MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 40/1988

4-dimensional Manifolds

11.9. bis 17.9.1988

The meeting was organized by S. Donaldson (Oxford) and M. Kreck (Mainz). About 40 participants from the United States, USSR, Japan and Western Europe attended the conference. In about 20 talks reports were given on major developments in the modern theory of 4-manifolds. Most talks were centered around applications of the fundamental theories of Donaldson and Freedman. A very fruitful problem session was organized by Rob Kirby leading to a list which is attached to this Tagungsbericht. The organizers would like to thank Rob Kirby for his excellent job.

Vortragsauszüge

T. TOM DIECK

Acyclic affine surfaces and arrangements of lines

A non-singualr affine surface V over $\mathbb{C}$ is called a homology plane if $\tilde{H}_*(V;\mathbb{Z}) = 0$. Gurjar and Shastri have shown that homology planes are rational: They arise as complements $V = X\setminus D$ of divisors D with ratianal components in rational surfaces X. Let $b\colon X \longrightarrow P^2$ be a contraction and $C = b(D)$ the image divisor, called a plane divisor of V. We classify all arrangements of lines which are plane divisors of homology planes. If a homology plane has a linear plane divisor, then it arises from one of the following arrangements $C(n;t_2,t_3,\dots,t_n)$ of n lines with t_r points of multiplicity r: $C(n;n-1,0,\dots,1,0), n \geq 4$. $C(4;4)$. $C(5;4,2)$. $C(7;3,6)$. $C(9;6,6,2)_1$. $C(9;6,6,2)_2$. $C(10;8,7,1,1)$.

F.QUINN

A guided tour of "Topology of 4-manifolds" by Freedman and Quinn

This lecture gave a survey of topics covered in this recently completed book, with emphasis on new material. In particular an embedding theorem, and the structure of ends of 4-manifolds were described.

The embedding theorem was described in the special case of connected sum decompositions. Suppose N is a closed 1-connected 4-manifold, and W any 4-manifold. Embedding the punctured manifold N^o in W corresponds to a decomposition $W \cong N \# W'$. A neccessary condition is that $\pi_2 W$ with its intersection and self-intersection forms decomposes as $(\pi_2 N \otimes \mathbb{Z}\pi_1 W) \oplus A$. The theorem asserts that if this happens, $\pi_1 W$ is poly-(finite or cyclic), and either $w_2 : \pi_2 W \longrightarrow \mathbb{Z}/_2$ is trivial or w_2 is nontrivial on A, then W does decompose on N # W'. If the w_2 conditions are not satisfied then either W decomposes, or a manifold homotopy equivalent to W with opposite Kirby-Siebenmann invariant decomposes.

Suppose W is a 4-manifold with a connected tame end with poly (finite or cyclic) fundamental group, π. Then Siebenmann's invariant in $\tilde{K}_0(\mathbb{Z}\pi)$ vanishes if and only there is a "weak collar" of the end. A weak collar is a codimension 0 submanifold $V \subset W$ with compact boundary, and such that $\partial V \longrightarrow V$ is a $\mathbb{Z}\pi_1 V$ homology equivalence. If $\pi_1 \partial V \longrightarrow \pi_1 V$ is also an isomorphism then V is a real collar $\partial V \times [0,\infty]$, but this only rarely occurs. This and other results are used to classify ends up to homeomorphism of neigbourhoods in terms of surgery structure sets. This in turn is used to classify isolated fixed points of actions of finite groups on 4-manifolds.

R. FINTUSHEL

Instanton homology of Seifert fibrations

In this talk I described the calculation of R. Stern and myself of Floer's instanton homology $I_*(\Sigma)$ for Seifert homology spheres. The calculation breaks into three parts. First- compute the space $R(\Sigma)$ of conjugacy classes of representations $\pi_1 \Sigma \longrightarrow SU(2)$ for a Seifert homology sphere Σ. This can be carried out explicitly for Brieskorn spheres $\Sigma(p,q,r)$ and for $\Sigma(a_1,a_2,a_3,a_4)$. For fibrations with more singular fibers, advances have recently been made by Bauer and Okonek.

Next, one needs to compute the spectral flow of the operator $*d_a$ from the flat connection α over Σ corresponding to a representation to the product connection. This is equivalent to computing the L^2-index of the self-duality operator on $\Sigma\times\mathbb{R}$ with corresponding asymptotics. The key step is to compactify the end of $\Sigma\times\mathbb{R}$ corresponding to α by attaching the mapping cylinder of the orbit map $\Sigma \longrightarrow S^2$ and to work with the resulting orbifold. One finds that the spectral flow is always odd. This means that when all representations are regular, the ∂-map vanishes, so there is no third step.

This process explicitly calculates $I_*(\Sigma(p,q,r))$. It can be used to calculate any $I_*(\Sigma(a_1,\ldots,a_n))$ where each connected component of $R(\Sigma)$ has a Morse function with critical points only in even degrees. We conjecture this to be true whenever Σ is a Seifert fibred homology sphere.

B. MOISHEZON

<u>Finite quotients of braid groups and algebraic surfaces</u>

Let V be an algebraic surface of general type and $V \xrightarrow{\varphi} \mathbb{C}P^N$ be a 5-canonical embedding of V. Consider the following diagram:

$$\begin{tikzcd} \mathbb{C}P^N \arrow[r, "\alpha"] \arrow[dr, "\gamma"'] & \mathbb{C}P^3 \arrow[d, "\beta"] \\ & \mathbb{C}P^2 \end{tikzcd}$$

where α,β,γ are generic projections. Let $\tilde{V}$ be the image of V in $\mathbb{C}P^3$, $\tilde{D}$ be the singularity set of $\tilde{V}$, D be the image of $\tilde{D}$ in $\mathbb{C}P^2$. Let $f = \gamma|_V$ and $S \subset \mathbb{C}P^2$ be the branch curve of f. Denote by $\Sigma = D \cup S$ and by Σ' an affine part of Σ corresponding to a generic choice of the line at ∞ in $\mathbb{C}P^2$. Let $G = \pi_1(\mathbb{C}^2-\Sigma',*)$. Then $\exists$ a canonical <u>epimorphism</u> $\varphi\colon G \twoheadrightarrow B_n$ where $n = \deg f$ and B_n is the braid group of degree n. There exists also an epimorphism

ψ: $B_n \twoheadrightarrow Sp(n-1,\mathbb{Z}_3)$. Denote by $\tilde{G} = G/_{\{\Gamma_i^3\}}$ where $\{\Gamma_i\}$ are geometrical generators of G. We can factor $\psi \circ \varphi$ through $\tilde{G}$ and get an exact sequence $1 \longrightarrow \mathrm{Ker}\ \pi \longrightarrow \tilde{G}_r \xrightarrow{\pi} Sp(n-1,\mathbb{Z}_3) \longrightarrow 1$. Replacing Ker π by AbKer π we can ask about rk(AbKerπ) and about the corresponding finite invariants. Thus some finite invariants of V can be obtained, which possibly can distinguish components of moduli space of V.

R. STERN

$\mathbb{Z}$-graded instanton homology groups

The computations of the Floer homology groups for the Brieskorn homology spheres $\Sigma(p,q,r)$ given by Fintushel-Stern indicate that certain classical invariants of flat connections for homology 3-spheres Σ^3 are hidden in the Floer instanton groups $\tilde{I}_*(\Sigma)$, $* \in \mathbb{Z}_8$. In this talk we take a closer look at these groups $\tilde{I}_*(\Sigma)$, and define a complex $C_*(\Sigma)$, $* \in \mathbb{Z}$ with $\tilde{C}_k(\Sigma) = \oplus_{j\equiv k(8)} C_j(\Sigma)$. ($\tilde{C}_k$ the Floer chain groups.)
Furthermore, there is a map $\partial: C_j \longrightarrow C_{j-1}$ with $\partial\partial = 0$, thus defining groups $I_*(\Sigma)$, $* \in \mathbb{Z}$. Computations for $\Sigma = \Sigma(p,q,r)$ are discussed and it is shown that for any $n \in \mathbb{Z}$, there is an homology sphere Σ with $I_n(\Sigma) \neq 0$. It is conjectured that there are examples of homology spheres Σ^3 with $\tilde{I}_k(\Sigma) \neq \oplus_{j\equiv k(8)} I_j(\Sigma)$.

A. EDMONDS

Topological Realization of Equivariant Intersection Forms

(joint work in progress with John Ewing)

Let G be a cyclic group of odd prime order p, with chosen generator g. Suppose that G acts locally linearly on a closed, simply connected 4-manifold M with nonempty fixed point set consisting of $n+2$ points with local (complex) representations (a_i, b_i), $i=0,\ldots, n+1$. Consider $V = H_2(M)$ as a ZG-module with nonsingular G-invariant symmetric bilinear form Φ given by intersection numbers. Then:

REP: As a ZG-module V is isomorphic to $T \oplus R$, where $T=Z^n$ and $R=ZG^m$.

GSF: The g-signature of g on (V, Φ) is given in the usual way as a sum of algebraic numbers associated with the fixed point data.

RTOR: The product of the Reidemeister torsions associated with the linear actions on the 3-spheres around each of the fixed points equals the determinant of the Hermitianized intersection form , modulo squares and "trivial units".

If the action arises from an equivariant "almost handle decomposition", with a 0-handle and a collection of 2-handles, capped off by an action on a contractible 4-manifold, then

DET: $\det(\Phi|T) \equiv \dfrac{-b_1\ldots b_n}{a_1\ldots a_n} \times \dfrac{a_o b_o}{a_{n+1}, b_{n+1}}$ (mod p) for some ordering of the fixed point data.

THEOREM. If fixed point data and an equivariant form satisfy the conditions REP, GSF, RTOR, and DET above, then there is a locally linear G action on a 4-manifold realizing the given data and form.

The condition DET may be a consequence of the other hypotheses. The theorem leads to a variety of examples of topological locally linear actions on certain smooth manifolds that are not obviously smoothable actions.

J. MORGAN

Applications of Donaldson's Invariants to Algebraic Surfaces

On an appropriate compactification of the modulispace of anti-self-dual connections one can interpret Donaldson's polynomial invariantes as being cohomology products evaluated on the top homology class. This allows one to define the full set of Donaldson polynomial invariants:

$$\gamma_k(M) \in S^{d(k)}[H^2(M;\mathbb{Z}[\tfrac{1}{2}]) \oplus H^0(M;[\mathbb{Z}\tfrac{1}{2}])]$$

Here M is a closed oriented smooth simply connected 4-manifold, S^* is the symmetric algebra and $d(k) = 8k-3(b_2^+(M) + 1)$. Using these invariants one can show that the moduli space of all algebraic surfaces on a given C^∞-manifold has finitely many components. This result was proved jointly with R. Friedman.

D. KOTSCHICK

Gauge theory on 4-manifolds with $b_2^+ = 1$

In his talk, John Morgan explained the definition and application of Donaldson's polynomial invariants for smooth manifolds with $b_2^+ \geq 3$. My talk covered the case $b_2^+ = 1$. I discussed the basic problem, namely that reducible connections appear in 1-parameter families of metrics. This gives rise to a chamber structure on the positive cone in $H_+^2(X,\mathbb{R})$, and the invariants defined using Donaldson's method will, in general, depend on the choice of a chamber, coming from the choice of Riemannian metric.

However, using SO(3)-bundles (rather than SU(2)), allows easier arguments. The advantage is twofold: firstly, a clever choice of second Stiefel-Whitney class sometimes makes reducible connections impossible; secondly, whenever $w_2 \neq 0$, the trivial connection does not appear in any moduli space. To illustrate this, a differential invariant for smooth, closed, oriented 4-manifolds with $\mathrm{sign}(X) \equiv 1 \bmod 8$ was defined. This was then used to prove the following:

Theorem ([K]): The Barlow surface is not diffeomorphic to $\mathbb{C}P^2 \# 8\, \overline{\mathbb{C}P}^2$.

The point of this is, of course, that the two manifolds are homeomorphic by Freedman's classification of 1-connected 4-manifolds. Recall that the Barlow surface is an algebraic surface of general type with $q = p_g = 0$, $K^2 = 1$.

We indicated the definition of more general SO(3)-invariants Φ_k, associated with moduli spaces of anti-self-dual connections on bundles with $p_1 = -3-k$. Two theorems about the behaviour of these invariants under connected summing with $\overline{\mathbb{C}P}^2$ were given. They imply:

Theorem [K2]: If two smooth manifolds are shown not to be diffeomorphic by a suitable Φ_k, then they are still non-diffeomorphic after connected summing with arbitrarily many copies of $\overline{\mathbb{C}P}^2$, and this is again detected by a suitable Φ-invariant.

This has many obvious corollaries, for example the Barlow surface is still exotic after arbitrary blow-ups. This is relevant in connection with a conjecture of Hambleton and Kreck mentioned in the talk of Ian Hambleton.

References

[K] D. Kotschick: On manifolds homeomorphic to $\mathbb{C}P^2 \# 8\overline{\mathbb{C}P}^2$, Invent.math. (to appear)

[K] - : Oxford thesis.

I. HAMBLETON

Topological Classification of 4-manifolds

(joint work with M. Kreck)

In the talk, I gave a survey of some results about the classification of closed, oriented topological 4-manifolds X, with $\pi_1 X$ finite.

Theorem 1

There are only finitely many homeomorphism types of such manifolds with given finite π_1 and given Euler characteristic.

To obtain a precise statement for a particular π_1, we first attempt to understand the stable classification (up to connected sums with $S^2 \times S^2$'s), and then solve a cancellation problem. For example,

Theorem 2

Let X^4 be closed, oriented 4-manifold with $\pi_1 X$ odd order cyclic. Then X is determined up to homeomorphism by the intersection form on $H_2(X;\mathbb{Z})/_{\text{Tors}}$ and the Kirby-Siebenmann invariant.

In general however, we can only solve the cancellation problem in the following context:

Theorem 3

Let X_o, X_1 be closed oriented 4-manifolds with π_1 finite and $X_o \# r (S^2 \times S^2) \approx X_1 \# r (S^2 \times S^2)$ for some $r \geq 0$. If $X_o \approx X_o' \# 2 (S^2 \times S^2)$, then $X_o \approx X_1$.

The stability assumption in Theorem 3 is not too restrictive for the study of algebraic surfaces. Since algebraic surfaces are known to be smoothly indecomposable as a connected sum unless one part is negative definite (Donaldson), a homeomorphism classification will often imply the existence of distinct smooth structures on algebraic surfaces.

Theorem 4

Let G be a finite group. There is a constant $c(G)$ such that for any algebraic surface X with $\pi_1 X \cong G$, $e(X) \geq c(G)$ and $c_1^2(X) \geq 0$, there is a smooth manifold Y, homeomorphic to X, but $Y \# s(\overline{\mathbb{C}P}^2)$ is not diffeomorphic to $X \# s(\overline{\mathbb{C}P}^2)$ for any $s \geq 0$.

P. TEICHNER

Rational homology 4-spheres

The manifolds under consideration are always closed, oriented, topological 4-manifolds.

Definition: Such a manifold M is called a rational homology 4-sphere if

$$H_*(M;\mathbb{Q}) \cong H_*(S^4;\mathbb{Q}).$$

Problem: Which groups are realized as fundamental groups of rat.hom.4-spheres?

The answer to this question is unknown in general, but for abelian groups I have shown the

<u>Theorem</u>: An abelian group is realized as the fundamental group of a smooth rat.hom. 4-sphere if and only if it is finite and can be generated by at most 3 elements.

If one looks for homology 4-spheres (not just rational!), the fundamental group π has to be superperfect, i.e. $H_1(\pi;\mathbb{Z}) = \{0\} = H_2(\pi;\mathbb{Z})$. One class of such groups are the 2x2-matrices over GF(p) with determinant 1, namely the groups $SL_2(p)$. By taking a suitable 3-manifold N and performing surgery on an embedded curve in $N \times S^1$ I constructed these groups as fundamental groups of homology 4-spheres. Moreover, for every $p \equiv 3,5 \pmod 8$ the two different framings of the embedded 1-sphere give in fact two homology 4-spheres Σ_o, Σ_1 such that for any r $\Sigma_o \# r(S^2 \times S^2)$ is <u>not</u> homoemorphic to $\Sigma_1 \# r(S^2 \times S^2)$. This makes it possible to apply a classification program of I. Hambleton and M. Kreck:

<u>Theorem</u>: Let M,N be 4-manifolds with $\pi_1 \cong SL_2(p)$, $p \equiv 3,5(8)$.

If M and N are both spin or non-spin and the signature, eulercharacteristic and a $Z/_2$-valued invariant (which is the Kirby-Siebenmann invariant in the non-spin case) agree, then M and N are homeomorphic, provided $\chi(M) \geq |\sigma(M)| + 6$ (resp. $\chi(M) \geq |+8$ if $w_2(M) = 0$ and $\sigma(M) = 0$.)

Another application of the existence of two stably non-homeomorphic (differentiable) homology 4-spheres can be proved by using S. Donaldson's result on the indecomposebility of algebraic surfaces as connected sum of diff. manifolds:

Theorem: An algebraic surface X with fundamental group $SL_2(p)$, $p \equiv 3,5\ (8)$ has an exotic structure, stable under blow up, i.e. under connected sum with $\overline{\mathbb{C}P}^2$, if $c_2(X) \geq 13$ and $c_1^2(X) \geq 0$.

R.E. GOMPF

Connected sums of algebraic surfaces

Mandelbaum and Moishezon showed that for M^4 a complete intersection or simply connected elliptic surface, $M \# \mathbb{C}P^2$ is diffeomorphic to a connected sum of $\pm\ \mathbb{C}P^2$. Similarily, for M simply connected, elliptic, and spin, $M \# S^2 \times S^2$ is diffeomorphic to $\#_k K3 \#_1 S^2 \times S^2$ (Mandelbaum). The following generalizations can now be proven:

Theorem 1. Let M_1 and M_2 be simply connected algebraic surfaces $\neq \mathbb{C}P^2$. If either is of general type, assume it is a complete intersection. If at least one of the manifolds is not spin, then $M_1 \# -M_2$ is diffeomorphic to $\# \pm \mathbb{C}P^2$.

Theorem 2. Let M_1 and M_2 be simply connected elliptic surfaces. Suppose both are spin. Then $M_1 \# -M_2$ is diffeomorphic to $\pm (\#_k K3 \#_1 S^2 \times S^2)$.

A similar decomposition result holds for fiber sums of elliptic surfaces with incompatible orientations.

In each case, the choice of orientations seems crucial. The sum operation introduces embedded spheres of positive square to an algebraic surface which is essentially "negative", destroying its structure, and yielding a sum of simple pieces. An open question is: What can be said about sums of irrational algebraic surfaces with compatible orientations?

G. MASBAUM

The integer cohomology algebra of the classifying space of the gauge group on a 1-connected punctured 4-manifold

Let $\mathcal{B}^*$ = moduli space of irred. connections on a SU(2)-bundle over X^4, $\pi_1(X^4) = 0$. Then it is well known that $H^*(\mathcal{B}^*,\mathbb{Q}) = \mathrm{Sym}(H_2(X;\mathbb{Q})) \otimes \mathbb{Q}\,[p]$. The "formal definition" of S. Donaldson's polynomial invariants pairs this cohomology with the moduli spaces of anti-self-dual connections. One may want to know the algebra $H^*(\mathcal{B}^*;\mathbb{Z})$. One step in this direction that is accomplished here, is the determination of the integer cohomology algebra of the classifying space of the gauge group over the 2-skeleton of X. We call this algebra A(L), where $L = H_2(X;\mathbb{Z})$.

We show: A(L) is the $\mathbb{Z}[p]$-algebra generated by elements $\mu_n(\alpha)$ of degree 2n, with relations determined by $\mu_0(\alpha) = 1, \mu_1$ is linear, and

$$\sum_{n=0}^{\infty} \mu_n(\alpha) = \exp(\mu_1(\alpha)\,\frac{\arctan\sqrt{P}}{\sqrt{P}}).$$

Moreover, if $\alpha \in L$ is indivisible, then $\mu_1(\alpha)^n \in A(L)$ is divisible exactly by the power of 2 contained in n!. Since μ_1 corresponds to Donaldson's map $\mu\colon L \longrightarrow H^2(\mathcal{B}^*;\mathbb{Z})$, it follows that any divisibility of the invariant $q(\alpha) = \langle \mu(\alpha)^n, [M] \rangle$ by odd primes cannot come from the 2-skeleton of X. This is because the subalgebra of $H^*(\mathcal{B}^*;\mathbb{Z}[\frac{1}{2}])$ generated by the image of μ, is naturally isomorphic to $A(L) \otimes \mathbb{Z}[\frac{1}{2}]$.

For more detail, see C.R.Acad.Sci. Paris, t.307, Serie I, p. 339-342, 1988.

T.D. COCHRAN

Homology 3-sphers with non-trivial Floer homology but vanishing Casson's invariant

In all previously computed cases, the chain groups for the Floer homology of a homology 3-sphere have been able to be made trivial in all even dimensions (or all odd).

R.E. Gompf and myself have exhibited homology 3-spheres with trivial Casson's invariant but which bound 4-manifolds with positive definite (non-standard) intersection forms.

It is a theorem (claimed by Floer, Donaldson, Taubes) that Donaldson's original theorem works for 4-manifolds with boundary Σ as long as $HF_*(\Sigma) \equiv 0$.

As a consequence of the two previous paragraphs, the homology spheres Σ (of Gompf and myself) have $HF_*(\Sigma) \neq 0$ but Casson's inv. = 0, hence are different from the previously observed examples.

T. MATUMOTO

Riemannian metrics on the moduli space of 1-instantons over a 4-manifold

We have three Riemannian (semi-)metrics of type I, II and I-II on the moduli space $\mathcal{M}$ of 1-instantons over a Riemannian 4-manifold M. Over S^4 type II is the hyperbolic metric with negative constant curvature $-5/32\pi^2$, type I-II is complete and has negative sectional curvatures and type I is bounded and has positive sectional curvatures everywhere (Matumoto, Matumoto, Doi-Matsumoto-Matumoto (resp.)). Over CP^2 with the Fubini-Study metric type II and type I-II are asymptotically $(32\pi^2/5)(dx^2 + cg_M)/x^2$ near the collar and type I is bounded and asymptotic behavior is also studied (K. Kobayashi). Type I metrics over S^4 and CP^2 are also studied by Groisser-Parker. Moreover over a simply-connected Riemannian 4-manifold with positive definite intersection form it is proved that the metrics are asymptotically similar to the one for the standard case in the C^0 sense near the collar (Groisser-Parker (type I), Doi-K.Kobayashi).

O. VIRO

(joint work with S. Finashin and M. Kreck)

Exotic knottings of surfaces in the 4-spher

In the talk a proof of the following result was sketched

Theorem: There exists an infinite series S_1, S_2,... of smooth submanifolds of S^4 such that:

(1) for any i,j the pairs (S^4,S_i), (S^4,S_j) are homeomorphic via a map restricting to a diffeomorphism between appropriate neighborhoods of the surfaces.

(2) for any $i \neq j$ the pairs (S^4,S_i), (S^4,S_j) are not diffeomorphic:

(3) each S_n is homeomorphic to the connected sum $\#\mathbb{RP}^2$ of 10 copies of the real projective plane;

(4) $\pi_1 (S^4,S_n) = \mathbb{Z}_2$

(5) the normal Euler number (with local coefficients) of S_n in S^4 is 16.

The proof uses recent results of Donaldson, Friedman and Morgan and independently Okonek and van de Ven about Dolgachev surfaces implying that for odd q and q', $q \neq q'$, $D_{2,q}$ is not diffeomorphic to $D_{2,q'}$. We construct antiholomorphic involutions such that the orbit space is S^4 and the fixed point set is $\#_{10} \mathbb{RP}^2$. These are knottings (S^4, S_q). On the other hand we prove that the number of homeomorphism types in the sense of (1) of these knottings is finite implying the theorem.

St. BAUER

(joint with: Christian Okonek)

Moduli spaces of flat SO(3)-bundles on Dolgachev surfaces

Let $\Sigma = \Sigma(a_1,\ldots,a_n)$ denote a Seifert-fibred homology 3-sphere with multiple fibres of multiplicities $a_1,\ldots,a_n$. There exist algebraic Dolgachev surfaces $X = X(a_1,\ldots,a_n)$ with fundamental group $\pi_1(X(a_1,\ldots,a_n) \cong \pi_1(\Sigma(a_1\ldots a_n))/_{\text{center}}$. We are interested in the space $R(\Sigma) = \mathrm{Hom}^*(\pi_1(\Sigma), SO(3))/_{SO(3)}$ of irreducible representations of $\pi_1(\Sigma)$. Using a translation into antiselfdual connections, lifting those to Hermite-Einstein-connections on appropriate U(2)-bundles over X and finally applying the solution of the Kobayashi-Hitchin conjecture by Donaldson, we get:

Theorem 1: $R(\Sigma) \cong M_X^H(K_X,0) \amalg M_X^H(0,0) = \mathcal{M}$.

Here $M_X^H(c_1,c_2)$ denotes the moduli space of H-stable bundles with Chern classes c_1 and c_2 with respect to an ample line bundle H.

Theorem 2: $\mathcal{M}$ is a finite disjoint union of complex projective algebraic varieties, each of which admits a stratification by Zariski-open subsets of projective spaces..

Theorem 3: Each component of $\mathcal{M}$ is smooth and rational.

These results apply to a

Conjecture (Fintushel and Stern): $R(\Sigma)$ admits a Morse function with even indices only.

Theorem 3 proves the conjecture in the case $n \leq 5$. Since $\pi_1(R(\Sigma),*) = 0$ by Thm. 3, an old result of Smale shows that the conjecture is equivalent to:

Conjecture': $H^{2*+1}(R(\Sigma);\mathbb{Z}) = 0$.

There is also a method of computing some of the components of $R(\Sigma)$.

L.R. TAYLOR

PIN structures on low dimensional manifolds

This talk describes joint work with R. Kirby. We define a Spin structure, roughly, as a way of giving a prefered class of framings to the normal bundle of embedded circles.

A Pin structure on a manifold is a Spin structure on the total space of the determinant line bundle for the tangent bundle. This is not the only type of Pin structure that there is and we have results on the other, but this was the case described in the talk.

There are two cases in which we can put a Pin structure naturally on a codimension 1 submanifold. If the normal bundle is trivial there is no problem: Pin structures on the total space correspond to Pin structures on the codimensional 1 submanifold. If the big manifold is oriented, then Spin structures on the big manifold correspond to Pin structures on the submanifold.

In dimension 3 we can give a geometric formulation of some results of Turaev (Math. USSR Sbornik (1983) 120 p. 68-83, or # 48 (1984) 65-74 (translation)). In particular we give a geometric description of the function which describes how the Rochlin invarinat changes when the Spin structure changes on an oriented 3 manifold. This involves an explicit identification of the 2 dimensional Pin bordism group with Z/8Z which we do via Brown's Arf invariant formula.

In dimension 4 we give an explicit way to put a Pin structure on a surface dual to $w_2 + w_1^2$. We can use this to recover and improve results of Freedman-Kirby and Guillou-Marin. This was discussed.

M. UE

Geometric 4-manifolds in the sense of Thurston and Seifert 4-manifolds

We will characterize the closed orientable geometric 4-manifolds of eight types in terms of the Seifert 4-manifolds. Here by Seifert 4-manifolds we mean the fibred orbifolds over some 2-orbifolds with general fiber T^2 whose total spaces are nonsingular. They are described by Seifert invariants analogous to those for Seifert 3-manifolds. The correspondence between them and the geometries are given by the following table which generalizes the one for elliptic surfaces with $c_2 = 0$ by C.T.C. Wall.

The types of the bases				
spherical or bad	$S^2 \times E^2$	$S^3 \times E^1$		
euclidean	E^4	$Nil^3 \times E^1$	Nil^4	$Sol^3 \times E^1$
hyperbolic	$H^2 \times E^2$	$\widetilde{SL}_2 \times E^1$	non-geometric	

(There is one closed orientable euclidean 4-manifold which is not a Seifert 4-manifold in the above sense.)

D. WILCZYNSKI

Embedding 2-spheres in 1-connected 4-manifolds

(joint with Ronnie Lee)

When is a homology class $x \in H_2(M;\mathbb{Z})$ represented by a locally flat, topological embedding $f: S^2 \longrightarrow M$? If x is so represented, how many different embeddings are there?

We give fairly complete answers to both questions for simple embeddings representing homology classes of odd divisibility. (An embedding $f: S^2 \longrightarrow M$ is called simple if $\pi_1(M - f(S^2))$ is abelian.)

Theorem 1 Let $x \in H_2(M)$, $x = ny$ (y primitive, $n \in \mathbb{Z}$ odd). Assume that the intersection form λ of M is indefinite. Then x is represented by a locally flat, simple embedding $f: S^2 \longrightarrow M$ iff

(i) $\frac{1}{8}[\lambda(x,x)-\mathrm{sign}(M)] \bmod 2 = KS(M)$ for characteristic class x,

(ii) $b_2(M) \geq \max\limits_{0\leq j<n} |\mathrm{sign}(M) - 2j(n-j)m|$, $m = \lambda(y,y)$.

The assumption of indefinite λ is not always needed. This is the case (for instance), when M has a smooth structure.

Theorem 2. On the situation of Theorem 1, assume that (n odd)

$$b_2(M) \geq \max_{0\leq j<n} |\mathrm{sign}(M) - 2j(n-j)m| +2.$$

Then any two simple embeddings $KS^2 \longrightarrow M$ representing x are isotopic (through simple embeddings).

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4-MANIFOLD PROBLEMS

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PROBLEM 1 (Arnol'd): Can two homeomorphic but not diffeomorphic 4-manifolds be distinguished by the homotopy types of strata in the spaces of smooth functions on them? Can the homotopy types be determined by link calculus data?

PROBLEM 2 (Stoltz): Conjecture: the stable diffeomorphism class of a closed M^4 for fixed π_1 is determined by the spectra of certain differential operators on M.

PROBLEM 3 (Viro): Any smooth, oriented, closed surface in R^4 can be isotoped so that the restriction to the surface of the standard projection $R^4 - R^1 \to S^2$ will be a branched covering. (This presentation of 2-links is an analogue of the closed braid presentation for classical links.) Find analogues of Markov's moves.

PROBLEM 4 (Viro): Does there exist a real algebraic, plane, projective, nonsingular curve of degree $m > 3$ with maximal number of components ($= \frac{(m-1)(m-2)}{2} + 1$) such that one of its ovals encircles all the others? For $4 \leq m \leq 8$ the answer is known to be no.

PROBLEM 5 (Viro): Find a combinatorial or link calculus version of Donaldson's invariants.

PROBLEM 6 (Viro): Find a lower bound for the genus of a smooth, embedded surface realizing $\alpha \in H_2(M^4; Z)$ in terms of the intersection form and Donaldson's invariants. Just conjecturing a lower bound is an interesting problem.

PROBLEM 7 (Viro): Let $L(p,q)$ denote the 4-manifold obtained from $S^2 \times S^2$ by a pair of logarithmic transforms of type p and q, (p,q)=1, along tori realizing $(1,0) \in H_2(S^2 \times S^2)$. Is $L(p,q)$ diffeomorphic to $S^2 \times S^2$? Diffeomorphic to $L(p',q')$ The same for $CP^2 \# \overline{CP}^2$ and the class (1,1)? Are these manifolds algebraic?

PROBLEM 8 (Kreck): "See" the homology 3-spheres which split a simply connected Dolgachev surface according to the splitting of the intersection form as $\langle 1 \rangle + 9\langle -1 \rangle$.

PROBLEM 9 (Fintushel): Let Σ be a Seifert fibered, homology 3-sphere and let $\mathcal{R}(\Sigma)$ be the space of conjugacy classes of representations $\pi_1(\Sigma) \to SU(2)$. Conjecture: Each connected component of $\mathcal{R}(\Sigma)$ admits a Morse function having critical points only in even degrees.

This was just proved (November, 88) by Kirk and Klassen at Cal. Tech.

Remarks:- 1. This would facilitate computations of the Floer homology of Σ.

2. When Σ has 3 exceptional fibers, $\mathcal{R}(\Sigma)$ is finite. S. Bauer and C. Okonek have shown that the components of $\mathcal{R}(\Sigma)$ are smooth, rational, algebraic varieties (and therefore satisfy the conjecture) when Σ has 4 or 5 exceptional fibers.

PROBLEM 10 (Morgan): Let Σ be a homology 3-sphere. Suppose all representations of $\pi_1(\Sigma)$ to $SU(2)$ are regular (i.e. smooth points of the representation variety). Does the Casson invariant of Σ depend, up to sign, only on $\pi_1(\Sigma)$? For example, if H is a homotopy sphere, are the Casson invariants of Σ and $\Sigma \# H$ equal?

PROBLEM 11 (Fintushel): Must the Floer homology of a nontrivial, connected sum contain 2-torsion? (assuming each side has nontrivial SU(2)-representations).

PROBLEM 12 (Akbulut): Is there a Lefchetz fixed point theorem for Floer homology?

PROBLEM 13 (Viro): Conjecture: Let X be a simply connected, complex surface and $\tau : X \to X$ be an antiholomorphic involution. Then $dim H_1(fix(\tau) : Z/2) \leq h^{1,1}(X)$.

This conjecture generalizes an old Ragsdale-Petrovsky conjecture: if A is a real non-singular plane projective curve of an even degree $m = 2k$, and B_+ is an orientable domain in RP^2 with $\partial B_+ = A$, then

$$dim H_0(B_+; Z/2) \leq \frac{3k(k+1)}{2} + 1$$

$$dim H_1(B_+; Z/2) \leq \frac{3k(k+1)}{2} + 1$$

PROBLEM 14 (Donaldson): Exotic structures:

1. Are there exotic structures on simply connected 4-manifolds with b_2^+ even?

2. Are there exotic structures on 4-manifolds of the form S/σ where S is a complex, algebraic surface and $\sigma : S \to S$ is a real involution?

Remarks: There are no exotic structures in the case that S is a $K3$ surface. For there is (Yau) a Kahler-Einstein metric which is invariant under σ which gives a family of complex structures on S (an action of the quaternions I, J, and K on TS). I is the original complex structure, and with respect to one of the new structures, say J, σ is a holomorphic map. So $(S, J) \to (S/\sigma)$ is a holomorphic branched cover over a complex surface. From surface theory, S/σ is a rational surface (if σ is not free) diffeomorphic to $CP^2 \sharp s(\overline{CP}^2)$, or an Enriques surface (the Habegger manifold) (if σ is free). This problem is related to the previous problem since the genus of $fix(\sigma)$ is related to the homology of S/σ.

PROBLEM 15 (Donaldson): Symplectic structures:

1. First recall that homotopy theoretically almost complex structures and almost symplectic structures are equivalent. The only known compact simply connected symplectic 4-manifolds arise from complex Kahler surfaces by using the Kahler 2-form to define the symplectic 2-form ω. Are there any others? In particular, are there symplectic structures on the almost complex manifold $CP^2 \sharp CP^2 \sharp CP^2$?

A more precise version is, if X is a Kahler surface and ω is a symplectic structure on X then is ω equivalent to a Kahler structure, i.e. is there a diffeomorphism $f : X \to X$ such that $f^*(\omega)$ is the 2-form corresponding to the Kahler metric. Or, weaker, is the first Chern class of the almost complex structure defined by ω equal to the first Chern class of a complex structure? (This is related to the genus of embedded surfaces and diffeomorphisms of X).

2. If (M, ω) is a symplectic manifold and $[\omega] \in H^2$ is integral, is it true that for $N \gg 0$ the Poincare dual of $N[\omega]$ is represented by a symplectic (i.e. $\omega \mid \Sigma$ is non-degenerate) surface Σ? Note that an affirmative answer means we can predict $genus(\Sigma)$.

PROBLEM 16 (Donaldson): Invariants and surgery:

1. Can one give formulae for the effect of generalized "logarithmic transforms" on the Donaldson invariants (via extension of Floer homology etc.)?

2. If $\tilde{X} \to X$ is a branched cover branched over a 2-manifold B, can one give formulae for the Donaldson invariants of $\tilde{X}$ in terms of those of X and the restriction map from connections on X to connections on B? (This problem should be more tractable in case B can degenerate into a "double surface" in which case $\tilde{X}$ degenerates into $X \cup_B X$.)

PROBLEM 17 (Hambleton and Kreck): Conjecture: Let X be an algebraic surface. Then there exists a smooth 4-manifold Y with

(i) Y homeomorphic to X, and

(ii) $Y \sharp s(\overline{CP}^2)$ not diffeomorphic to $X \sharp s(\overline{CP}^2)$ for any $s \geq 0$.

Remarks 1. Work of Donaldson, Freedman, Friedman-Morgan and Kotschick established the conjecture for 1-connected surfaces except for $X = CP^1 \times CP^1, CP^2 \sharp s(\overline{CP}^2)$ for $s \leq 7$.

2. Hambleton and Kreck have shown that for any finite group G, the conjecture is true for minimal, algebraic surfaces X with $\pi_1(X) = G$, and sufficiently large Euler characteristic $e(X) \geq c(G)$ where $c(G)$ depends only on G. The conjecture is true for X such that $\pi_1(X) \cong Z/k$, k odd, $k \neq 1$.

PROBLEM 18 (Stern): Which $\Sigma(p,q,r)$ embed in the standard $K3$ surface? Prediction: only finitely many embed.

PROBLEM 19 (Stern): (a) Find an example of a homology 3-sphere Σ^3 with vanishing Floer homology groups.

(b) Do all $\pi_1(\Sigma^3)$ represent in $SO(3)$?, $SO(n)$?, $PSL(2,C)$?

(c) If K is a knot, do all $\pi_1(S^3 - K)$ represent (non-Abelian) in these groups?

PROBLEM 20 (Stern): 1. Which groups can occur as fundamental groups of homology 4-spheres?

2. Find an example of a group G with G realized as the fundamental group of a TOP homology 4-sphere, but no DIFF homology 4-sphere.

3. More generally, having fixed the intersection form I, what restrictions are placed on $\pi_1(M^4)$ with M^4 having intersection form I and M^4 either TOP or DIFF?

PROBLEM 21 (Stern): Does $CP^2 \sharp n\overline{CP}^2$, $-\infty < n \leq 7$ possess more than one smooth structure?

PROBLEM 22 (Stern): Find an example of a homology 3-sphere Σ^3 with no solutions to the anti-self-dual equation over $\Sigma^3 \times R$ with finite action where the virtual dimension is positive.

PROBLEM 23 (Stern): Does there exist a homology 3-sphere Σ^3 with $\mu(\Sigma^3) = 1$ and such that $\Sigma \sharp \Sigma$ bounds an acyclic 4-manifold?

Comments: (i) Yes iff all TOP n-manifolds, $n \geq 5$, are simplicial complexes.

(ii) One can rule out any $\Sigma(p,q,r)$ with $R(p,q,r) \geq 1$ (see Fintushel-Stern, Pseudo-free orbifolds, for the definition of $R(p,q,r)$).

PROBLEM 24 (Stern): Does Θ_3^H contain any torsion elements?

PROBLEM 25 (Hillman): Which smooth structures on R^4 have compatible Stein structures?

Remarks: Recall that a Stein manifold is a proper, nonsingular, complex submanifold of some C^n, e.g. an affine, algebraic manifold over C. All exotic R^4's admit complex structures (and even Kahler metrics) since any contractible 4-manifold immerses smoothly in $R^4 = C^2$. On the other hand, an affine, algebraic surface which is contractible and 1-connected at ∞ must have the standard C^∞-structure by Ramanujam, Annals 1973.

PROBLEM 26 (Hillman): Can one obtain "exotic pairs" of TOP equivalent, but DIFF inequivalent smooth 2-knots in homotopy 4-spheres by surgery on simple closed curves in (homotopy equivalent, but non-diffeomorphic) complex, analytic surfaces? (e.g. $S^1 \times S^3$ or T^3 bundles over S^1).

Remark: There are knots with solvable groups which arise from such a construction on certain Hopf, Inoue, secondary Kodaira . . . surfaces (these are never algebraic since we need $\beta_1 = 1$).

PROBLEM 27 (Ruberman): Which rational homology 3-spheres have a TOP (locally flat) embedding in S^4?

Remark: The known necessary conditions are that the linking pairing be hyperbolic and that α-invariants associated to certain coverings of prime-power degree vanish (Gilmer-Livingston, Topology, 22 (1983) 241-252). Are these conditions sufficient? Analogous conditions in high dimensions suffice (Cappell-Ruberman, Comm. Math. Helv. 63 (1988) 75-88). By work of Fintushel-Stern (Topology, 26 (1987), 385-394), Matic (J. Diff. Geom. 28 (1988) 277-308) and Ruberman (Topology, to appear), in the smooth case one gets restrictions on α-invariants of all orders.

PROBLEM 28 (Taylor): Let $S(k)$ denote the connected sum of k copies of $S^2 \times S^2$. For each integer $k \geq 3$, find an example of a smoothing of R^4 which embeds smoothly in $S(k+1)$ but not in $S(k)$.

Remark: For $k = 1$, we can represent two hyperbolic pairs in the Kummer surface by two pairs of Casson handles which also smoothly imbed in $S(2)$: the complement of the TOP cores of these Casson handles is an exotic R^4 in $S(2)$ which cannot smoothly imbed in $S^2 \times S^2$ because this would allow us to split one hyperbolic off the Kummer surface, contradicting Donaldson. Similarly for $k = 2$.

PROBLEM 29 (Taylor): Let X^4 denote either $S^3 \times S^1$ or the twisted S^3 bundle over S^1. In both cases, $H^3(X; Z/2) = Z/2$ so there is a smoothing, $(X \times R^1)_\Sigma$ of $X \times R^1$ and a homeomorphism $h : (X \times R^1)_\Sigma \to X \times R^1$ whose obstruction to being isotopic to a diffeomorphism is the non-zero element of $Z/2$. Find such a smoothing of X itself or prove that there does not exist one.

Remarks: Scharlemann (for $X = S^3 \times S^1$) and Akbulut and Fintushel-Stern (Contemp. Math. vol. 35, 269-276) (for $X = S^3 \tilde{\times} S^1$) have constructed such smoothings on $X \# S^2 \times S^2$.

If such smoothings exist, then all the smoothings on M^4 predicted by $H^3(M;Z/2)$ would occur.

PROBLEM 30 (Quinn): Is the uniqueness obstruction for topological connected sums realized? More precisely, suppose we are given a 1-connected N and a homeomorphism $h : N\sharp M \to N\sharp M'$ such that the inclusions of the punctured N are homotopic. Then there is an obstruction to homotoping h to a homeomorphism which is the identity on the punctured N, thereby giving a homeomorphism from M to M'. The obstruction lies in $H^2(N;Z[T_+])$, where

$$T_+ = \{g \in \pi_1(W^n) | g^2 = 1, g \neq 1, w_1(g) = 1 \in Z/2 = \{\pm 1\}\}.$$

For example, is the decomposition of $X^4 = N\sharp RP^3 \times S^1$ determined by the homotopy class of $N_0 \to X$? (here T_+ has one element). (See Chapter 10 of Freedman-Quinn, "Topology of 4-manifolds".)

PROBLEM 31 (Gompf): Conjecture: If the connected sum (or fiber connected sum of elliptic surfaces) of two complex, algebraic surfaces (other than CP^2), one with its natural orientation reversed, is simply connected, then it always decomposes as a connected sum into copies of $\pm CP^2$ in the odd (non-spin) case and copies of $K3$ and $S^2 \times S^2$ in the even (spin) case.

Remark: The conjecture is true if both surfaces are elliptic; it is also true if one of the surfaces is not spin except possibly when at least one surface has general type and is not a complete intersection (see Gompf, Inv. Math.).

PROBLEM 32 (Morgan): Let X and Y be non-rational algebraic surfaces, e.g. elliptic surfaces. What kind of connected sum decompositions does $X\sharp Y$ have?

PROBLEM 33 (Gompf): Let M be a simply connected, algebraic surface. Is $M\sharp CP^2$ diffeomorphic to a connected sum of copies of $\pm CP^2$? (This is trivially true for M rational, and true for M elliptic or a complete intersection by Mandlebaum-Moishezon.) If M is also spin, is $M\sharp S^2 \times S^2$ diffeomorphic to a connected sum of copies of $\pm K3$ and $S^2 \times S^2$? (True for M elliptic by Mandlebaum.)

PROBLEM 34 : Does there exist a non-trivial, algebraic surface without a smoothly imbedded (holomorphic?) 2-sphere with self intersection -2?

PROBLEM 35 (Morgan): Find an algebraic surface X and an orientation-preserving diffeomorphism $\phi : X \to X$ such that $\phi^*(k_X) \neq \pm k_X$ where k_X is the canonical class of X. Can this be done with the extra condition that $b_2^+(X) > 1$? Can a ϕ be found such that $\phi^*(k_X) \neq mk_X$ for any $m \in Q^*$?

PROBLEM 36 (Moishezon): Let $f : X \to CP^2$ be a stable morphism (generic projection) of an algebraic surface X to CP^2. Denote by $\tilde{f} : \tilde{X} \to CP^2$ the Galois covering. We know an important infinite series of examples when $\pi_1(\tilde{X})$ is a finite, abelian group for $\pi_1(X) = 0$.

Question: for $\pi_1(X) = 0$, is it true that $\pi_1(\tilde{X})$ is always abelian? (We know that when $X = V_2$ (the Veronese surface of order two and f a generic projection to CP^2) then $\pi_1(\tilde{X}) = Z \oplus Z \oplus Z \oplus Z$.)

Question: are there any other examples of X with $\pi_1(X) = 0$ and $\pi_1(\tilde{X})$ infinite?, or is $\pi_1(\tilde{X})$ finite but for a few exceptional cases?

Remark: surfaces like $\tilde{X}$ above almost always have $\tau(\tilde{X}) > 0$.

PROBLEM 37 (Fintushel): Is there a topological 4-manifold with nontrivial Kirby-Siebenmann invariant on which S^1 can act? (Such an action cannot be locally smooth.) If so, can S^1 act on the $*CP^2$ (the Chern manifold, i.e. the non-smoothable, homotopy CP^2)?

PROBLEM 38: Are there any "exotic" smooth, finite group actions (orientation-preserving) on any 4-manifolds? Here "exotic" can have many different meanings. For example:

(a) If X is a compact, algebraic surface and G acts smoothly on X inducing the identity on homology, is G isomorphic to a subgroup of the algebraic automorphism group of X? (Hambleton-Lee).

(b) Let X be a smooth, simply connected, negative definite 4-manifold. If G acts smoothly on X inducing the identity on homology, then is G isomorphic to a subgroup of $PGL_3(C)$? (Hambleton-Lee). (Yes if X is homeomorphic to $\overline{CP}^2$ by Hambleton-Lee and Wilczynski.)

(c) Are there any smooth, cyclic group actions on a connected sum of CP^2 which are not equivalent to a connected sum of linear actions? Edmonds and Ewing have TOP locally linear examples. (Fintushel).

PROBLEM 39 (Edmonds): Conjecture: None of the natural periodic maps on Brieskorn homology 3-spheres $\Sigma(p,q,r)$ extend to smooth periodic maps of contractible 4-manifolds.

Remarks: (1) Those with fixed points do not, for signature reasons (otherwise some torus knot would be homotopically slice).

(2) These natural actions do extend smoothly to the plumbing manifolds with boundary Σ. Look for "minimal" non-plumbing manifolds over which the actions extend.

(3) For the fixed point free case, a TOP locally linear extension to a contractible 4-manifold exists iff the associated $\alpha - invariant$ and Reidemeister torsion correspond with those of some linear action on S^3. As a specific question then, does the free involution on $\Sigma(3,5,19)$ extend smoothly to a contractible 4-manifold? (A computer search does turn up other examples of periodic maps with appropriate invariants.)

PROBLEM 40 (Edmonds): Conjecture: Any Z/p action on a homology 3-sphere Σ extends to a TOP Z/p action on a contractible 4-manifold.

Remarks: If the action is free, then the extension exists and is unique (an application of topological surgery for finite fundamental groups). If the action fixes a knot K, the extension often must be nonlocally linear, for signature reasons. A surgical approach, in analogy with the fixed point free case, runs into fundamental group difficulties, since $\pi_1(\Sigma - K)$ may not be "good". For similar reasons, the non-equivariant version (does K bound an imbedded, non-flat disk in a contractible manifold?) is also unsolved.

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PROBLEM 41 (Edmonds): Find examples of smooth Z/p actions on a connected sum of copies of CP^2 that do not split as connected sums of actions on the individual summands. Edmonds and Ewing have TOP locally linear examples.

PROBLEM 42 (Kirby): Construct directly an exotic R^4. Known constructions involve complements of Casson handles with an unknown amount of replication, so a direct construction might be done in one of the following ways.

1. Construct a smooth, proper imbedding of the punctured Poincare homology sphere, P, into an exotic R^4.

Remark: (Gompf) P does not imbed, smoothly and properly, in R^4 by Taubes' end theorem (if so, then closureP imbeds in S^4 and a 2-sphere S in P linking the singular point has knot group Z, so surgery on S produces a homotopy $S^3 \times S^1$ containing closureP as a slice, contradicting Taubes' theorem). Hence, if one constructs a smooth, contractible, 1-connected-at-∞ 4-manifold containing P properly and smoothly, then one has constructed an exotic R^4.

2. (Gompf): Let H be the handlebody obtained by adding a 2-handle to the 4-ball with framing zero along the double of the trefoil knot (or any TOP slice but DIFF non-slice link L). Construct an explicit description of a smooth, contractible, 1-connected-at-∞ 4-manifold smoothly containing H.

Remarks: Such a manifold would necessarily be an exotic R^4, since if it were standard, it would exhibit a DIFF slicing of L. Such a manifold must exist: H embeds in R^4 since L is TOP slice. Define a smooth structure on R^4 by first smoothing the image of H in R^4 via the embedding, and then extending the smoothing over the rest of R^4 using Quinn's work.

Related to 2. is the problem of drawing an explicit picture of a flat, TOP slicing for the double of the trefoil knot, or any L as above.