MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 39/1988

Topology

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The meeting was organized by M. Kreck (Mainz), A. Ranicki (Edinburgh) and L. Siebenmann (Orsay). About 55 participants from Europe, Asia and North America attended the conference. The twenty talks dealt with a wide range of topics in algebraic and geometric topology.

Vortragsauszüge

C.F. BÖDIGHEIMER

Homology Operations for Moduli Spaces of Riemann Surfaces

Let $\vec{\mathcal{M}}(g)$ denote the moduli space of conformal classes of triples $(Z, z_o, \vec{z}_o)$ where Z is a Riemann surface of genus g, $z_o \in Z$ and $\vec{z}_o$ a direction at z_o. By a result of Harer, $H_*\vec{\mathcal{M}}(g) \cong H_*\Gamma(g,1)$, where $\Gamma(g,1)$ is the mapping class group of a surface of genus g with one boundary curve.

We intruduce homology operations

$$\Theta_\alpha : H_*\vec{\mathcal{M}}(g) \longrightarrow H_{*+|\alpha|}\vec{\mathcal{M}}(g+kg')$$

for any $g,g' \geq 0$, $k \geq 1$ and $\alpha \in H_*[B\tilde{B}(k) \times_{\Sigma_k} \vec{\mathcal{M}}_o(g')]$ where $\tilde{B}(k)$ is the pure braid group, Σ_k is the symmetric group, and $\vec{\mathcal{M}}_o(g')$ is a certain subspace of $\vec{\mathcal{M}}(g)$. These operations are (i) associative, (ii) commute with Dyer-Lashof operations, and (iii) for $k=1$, $g'=1$, $\alpha \in H_o$, $\Theta_\alpha : H_*\vec{\mathcal{M}}(g) \longrightarrow H_*\vec{\mathcal{M}}(g+1)$ corresponds to Harer's stabilization map.

To define these operations we described $\vec{\mathcal{M}}(g)$ as a space of Hilbert slit domains, arising from extremal harmonic maps.

A.V. CHERNAVSKII

On the Peano-Keldysh construction

A generalisation of a result by L.V. Keldysh is proved:

Theorem: $\forall\, k \geq 0$, $p \geq 1$, $n \geq 2k+3$, $\exists\, f: I^n \longrightarrow I^{n+p}$, s.t. $\forall\, y \in I^{n+p}$ and a neighbourhood U of $f^{-1}(y)$ $\exists$ a neighbourhood $V(f^{-1}(y))$ s.t. $\forall\, g: S^j \longrightarrow V$ $\exists$ $\bar{g}: D^{j+1} \longrightarrow U$, where D^{j+1} is the unit disk, $S^j = \partial D^{j+1}$ and $\bar{g}_{|S^j} = g$, $0 \leq j \leq k$.

(It is sufficient to consider the case $n = 2k+3$ and $p = 1$.)

Let $\Sigma = (p \amalg q)^* I^n$. We take Σ in place of I^{n+1}. $P = p^* \partial I^n$, $Q = q^* \partial I^n$.

The proof follows by inductive construction of a sequence of imbeddings $q_i : I^n \longrightarrow \Sigma$ for any given decreasing sequence $\varepsilon_i \longrightarrow 0$, $\varepsilon_o = 2$, with the properties:

1. $q_i I^n$ is ε_i-dense in Σ;
2. $q_i I^n$ divides Σ into two pd-manifolds U_i^P and U_i^Q with boundary, each with a spine $P_i = P \cup \overline{P}_i$, $Q_i = Q \cup \overline{Q}_i$ respectively, $\dim \overline{P}_i = k+1 = \dim \overline{Q}_i$, and with ε_i-pushes $s_i : U_i^P \longrightarrow P_i$, $t_i : U_i^Q \longrightarrow Q_i$;
3. $\forall\, g : S^j \longrightarrow q_i I^n$, $j \leq k$, $\exists\, \overline{g} : D^{j+1} \longrightarrow q_i I^n$, $\overline{g}|_{S^j} = g$, and $\operatorname{diam}(\overline{g}(D^{j+1})) \leq \operatorname{diam}(g(S^j)) + 2\varepsilon_i$.

Then $f = \lim q_i$. The property 3 for $k=0$ was founded first by L. Keldysh. By a theorem of R. Frum-Ketkov the restriction on k is sharp. By another construction of L. Keldysh the mapping f may be converted into an open one.

If $n < 2k+3$ the following "Keldysh constants" are possibly (in view of our result) non-zero: $K(n,m,k) = \lim\limits_{\varepsilon \to 0_+} \inf\limits_{q} \sup\limits_{g} \inf\limits_{\overline{g}} (\operatorname{diam} \overline{g}(D^{k+1}) - \operatorname{diam} g(S^k))$,

where q runs over ε-dense imbeddings I^n into I^{n+1}, g are mappings of the sphere S^k into $q(I^n)$, $\overline{g}$ are then extensions of g to the disk D^{k+1}.

It would be interesting to have a standard form of this construction analogous to the usual constructions of Peano curves.

J. CUNTZ

KK-theory and cyclic cohomology

This was a survey of certain aspects of KK-theory and cyclic cohomology for C^*-algebras. In particular, we described the construction of an algebra qA for a given algebra A. This algebra can be regarded as being dual to the algebra $M_\infty(A)$ of matrices over A and leads naturally to the definition of the Kasparov group KK(A,B) as the group of homotopy classes of homomorphisms from qA to

$M_\infty(B)$, and also to the Kasparov product.

On the other hand, in joint work with A Connes we use an interpretation of qA as an algebra of "K-theory differential forms" to obtain the cyclic cohomology of A from traces on qA, to construct a Chern character from certain elements of $K^0(A)$ to $H_\lambda^{even}(A)$ and to prove a partial Chern isomorphism theorem.

A.N. DRANISHNIKOV

CE-problem and K-theory

A proper map $f: X \longrightarrow Y$ is called cell-like (briefly CE-map) if each fiber $f^{-1}(y)$ can be embedded in some euclidean space $\mathbb{R}^m$ as a cellular subset. This means that $f^{-1}(y) = \bigcap_{i=1}^{\infty} B_i$ where $B_{i+1} \subset B_i$ and each B_i is an closed n-cell in $\mathbb{R}^m$.

CE-problem. Can CE-maps raise the dimension on $\mathbb{R}^n$?

It is easy to see that CE-maps cannot raise the cohomological dimension. By the Alexandroff theorem this means that if the dimension rises under a CE-map then it must rise to infinity.

The main result. For any $n \geq 7$ there exists a CE-map of $\mathbb{R}^n$ that raises the dimension.

The proof uses the well-known Edwards theorem on resolution of compacta with finite cohomological dimension and the following theorm in K-theory:

Mishchenko theorem. $\tilde{K}_{\mathbb{C}}^*(K(\mathbb{Z},3);\mathbb{Z}_p) = 0$ for any p.

Problem. Does there exist a non trivial generalized cohomology theory E such that $E^*(\mathbb{CP}^\infty) = 0$?

M. FARBER

Classification of multidimensional links

Ambient isotopy types of stable multidimensional links are described in terms of stable homotopy.

An n-dimensional m-component link is an oriented smooth submanifold l^n of S^{n+1}, where $l^n = l_1^n \cup \ldots \cup l_m^n$ is the ordered disjoint union of m submanifolds of S^{n+2}, each homeomorphic to S^n. l is a stable link if there is an oriented smooth submanifold V^{n+1} of S^{n+2}, $V^{n+1} = V_1^{n+1} \cup \ldots \cup V_m^{n+1}$ the disjoint union of the submanifolds V_i, such that $\partial V_i = l_i$ and each V_i is $[(n+3)/3]$-connected ($n \geq 5$). The classifying object is called stable isometry structure and is a collection (X,u,y,Π), where X is a finite CW complex with base point, $u: X \wedge X \longrightarrow S^{n+1}$, $z: X \longrightarrow X$ are stable maps, and $\Pi = \{\pi_1,\ldots,\pi_m\}$ is a decomposition of unity:

$$\pi_i \pi_j = \delta_{ij} \pi_j \quad \text{and} \quad \Sigma \pi_i = 1.$$

For the class of odd dimensional simple links (admitting Seifert surfaces homotopy equivalent to a bouquet of spheres) this homotopy classification gives a complete algebraic description.

TH. FISCHER (joint work with M. Kreck)

Rohlin Invariants, Bordism of Spin Diffeomorphisms and the Integrality of some Spectral Invariants

According to a theorem by Rohlin and Ochanine, the signature of a spin manifold of dimension 8k+4 is divisible by 16. This allows the following

Definition: Let (M,ω) be the spin boundary of the spin manifold (N,ν) of dimension 8k+4. Then the Rohlin invariant of (M,ω) is defined as

$$R(M,\omega) := \operatorname{sign}(N,\nu) \bmod 16 \in \mathbb{Z}/16 .$$

Miller and Lee showed that this is a spectral invariant: there is a linear combination D of Dirac operators such that

$$S(M,\omega) := -\eta(M;\text{Hirtz}) + h(M;D) + \eta(M,D) \bmod 16$$

agrees with $R(M,\omega)$ whenever the latter is defined. $S(M,\omega)$ is defined for any spin manifold. We show:

Theorem 1: $S(M,\omega) \in \mathbb{Z}/16$.

For the proof we consider the bordism group of spin diffeomorphisms and define for a spin diffeomorphism (M,ω,f,h) ithe Rohlin invariant by $R(M,\omega,f,h) := R(M_f,\omega_h)$, where (M_f,ω_h) is the mapping torus of the spin diffeomorphism. This is not a bordism invariant, but can be made into one by subtraction of an appropriate invariant φ. We give a splitting of the sequence given by Kreck:

$$0 \longrightarrow K \longrightarrow \Delta^{spin}_{8k+2} \longrightarrow \Omega^{spin}_{8k+2} \oplus \Omega^{spin}_{8k+3} \otimes W_-(\mathbb{Z},\mathbb{Z}) \longrightarrow 0$$

Theorem 2: For $k>0$, $\Delta^{spin}_{8k+2} \cong K \oplus \Omega^{spin}_{8k+2} \otimes \Omega^{spin}_{8k+3} \otimes W_-(\mathbb{Z},\mathbb{Z})$ with $R-\varphi : K \xrightarrow{\cong} \mathbb{Z}/16$.

We give formulas for the dependence of $S(M,\omega,f,h)$ on the homotopy h an for the Rohlin invariant of a composition of diffeomorphisms.

For $H_{4k+1}(M,\mathbb{Z})$ torsion free, Lee, Miller and Weintraub gave a formula for $R(M,\omega,f,h)$ in terms of the automorphism f_* on $H_{4k+1}(M,\mathbb{Z})$ and the spin structure ω. We give an example that such a formula canot hold if $H_{4k+1}(M,\mathbb{Z})$ is allowed to have torsion.

C. GORDON and J.LUECKE

The Knot Complement Conjecture

We prove that two knots in the 3-sphere are equivalent if and only if their complements are homeomorphic. The proof involves a combinatorial analysis of the intersection of a certain pair of planar surfaces which would exist in the complement of a counterexample.

A.F. HARSHILADZE

Splitting along 1-sided submanifolds

Let Y be a manifold and X c Y a 1-sided submanifold. The best way is to consid er a map $\varphi : Y \longrightarrow \mathbb{RP}^N$ with N>2dimY and $\varphi_* : \pi_1(Y) \longrightarrow \pi_1(\mathbb{RP}^N)$ epimorphic, and let $X = \varphi^{-1}(\mathbb{RP}^{N-1})$. If the inclusion X c Y induces an isomorphism of π_1, splitting of homotopy equivalences $f : N \longrightarrow Y$ is possible if and only if the obstruction in the Browder-Livsey group $LN_n(\pi \to G)$ vanishes, n = dimX, G = $\pi_1(Y)$, $\pi = \ker\varphi_*$. If the inclusion X c Y induces only an epimorphism of π_1 then the obstruction group for splitting is no longer LN_n, but the group $LSA_n(\Phi)$ constructed by Pete Akhmetev, where Φ is a pushout square of fundamental groups for X c Y. The definition for n = 2k is as follows. Let

$$\Phi = \begin{array}{ccc} A & \xrightarrow{s} & C \\ {\scriptstyle r}\downarrow & & \downarrow{\scriptstyle l} \\ B & \xrightarrow[j]{} & D \end{array}$$

be a square of rings with involution, for which the vertical arrows are quadratic extensions and the horizontal ones epimorphisms. For a D-module P we call the submodule L c $l^!P$ over C representable if the composition $l_*L \longrightarrow l_*l^!P \longrightarrow P$ becomes an isomorphism. The objects of $LSA_{2k}(\Phi)$ are the pairs $((P,\varphi),L)$ consisting of the nonsingular quadratic $(-1)^k$-form (P,φ) over B and a representable Lagrangian $L \subset l^!j_*P = s_*r^!P$ of the form $s_*r^!(P,\varphi)$ over C. The object is zero, if there is a Lagrangian M of (P,φ) for which $s_*r^!M + L$ is a direct summand of $s_*r^!P$ and $L \cap s_*r^!M$ is representable in $s_*r^!M = l^!l_*M$.

Now I describe the map $\theta : L_{2k}(D) \longrightarrow LSA_{2k-2}(\Phi)$ which corresponds to passing to the boundary (or is induced by the action of $L_{2k}(D)$ on the homotopy smoothings). Let (Q,Ψ) be a nonsingular quadratic $(-1)^k$-form over D. We have to construct the pair $((P,\varphi),L)$ over Φ. As the D-module Q is free there exists an A-module Q_0 for which $Q = j_*r_*Q_0 = l_*s_*Q_0$. For (P,φ) we take $H^{\mp}(r_*Q_0)$ here $\mp$ means $(-1)^{k+1}$. Now $l^!Q = l^!l_*s_*Q_0$ and $s_*Q_0 \subset l^!l_*s_*Q_0$ is a direct summand, so take any projection $l^!Q \longrightarrow s_*Q_0$ and denote by $\mathcal{P}$ the composition $l^!Q \longrightarrow s_*Q_0 \subset l^!Q$. Let $\theta := (l^!\Psi)\circ\mathcal{P} : l^!Q \longrightarrow l^!Q^*$. For L we take $\Gamma_{\theta\pm\theta^*}$, that is the graph

of the map $\theta \pm \theta^*$. One can prove that this is a representable Lagrangian. So the map Θ is $[(Q,\Psi)] \longrightarrow [((P,\varphi), L)]$.

H.-W. HENN (joint work with Lionel Schwartz)

Summands in $H^*(B(\mathbb{Z}/p)^d, \mathbb{Z}/p)$ which are unstable algebras

Let p be a prime and denote by A the mod p Steenrod algebra. We abbreviate $H^*(X;\mathbb{Z}/p)$ by $H^*(X)$.

We determine the indecomposable A-module summands of $H^*(B(\mathbb{Z}/p)^d)$ which admit the structure of an unstable A-algebra (without unit). This is done by making use of the classification of subgroups of $GL(d,\mathbb{Z}/p)$ acting transitively on $(\mathbb{Z}/p)^d \setminus \{0\}$ (Hering) and hence of the classification of finite simple groups.

It turns out that there are very few examples of such K and all examples arise as $\tilde{H}^*(BG)$ for a suitable finite group G with p-Sylow subgroup $(\mathbb{Z}/p)^d$ but of course not all such G will have $\tilde{H}^*(BG)$ indecomposable as A-module. In fact, a consequence of our main result is that there are very few finite groups G with p Sylow subgroup $(\mathbb{Z}/p)^d$ such that BG is stably indecomposable at p.

G.G. KASPAROV

Novikov's conjecture and KK-theory

Novikov's conjecture on the homotopy invarinace of higher signatures of closed smooth manifolds with fundamental group π can be formulated in a stronger form as the conjecture that the natural homomorphism β: $RK_*(B\pi)\otimes\mathbb{Q} \longrightarrow K_*(C^*(\pi))\otimes\mathbb{Q}$ is injective. This Strong Novikov Conjecture can be proved in the case when $E\pi$, the universal covering space of the classifying space $B\pi$, is a special π-manifold. This condition means that $E\pi$ must be a complete Riemannian manifold, and if one associates to it its geometrically related C^*-algebra A

defined by the natural Clifford bundle on $E\pi$, one will have that the natural Dirac element d in the equivariant K-group $K^0_\pi(A)$ has a one-sided inverse $\eta \in K^\pi_0(A)$: $d\circ\eta = 1_A \in KK^\pi(A,A)$. Details may be found in Inventiones Mathematicae, 91(1988), 147-201.

The most interesting case outside the situation of special Riemannian $E\pi$ is the case when $E\pi$ is a Bruhat-Tits building. This is very much in parallel with the previous case. In this situation the natural algebra A and the element d can be constructed as well, and also the construction of η is performed in some cases. This work is in progress now.

W. LÜCK

Reidemeister torsion, G-homotopy representations and spectral theory

A round structure on a G-CW-complex is a collection of $F[WH]$-isomorphisms $\mathcal{F}(H)\colon H(X^H;F)_{odd} \longrightarrow H(X^H;F)_{ev}$ for $H \subset G$ and F a field of characteristic 0. For a round finite G-CW-complex X there is defined an equivariant Reidemeister torsion

$$\rho^G(X) \in Wh(QOr(G)) = \bigoplus_{(H)} Wh(Q[WH])$$

If X is finitely dominated and round, we obtain a reduced version

$$\overline{\rho}^G(X) \in K_1(QOr(G))/K_1(\mathbb{Z}_{(|G|)}Or(G)).$$

The unit sphere SV of a complex G-representation carries a canonical round structure so that $\rho^G(SV)$ is defined. This is also true for a compact smooth closed Riemannian G-manifold M if $\dim M^H$ is odd, M^H is oriented and WH acts orientation preserving for $H \subset G$.

We prove

Theorem: $\rho^G_{\mathbb{R}}\colon Rep_{\mathbb{R}}(G) \longrightarrow Wh(QOr(G))$, $[V] \longmapsto \rho^G(SV\otimes_{\mathbb{R}}\mathbb{C})$ is an injective homomorphism.

This implies de Rham's theorem that two G-representations are linearly isomor-

phic if and only if their unit spheres are G-diffeomorphic.

Theorem: Let X and Y be oriented G-homotopy representations with the same dimension function.

a) They are oriented G-homotopy equivalent if $\bar{\rho}^G(X) - \bar{\rho}^G(Y) = 0$.

b) Let $\partial : K_1(\mathbb{Q}Or(G))/K_1(\mathbb{Z}_{(|G|)}\, Or(G)) \longrightarrow K_0(\mathbb{Z}Or(G))$ be given by the localization square. Then $\partial(\bar{\rho}^G(X))$ is the finiteness obstruction.

Let M be a compact closed smooth Riemannian G-manifold. If Δ_n is the Laplace operation in dimension n and E_λ the eigenspace of the eigenvalue λ, define

$$\xi_n(s) = \sum_{\lambda>0} \lambda^{-s} \cdot E_\lambda \in \mathbb{C} \otimes_{\mathbb{R}} Rep_{\mathbb{R}}(G).$$

This is a meromorphic function on $\mathbb{C}$, analytic in 0. Define the analytic Reidemeister torsion

$$\rho_{an}(M) = \sum_{n\geq 0} (-1)^n \cdot n \cdot \xi_n'(0) \in \mathbb{R} \otimes_{\mathbb{Z}} Rep_{\mathbb{R}}(G)$$

Consider the following composition

$$f : Wh(\mathbb{R}Or(G)) \xrightarrow{(1)} Wh(\mathbb{R}G) \xrightarrow{(2)} K_1(\mathbb{R}G)^{\mathbb{Z}/2} \xrightarrow{(3)}$$

$$\xrightarrow{(3)} \prod_{Irr_{\mathbb{R}}(G)} \mathbb{R}^* \xrightarrow{(4)} \prod_{Irr_{\mathbb{R}}(G)} \mathbb{R} \xrightarrow{(5)} \mathbb{R} \otimes_{\mathbb{Z}} Rep_{\mathbb{Z}}(\mathbb{R})$$

where (1) is the projection, (2) sends $[u]$ to uu^*, (3) is induced from Wedderburn's theorem, (4) is given componentwise by $x \longmapsto \ln(x)$ and (5) is the obvious isomorphism.

Conjecture: $f(\rho^G(M)) = \rho_{an}(M)$.

T. MATUMOTO

Any group is represented by an outer automorphism group

S. Kojima shows that any finite group G is isomorphic to the outer automorphism class group $Out(\pi) = Aut(\pi)/Inn(\pi)$ of some discrete subgroup π of $PSL(2,C)$. I will show the following theorem.

THEOREM. For any group G there is a group π such that G is isomorphic to

the outer automorphism class group $Out(\pi)$ of π.

In terms of the homotopy theory our theorem says that G is isomorphic to the group $\mathcal{E}(K(\pi,1))$ of free homotopy classes of homotopy self-equivalences of the Eilenberg-MacLane space $K(\pi,1)$. In 1959 de Groot showed that any group G is isomorphic to the automorphism group of some connected oriented graph Γ and then to the homeomorphism group $Homeo(X)$ of some metric space X. I construct a suitable graph of groups over Γ and prove that its fundamental group π is a repuired one using the informations on Mathieu groups given by the atlas of finite groups.

A.S. MIŠČENKO

Survey of the application of noncommutative algebras to problems of the topology of manifolds

In this lecture a survey was given of some results obtained in Moscow in the investigations of connections between the invariants of noncommutative algebras and several problems of the topology of manifolds. These algebras arise by means of actions of fundamental groups or of monoids of loops of (smooth) manifolds. There are at least two ways of applications of noncommutative algebras. The first is the reduction to homology invariants such as K-theory. The second consists of the introduction of a reasonable topology with passing to the theory of C^*-algebras and then also to K-theory. Both ways are usefull and have close anologies.

The most interesting results are:

1. Description of $\pi_k(H(M^n)/Homeo(M^n))$, $k \leq [(n+15)/7]$ in terms of dihedral homology.
2. Existence of the topology of the space of Fredholm operators between strong and uniform topologies.
3. Independence of the spectrum of elliptic operators with almost periodical

coefficients of the function space under action.

4. The index formula for elliptic operators over a C^*-algebra in terms of cyclic homology.

W.D. NEUMANN

The Topology of Complex Algebraic Plane Curves

ABSTRACT: Surprisingly little is known about the topology of algebraic plane curves $V \subset \mathbb{C}^2$. For example, the classification when V is contractible was only completed in 1982 by Zaidenberg and Lin. The *link at infinity* of an algebraic plane curve $V \subset \mathbb{C}^2$ is the pair $\mathfrak{L}(V,\infty) = (S^3, S^3 \cap V)$, where S^3 is a sufficiently large sphere in $\mathbb{C}^2$. We show that if V is regular (i.e. it is "typical" fiber of its defining polynomial), then $\mathfrak{L}(V,\infty)$ determines the topology of $V \subset \mathbb{C}^2$ (as an embedded smooth real 2-manifold up to isotopy). Even in the irregular case, $\mathfrak{L}(V,\infty)$ determines the set $\mathrm{DEG}(V)$ of all possible values of the degree $\deg(V)$ (the degree $\deg(V)$ is only well defined relative to a chosen affine structure on $\mathbb{C}^2$ - if one changes the affine structure by a polynomial biholomorphic map $\sigma\colon \mathbb{C}^2 \longrightarrow \mathbb{C}^2$ then $\deg(V)$ changes). Also, $\mathfrak{L}(V,\infty)$ determines the Euler characteristic of V (corrected by the Milnor numbers of the singularities of V if V is singular). One can classify a class of links called "RPI links", which includes all links at infinity of algebraic plane curves, in terms of splice diagrams, and in terms of this classification, the above mentioned invariants are extremely easy to compute. Together these results enable algebraic classifaction of algebraic plane curves with given topology in many cases; we described this for low genus nonsingular curves with one point at infinity.

V. PUPPE

On rather ugly manifolds

We call a manifold "rather ugly" if it admits nontrivial topological Z/p-actions for at most finitely many primes p. It has been known for some time that there even exist "really ugly" manifolds, i.e. compact, oriented differentiable manifolds that do not admit any nontrivial topological action of a compact Lie group, but all the examples - given by several authors - have nontrivial fundamental group. It is still an open question whether there exist simply connected really ugly manifolds. In an attempt to answer this question the following theorem is proved using the connection between P.A. Smith-theory and deformations of algebras.

Theorem: There exist simply connected, compact, oriented, 6-dimensional, differentiable manifolds M such that any compact, orientable manifold N with $H^*(N;\mathbb{Q}) \cong H^*(M;\mathbb{Q})$ (as graded $\mathbb{Q}$-algebras) is rather ugly.

This gives counterexamples to a conjecture of P. Löffler and M. Raußen.

F. QUINN

Stratified simple homotopy theory

The objective of the lecture was to describe a version of the topological h-cobordism theorem for sets which are stratified in a weak sense.

Strongly (geometrically) stratified sets have neighborhoods of strata which are mapping cylinders of fiber bundles. Examples of these are Whitney stratified sets, polyhedra with the intrinsic skeleton filtration, and M/G where the compact Lie group G acts smoothly (or PL with local slices) on M. In these the mapping cylinder neighborhoods allow separation of the problems on different strata, so the h-cobordism obstruction group is just a direct sum of Whitehead groups of fundamental groups of strata.

However if the stratified sets are only topological, or only topological pro-

duct structures are required on h-cobordisms, then the situation changes. Infinite constructions become possible which converge to lower strata. A class of "homotopically" stratified sets was described which forms a very general and satisfactory setting for the theory, and the obstruction group for topological h-cobordisms of these was described. Again it is a sum over strata, but now the terms are relative homology groups of cones, with cosheaves of spectra obtained from Whitehead spectra. The description is complicated, but in fact the groups are trivial in a number of interesting geometric situations. They appear as the third term in a long exact sequence with a "assembly" homomorphism, so their calculation is related to the study of assemblies.
This material, as well as stratified versions of "finite complexes" and simple homotopy of these is contained in the paper "Homotopically stratified sets" J.Amer.Math Soc. 1 (1988) 244-299.

E. REES (joint work with FEA Johnson)

Fundamental Groups of Compact Kähler Manifolds

Let $\mathfrak{K}_n$ denote the class of groups Γ such that there is an n-dimensional compact Kähler manifold M whose fundamental group is isomorphic to Γ. Define $\mathfrak{C}_n$ and $\mathfrak{P}_n$ similarly for complex manifolds and smooth projective varieties respectively. Let $\mathfrak{F}_n$ denote any one of these classes. The following facts are immediate:

$$\mathfrak{P}_n \subset \mathfrak{K}_n \subset \mathfrak{C}_n \quad \text{for } n \geq 1,$$

$$\mathfrak{P}_1 = \mathfrak{K}_1 = \mathfrak{C}_1,$$

$$\mathfrak{F}_n \subset \mathfrak{F}_{n+1} \quad \text{for } n \geq 1.$$

In 1956 J.-P. Serre showed that every finite group belongs to $\mathfrak{P}_2$, so $\mathfrak{P}_1 \neq \mathfrak{P}_2$. It is a consequence of the Hodge decomposition that if $\Gamma \in \mathfrak{K}_n$ then the first Betti number $b_1(\Gamma)$ is even. Hence

$$\mathbb{Z}^{2m+1} \notin \mathfrak{K}_n.$$

A theorem due to K. Kodaira (1964) implies that if $\Gamma \in \mathfrak{C}_2$ and $b_1(\Gamma)$ is even

then $\Gamma \in \mathcal{F}_2$. Hence $\mathcal{F}_2 = \mathcal{K}_2$.

By considering the Hopf surface, one sees that $Z \in \mathcal{C}_2$ but $Z \notin \mathcal{K}_2$. Hence $\mathcal{C}_2 \neq \mathcal{K}_2$.

Using Bertini's Theorem and Lefschetz's hyperplane section Theorem, one has that $\mathcal{F}_n = \mathcal{F}_{n+1}$ for $n \geq 2$.

Using rational homotopy theory and Kodaira's Theorem one has that $\Gamma_3 \notin \mathcal{C}_2$ where Γ_3 denotes the 3×3 Heisenberg group over the Gaussian Integers, clearly $\Gamma_3 \in \mathcal{C}_3$ so $\mathcal{C}_2 \neq \mathcal{C}_3$.

No other relations amongst the $\mathcal{F}_n$ are known.

If $\Gamma \in \mathcal{K}_n$ then, using the Hard Lefschetz Theorem, one has a homomorphism $\lambda: H^2(\Gamma) \longrightarrow \mathbb{R}$ such that

$$H^1(\Gamma) \otimes H^1(\Gamma) \xrightarrow{\cup} H^2(\Gamma) \xrightarrow{\lambda} \mathbb{R}$$

is a skew, non-degenerate form. From the fact that a finite cover of a Kähler manifold is also Kähler, any subgroup of finite index in Γ also has such a form. Using this one can prove:

Theorem: Let G_1, G_2 be groups, both of which have non-trivial finite quotients and let H be any group. then

$$(G_1 * G_2) \times H \notin \mathcal{K}_n \text{ for any } n.$$

It follows that the only direct product of free groups that is in $\mathcal{K}_n$ is free abelian of even rank.

J.B. RUDYAK

K-orientability of $\mathbb{R}^n$-bundles and spherical fibrations

It is a well-known fact (Atiyah-Bott-Shapiro) that a vector bundle ξ is K-orientable iff $w_1(\xi) = 0 = \delta w_2(\xi)$. However, that (necessary) condition is not sufficient even for K-orientability of PL $\mathbb{R}^n$-bundles, and we indicate the obstruction theory for the K-orientability of PL, TOP $\mathbb{R}^n$-bundles and spherial fibrations. Note that Vect $\subset$ PL $\subset$ TOP $\subset$ G, where the symbols denote the classes of bundles (fibrations).

For every HZ-oriented spherical fibration ξ over X we define the higher characteristic classes $\kappa_n(\xi) \in H^{2n+1}(X)$. Here $\kappa_n(\xi)$ is defined iff $\kappa_i(\xi) \ni 0$ for all $i < n$, and ξ is K-orientable iff $\kappa_n(\xi) \ni 0$ for all n. Further, $\kappa_1(\xi) = \delta w_2(\xi)$. We say that the κ_n is realizable by an object of one of the four classes above, iff there exists ξ from the class with $\kappa_n(\xi) \not\ni 0$.

Theorem A.

1. If $\kappa_1(\xi) = 0$ for a vector bundle ξ, then $\kappa_n(\xi) \ni 0$ for all n. Hence none of the classes κ_n, $n>1$, may be realized by vector bundles. (Of course, this is a reformulation of the Atiyah-Bott-Shapiro Theorem)
2. (joint with A. Hohlov). Every class κ_n may be realized by sperical fibration.
3. If $\kappa_1(\xi) = 0$ for a TOP $\mathbb{R}^n$-bundle ξ, then $\kappa_2(\xi) \ni 0$. Hoever, all the classes κ_n, $n>2$, may be realized by TOP $\mathbb{R}^n$-bundles.
4. If $\kappa_1(\xi) = 0$ for a PL $\mathbb{R}^n$-bundle ξ, then $\kappa_2(\xi) \ni 0$, $\kappa_3(\xi) \ni 0$. However, there exists r such that all κ_n, $n \geq r$, may be realized by PL $\mathbb{R}^n$-bundles.

Another topic is connected with the Sullivan result on the K[½]-orientability of STOP $\mathbb{R}^n$-bundles and is concerned with the realization of the homology classes.

Theorem B: The spectrum $MSPL\langle\mathbb{C}P^{p-1}\rangle_{(p)}$ is the spectrum of the ordinary (co)homology (with the corresponding coefficients). Here the subscript (p) is the p-localization, p is an odd prime, and the superscript $\mathbb{C}P^{p-1}$ (complex projective space) is the Sullivan singularity.

Corollary 1. Every homology class may be realized by the sum of polyhedra of the form $V \cup_\partial c(\mathbb{C}P^{p-1}) \times A$ with PL V and A, where $V = \mathbb{C}P^{p-1} \times A$, c denotes a cone and p runs over all odd primes.

Corollary 2. In the ring $\pi_*(MSPL\langle\mathbb{C}P^{p-1}\rangle_{(p)})/tors$, $(n+1) \mid [\mathbb{C}P^n]$ for all n.

Corollary 3. Let $x \in \pi_{2p^2-2p-1}(MSPL)$ be a lower-dimensional p-torsion element. Then $x \cdot \mathbb{C}P^{p-1} = 0 \in \pi_*(MSPL)$.

E. VOGT

A Foliation of $\mathbb{R}^3$ by circles

One of the problems collected by P. Schweitzer during the conference on Gelfand-Fuks cohomology and foliations which took place in Rio de Janairo in 1976 is the following question attributed to D.B.A. Epstein:

Can $\mathbb{R}^3$ be foliated by circles?

We present an example of such a foliation by smooth circles which is differentiable in the sense that coordinate changes of foliation charts have a derivative everywhere, but the derivatives do not vary continuously. Thus the question of Epstein should now be replaced by:

Does there exist a C^1-foliation of $\mathbb{R}^3$ by circles?

The key step in our example is a foliation by circles on a solid torus V of whose boundary the complement of two open annuli A_1 and A_2 has been removed, where the core of A_1 is a longitude of V while, and that is the crucial point, the core of A_2 bounds a disk in V.

Berichterstatter: Thomas Fischer, Mainz

Tagungsteilnehmer

H. Azcan
Mathematics Division
University of Sussex
Falmer

GB- Brighton , BN1 9QH

Prof. Dr. J. Cuntz
Institut für Angewandte Mathematik
der Universität Heidelberg
Im Neuenheimer Feld 294

6900 Heidelberg 1

Dr. C.-F. Bödigheimer
Mathematisches Institut
der Universität Göttingen
Bunsenstr. 3-5

3400 Göttingen

Prof. Dr. T. tom Dieck
Mathematisches Institut
der Universität Göttingen
Bunsenstr. 3-5

3400 Göttingen

Prof. Dr. G. Burde
Fachbereich Mathematik
Mathematisches Seminar
der Universität Frankfurt
Robert-Mayer-Str. 6-10

6000 Frankfurt 1

Prof.Dr.Dr.h.c. A. Dold
Mathematisches Institut
der Universität Heidelberg
Im Neuenheimer Feld 288

6900 Heidelberg 1

Prof. Dr. A. V. Cernavski
Institute for Problems of Information Transmission, Academy of Sciences, Steklov Mathem. Institute
ul. Ermolova 19

Moscow 117 966 GSP-1
USSR

Prof. Dr. M. Domergue
U. E. R. de Mathematiques
Universite de Provence
3, Place Victor Hugo

F-1331 Marseille Cedex 3

Prof. Dr. H. Chaltin
Institut de Mathematiques
Universite Louis Pasteur
7, rue Rene Descartes

F-67084 Strasbourg Cedex

Prof. Dr. A.N. Dranisnikov
Steklov Mathematical Institute
Academy of Sciences of the USSR
42, Vavilova str.

Moscow 117 966 GSP-1
USSR

Prof. Dr. M. Farber
Dept. of Mathematics
Tel Aviv University
Ramat Aviv
P.O. Box 39040

Tel Aviv , 69978
ISRAEL

Prof. Dr. I. Hambleton
Department of Mathematics and
Statistics
Mc Master University

Hamilton, Ontario L8S 4K1
CANADA

Prof. Dr. R. Fenn
Mathematics Division
University of Sussex
Falmer

GB- Brighton , BN1 9QH

Prof. Dr. A. Ph. Harshiladze
IZMIRAN
Troitsk

Moscow Region 142092

USSR

Dr. Th. Fischer
Fachbereich Mathematik
der Universität Mainz
Saarstr. 21

6500 Mainz

Prof. Dr. J.-C. Hausmann
Section de Mathematiques
Universite de Geneve
Case postale 240

CH-1211 Geneve 24

Prof. Dr. C. M. Gordon
Dept. of Mathematics
University of Texas at Austin

Austin , TX 78712
USA

Dr. H.-W. Henn
Mathematisches Institut
der Universität Heidelberg
Im Neuenheimer Feld 288

6900 Heidelberg 1

Prof. Dr. A. Haefliger
Section de Mathematiques
Universite de Geneve
Case postale 240

CH-1211 Geneve 24

Dr. J. Huebschmann
Mathematisches Institut
der Universität Heidelberg
Im Neuenheimer Feld 288

6900 Heidelberg 1

Prof. Dr. S. Jackowski
Instytut Matematyki
Uniwersytet Warszawski
Palac Kultury i Nauki IX p.

00-901 Warszawa
POLAND

Prof. Dr. U. Koschorke
Lehrstuhl für Mathematik V
FB 6 - Mathematik
der Universität Siegen
Hölderlinstr. 3

5900 Siegen 21

Prof. Dr. T. V. Kadeishvili
Institute of Mathematics
Academy of Sciences of the Georgian SSR
Z. Rukhadze Str. 1

380093 Tbilisi

USSR

Prof. Dr. T. Kozniewski
Sonderforschungsbereich 170
Geometrie und Analysis
Mathematisches Institut
Bunsenstr. 3 - 5

3400 Göttingen

Prof. Dr. G. G. Kasparov
Institute of Chemical Physics
Academy of Sciences of the USSR
Chernogolovka

Moscow obl., 142432
USSR

Prof. Dr. M. Kreck
Fachbereich Mathematik
der Universität Mainz
Saarstr. 21

6500 Mainz

Dr. C. Kearton
Dept. of Mathematics
The University of Durham
Science Laboratory
South Road

GB- Durham , DH1 3LE

Dr. W. Lück
Mathematisches Institut
der Universität Göttingen
Bunsenstr. 3-5

3400 Göttingen

Prof. Dr. R. C. Kirby
Dept. of Mathematics
University of California

Berkeley , CA 94720
USA

Prof. Dr. T. Matumoto
Dept. of Mathematics
Faculty of Science
Hiroshima University

Hiroshima 730
JAPAN

Prof. Dr. W. Metzler
Fachbereich Mathematik
Mathematisches Seminar
der Universität Frankfurt
Robert-Mayer-Str. 6-10

6000 Frankfurt 1

Prof. Dr. B. Morin
Institut de Mathematiques
Universite Louis Pasteur
7, rue Rene Descartes

F-67084 Strasbourg Cedex

Prof. Dr. H. Miller
Dept. of Economics
Massachusetts Institute of
Technology

Cambridge , MA 02139
USA

Prof. Dr. W.D. Neumann
Dept. of Mathematics
The Ohio State University
231 W. 18th Ave.

Columbus , OH 43210-1174
USA

Prof. Dr. K.C. Millett
Dept. of Mathematics
University of California

Santa Barbara , CA 93106
USA

Prof. Dr. Nguyen H.V. Hung
Dept. of Mathematics
University of Hanoi
Dai hoc Tong hop

Hanoi
VIETNAM

Prof. Dr. Z. R. Miminoshvili
Institute of Mathematics
Academy of Sciences
of the Georgian SSR
Z. Rukhadze Str. 1

380093 Tbilisi

USSR

Prof. Dr. R. Oliver
Matematisk Institut
Aarhus Universitet
Ny Munkegade
Universitetsparken

DK-8000 Aarhus C

Prof. Dr. A. S. Miscenko
c/o Professor M. M. Postnikov
Steklov Mathematical Institute
Academy of Sciences of the USSR
ul. Vavilova 42

Moscow 117 966 GSP-1
USSR

Prof. Dr. A. Papadopoulos
Institut de Mathematiques
Universite Louis Pasteur
7, rue Rene Descartes

F-67084 Strasbourg Cedex

Prof. Dr. F. Paulin
Mathematiques
Universite de Paris Sud (Paris XI)
Centre d'Orsay, Bat. 425

F-91405 Orsay Cedex

Prof. Dr. A.A. Ranicki
Dept. of Mathematics
University of Edinburgh
James Clerk Maxwell Bldg.
Mayfield Road

GB- Edinburgh , EH9 3JZ

Prof. Dr. E.K. Pedersen
Matematisk Institut
Odense Universitet
Campusvej 55

DK-5230 Odense M

Prof. Dr. E. G. Rees
Dept. of Mathematics
University of Edinburgh
James Clerk Maxwell Bldg.
Mayfield Road

GB- Edinburgh , EH9 3JZ

Prof. Dr. D. Puppe
Mathematisches Institut
der Universität Heidelberg
Im Neuenheimer Feld 288

6900 Heidelberg 1

Prof. Dr. J. B. Rudjak
MISI im V.V. Kuibyshev
Jaroslavskoe sosse, 26

Moscow 129 337
USSR

Prof. Dr. V. Puppe
Fakultät für Mathematik
der Universität Konstanz
Postfach 5560

7750 Konstanz 1

Prof. Dr. L.C. Siebenmann
Mathematiques
Universite de Paris Sud (Paris XI)
Centre d'Orsay, Bat. 425

F-91405 Orsay Cedex

Prof. Dr. F.S. Quinn
Dept. of Mathematics
Virginia Polytechnic Institute and
State University
460 McBryde Hall

Blacksburg , VA 24061-4097
USA

Prof. Dr. W. Singhof
Mathematisches Institut
der Universität Düsseldorf
Universitätsstraße 1

4000 Düsseldorf 1

Prof. Dr. P. Vogel
Mathematiques
Universite de Nantes
2, rue de la Houssiniere

F-44072 Nantes Cedex 03

Prof. Dr. E. Vogt
Institut für Mathematik I
der Freien Universität Berlin
Arnimallee 3

1000 Berlin 33

Prof. Dr. R. Vogt
Fachbereich Mathematik/Informatik
der Universität Osnabrück
PF 4469, Albrechtstr. 28

4500 Osnabrück

Prof. Dr. F. Waldhausen
Fakultät für Mathematik
der Universität Bielefeld
Postfach 8640

4800 Bielefeld 1

Dr. J. Weidner
Mathematisches Institut
der Universität Heidelberg
Im Neuenheimer Feld 288

6900 Heidelberg 1