MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 37/1988

Gruppentheorie (auflösbare Gruppen)

21.8. bis 27.8.1988

Die Tagung fand unter der Leitung der Herren T.O.Hawkes (Coventry) und O.H.Kegel (Freiburg) statt. Es wurden 32 Vorträge gehalten, wobei die Themenliste recht breit gefächert war. Schwerpunkte bildeten die Darstellungstheorie und die Theorie der Fittingklassen und Injektoren in endlichen, auflösbaren Gruppen. Ebenso wurden verbandstheoretische Charakterisierungen auflösbarer Gruppen sowie Bedingungen an die Subnormalteiler vorgestellt. Aber es gab auch Vorträge über unendliche (auflösbare und nicht-auflösbare) Gruppen. Großes Interesse fand die von Herrn P.Schmid vorgenommene Analyse des Beweises des wohlbekannten Satzes von Šafarevič, der gegenwärtig unvollständig zu sein scheint. Leider war es den eingeladenen Kollegen aus der Sowjetunion wiederum nicht möglich, an der Tagung teilzunehmen.

Vortragsauszüge

Z. Arad:

An application of Ramsey theory to partitions of groups

We prove a version of Schur's theorem on 2-colourings of $\mathbb{Z}$ for arbitrary groups (joint work with G.Ehrlich, J.Lennox and O.H.Kegel).

Theorem. For any n-colouring of a group G there exists a monochrome solution to the equation $xy = z$, where x,y,z are distinct elements $\neq 1$, if one of the following holds: (a) G is infinite; (b) G is finite of order $> R(2,8,(n^2+n)/2) + 1$; (c) $n=2$ and $|G| > 7$, but G not elementary-abelian of order 9; (d) $n=3$, and $|G| = 17$ or > 18; (e) $n=4$, and G cyclic of order 46,47 or > 48.

The numbers $R(a,b,c)$ appearing here are Ramsey numbers and turn out to be extremely large - in fact they have not been computed yet. The bounds on group order appearing in (b),(c),(d) are smaller if we do not require x,y,z to be distinct. Indeed, if $f(t)$ denotes the largest n such that $\{1,\dots,n\}$ can be t-coloured without a monochrome solution to $x + y = z$, then $f(1) = 1$, $f(2) = 4$, $f(3) = 13$, $f(4) = 44$. The evaluation of $f(5)$ appears to be a difficult combinatorial problem. If $f^*(t)$ is defined in the same way as $f(t)$ except that we insist that there is no monochrome solution to $x+y=z$ for distinct x,y,z, then $f^*(1)=2$, $f^*(2)=8$, $f^*(3)=23$ and $f^*(4)=66$.

J.C.Beidleman:

Lockett classes of $\underline{\underline{S}}_1$-groups

Let $\underline{\underline{S}}_1$ be the class of all groups with a finite normal series in which the factors are abelian groups of finite rank whose torsion subgroups are Černikov groups. Trying to extend the well-known theorem of Fischer-Gaschütz-Hartley about injectors of Fitting classes in finite soluble groups we can show the existence and conjugacy of injectors of Fitting classes in $\underline{\underline{S}}_1$-groups in certain cases, but there do also exist counterexamples. Moreover, we can generalize Lockett's *-construction to Fitting classes of $\underline{\underline{S}}_1$-groups. Some of his results carry over to $\underline{\underline{S}}_1$-groups, and some informations about Lockett classes of $\underline{\underline{S}}_1$-groups can be obtained. (joint work with M.J.Karbe and M.J.Tomkinson).

R.Brandl:

CLT-groups and character degrees

A p-CLT group in the sense of J.F.Humphreys and D.L.Johnson is a finite group G such that for every divisor p^a of $|G|$ there exists a subgroup of index p^a in G. Natural examples are permutation groups of degree p. Theorem. Let G be a p-CLT group and assume $O_{p'}(G)=1$. Then the wreath product $P \operatorname{wr} G$ is a p-CLT group for every p-group P. For the proof, some results concerning the distribution of the irreducible character degrees of G are established. (joint work with P.A. Linnell).

B.Brewster:

Monomial groups and related classes

It is an open problem to find a group theoretic characterization of the class of monomial groups. One can ask what abstract properties such a class needs to have, and whether some closely related class can be so characterized. Many such results exist in the literature and various collections of notes. Some new twists have been studied by me and my doctoral student G.Yeh.

R.M.Bryant:

Automorphisms of free metabelian groups

Theorem 1. If E is a free group of infinite rank and V(E) is a verbal subgroup of E such that E/V(E) is nilpotent, then every automorphism of E/V(E) is induced by an automorphism of E. (joint work with O.Macedońska). Theorem 2. If E is a free group of infinite rank then every automorphism of the free metabelian group E/E" is induced by an automorphism of E. (joint work with J.R.J.Groves).

C.Casolo:

Groups with restrictions on subnormal subgroups

We consider classes of soluble groups satisfying the following conditions: - every subnormal subgroup of G has a finite (boundedly finite) number of conjugates in G; - every subnormal subgroup of G has finite (boundedly finite) index in its normal closure.
Some structural properties of these groups are given. The main result is: If in the soluble group G every subnormal subgroup has at most n conjugates, then the Wielandt subgroup of G (that is, the intersection of the normalizers of all subnormal subgroups of G) has finite bounded index.

J.Cossey:

The Wielandt subgroup of soluble groups

The Wielandt subgroup of a group is the intersection of the normalisers of all subnormal subgroups. There are three basic questions about the Wielandt subgroup of a group: its structure, how it is embedded in the group, and its influence on the structure of the group. I did discuss each of these questions briefly for polycyclic groups.

A.Espuelas:

An extension of a conjecture of J.G.Thompson

The following conjecture was made by the speaker at Oberwolfach in May 1987, extending an old conjecture of J.G.Thompson: Let A be a group of order p^n acting fixed-point-freely on every A-invariant p'-section of the p-solvable group G. Then $l_p(G) \leqq n+1$. The bound is best possible. Definition. Let A be a finite group acting on the finite group G. Then G is said to be of p-splitting type w.r.t. A, if the following holds: (a) $G = P_1 .. P_n$ where P_{i+1} is $AP_1 .. P_i$-invariant for $i = 0,..,n-1$. (b) P_i is either a p-group or a p'-group and $(|P_i|,|P_{i+1}|) = 1$ for $i = 1,..,n-1$. (c) P_i acts faithfully on P_{i+1} for $i = 1,..,n-1$. (d) $AP_1 .. P_i$ normalizes a Hall system of P_{i+1} for $i = 0,..,n-1$. Theorem. Let A be a group of order p^n acting f.p.f. on every A-invariant p'-section of the p-solvable group G. Assume that G is of p-splitting type w.r.t. A, and $p \geq 5$. If $2 \mid |G|$ and p is a Mersenne prime suppose, in addition, that A is $C_p \wr C_p$-free. Then $l_p(G) \leq n+1$.

R.Gow:

Characters of soluble groups induced by characters of degree 2

Let G be soluble, let S be a Sylow 2-subgroup of G and let $T \triangleleft S$ with S/T dihedral, semi-dihedral or generalized quaternion. Let θ be an irreducible faithful character of S/T (with $\theta(1) = 2$). Assume that S/T is not dihedral of order 8, generalized quaternion of order 8 or 16, semi-dihedral of order 16. Then θ^G contains a uniquely described character χ with $(\theta^G,\chi) = 1$ and $(\chi_M, 1_M) = 2$, where M is a Sylow 2-complement of G. We have $\chi(1) = 2m$, m odd, and χ is monomial.

P.Hauck:

Injector-like subgroups in finite soluble groups

A Fitting functor f assigns to each finite soluble group G a non-empty set f(G) of subgroups such that $f(\tilde{G}) = \{\alpha(X) \mid X \in f(G)\}$ for any isomorphism $\alpha: G \longrightarrow \tilde{G}$, and $f(N) = \{X \cap N \mid X \in f(G)\}$ for all $N \trianglelefteq G$. The most prominent examples of conjugate Fitting functors are provided by injectors for Fitting classes. In this talk we report on some of the main results about Fitting functors f having the property that, for any group G, all members of f(G) satisfy the Frattini argument or, at least, have the cover-avoidance property w.r.t. any chief series of G. We finally discuss Fitting functors with certain additional "closure"-properties with particular emphasis on Fischer functors.

H.Heineken:

Hypernormalizing groups

A group is called hypernormalizing, if the normalizer of a subnormal subgroup is always itself subnormal. A.Camina considered this class of finite groups first in 1967. He showed that soluble such groups have p-length one for each prime p and that there are groups of nilpotent length three. Further analysis now shows: Soluble finite hypernormalizing groups are contained in the class $\underline{\underline{NNR}} \cap \underline{\underline{NN}}_2\underline{\underline{N}} \cap \underline{\underline{NANA}} \cap \underline{\underline{ANAN}}$ where $\underline{\underline{N}}$ denotes the class of nilpotent, $\underline{\underline{N}}_2$ of nilpotent class two, $\underline{\underline{R}}$ of nilpotent of squarefree exponent, $\underline{\underline{A}}$ of abelian groups.

M.Herzog:
<u>Class lengths in finite groups</u>

Let Con(G) be the set of all conjugacy classes of a finite group G. Our purpose is to study the following question: How do arithmetic restrictions on $\{|C|\ ;\ C \in \mathrm{Con}(G)\}$ influence the structure of G ? The restrictions we impose are analogous to the ones used in the recent studies of the character degrees of G. Some of the results are: (1) If $|C|$ is a prime power for every $C \in \mathrm{Con}(G)$, then there are two primes p,q such that $G = P \cdot O_q(G) \times A$, where $P \in \mathrm{Syl}_p(G)$ and A is a Hall $\{p,q\}'$-subgroup. In particular, G is solvable. (2) If $|C|$ is squarefree for every $C \in \mathrm{Con}(G)$, then G is supersolvable with derived length ≤ 3. Also, $|\mathrm{Fit}(G)'|$ is squarefree, and $G/\mathrm{Fit}(G)$ is cyclic of squarefree order. (3) In general, the derived length and nilpotency length of a finite solvable group G is bounded in terms of the highest exponent of a prime power occuring in $\{|C|\ ;\ C \in \mathrm{Con}(G)\}$. Explicit bounds are given. We also consider arithmetic conditions on the set $\{|G|/\chi(1)\ ;\ \chi$ is an irreducible character of $G\}$ and on the character degrees in the principal p-block of G. (joint work with D.Chillag).

B.Huppert:
<u>A characterization of $SL_2(5)$ and $GL_2(3)$ by the degrees of their representations</u>

Seien 1,2,..,k die Grade der irreduziblen Darstellungen von G (beliebige Vielfachheiten). Dann gilt: (a) $k \leq 6$ und $k \neq 5$.
(b) Ist $k = 5$, so $G = SL(2,5) \cdot Z(G)$. (c) Ist $k = 4$, so $G/N \cong S_4$. Ist $M \triangleleft G$ und $A \triangleleft G$ mit $|M/N| = 4$, $|G/A| = 2$, so gilt $N \leq Z(A)$, $|M'| = 2$ und $N/M' \leq Z(G/M')$.

Gefördert durch DFG Deutsche Forschungsgemeinschaft

M.-J.Iranzo:

The p^*p-injectors of a finite group

H.Bender in "On the normal p-structure of a finite group and related topics", Hokkaido Math. J. 7 (1978), 271-288 gave the following Definition. Given a fixed prime p, a group G is a p^*p-group if $G = N \cdot C_G^*(N_p)$ for every $N \trianglelefteq G$, where N_p denotes a Sylow p-subgroup of N and $C_G^*(N_p)$ is the largest normal subgroup of $N_G(N_p)$ nilpotent on N_p. It is our aim to give a description of the p^*p-injectors of a finite group G. These injectors are the subgroups of the conjugacy class $O_{p^*}(G) \cdot T$ where T describes the set of all Sylow p-subgroups of $C^*_{O_{p^*}(G)P}(L)$ where P describes the set of such subgroups of G and L those of $O_{p^*}(G)$.

L.-C.Kappe:

Graphs associated with commutator words and a problem of Erdös and Neumann

We consider various generalizations arising from characterizations of central-by-finite groups. Starting from Baer's characterization of central-by-finite groups as those groups which are a finite union of abelian subgroups we obtain Theorem 1. Let G be a group and L(G) the characteristic subgroup of right 2-Engel elements. Then G is the finite union of 2-Engel subgroups iff G/L(G) is finite. Obviously, $G/Z_2(G)$ finite implies that G is the finite union of class 2 subgroups. However, the converse is not true.
Let the elements of a group G be the vertices of a graph $\Gamma_k(G)$, k a nonnegative integer, defined as follows: Join a and b in $\Gamma_k(G)$ by an edge if $[a,b] \notin Z_k(G)$. The following theorem generalizes characterizations of central-by-finite groups due to B.H.Neumann (k=0).
Theorem 2. Every complete subgraph of $\Gamma_k(G)$ is finite iff $G/Z_{k+1}(G)$ is finite. Various other generalizations are explored arising from other characterizations of central-by-finite groups.

P.H.Kropholler:

The Goldie rank problem for group rings of soluble groups

Using a K-theoretic result of J.Moody, it is easy to establish the zero divisor conjecture for group rings of soluble groups. With only a little more work one can solve a form of the Goldie rank problem and show that, for G soluble and k a field, kG is a prime Goldie ring if and only if $\Delta^{+}(G) = 1$ and $\{|F| ; F$ a finite subgroup of $G\}$ is bounded. When these conditions hold kG has Goldie rank = $= \text{l.c.m.}\{|F| ; F$ a finite subgroup of $G\}$. (joint work with P.Linnell and J.Moody).

H.Laue:

Neatly embedded direct decompositions of normal subgroups

Let $H \trianglelefteq G$ such that both chain conditions on G-invariant subgroups of H hold. Given a G-direct decomposition $\mathfrak{D}$ of H into G-indecomposable factors, let $\mathfrak{D}^*$ be the set of all $M \in \mathfrak{D}$ which have a normally intersecting complement S in G (i.e., $S \cap H \trianglelefteq G$). $\mathfrak{D}$ is called <u>neatly embedded</u> in G if even $\text{Rad}\,\mathfrak{D} = \prod\{M \mid M \in \mathfrak{D}^*\}$ has such a complement. Then $\text{Rad}\,\mathfrak{D}$ is maximal among all G-invariant subgroups of H with normally intersecting complement. <u>Theorem</u>. (Generalized Krull-Schmidt th.) Let $\mathfrak{D}, \mathfrak{E}$ be n.e., S a normally intersecting complement of $\text{Rad}\,\mathfrak{D}$ in G. Then there exists $\alpha \in C_{\text{Aut}(G)}(G/Z(H), S)$ such that $\mathfrak{D}^{*\alpha} = \mathfrak{E}^*$. In particular, $\text{Rad}\,\mathfrak{D}$ and $\text{Rad}\,\mathfrak{E}$ are G-isomorphic and have the same normally intersecting complements in G.
We describe in general $C_{\text{Aut}(G)}(G/A)$ where A is normal and abelian, and we study the action of a nilpotent normal subgroup N of $C_{\text{Aut}(G)}(G/Z(H))$ on the set of sets $\mathfrak{D}^*$. We thus prove a refinement of the above transitivity theorem. This enables us to derive a rather general theorem on the invariance of certain complemented G-invariant subgroups of H under the action of a finite operator group. A typical special case is <u>Corollary</u>. Let G be finite, H abelian, $W \trianglelefteq X \leq$ $\leq N_{\text{Aut}(G)}(H)$, $(|X/W|, |H|) = 1$. Then among all maximal W-invariant normal subgroups of G which are contained in H and have a W-invariant complement in G there is one which is X-invariant and has an X-invariant complement in G. (The results were presented in a more general form, considering Ω-subgroups where Ω is a certain set of endomorphisms.)

F.Leinen:

Countable closed LFC-groups with p-torsion

Let LFC be the class of all locally FC-groups. F.Haug has studied the existentially closed (e.c.) LFC-groups extensively. Some of his re-

sults are as follows: (1) Let G be a countable e.c. LFC-group. Then $G' = T(G)$ (torsion subgroup), and G splits over $T(G)$. Moreover, $T(G)$ is P.Hall's countable universal locally finite group ULF, and $G/T(G)$ is divisible. (2) For every $\rho \leq \omega$ there exists precisely one countable e.c. LFC-group G_ρ with $r_o(G_\rho/T(G_\rho)) = \rho$.
With different methods we are able to illuminate the situation in the class LFC_p of all LFC-groups whose torsion subgroup is a p-group. There exist $2^{\aleph_0}$ countable e.c. LFC_p-groups H. All these groups are even closed in LFC_p. (In particular, $r_o(H/T(H)) = \omega$.) Furthermore, (1) holds correspondingly with a locally finite p-group T_p in place of ULF, where T_p has similar properties as the countable e.c. locally finite p-group E_p. And every H acts via conjugation on its torsion chief factors as $K^\times$ acts regularly on K^+, where K denotes the algebraic closure of $GF(p)$.

O.Manz:

Special module actions of solvable groups

In the talk, the following theorem was discussed.
Theorem. (T.Wolf, O.M.) Let G be a finite solvable group, q a prime and $G = O^{q'}(G) \neq 1$. Suppose that G acts faithfully and irreducibly on a finite vector space V such that $q \nmid |G:C_G(v)|$ for all $v \in V$.
(a) If V_N is homogeneous for all N char G, then $O^q(G)$ is either cyclic, quaternion of order 8 or extra-special of order 27 and exponent 3. (In the second and third case, $q = 3$ resp. 2.)
(b) If $C \triangleleft G$ is maximal such that $V_C = V_1 \oplus \ldots \oplus V_n$ is non-homogeneous with homogeneous components V_i, then either G/C is dihedral of order 6 or 10 (where $q=2$), or G/C is the affine semilinear group over $GF(8)$ (where $q=3$). Moreover, $C/C_C(V_i)$ acts transitively on $V_i^\#$; in particular, C is metabelian (up to some exceptions).
As a consequence, we obtain a modular analogue of Ito's Theorem on character degrees.

P.P.Pálfy:

Bounds for permutation groups of odd order

T.R.Wolf and I published the following result independently in 1982. If $G \leq S_n$ is a solvable primitive permutation group then $|G| \leq$ $\leq 24^{-1/3}n^{1+c}$, where $1+c = 1 + \log(48 \cdot 24^{1/3})/\log 9 = 3.24\ldots$.

This, in fact, is equivalent to the bound $|G| \leqq 24^{-1/3}|V|^c$ for any solvable irreducible linear group $G \leqq GL(V)$, where V is a finite vector space. For irreducible linear groups of odd order we get $|G| \leqq 3^{-1/2}|V|^{c'}$ with $c' = 1 + \log((21\sqrt{3}/8)^{1/3})/\log 2 = 1.72...$, which improves the bound found by Wolf. For primitive permutation groups $G \leqq S_n$ of odd order the bound $|G| \leqq 3^{-1/2}n^{1+c''}$ holds with $1+c'' = 2 + \log((13\sqrt{3}/9)^{1/3})/\log 3 = 2.27...$. For arbitrary permutation groups $G \leqq S_n$ of odd order the exponential bound $|G| \leqq 3^{(n-1)/2}$ follows quite easily.

J.Pense:
Outer Fitting pairs

A Fitting class is known as a nonempty class of finite groups which is closed under isomorphic copies, normal subgroups and normal products; similarly, a Fitting set is a nonempty set of finite subgroups of a fixed group which is closed under conjugates, normal subgroups and normal products. I define an outer Fitting pair (d,A) to be a family of homomorphisms $d_G: G \longrightarrow A$ (where G runs over all finite groups and A is one fixed group) s.t. for every normal embedding $\nu: N \longrightarrow G$ there is an inner automorphism α_ν of A s.t. $d_G \nu = \alpha_\nu d_N$. Theorem 1 says that the preimage of a Fitting set under an outer Fitting pair is a Fitting class. Take a finite simple group J and let $A_J = \lim_n(\mathrm{Aut}(J^n))$. ($A_J$ is the stable linear group GL(p) if $J = Z_p$.) For each finite group G select a chief series H and let D_G^J be the direct product of the J-chief factors from H. From the obvious action of G on D_G^J we get a homomorphism $d_G^J: G \longrightarrow A_J$. Theorem 2 states that (d^J, A_J) is an outer Fitting pair. If $\underline{\underline{F}}$ is a Fitting class, the Fitting class $(d^J)^{-1}(\{G \leqq A_J \mid G \in \underline{\underline{F}}\})$ is a generalization of the locally defined Fitting formations. If $N \trianglelefteq GL(p)$, then $(d^{Z_p})^{-1}(\{G \leqq N \mid G \text{ finite}\})$ is a Fitting class originally introduced by Blessenohl and Gaschütz (1970). Theorem 3 says that a Fitting set of q-groups (q a prime $\neq p$) in GL(p) essentially consists of all q-subgroups of GL(p). Theorem 4 computes Fitting sets of GL(p) generated by "transitive" subgroups of GL(n,p); from this one gets a unification and clarification of the Fitting class constructions of Dark (1972) and McCann (1987).

Gefördert durch DFG Deutsche Forschungsgemeinschaft

F.Pérez Monasor:

Existence and conjugacy of $\underline{\underline{F}}$-injectors in finite groups

Let $\underline{\underline{Y}}, \underline{\underline{F}}$ be Fitting classes, $\underline{\underline{Y}} \subseteq \underline{\underline{S}}$. Let $\overline{Q}_1, \ldots, \overline{Q}_r$ be the components of $\overline{G} = G/G_{\underline{\underline{Y}}}$ and $M/G_{\underline{\underline{Y}}} = \overline{Q}_1 \ldots \overline{Q}_r$. We denote $L(G) = M^{(\infty)}$ and $E_i = Q_i^{(\infty)}$. The subgroups E_i are called the $\underline{\underline{Y}}$-components of G and $L(G) = E_1 \ldots E_r$. The $\underline{\underline{YF}}$-injectors I_i of E_i are its $\underline{\underline{YF}}$-maximal subgroups. Set $I = I_1 \ldots I_r$, then we have that the $\underline{\underline{YF}}$-injectors of $N_G(I)$ are $\underline{\underline{YF}}$-injectors of G. (Note that in general I is not an $\underline{\underline{YF}}$-injector of L(G).) As a consequence we obtain Theorem. Assume that char $\underline{\underline{F}} = \mathbb{P}$. Then if all $\underline{\underline{F}}$-constrained groups have $\underline{\underline{F}}$-injectors, all groups have $\underline{\underline{F}}$-injectors. With respect to the conjugacy of $\underline{\underline{F}}$-injectors, when char $\underline{\underline{F}} = \sup \underline{\underline{F}}$ we have Theorem. (a) If a group G has a unique conjugacy class of $\underline{\underline{F}}$-injectors, then G is $\underline{\underline{F}}$-constrained. (b) If all finite groups have a unique conjugacy class of $\underline{\underline{F}}$-injectors, then either $\underline{\underline{F}} = \{1\}$ or $\underline{\underline{F}} = \underline{\underline{S}}_p$ or $\underline{\underline{N}}^* \subseteq \underline{\underline{F}}$.

R.E.Phillips:

Groups of finitary transformations

A group G of linear transformations on a $\mathbb{K}$-space V is finitary if for each $g \in G$, $[V,g] = \{v(g-1) \mid g \in G\}$ has finite dimension. Considerable progress on the structure of such groups has been made and several interesting questions remain. In particular, if $\mathrm{char}(\mathbb{K}) = 0$, the structure of G is known and if $\mathbb{K}$ is finite, the structure is nearly known. Finitary groups over infinite fields of prime characteristic are currently being studied.

G.R.Robinson:

Integral permutation modules

We use the poset of non-trivial solvable subgroups of a finite group G to prove: Theorem. Let G be a finite insoluble group, m,n be distinct non-negative integers. Then there are finite G-sets Ω_1, Ω_2 with $|\Omega_1^G| = m$, $|\Omega_2^G| = n$, such that $\mathbb{Z}_{(p)}\Omega_1 \cong \mathbb{Z}_{(p)}\Omega_2$ for all primes $p \in \mathbb{Z}$.

P.Rowley:

(S_3, S_6)-amalgams

An (S_3, S_6)-amalgam in a group G is a pair of finite subgroups P_1, P_2

of G which satisfy: (i) $G = \langle P_1, P_2 \rangle$, $\mathrm{core}_G\, P_1 \cap P_2 = 1$; (ii) $P_1/O_2(P_1) \cong S_3$ and $P_2/O_2(P_2) \cong S_6$; (iii) $P_1 \cap P_2 \in \mathrm{Syl}_2\, P_i$ for i=1,2; (iv) $C_{P_i}(O_2(P_i)) \leqq O_2(P_i)$ for i=1,2.

By reinterpreting (S_3,S_6)-amalgams in terms of a certain group acting upon an infinite tree the structures of P_1 and P_2 may be studied. In this talk some of the arguments which may be carried out using the tree were presented.

E. Salomon:

Structure-preserving subgroups and Schunck classes of finite groups

We introduce two subgroup functors for arbitrary finite groups which in soluble groups coincide with the functors assigning to a group its subgroups $D_1(G) = \{H \mid S \trianglelefteq T \leqq G,\ T/S \text{ semisimple} \Rightarrow (T \cap H)S \trianglelefteq T\}$ resp. $D_2(G) = \{H \mid S \trianglelefteq T \trianglelefteq\trianglelefteq G,\ T/S \text{ semisimple} \Rightarrow (T \cap H)S \trianglelefteq T\}$. We show various inheritage properties of these functors. Then we introduce D_i-primitive groups, which are special cases of the ordinary primitive groups and have properties resembling those of soluble primitive groups. This is used to define D_i-Schunck classes. (A class $\underline{\underline{X}}$ is a D_i-Schunck class iff: (i) $\underline{\underline{X}}$ is closed under epimorphisms; (ii) if $G/N \in \underline{\underline{X}}$ for all D_i-primitive factor groups G/N of G, then $G \in \underline{\underline{X}}$.) Also D_i-$\underline{\underline{X}}$-projectors are introduced, and these generally differ from usual projectors of the same classes. We show the Theorem. $\underline{\underline{H}}$ is a D_i-Schunck class iff in every finite group there exist D_i-$\underline{\underline{H}}$-projectors.

P. Schmid:

Free elementary p-groups and the inverse problem of Galois theory

Let p be a prime. For any group F define inductively $\lambda_1(F) = F$ and $\lambda_{i+1}(F) = [\lambda_i(F), F] \cdot \lambda_i(F)^p$. One can associate to this (descending) p-central series a graded Lie algebra over $\mathbb{F}_p$. If F is the free group of rank d, $P(c,d) = F/\lambda_{c+1}(F)$ is the free elementary p-group of rank d and class c. In fact, $P = P(c,d)$ has centre $\lambda_c(P)$ and for any $x \in P - \lambda_2(P)$, $o(x) = p^c$ and $C_P(x) = \langle \lambda_c(P), x \rangle$. The (module) structure of $\lambda_c(P)$ can be described quite explicitly (in terms of $P/\lambda_2(P)$), by virtue of results by Skopin, Lazard, and Bryant-Kovács. A corresponding statement made by H.Koch is false (for p=2). This is troublesome in that the work of Šafarevič (and Išhanov) on the inverse problem of Galois theory relies on such arguments. So at the moment it seems to be not clear whether the theorem of Šafarevič is valid for solvable groups of even order.

R.Schmidt:

Lattice theoretical characterizations of classes of infinite soluble and locally nilpotent groups

We present a number of lattice theoretical characterizations of classes of infinite groups. For example, we discuss the following Theorem. The group G is polycyclic if and only if there exists a chain $1 = M_0 \leqq M_1 \leqq \dots \leqq M_n = G$ of subgroups M_i of G such that M_i is permodular in G and $[M_{i+1}/M_i]$ is distributive and satisfies the maximal condition for all $i = 0,\dots,n-1$. Here, a subgroup M of a group G is called permodular in G if it is modular in G (that is if $(U \cup M) \cap V = U \cup (M \cap V)$ for all $U,V \leqq G$ such that $U \leqq V$ and $(U \cup M) \cap V = M \cup (U \cap V)$ for all $U,V \leqq G$ such that $M \leqq V$) and satisfies that for all $g \in G$ and $\langle M,g\rangle \geqq Y \geqq M$ the finiteness of $[\langle M,g\rangle/Y]$ implies the finiteness of $|\langle M,g\rangle : Y|$. (Note that by results of Zacher this is a lattice property.) Similar characterizations were given for the classes of hyperabelian, finitely generated soluble, torsionfree nilpotent, and locally nilpotent non-periodic groups.

R.Stöhr:

Gruppentheoretische Interpretation von höheren Homologiegruppen

Wir betrachten die ganzzahligen Homologiegruppen H_nG $(n \geqq 1)$ einer abstrakten Gruppe G und fragen nach einer rein gruppentheoretischen Beschreibung dieser homologischen Invarianten. Für $n=1$ ist die Antwort einfach: $H_1(G)$ ist die abelsch gemachte Gruppe G. Gruppentheoretische Interpretationen der 2. Homologiegruppe werden durch die Hopfsche Formel (H.Hopf, 1941/42) und den Schurschen Multiplikator (I.Schur, 1904) geliefert. In letzter Zeit wurden nun von mehreren Autoren unabhängig voneinander verschiedene Interpretationen der H_nG für höhere Dimensionen angegeben (B.Conrad, 1985; A.Rodicio, 1986; R.Brown und G.Ellis, 1987; sowie vom Verf., 1987). Das Ergebnis des Verf. ist (wie übrigens auch die Resultate von Rodicio und Brown/Ellis) eine Verallgemeinerung der Hopfschen Formel, d.h. H_nG wird, ausgehend von einer freien Präsentation von G, als Subquotient einer freien Gruppe dargestellt.

A.Turull:

Groups of automorphisms and centralizers

Let G be a finite solvable group and A a group of automorphisms of G such that $(|A|,|G|)=1$. We denote by $h(G)$ the Fitting height of G and by $l(A)$ the length of the longest chain of subgroups of A. Then under certain additional hypotheses, it is known that $h(G) \leqq 2 \cdot l(A) + h(C_G(A))$ and that, when $C_G(A)=1$, $h(G) \leqq l(A)$, both results being best possible. I attempted to explain the difference in coefficient in the two results (from 2 to 1) by presenting some results which generalize the $C_G(A)=1$ case into the $C_G(A) \neq 1$ case but keeping the coefficient 1. The key is that although examples with $h(G) = 2 \cdot l(A) + h(C_G(A))$ exist for all solvable A and all $h(C_G(A)) > 0$, they necessarily involve large $C_G(A)$. When $h(C_G(A))=0$ then $C_G(A)$ is necessarily small, but this does not happen for any other value of $h(C_G(A))$. We show, for example, that for $C_G(A)$ small compared to A, $h(G)/l(A)$ is at most $1+\varepsilon$.

R.W.van der Waall:

A characterization of the groups $\mathbb{Z}/2\mathbb{Z}$ and S_3

We give a proof of the following Theorem. Let A be the set of all those finite groups G, in which elements of equal order are conjugate. Then $A = \{\{1\}, C_2, S_3\}$. The proof uses the classification of the finite simple groups. (There are earlier proofs known: Fitzpatrick ±1984, not complete; later restored.)

J.S.Wilson:

Products of infinite soluble groups

Let $G = HK$, where H,K are minimax groups and G is soluble: then G too is a minimax group. The proof of this result was discussed briefly and some related results were described.

G.Zacher:

Complete lattice homomorphisms in non-periodic groups

Given a group G and a complete lattice $\mathcal{L}$, a mapping σ of the subgroup lattice $\ell(G)$ of G into $\mathcal{L}$ is called a complete lattice-homomorphism if for every subset $\mathcal{F}$ of $\ell(G)$ we have

$$(*) \qquad (\bigwedge_{X\in\mathcal{F}} X)^{\sigma} = \bigwedge_{X\in\mathcal{F}} X^{\sigma} \quad \text{and} \quad (\bigvee_{X\in\mathcal{F}} X)^{\sigma} = \bigvee_{X\in\mathcal{F}} X^{\sigma}.$$

If (*) holds for finite subsets $\mathcal{F}$ we speak of a lattice-homomorphism. A lattice-isomorphism σ of a group G onto a group $\bar{G}$ is called index-preserving if for every $H \leq K \leq G$ with $|K:H| < \infty$ we have $|K:H| = |K^{\sigma}:H^{\sigma}|$. We give some general conditions on a non-periodic group $\bar{G}$ such that: (1) every non-trivial complete lattice-homomorphism σ of G in a complete lattice $\mathcal{L}$ is necessarily injective; (2) every complete lattice-epimorphism of a group G onto $\bar{G}$ is necessarily an index-preserving lattice-isomorphism. As a corollary we get: Let $G \neq 1$ be locally a non-periodic group with an ascending normal series whose factors are either locally finite or abelian and $1 \neq H \lneq G$. Then the map defined by $X \longmapsto X \wedge H$ is a lattice-epimorphism (of G onto H) iff G is locally cyclic. (joint work with S. Stonehewer).

Berichterstatter: F.Leinen

Tagungsteilnehmer

Prof. Dr. Z. Arad
Dept. of Mathematics
Bar-Ilan University

52 100 Ramat-Gan
ISRAEL

Dr. R.M. Bryant
Dept. of Mathematics
UMIST (University of Manchester
Institute of Science a. Technology)
P. O. Box 88

GB- Manchester , M60 1QD

Prof. Dr. J.C. Beidleman
Dept. of Mathematics
University of Kentucky
715 POT

Lexington , KY 40506-0027
USA

Prof. Dr. C. Casolo
Dipartimento di Matematica e
Informatica
Universita degli Studi di Udine
Via Zanon, 6

I-33 100 Udine

Prof. N. Blackburn
Dept. of Mathematics
University of Manchester
Institute of Science and Technology

GB- Manchester M13 9PL

Prof. Dr. K. N. Cheng
Department of Mathematics
National University of Singapore
Kent Ridge

Singapore 0511
SINGAPORE

Dr. R. Brandl
Mathematisches Institut
der Universität Würzburg
Am Hubland 12

8700 Würzburg

Prof. Dr. D. Chillag
Department of Mathematics
Technion
Israel Institute of Technology

Haifa 32000
ISRAEL

Prof. Dr. B. Brewster
Dept. of Mathematical Sciences
State University of New York
at Binghamton

Binghamton , NY 13901
USA

Dr. J. Cossey
Department of Mathematics
Faculty of Science
Australian National University
GPO Box 4

Canberra ACT 2601
AUSTRALIA

Dr. R. S. Dark
Faculty of Mathematics
University College

Galway
IRELAND

Dr. R. B. S. Gow
Dept. of Mathematics
University College
Belfield

Dublin 4
IRELAND

Prof. Dr. K. Doerk
Fachbereich Mathematik
der Universität Mainz
Saarstr. 21

6500 Mainz

Prof. Dr. P. Hauck
Mathematisches Institut
der Universität Freiburg
Albertstr. 23b

7800 Freiburg

Prof. Dr. A. Espuelas
Departamento de Algebra
Facultad de Ciencias
Universidad de Zaragoza

E-50009 Zaragoza

Prof. Dr. T. O. Hawkes
Mathematics Institute
University of Warwick

GB- Coventry , CV4 7AL

Dr. P. Förster
Department of Mathematics
Faculty of Science
Australian National University
GPO Box 4

Canberra ACT 2601
AUSTRALIA

Prof. Dr. H. Heineken
Mathematisches Institut
der Universität Würzburg
Am Hubland 12

8700 Würzburg

Prof. Dr. W. Gaschütz
Mathematisches Seminar
der Universität Kiel
Olshausenstr. 40

2300 Kiel 1

Prof. Dr. M. Herzog
Dept. of Mathematics
Tel Aviv University
Ramat Aviv
P.O. Box 39040

Tel Aviv , 69978
ISRAEL

Prof. Dr. B. Huppert
Fachbereich Mathematik
der Universität Mainz
Saarstr. 21

6500 Mainz

Dr. H. Laue
Dipartimento di Matematica
Universita degli Studi di Lecce
Via Arnesano

I-73100 Lecce

Prof. Dr. M. J. Iranzo
Departamento de Algebra y Fundamentos
Universidad de Valencia
Doctor Moliner, 50, Burjasot

E-46071 Valencia

Dr. F. Leinen
Fachbereich Mathematik
der Universität Mainz
Saarstr. 21

6500 Mainz

Prof. Dr. L.-C. Kappe
Dept. of Mathematical Sciences
State University of New York
at Binghamton

Binghamton , NY 13901
USA

Dr. O. Manz
Fachbereich Mathematik
der Universität Mainz
Saarstr. 21

6500 Mainz

Prof. Dr. O.H. Kegel
Mathematisches Institut
der Universität Freiburg
Albertstr. 23b

7800 Freiburg

Prof. Dr. M.L. Newell
Faculty of Mathematics
University College

Galway
IRELAND

Dr. P.H. Kropholler
St. John's College

GB- Cambridge

Dr. P. P. Palfy
Mathematical Institute of the
Hungarian Academy of Sciences
Realtanoda u. 13 - 15
P. O. Box 127

H-1364 Budapest

J. Pense
Fachbereich Mathematik
der Universität Mainz
Saarstr. 21

6500 Mainz

Prof. Dr. P. Rowley
Dept. of Mathematics
UMIST (University of Manchester
Institute of Science a. Technology)
P. O. Box 88

GB- Manchester , M60 1QD

Prof. Dr. F. Perez Monasor
Departamento de Algebra y
Fundamentos
Universidad de Valencia
Doctor Moliner, 50, Burjasot

E-46071 Valencia

Dr. E. Salomon
Fachbbereich Mathematik der
Universität Mainz
Saarstr. 21

6500 Mainz

Prof. Dr. R.E. Phillips
Dept. of Mathematics
Michigan State University

East Lansing , MI 48824-1027
USA

Prof. Dr. P. Schmid
Mathematisches Institut
der Universität Tübingen
Auf der Morgenstelle 10

7400 Tübingen 1

Dr. G. R. Robinson
Dept. of Mathematics
UMIST (University of Manchester
Institute of Science a. Technology)
P. O. Box 88

GB- Manchester , M60 1QD

Prof. Dr. R. Schmidt
Mathematisches Seminar
der Universität Kiel
Olshausenstr. 40

2300 Kiel 1

Dr. J. E. Roseblade
Jesus College
15 New Square

GB- Cambridge CB5 8BL

Prof. Dr. U. Schoenwaelder
Lehrstuhl D für Mathematik
der RWTH Aachen
Templergraben 64

5100 Aachen

Dr. R. Stöhr
Akademie der Wissenschaften
Institut für Mathematik
Mohrenstr. 39

DDR-1086 Berlin

Prof. Dr. J.S. Wilson
Dept. of Pure Mathematics and
Mathematical Statistics
University of Cambridge
16, Mill Lane

GB- Cambridge , CB2 1SB

Prof. Dr. S. Tobin
Faculty of Mathematics
University College

Galway
IRELAND

Prof. Dr. G. Zacher
Dipartimento di Matematica
Universita di Padova
Via Belzoni, 7

I-35131 Padova

Prof. Dr. A. Turull
Dept. of Mathematics
University of Miami

Coral Gables , FL 33124
USA

Prof. Dr. G. Zappa
Istituto Matematico "U.Dini"
Universita di Firenze
Viale Morgagni 67/A

I-50134 Firenze

Prof. Dr. R. W. van der Waall
Mathematisch Instituut
Fakulteit Wiskunde en Informatica
Universiteit van Amsterdam
Roetersstraat 15

NL-1018 WB Amsterdam