$$d\phi_t = -\frac{1}{2}\left((-\Delta+1)^\rho\phi_t + \lambda(-\Delta+1)^{-1+\rho}:\phi_t^3:\right)dt + \varepsilon dW_t$$

where $\rho<1$, $:\phi^3:=\phi^3-3\varepsilon^2C(0)\phi$, $C(x,y)=(-\Delta+1)^{-1}(x,y)$, and $E(W(t,x)W(t',x'))=\min(t,t')(-\Delta+1)^{-1+\rho}(x,x')$, $x\in\mathbb{R}^2$. Due to the presence of infinite renormalization ($C(0)=\infty$) the usual large deviation techniques do not apply immediately and a new strategy has to be developed. We prove some estimates analogous to the Ventcel-Freidlin inequalities. From these it follows that the field trajectories suitably smeared in space over a scale r_0 are close in probability to the projection on the same scale of a field obeying a regularized stochastic equation with a sufficiently large cut-off. A new feature of this problem is that the value of ε for which the noise can be considered to be small depends on the scale over which the field is considered. (Joint work with P. K. Mitter)

Wilbert Kallenberg

Limiting exact efficiency of quadratic statistics

Application of exact Bahadur efficiencies in testing theory or inaccuracy rates in estimation theory needs evaluation of large deviation probabilities. There are many results available in the literature expressing large deviation probabilities in terms of Kullback-Leibler information numbers or moment generating functions. However, in many cases the obtained expressions are very complicated and therefore many authors consider a local limit of the nonlocal measure, which

is important from a statistical point of view. In the March 1986 meeting in Oberwolfach general results were presented on local limits of exact Bahadur efficiencies and exact inaccuracy rates for statistics, which are in a local sense asymptotically normal. The method is now extended to general quadratic statistics. The involved large deviation probabilities are related to large deviation probabilities of k-dimensional i. i. d. random vectors. Local limits of the latter large deviation probabilities are easily obtained. The results are applied to several examples, including generalized Cramér-von Mises statistics (e. g. Anderson-Darling statistic) and likelihood ratio statistics.
(This work is joint work with Gérard Jeurnink.)

S. Kusuoka

The large deviation principle for hypermixing processes

Let X be a Polish space and $\Omega = D([0,\infty)\to X)$. We define $\omega_s \in \Omega$ by $\omega_s(t) = \omega(t-[t/s]s)$, $t \geq 0$, for any $\omega \in \Omega$ and $s>0$. Also, we define $R_s(\omega) \in \mathcal{P}_{st}$ by $R_s(\omega) = 1/s \int_0^s \delta_{\omega_s(\cdot+t)} dt$. Here $\mathcal{P}_{st}$ denotes the set of stationary probability measures on Ω. We say that a stationary probability measure P or Ω satisfies large devaiation principles, if there is a good (lower semicontinuous, convex or affine) functional $H: \mathcal{P}_{st} \to [0,\infty]$ such that

$$\overline{\lim_{s\to\infty}} \frac{1}{s} \log P[R_s \in K] \leq -\inf\{H(Q) : Q \in K\}$$

for all closed $K \subset \mathcal{P}_{st}$ and

$$\underline{\lim_{s\to\infty}} \frac{1}{s} \log P[R_s \in G] \geq -\inf\{H(Q) : Q \in G\}$$

for all open $G \subset \mathcal{P}_{st}$. The large deviation principle is well studied in the case when P is Markov. However, a few results are known in the case when P is non-Markovian. We introduced a new notion of mixing property, "hypermixing", and showed that the hypermixing property implies the large deviation principle. Also, we showed that hypercontractive ε-Markov processes and good Gaussian processes are examples of hypermixing processes. The definition of the hypermixing property is the following:
Notion: We say that $f_1, \ldots, f_n$ are l measurably separated if there exist $0 \leq a_1 < b_1 < a_2 < b_2 < \ldots < a_n < b_n$ such that f_i is $\mathcal{F}[a_i, b_i]$-measurable, $i = 1, \ldots, n$, and $a_{i+1} - b_i \geq l$, $i = 1, \ldots, n-1$.
Definition: We say that P satiesfies the hypermixing condition if there exist functions $\alpha : [0,\infty) \to (1,\infty)$ and $\beta : [0,\infty) \to [0,1)$ such that α, β are non-increasing, $\lim_{l\to\infty} l(\alpha(l)-1) = 0$, $\lim_{l\to\infty} \beta(l) = 0$,

$$\| \prod_{k=1}^{n} f_k \|_{L^1(dP)} \leq \prod_{k=1}^{n} \|f_k\|_{L^{\alpha(l)}(dP)}$$

if $f_1, \ldots, f_n$ are l measurably separated, and

$$\left| \int_\Omega (f - \int_\Omega f dP) g \, dP \right| \leq \beta(l) \|f\|_{L^{\alpha(l)}(dP)} \|g\|_{L^{\alpha(l)}(dP)}$$

if f, g are l measurably separated. (This is joint work with T. Chiyonobu.)

Remi Léandre

Degenerate little perturbations of dynamical systems

Let $dx^\varepsilon_s = \varepsilon \Sigma X_i(x^\varepsilon_s) dw^i_s + X_0(x^\varepsilon_s) ds$ be a small random perturbation of a dynamical system. Under hypoelliptic conditions, you need to change the functional in order to give a type of Varadhan's estimate of the density $\lim_{\varepsilon \to 0} 2\varepsilon^2 \log p_\varepsilon(x, y) = -d_R{}^2(x, y)$, where $d_R(x, y)$ is the regularized distance and larger than the distance of the large deviation theory. (This is taken from joint work with Benarous.)

Teresa Ledwina

On probabilities of excessive deviations for Kolmogorov-Smirnov, Cramér-von Mises and chi-square statistics

Let $T: D[0, 1] \to \mathbb{R}$, α_n be the classical empirical process, and B_1 the Brownian bridge on [0, 1]. Assume that T satisfies the Lipschitz condition and that there exists a positive constant a such that $\log P(T(B_1) \geq y) = -(a/2)y^2(1 + o(1))$ as $y \to \infty$. Some explicit bounds of $P((T(\alpha_n) \geq x_n n^{1/2})$ are obtained for every n and $x_n > 0$ by using the Komlós-Major-Tusnády inequality. These bounds imply some results for the whole scope of x_n. In particular, expansions for large deviations as well as some moderate and Cramér-type large deviations results for $T(\alpha_n)$ are established. This unifies and generalizes some earlier results on deviations for Kolmogorov-Smirnov, Cramér-von Mises and chi-square statistics. Some new results for some quadratic statistics are derived also. (This is joint work with T. Inglot)

H. R. Lerche

Statistical aspects of the LIL or on the hazard that a Brownian path is killed at a curved boundary

Let W(t) denote Brownian motion with drift θ and $\psi(t)$ denote a nonnegative function on $\mathbb{R}_+$. Let $T = \inf\{t > 0 : W(t) \geq \Psi(t)\}$ and $\theta_T = \psi(T)/T$. In sequential statistics good approximations of the densities of T, $p_\theta(t)$, and of θ_T, $\hat{q}_\theta(\mu)$ are of interest. Let $\hat{q}_\theta(\mu) = t(\mu)\,\phi(t(\mu)^{1/2}(\mu-\theta))$ with $t(\mu)$ defined by $\mu=\psi(t)/t$ denote the formal saddlepoint approximation and let $\tilde{p}_\theta(t) = t^{-3/2}\Lambda(t)\,\phi(t^{-1/2}(\psi(t)-t\theta))$ with $\Lambda(t) = \psi(t)-t\psi'(t)$ and $\phi(y) = (2\pi)^{-1/2}\exp(-y^2/2)$ denote the tangent approximation. Discussing the range where the approximations $q_\theta(\mu) = \hat{q}_\theta(\mu)(1+o(1))$ and $p_\theta(t) = \tilde{p}_\theta(t)(1+o(1))$ hold when the boundary ψ recedes to infinity, one is led to the following result on the hazard function of T.

Theorem: Let ψ be a fixed nonnegative function ψ with the property that $P(T>t)>0$ for all $t\geq 0$. Assume further that $t^{-1/2}\psi(t)\to\infty$ as $t\to\infty$, that there exists an $\alpha\in(\frac{1}{2},1)$ with $\psi(t)/t^\alpha$ monotonely decreasing, and smoothness of the tangent intercepts. Then $p(t)/P(T>t)=t^{-3/2}\Lambda(t)\phi(t^{-1/2}\psi(t))(1+o(1))$ as $t\to\infty$.

Corollary 1: Is $t^{-1/2}\psi(t)$ monotonely decreasing on $(0,\varepsilon)$ and $P(T<\infty)=1$, then $P(T>t)=\exp(-\int_0^t u^{-3/2}\Lambda(u)\,\phi(u^{-1/2}\psi(u))du\,(1+o(1)))$ as $t\to\infty$.

Corollary 2: $P(T<\infty)<1$ if and only if $\int_1^\infty t^{-3/2}\Lambda(t)\,\phi(t^{-1/2}\psi(t))dt<\infty$.

J. T. Lewis

Second level large deviation results in statistical mechanics and applications

Consider a typical situation in statistical mechanics: $\{\Lambda_l : l=1,2,\ldots\}$ is a sequence of regions in $\mathbf{R}^d$, $V_l=\mathrm{vol}(\Lambda_l)$; associated with Λ_l is a countable set Ω_l, the configurations of the system in Λ_l; $H_l(\omega)$ is the energy of the configuration ω and $N_l(\omega)$ is the number of particles in ω. To each configuration ω we assign a probability $P^\mu{}_l[\omega]$ given by $P^\mu{}_l[\omega]=\exp(\beta\{N_l(\omega)\mu-H_l(\omega)\})/\exp(\beta V_l p_l(\mu))$; this is the grand canonical Gibbs measure in volume V_l and $p_l(\mu)$ is the grand canonical pressure; the canonical free energy $f_l(x)$ satisfies $\Sigma_{\{\omega:N_l(\omega)=n\}}\exp(-\beta H_l(\omega))= \exp(-\beta V_l f_l(n/V_l))$. The existence of the pressure in the thermodynamic limit ($p(\mu) =\lim_{l\to\infty}p_l(\mu)$), or of the free-energy ($f(x)=\lim_{l\to\infty}f_l(x)$), is a natural condition of weak dependence in statistical mechanics. Let $K^\mu{}_l$ be the distribution of the particle number density $X_l=N_l/V_l : K^\mu{}_l=P^\mu{}_l\circ X^{-1}{}_l$; then $\{K^\eta{}_l : l=1,2,\ldots\}$ satisfies the LDP with constants $\{V_l\}$ and rate-function $I^\mu{}_1(x)=p(\mu)+p^*(x)-\mu x$ whenever $p(\mu)$ exists and is differentiable and the range of p' is $[0,\infty)$. With weak conditions on the f_l,$\{K^\eta{}_l\}$ satisfies the LDP with rate-function $I^\mu{}_2(x)=f^*(\mu)+f(x)-\mu x$ whenever $f(x)$ exists and $\mu<\mu_\infty=\lim_{l\to\infty}\underline{\lim}_{x\to\infty}x^{-1}\inf\{f_k(x): k\geq l\}$.

Define the sequence spaces $\Omega^F=\{\omega : N\to\{0,1\} \mid \Sigma_{j\geq 1}\,\omega(j)<\infty\}$, $\Omega^B=\{\omega : N\to\{0,1, 2,\ldots\} \mid \Sigma_{j\geq 1}\,\omega(j)<\infty\}$; Ω^F describes the configurations of a system of free fermions, Ω^B of free bosons. Let $\sigma_j(\omega)=\omega(j)$ and $N_l=\Sigma_{j\geq 1}\,\sigma_j$, $H_l=\Sigma_{j\geq 1}\,\lambda_l(j)\sigma_j$, where $0=\lambda_l(1) \leq\lambda_l(2)\leq\ldots$. Suppose that (S1) $\phi_l(\beta)=\Sigma_{j\geq 1}\exp(-\beta\lambda_l(j))\to\phi(\beta)$ for all β in $(0,\infty)$ and

(S2) $\phi(\beta)\neq 0$ for some β in $(0,\infty)$; then $p(\mu)=\lim_{l\to\infty}p(\mu)$ for all μ (in the fermion case) and for $\mu<0$ in the boson case, and $\{K^{\eta}_{l}\}$ satisfies the LDP with rate-function $I^{\mu}_{1}=I^{\mu}_{2}$ in both cases. Mean-field models of interacting gases are treated using these results. To treat more realistic models, second-level results are required.

Let $L_l(\omega; A)=V_l^{-1}\Sigma_{j\geq 1}\sigma_j(\omega)\delta_{\lambda_l(j)}[A]$ be the density of particles with energy in the set A; then $L_l:\Omega\to M^+_b(\mathbb{R}^+)$, the positively bounded measures on $[0,\infty)$. Let $K^{\mu}_{l}\circ L^{-1}_{l}$; then $\{K^{\mu}_{l}\}$ satisfies the LDP with rate-function $I^{\mu}[m]$ given by $I^{\mu}[m]=p(\mu)+p^*[m]-\mu\|m\|$ where $p^*[m]=\int_{[0,\infty)}\lambda\, m(d\lambda)-\beta^{-1}s[m]$ and $s[m]$ is the entropy functional. Put $m(d\lambda)\ =\ m_s(d\lambda)+\rho(\lambda)dF(\lambda)$ where F is the integrated density of states $(\phi(\beta)=\int_{[0,\infty)}e^{\beta\lambda}\,dF(\lambda))$; then $s[m]=\int_{[0,\infty)}[(1+\rho(\lambda))\ln(1+\rho(\lambda))]\,dF(\lambda))\ -\int_{[0,\infty)}\rho(\lambda)\ln\rho(\lambda)\,dF(\lambda)$ in the boson case and $s[m]=-\int_{[0,\infty)}\{\rho(x)\ln\rho(x)-(1-\rho(x))\ln(1-\rho(\lambda))\}dF(\lambda)$ in the fermion case.

Anders Martin-Löf

Large deviations in risk theory

The object of the study is the "standard model" in risk theory: $U(t)=u+pt-S(t)$, where $U(t)$ is the surplus, $S(t)$ the total payments. $S(t)$ is a compound process defined by

$$E(e^{z\,S(t)})\ =\ \exp\Big(\lambda t\int_0^\infty (e^{zx}-1)F(dx)\Big)\equiv\exp(\lambda t g(z)).$$

It is required to derive LDE for $T=\min\{t: S(t)-pt>u\}$. The entropy function associated with $g(z)$ is the Legendre transform: $h(x)=\min(g(z)-zx)$. In terms of it we have the well known Chernoff bound $P(S(t)\geq x)\leq\exp(\lambda t h(x/\lambda t))$ if $x/\lambda t\geq\mu\equiv g'(0)$ and the famous Esscher-Cramér asymptotic formula $P(S(t)\geq x)\approx\exp(\lambda t h(x/\lambda t))/R(2\pi\lambda t g''(R))^{-1/2}$ for $x/\lambda t=g'(R)$ and $R\geq 0$. In terms of $h(x)$ we have an entropy functional for $S_\lambda(t)\equiv S(t)/\lambda$ in path space:

$$P(S_\lambda(s)\approx x(s),\,0\leq s\leq t)\ \approx\ \exp\Big(\lambda\int_0^t h(x'(s))\,ds\Big),$$

where $x(\cdot)$ is a continuous path with $x(0)=0$. In terms of it we ought to have the following LDE for the time of ruin T (putting $u=a\cdot\lambda$, $p=b\cdot\lambda$) as $\lambda\to\infty$:

$$P(T\leq t)\approx\exp\Big(\lambda\max_{x(.)\in R_t}\int_0^t h(x'(s))\,ds\Big)$$

where R_t is the set of paths, leading to ruin before t. This max problem has a simple solution and the entropy function for the random variable T is given by $H(\tau)\equiv\tau\cdot h(b+a/\tau)$. Like $h(.)$, $H(.)$ is a concave function, and $\max_\tau H(\tau)$ is attained at a value $\tau=\bar{T}$. $\bar{T}$ and $H(\bar{T})\equiv -aR$ are determined by the equations $g(R)/R=b$, $g'(R)=b+a/\bar{T}$. We have a situation similar to that in thermodynamics.

The state variables are determined in terms of the fundamental intensive parameter R analogous to the temperature. Ref. Anders Martin-Löf, Entropy, a useful concept in risk theory, Scand. Act. J. 1986, No. 3 - 4, p. 223 - 235.

A. Mogulskii

Large deviations and classes of asymptotically optimal tests

This talk deals with the problem of testing two compound hypotheses (parametric). The main statistical result is: In the general problem of testing two compound hypotheses, the likelihood ratio test remains to be an asymptotically optimal one in the sense, presented by partially Bayisian and partially minimax approaches. (Joint work with A. A. Borovkov.)

Péter Major

Large deviation results for Dyson's hierarchical model

In this talk I have discussed large deviations type results for a model in statistical physics. My main aim was to show how these results give an explanation for such phenomena as critical behaviour and universality in statistical physics.

Masar Nagasawa

The least action principle of Schrödinger processes

A Schrödinger process is a diffusion process satisfying

$$X_t = X_0 + B_t + \int_0^t \left\{ a(t, X_v) + \frac{\nabla\phi}{\phi}(v, X_v) \right\} dv$$

on (Ω, Q) with $\phi = e^{\alpha+\beta}$, where $\psi = e^{\alpha + i\beta}$ is a solution to the Schrödinger equation

$$i\frac{\partial\psi}{\partial t} + \frac{1}{2}\Delta\psi + i a(t,x)\cdot\nabla\psi - V(t,x)\psi = 0$$

with $\nabla\cdot a(t, x) = 0$. The process $(X_t, Q_{(s,x)})$ can be obtained by a transformation with

$$N_s^t = \exp\left(\int_s^t c(v, X_v)\, dv\right) \frac{\phi(t, X_t)}{\phi(s, X_s)} 1_{\{t < T_s\}}, \quad T_s = \inf\{v \geq s : \phi(v, X_v) = 0\}$$

of a space-time diffusion process $(X_t, P_{(s,x)})$ with the generator

$$L = \frac{\partial}{\partial s} + \frac{1}{2}\Delta + a\cdot\nabla.$$

c(t, x) is defined by

$$c(t,x) = -\frac{L\phi(t,x)}{\phi(t,x)}, \qquad (t,x)\in D = \{(t,x):\phi(t,x)>0\}.$$

Consider a class of semimartingales Y_t on (Ω, Q)

$$Y_t = Y_0 + B_t + \int_a^t \{a(v,Y_v)+b(v,.)\}dv, \quad Q\left[\int_a^b \|b\|^2 dv\right] < \infty$$

and its time reversal $\hat{Y}_t$ with $\hat{b}(v,.)$. Define an action functional

$$I(Y) = Q\left[\int_a^b \frac{1}{2}\left\{\frac{b^2+(\hat{b}*)^2}{2} - c\right\} dv + \beta(a,Y_a) - \beta(b,Y_b)\right].$$

Then the Schrödinger process attains the minimum $\min\{I(Y): Y\in H\} = I(X) = 0$.

Peter Ney

Large deviations and the composition of a branching population

Let $\{Z_{ij}^{(n)}(\Gamma): i,j=1,\dots,d; \Gamma\subset \mathbf{R}\}$ the number of particles of "type" j in the set Γ, at the n-th generation (time), given that there was one type i particle at the origin at time 0, in a population evolving according to a standard Galton-Watson multi (d) type branching random walk. Let $m_{ij}(\Gamma) = EZ_{ij}^{(n)}(\Gamma)$, $\hat{m}_{ij}(\alpha) = \int e^{\alpha s} m_{ij}(ds)$, $\lambda(\alpha)$ the maximal eigenvalue of $\hat{M}(\alpha) = \{\hat{m}_{ij}(\alpha): i,j=1,\dots,d\}$, and $r_i(\alpha)$, $l_i(\alpha)$ be the associated right and left eigenvectors. Write $Z_{ij}^{(n)}(-\infty, a) = Z_{ij}^{(n)}(a)$ etc. Let $\Lambda = \log\lambda$ and Λ^* the convex conjugate of Λ. Let $\rho = \lambda(0)$ and assume $\rho > 1$. Choose $\underline{a}$ so that $\Lambda^*(\underline{a}) < 0$. Assume that $\Lambda'(\alpha) = \underline{a}$ has a solution α_a. Then $Z_{ij}^{(n)}(a_n)/Z_{hk}^{(n)}(a_n)$ converges in probabitity to $l_j(\alpha_a)/l_k(\alpha_a)$, where $i,j,h,k\in\{1,\dots,d\}$. The asymptotics of the population vector $(Z_{i1}^{(n)}(an),\dots,Z_{id}^{(n)}(an))$ are also determined. These results are subject to suitable moment conditions on the particle production and motion.

Esa Nummelin

Renewal representations for Markov operators on general vector lattices

We look after conditions under which the iterates of a given Markov operator can be represented as a (possibly) delayed renewel sequence. The results generalize the existing similar representation results from the class of Harris recurrent Markov chains. As a new example we are able to deal with the class of so called Doeblin-Forster operators.

Stefano Olla

Large deviations from the hydrodynamic limit for the simple exclusion

We prove the hydrodynamical limit for weakly asymmetric simple exclusion processes. A large deviation property with respect to this limit is established for the symmetric case. We treat also the situation where a slow reaction (creation and annihilation of particles) is present.
(Joint work with C. Kipnis and S. R. S. Varadhan.)

Ross G. Pinsky

Large deviations for the exit time of conditional diffusions and a conditional gauge theorem

Using only a fundamental large deviation result of Donsker and Varadhan for conditional diffusions and a couple basic pde facts - namely Harnack's inequality and the Hopf maximum principle - we prove a large deviation theorem for the exit time of conditional diffusions. From this we derive a conditional gauge theorem.

Uwe Schmock

Convergence of the normalized one-dimensional Wiener sausage path measures to a mixture of Brownian taboo processes

Let $\Omega := \{\beta \in C([0,\infty), \mathbb{R}) : \beta(0) = 0\}$. For $T \in [0,\infty]$ let $\mathcal{M}_T$ be the σ-algebra on Ω generated by the paths up to time T. For every $T \in [0,\infty)$ denote by $C_T(\beta) := \{\beta(t) : t \in [0,T]\}$ the image of the path $\beta(t)$ for $t \in [0,T]$. $C_T(\beta)$ is a compact interval containing the origin. Its length is denoted by $|C_T(\beta)|$. Let v be a positive real parameter and denote by P the Wiener measure on $(\Omega, \mathcal{M}_\infty)$. A theorem of Donsker and Varadhan implies $\lim_{T\to\infty} T^{-1} \log E(\exp(-vT|C_T(\beta)|)) = -(3/2)(v\pi)^{2/3}$. The aim is to characterize the Brownian motion paths which determine this asymptotic behaviour of $E(\exp(-vT|C_T(\beta)|))$ as $T\to\infty$. For every $T \in [0,\infty)$ define the transformed measure P_T on Ω by $P_T(A) = E(1_A \exp(-vT|C_T|))/E(\exp(-vT|C_T|))$ for all $A \in \mathcal{M}_\infty$. These measures favour those Brownian motion paths which spread over a small interval up to time T without being too improbable (with respect to P).

Theorem: The measures $(P_T)_{T\in[0,\infty)}$ converge weakly to P_∞ as $T\to\infty$. P_∞ is given by

$$P_\infty(A) = \int_{(0,c_o)} \frac{\pi}{2c_o} \sin\frac{\pi b}{c_o}\; P_{(b-c_o,\,b)}(A)\; db$$

for all $A \in M_\infty$, where $c_0 := (\pi^2/v)^{1/3}$ and $P_{(b-c_0,\, b)}$ denotes the path measure of a Brownian taboo process (starting at zero) with taboo set $\{b-c_0, b\}$, i.e. the unique diffusion process on $(b-c_0, b)$ with generator of the form

$$\frac{1}{2}\frac{d^2}{dx^2} - \frac{\pi}{c_0}\,\mathrm{ctg}\,\frac{\pi(b-x)}{c_0}\,\frac{d}{dx}.$$

Herbert Spohn

An example for constrained dynamics

We consider a spin flip dynamics, σ_t, in one spatial dimension. The flip rates depend only of the nearest neighbors. The energy of the model is the nearest neighbor Ising, $H(\sigma) = -\Sigma_x \sigma(x)\sigma(x+1)$. The spin flip dynamics is reversible with respect to the Gibbs measure $Z^{-1}e^{-\beta H}$. We are interested in the spin-spin correlation function at the origin, $S(t) = E(\sigma_t(0)\sigma_0(0))$, average in the stationary process.

R. Holly proves that S(t) decays exponentially provided the flip rates are bounded away from zero. We study the case of only energy conserving flips, where some rates are zero. We prove $-c_- \le \liminf_{t\to\infty} t^{-1/2} \log S(t) \le \limsup_{t\to\infty} t^{-1/2} \log S(t) \le -c_+$. c_- is given through a complicated variational problem.

The model is an example for the Kohlrausch-Williams-Watts law.

Josef Steinebach

On convergence rates in Erdös-Rényi type laws based upon large deviation asymptotics

Erdös-Rényi type laws of large numbers deal with an a.s. limiting behaviour of maximum increments of random sequences of stochastic processes such as partial sums, sample quantiles, renewal processes etc. Some rather general versions have been provided by S. Csorgö (1979) and Steinebach (1981) making use of first order large deviation asymptotics together with certain independence and stationarity properties of the underlying processes. In view of recent second order large deviation asymptotics due to Dersch (1986), corresponding refinements can be obtained. The latter results provide convergence rate improvements of their earlier counterparts, typically giving the best rates, but not necessarily the best constants. Some specific examples are discussed.

S. L. Zabell

Large deviations for exchangeable random variables

Let $X_1, X_2, \ldots, X_n, \ldots$ be an infinite exchangeable sequence of 0-1 valued random variables. If $\lambda(p, x)$ denotes the rate function for i. i. d. Bernoulli random variables with success probability p, then $X_1, X_2, \ldots$ satisfies the large deviation property with rate function $\lambda(x) = \inf\{\lambda(p, x) : p \in S\}$, where S is the support of the mixing measure in the de Finetti representation for the sequence. For example, if the mixing measure assigns mass $\frac{1}{2}$ to $\frac{1}{3} = p_1$ and mass $\frac{1}{2}$ to $\frac{2}{3} = p_2$, then the rate function is not convex. This result follows from a more general result for upper and lower bounds for Banach space valued exchangeable random variables. These more general results include mixtures of other families of random variables such as the multivariate normal. These results are joint work with Ian Dikwoodie.

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