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The Navier-Stokes Equations:

Theory and Numerical Methods

18.9. bis 24.9.1988

The organizers of this first conference on the Navier-Stokes equations at Oberwolfach were J.G. Heywood (Vancouver), K. Masuda (Tokyo), R. Rautmann (Paderborn) and V.A. Solonnikov (Leningrad). Forty-four participants (including 7 from Japan and 9 from the USSR) discussed new results which were presented in 43 lectures, additional talks, and movies.

These results concerned important contributions to the theory of the Navier-Stokes equations in unbounded domains, to the theory of free boundary problems, to the theory of viscoelastic fluids, to the understanding of vanishing viscosity, and to recent advances in numerical methods of flow computation which are based on the new theoretical approaches. The joint work of various groups having their main interest in different directions of mathematics and its application was extremely fruitful, and stimulating further progress.

Abstracts

Ch. T. AMICK:

On Leray's problem of steady Navier-Stokes flow past a body in the plane

We consider a classical problem due to Leray (1934) concerning the existence of steady Navier-Stokes flow past a body in the plane. There ist assumed to be no body force (or at least an irrotational one), the fluid velocity is zero on the body, and is to approach a prescribed constant vector at infinity. The viscosity has an arbitrary positive value. Other than some results for small Reynolds numbers, this problem remained largely untouched until the important work of Gilbarg and Weinberger a decade ago. They showed that various physical quantities satisfy elliptic equations, and thereby maximum principles. In this

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lecture, we introduce additional quantities satisfying maximum principles. We use them to show that Leray's solution is non-trivial and has a pointwise limit at infinity.

K. ASANO:

Non-viscous Limit of the incompressible Navier-Stokes equations in $\mathbf{R}^n_+$

We consider the initial boundary value problem for the N-S equation

$$
(P)\begin{cases}
\partial_t u + u\cdot\nabla u - \nu\Delta u + \nabla p = 0 \quad , \quad t > 0 \quad , \quad x = (x',x_n) \in R^n_+ \ , \\
\nabla\cdot u = 0 \quad , \\
u|_{t=0} = u_0 \quad , \quad (\nabla\cdot u_0 = 0,\ \gamma u_0 = 0) \quad , \\
\gamma u \equiv u|_{x_n=0} = 0.
\end{cases}
$$

Here $R^n_+ = \{x = (x',x_n) \in R^{n-1} \times R_+\}$, $u = {}^t(u_1,\ldots u_{n-1},u_n) = {}^t(u',u_n) =$ $u(\nu,t,x)$ is the velocity of the fluid at the time $t > 0$ and the point $x \in R^n_+$, and $\nu \in (0,1]$ means the viscosity coefficient.

Under some analyticity conditions on u_0, there exists a (unique) solution $u(\nu,t,x) = u^0(\varepsilon,t,x) + \varepsilon u^1(\varepsilon,t,x) + \ldots + \tilde{u}^0(\varepsilon,t,x',x_n/\varepsilon) + \varepsilon\tilde{u}^1(\varepsilon,t,x',x_n/\varepsilon) + \ldots$, $\varepsilon = \sqrt{\nu}$, in $[0,T]$, where T does not depend on $\nu \in (0,1]$. $\tilde{u}^i$ is a solution of the equation of Prandtl type.

J. T. BEALE:

The existence of solitary water waves with surface tension

We will discuss a proof of the existence of solitary water waves of elevation, as steady solutions of the equations of inviscid, incompressible flow with a free surface above, taking into account the effect of surface tension. Such

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results are familiar for the case without surface tension. Recently Amick and Kirchgässner have given a result for waves of depression with surface tension. The present case has appeared more difficult because of a resonance which appears in the equations linearized about the equilibrium. This resonance suggests that there might be oscillations or "ripples" at infinity in a solitary wave with surface tension. Indeed, numerical calculations of long periodic waves with such behavior have strengthened this impression. We show that in fact there are solitary waves of elevation, with surface tension, which do not have this oscillation at infinity. The proof is based on an iteration of Nash-Moser type and is similar in structure to the speaker's earlier work on solitary and cnoidal waves.

H. BEIRÃO DA VEIGA:

On the one dimensional motion of general barotropic viscous fluids

One considers the one dimensional motion of a barotropic compressible viscous fluid in a bounded domain.

We are interested in long time behavior for solutions, under the effect of external forces which do not became small for large values of t. The main point is to determine conditions on the data under which the solution does not asymptotically develope vacuum or infinite density (in general, these phenomena occur). In particular, existence and stability of the stationary and periodic solutions are investigated.

Strong continuous dependence on the initial data is also proved.

W. BORCHERS:

The Stokes operator in exterior domains

On the basis of weak type a priori estimates for the Stokes operator in exterior domains, new kinds of Sobolev embedding theorems for the domains of fractional powers of the Stokes operator are established. It is shown that these embeddings imply the so called L^p-L^q decay estimates of the corresponding semigroup. Moreover, these results can be used to characterize the algebraic decay rates of the L^2-norm of weak Navier-Stokes flows in exterior domains.

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G.-H. COTTET:

Deterministic vortex methods and the problem of boundary conditions for the vorticity formulation of the N.S. equations

Deterministic vortex methods for N.S. equations are based on replacing the diffusion by integral operators and then discretizing this operator along particles. This method has proved to be an efficient alternative to random walk methods in the absence of boundary but raises the problem of finding a convenient formulation of boundary conditions on the vorticity.

This paper gives a simple and new approach to this problem and describes a complete deterministic code for solving a flow past an obstacle.

G.F.D. DUFF:

Derivative Estimates for the Navier Stokes Equations in Three Space Dimensions

The Navier Stokes equations

$$u_{i,t} + u_k u_{i,k} = -p_{,i} + \nu\Delta u_i$$

$$u_{i,i} = 0$$

describe the flow $u_i(x,t)$ of a viscous incompressible fluid. In the study of solutions of the initial and boundary value problem in a spatial domain Ω, $u_i(x,0) = u_i(x) \in L^2(\Omega)$ and $u_i(x,t) = 0$ for $x \in \delta\Omega$, the classical energy integrals $u \in L^\infty(0,\infty;\ L^2(\Omega))$; $\nabla u \in L^2(0,\infty;\ L^2(\Omega))$ play an essential role. It will be shown that all higher space and time derivatives of u satisfy related estimates of the form

$$D_t^r D_x^s u \in L^{\frac{2}{4r+2s-1}}(0,T;\ L^2(\Omega)) .$$

The significance of such estimates for the study of singularities of solutions will be discussed.

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T. FISCHER:

A Galerkin approximation for linear eigenvalue problems in two and three-dimensional boundary-layer flows

The eigenvalue problem for a fourth order ordinary differential equation (Orr-Sommerfeld equation) in a semi-infinite domain is investigated. Weighted spaces are used in order to analyse the discrete part of the spectrum. A Jacobi-Galerkin approximation scheme is investigated for convergence.

A.V. FURSIKOV:

Navier-Stokes equations from the viewpoint of ill-posed boundary value problems

For the three-dimensional Navier-Stokes equations

$$\dot{u} + Au + B(u) = f, \quad u|_{t=o} = u. \tag{1}$$

with periodic boundary values and fixed initial value $u_o \in H^{1/2}$ we consider the set $F_o \subset L_2(0,T;\, H^{-1/2})$ such, that $\forall\, f \in F_o$ the solution $u \in U \equiv \{v(t) \in L_2(0,T;\, H^{3/2}) : \dot{v} \in L_2(0,T;\, H^{-1/2})\}$ exists. The solution u of the problem (1) is unique in U. It's proved, that F_o is dense with respect to $L_p(0,T;\, H^{-l})$- topology, where $1/2 < l < 3/2$, $1 \leq p < 4/(5-2l)$.

For the chain of momenta equations

$$\dot{M}_k + A_k M_k + B_k M_{k+1} = 0, \quad M_k \Big|_{t=o} = m_k,\ k = 1,2,\ldots \tag{2}$$

corresponding to the three-dimensional Navier-Stokes equations the spaces H_R^α for initial values $m = (m_1, \ldots m_k, \ldots)$ and Y_R^α for solutions $M = (M_1 \ldots, M_k, \ldots)$ of (2) are introduced, where R is the Reynolds number, α is order of smoothness. It's proved that a) the solution M of the problem (2) is unique in Y_R^α if $\alpha > 2$, $R > 0$; b) the set $V_o \subset H_R^\alpha$ such, that $\forall\, m \in V_o$ there exists the solution $M \in Y_R^\alpha$ of (2), is dense with respect to H_R^α - topology.

Y. GIGA:

Three dimensional Navier-Stokes flow with measures as initial vorticity

This is my joint work with T. Miyakawa. We are concerned with the non-stationary three dimensional flow of a viscous incompressible fluid when the initial vorticity is very singular. The typical examples are vortex filaments and vortex rings. We introduce Morrey spaces of measures to describe them. We do not need any coordinates representation of filaments.

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We construct a unique global regular solution for such initial data when it is small in some sense. Physically, this means that weak vortex filaments or rings are regularized instantaneously. Our results generalizes corresponding two dimensional results obtained independently by T. Miyakawa, H. Osada and the author and Cottet.

V. GIRAULT:

Curl conforming finite element methods for 3-D Navier-Stokes equations with non-standard boundary conditions

Let Ω be a bounded, convex domain of R^3 with a polyhedral boundary Γ. For f given in $L^{4/3}(\Omega)$, we want to solve the boundary-value problem:

$$-\nu\Delta u + u\cdot\nabla u + \nabla p = f\ , \quad \operatorname{div} u = 0 \ \text{ in } \ \Omega \ , \quad u \times n|_\Gamma = 0\ , \quad \tilde{p}|_\Gamma = 0\ ,$$

where $$\tilde{p} = p + (1/2)u\cdot u\ ,$$

with the variational formulation:

$$u \in H_0(\mathrm{curl};\Omega) \cap L^4(\Omega)^3, \quad p \in W_0^{1,4/3}(\Omega) \ \text{ such that}$$

$$\nu(\mathrm{curl}\ u, \mathrm{curl}\ v) - (u\times \mathrm{curl} u, v) + (\nabla p, v) = (f,v) \quad \forall v \in H_0(\mathrm{curl};\Omega) \cap L^4(\Omega)^3\ ,$$

$$(\nabla q, u) = 0 \quad \forall q \in H_0^1(\Omega)\ .$$

We approximate u in the curl-conforming f.e. space introduced by Nédélec:

$$R_k = P_{k-1} \oplus \{p \in \tilde{P}_k;\ p(x)\cdot x = 0\} \quad \text{with} \quad k \geq 1$$

and p in the standard f.e. space P_k.

When the solution is sufficiently smooth, the error estimates are optimal:

$$\|\mathrm{curl}(u-u_h)\|_{0,\Omega} = O(h^k)\ , \quad \|p - p_h\|_{0,\Omega} = O(h^k)\ .$$

The analysis extends to other non standard boundary conditions.

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C. GUILLOPÉ:

Existence, uniqueness and stability results for flows of viscoelastic fluids with differential constitutive equation

This is a joint work with J.C. Saut.

We consider flows, in bounded domains, of viscoelastic fluids with a constitutive equation of Oldroyd type.

First we prove the local existence of solutions. Then we prove a global existence theorem: If the ratio of the Newtonian viscosity to the total viscosity of the fluid is large enough, then for small data, the solution exists and is bounded for all times.

For one-dimensional cases, we prove that, for all (bounded) data, the solution is actually bounded for all times. In the case of Couette flow, an analysis of the stability of the unique steady solution is carried out. In the case of Poiseuille flow, if the pressure gradient driving the flow is large enough, then the solution ceases to be unique and there is a continuum of continuous solutions (= velocity profiles), which are C^{∞} except at an even number of points.

F.K. HEBEKER:

On Lagrangean and Boundary Element Methods for some unsteady Navier-Stokes problems

Over the past decade boundary element methods (BEM) have been established as a useful tool to treat numerically the challenging problems of viscous hydrodynamics. In the present talk I investigated a subclass of barotropic viscous unsteady flows. A Lagrangean time stepping scheme decouples the terms of convection and viscosity. The resulting strongly parabolic system to be solved for each time step is treated by means of a time-space Galerkin-type BEM, resulting from a Volterra boundary integral equations systems of the first kind. For the latter one a relatively simple proof of the coercivity estimate in certain anisotropic Sobolev spaces has been presented, fundamental for quasioptimality of Galerkin schemes. This result has partly been obtained jointly with G.C. Hsiao (Delaware). It is supposed that it carries over to the corresponding integral equations of incompressible viscous unsteady flows.

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J.G. HEYWOOD:

On the role of regularity and stability theory in approximating the Navier-Stokes Problem

I report on some of the principal results and objectives of a series of joint papers with Rolf Rannacher devoted to the analysis of finite element approximations of the Navier-Stokes equations. We show that the smoothness of solutions that one might like to assume in such an analysis must not be assumed, as it depends on non-local compatibility conditions for the data. We prove sharp regularity estimates, and then, using them, optimal order error estimates. We formulate and investigate various notions of stability, and use them to extend local a priori error estimates, and local a posteriori error estimates, globally in time.

V.A KAZHIKHOV:

Boundary value problems for Navier-Stokes equations of viscous gas

This paper presents the investigation of the correctness of the boundary-value problems for the equations of one-dimensional motion of viscous gas.

K. KIRCHGÄSSNER:

Resonantly forced nonlinear surface waves

Solutions of the Euler equations for an inviscid fluid are constructed through external pressure p_0 waves reacting with nonlinear capillary - gravity surface waves. It is shown that these solutions lie on a low dimensional manifold in extended phase space. Two cases are discussed: p_0 having compact support and p_0 being periodic. All solutions of bounded amplitude are given. In the latter case there exists, for large periods, a transverse homoclinic point. Space like chaos exists.

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H. KOZONO:

Strong solutions for the Navier-Stokes equations in half space

In $Q := R^n_+ \times (0,\infty)$, $R^n_+ = \{(x^1,\ldots,x^n);\ x^n > 0\}$ $(n \geq 2)$, we consider the Navier-Stokes equations:

$$\partial_t u - \Delta u + u \cdot \nabla u + \nabla p = 0, \quad \nabla \cdot u = 0 \quad \text{in} \quad Q,$$

$$u\big|_{x^n = 0} = 0, \quad u(0) = a$$

for the initial data a. We show the existence and uniqueness of a global strong L^n-solution for small data and its decay properties. To prove such results, we make use of the well-known implicit function theorem combined with the fractional powers of the Stokes operator, which guarantees us the L^n-continuity of the solutions with respect to the initial data.

D. KRÖNER:

Flow with free boundaries and dynamic contact angle

We consider a container (in 2-D) which is uniformly filled with a fluid such that the column of the fluid in the container moves with a constant velocity upwards. The special feature of this problem is the fact, that the free interface between the fluid and the air touches the walls of the container and that the contact points move along the walls. Experiments show, that the contact angle is a monotone function of the

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velocity S of the contact points. For this physical situation we consider the mathematical model, which consists of the equations of Navier and Stokes and suitable boundary conditions. It turns out that the boundary condition at the walls of the container close to the contact-points plays the most important role.
The results of our work are an asymptotic expansion of the velocity of the fluid near the contact points and the regularity of the free boundary as well as an existence result.

K. MASUDA:

Compatibility conditions for Navier-Stokes equations

Consider the Navier-Stokes equations in a bounded domain Ω in $\mathbb{R}^3$:

$$(1) \quad \begin{cases} \frac{\partial}{\partial t}u - \nu\Delta u + (u\cdot\nabla)u + \nabla p = 0 \quad , \quad \operatorname{div} \ u = 0 \\ u|_{\partial\Omega} = 0 \quad , \quad u|_{t=0} = \phi \ . \end{cases}$$

Let $\phi \in C^{\infty}_{0,\sigma}(\Omega)$ (i.e. $\phi \in C^{\infty}_{0}(\Omega)$ with div $\phi = 0$). It is known:

$u(\cdot,t) \to \phi$ in $H^3(\Omega)$ as $t \downarrow o$ if and only if $P[-\nu\Delta\phi + (\phi\cdot\nabla)\phi]|_{\partial\Omega} = 0$ (P: the projection operator to the solenoidal space). I report about the more concrete characterization.

T. MIYAKAWA:

L^2 decay for the Navier-Stokes flow in halfspaces

Existence is proved on a weak solution of the Navier Stokes equations in halfspaces which has an algebraic rate of decay in L^2. The result is proved with the aid of the spectral decomposition and fractional powers of the Stokes operator.
The result is a complete generalization to the case of halfspaces of the result obtained previously for the Cauchy problem.

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R. MIZUMACHI:

Convergence and rate of the convergence of notions of incompressible fluids in R^2 as viscosity goes to zero

It is well known that solutions of the Navier-Stokes equations converge to the solution of the Euler equations (when the space domain is $\mathbb{R}^2$) under several assumptions on the initial velocity u_0. We assume:

$u_0 \in C^{1+\alpha}(\mathbb{R}^2)$, rot $u_0 \in L^1(\mathbb{R}^2)$ and there exists $\lim_{|x|\to\infty} u_0(x)$. We conclude the convergence and give the following estimates:

$$\|u^{(\nu)}(\cdot t)-u^{(o)}(\cdot t)\|_0 \leq C\ (\nu t)^{\frac{1+\alpha}{2}}$$

$$\|\mathrm{rot}\ u^{(\nu)}(\cdot t) - \mathrm{rot}\ u^{(o)}(\cdot t)\|_\beta \leq C\ (\nu t)^{\frac{\alpha-\beta}{2}}$$

where ν is the viscosity, $u^{(\nu)}$ and $u^{(o)}$ are solutions to Navier-Stokes and Euler equations respectively; $\|\cdot\|_0$ is $\mathbb{R}^2$-sup norm and $\|\cdot\|_\beta$ is the Hölder seminorm with exponent β .

W. NAGATA:

Convection in fluids, and bifurcations with symmetry

The onset of convection in a viscous fluid provides many examples for bifurcation theory. Idealizations made for a convection problem may in-

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troduce symmetries into the corresponding bifurcation problem, and affect qualitative predictions. As an example, we compare the onset of two-dimensional Rayleigh-Bênard convection in an infinite layer with the onset in a finite layer. We also discuss the effects of symmetry on the onset of oscillatory convection.

H. OKAMOTO:

Degenerate bifurcation equations and primary flow exchange mechanisms in the Taylor problem

The objective is to explain certain bifurcation equations which describe recent discovered phenomena in the Taylor-Couette problem of the fluid motion between two concentric cylinders. The equations to be considered here are polynomial equations whose zero point sets are easily drawn numerically. Some of them are of the following form:

$$x(\epsilon\lambda + \alpha + ax^2 + bz^2 + cx^2z) + (\beta + ez^2)xz = 0,$$

$$z(\delta\lambda + \hat{a}x^2 + \hat{b}z^2) \pm x^2 = 0.$$

Our aim is to show how the solutions to those equations fit the bifurcation diagrams which are recently given numerically by Tavener and Cliffe. What they computed are Taylor vortices of new type bifurcating from the Couette flow. The equations above are derived from a certain degeneration of the equations given in Fujii, Mimura and Nishiura. What connects them is the normal form theory.

K. PILECKAS:

Non-compact free boundary problems for the Navier-Stokes equations

The stationary flow of a heavy viscous incompressible capillary fluid which runs out of a slit and spreads down an inclined plane is considered. It is assumed that the bottom moves with a constant speed

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and the full flux of the fluid is prescribed. For small data of the problem the existence theorem for the corresponding non-compact free boundary problem for the Navier-Stokes system is proved, assuming that the free boundary tends to a straight line at the infinity. An example of nonuniqueness of the solution is constructed and its asymptotics at the infinity is found.

The flow down an inclined plate of a finite length is considered under the assumption that the domain Ω occupied by the fluid is thin.

The asymptotics with respect to a small parameter ε (the height of the domain Ω) of a solution of the free boundary problem for the Navier-Stokes system is constructed.

V.I. POLEZHAEV:

<u>The problems of mathematical modelling on the base of the unsteady Navier-Stokes equations</u>

The technology of the mathematical modelling on the base of the unsteady Navier-Stokes equations includes such steps as the construction of models, numerical discretization, developement of computer programs, analysis of results, the models' correction and applications. We present a short review of these steps including the results of the last 10 - 15 years.

G. PROUSE:

<u>A uniqueness criterion for the solution of the stationary Navier-Stokes equations</u>

It is well known that no uniqueness theorem has yet been proved for the weak solutions of the Dirichlet problem for the stationary Navier-Stokes equations, except under the special assumption that the external force is "small" or the viscosity is "large".

It can, however, be proved that, for a generic external force, there exists one, and only one, solution such that the mechanical power of the flow is maximum and that this maximum is "stable" in an appropriate sense.

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V.V. PUKHNACHOV:

The problem of momentumless flow for the Navier-Stokes equations

We consider stationary solutions of the system of Navier-Stokes equations which describe the momentumless flow past solid bodies and study their asymptotical properties.

R. RANNACHER:

On time stepping schemes for the nonstationary Navier-Stokes equations

Some basic criteria are discussed for choosing among the time stepping schemes for solving the nonstationary Navier-Stokes equations. These are particularly the "local smoothing property" and the "global regularity property"of the schemes. These desirable properties are discussed along simple model situations. In particular, the popular Crank-Nicolson scheme fails to have some of these properties if one uses it in the original form. Some ways for enhancing the stability of this scheme are described. These results were rigorously proven for the Navier-Stokes equations in bounded two- or three-dimensional domains.

R. RAUTMANN:

A convergent product formula approach to three dimensional flow computations

On a sequence of successive (small) time intervals, the solutions of suitable linearizations of the Navier-Stokes initial-boundary value problem can be represented by means of a convergent product formula. A numerical realization (due to W. Borchers, F.K. Hebeker and W. Varnhorn) of this general scheme leads e.g. to solutions of the "Kugelstop"-problem, which are in good agreement with experiments.

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V.J. RIVKIND:

Numerical methods in mechanics of viscous fluid with free boundary

A general theory of approximation methods in mechanics of viscous incompressible fluid, governed by the complete system of the Navier-Stokes equation, is considered. On the base of the constructed general theory some algorithms for solution of large number of problems (mesh methods, finite element methods, hybridized methods etc.) are proposed. Examples of calculation in particular cases (drops, bubbles, films etc.) are given. Algorithms for solution of unstable and non-stationary fluxes (rollie waves) are considered.

K.G. ROESNER:

Numerical solutions of the Navier-Stokes-equations for incompressible and compressible media

As an example the flow between two spherical shells is treated by an analytical approach using asymptotic series expansion for the flow quantities: velocity, pressure, density and specific energy. Reynolds number, Mach number, and the eccentricity are the expansion parameters. The ratio of the radii and the ratio of the angular velocities of the rotating boundaries are additional parameters which can be chosen arbitrarily while the Reynolds and Mach number and the eccentricity must be small compared to one.

The boundary conditions are as usual for the velocity, but the temperature is assumed to be equal for both the inner and outer sphere.

Numerical comparisons are made for the incompressible and the compressible case. The basis is a finite element approach due to Bar-Yoseph combining the booster-method initiated by Israeli.

New phenomena are discussed which arise, found in the large gap situation. Experiments are shown for different eccentricities and Reynolds numbers.

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B.L. ROZHDESTVENSKY:

Numerical simulation of turbulent incompressible fluid flows in channels and pipes by nonstationary solutions of Navier-Stokes equations

For a number of problems on incompressible fluid flows in a channel or in a pipe the classes of nonstationary solutions of Navier-Stokes equations, periodic in uniform coordinates, are introduced. In the statistical sence, however, the solutions are stationary. These solutions correspond to the so-called secondary flows.

The efficient techniques are developed for solving the corresponding mathematical problems numerically. It is shown that three dimensional secondary flows reproduce the basic integral characteristics of real turbulent flows rather accurately while the two-dimensional flows cannot, in principle, reproduce them correctly.

The numerical techniques and results obtained are discussed.

D. SOCOLESCU:

On the Navier-Stokes equations on manifolds

As it was pointed out by various scientists, for instance by J. Serrin in "Handbuch der Physik", it is not at all clear if the Navier-Stokes equations in their usual form used by analysts presently, are valid on manifolds. It is the aim of the present paper (a joint work with E. Binz from the University of Mannheim) to write the global equations of motion of the given deformable medium, for instance those of a soap bubble. Moreover our approach allows us to point out the intimate connection between geometry and mechanics of continua.

As an application of our investigations we obtain the correct Navier-Stokes equations on manifolds.

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H. SOHR:

Remarks on the Stokes equations in exterior domains

Let $\Omega \subset \mathbb{R}^n$ ($n \geq 3$) be an exterior domain with smooth boundary. Then the resolvent $(\lambda I + A_q)^{-1}$ of the Stokes operator $A_q = -P_q\Delta$ can be estimated in the following way: $\|(\lambda I + A_q)^{-1}\| \leq \frac{c(q,\varepsilon,\Omega)}{|\lambda|}$ for all $\lambda \in \mathbb{C}$ with $|\arg\lambda| < \pi - \varepsilon$ where $0 < \varepsilon < \frac{\pi}{2}$ (joint work with Borchers). From this it follows that the semigroup e^{-tA_q} is uniformly bounded for all $t > 0$. Moreover we obtain that $e^{-tA_q} f \to 0$ as $t \to \infty$ for all $f \in L^q_\sigma$. We can also construct the purely imaginary powers A^{iy} ($y \in \mathbb{R}$) and we can show that these operators are bounded (joint work with Giga). From this we obtain a new estimate for the Cauchy problem in L^q-spaces. This leads to a priori estimates for weak solutions of the Navier-Stokes equations.

V.A. SOLONNIKOV:

Free boundary problem for nonstationary Navier-Stokes equations

We consider the evolution of an isolated body of a viscous incompressible fluid taking into account the surface tension and the Newtonian attraction forces between the fluid particles. It is shown that the corresponding free boundary problem is solvable locally in time. Moreover, if the initial velocity vector field is small and the domain occupied by the fluid is close to a ball, the global solvability can be established. We also show that in the latter case the solution tends (as $t \to \infty$) to a stationary solution of the problem corresponding to the rotation of a fluid about a certain axis which is determined by initial data.

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S. SRITHARAN:

Invariant manifold theorems for the Navier-Stokes equations

Invariant manifold theory lays a bridge between the onset dynamics of turbulence and the theory of finite dimensional dynamical systems. In this talk we will describe the existence,uniqueness and analyticity of local invariant manifolds along with the characterization of the nonlinear hydrodynamic semigroup and the monodromy operator. These manifolds are constructed in the neighborhood of each periodic solution to the Navier Stokes equations establishing the hyperbolicity of these solutions. For a particular regularization of the Navier Stokes equations (for which we have global existence and uniqueness theorem) we are able to prove the existence of global invariant (also known as the inertial) manifolds. This result is significant since the (generalized) solution of the regularized system has a strong limit (as the regularization parameter approaches zero) to the weak solution of the Navier Stokes equations.

E.L. TARUNIN:

Investigation of implicit schemes in terms of the variables ψ, ω with the help of the Babenko-Gelfand Principle

We are concerned with the analysis of the stability of implicit schemes for two-dimensional problems of viscous flows using the stream function-vorticity formulation. Our approach is based on the so called Babenko-Gelfand principle. In particular, we propose several approximation procedures for the vorticity which make it possible to increase the order of the schemes' stability.

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E.S. TITI:

Numerical criteria for detecting stable stationary and time periodic solutions to the Navier-Stokes equations

Sufficient conditions for inferring the existence of stable stationary solutions to the Navier-Stokes equations (N.S.E.) from the apparent stability of their nonstationary Galerkin approximations were given by Constantin-Foias-Temam. An explicit uniform spectral bound for the linearized N.S.E. and their Galerkin approximations, which we provide here, enables us to give an explicit applicable form to the above criterion. We also introduce a similar criterion for detecting time-periodic solutions to the N.S.E.. We emphasize the fact that all the conditions in the above criteria must be verified by the actually computed Galerkin approximation only and no assumptions are made on the unknown exact solution.

A. VALLI:

A non-existence result for one-dimensional compressible Navier-Stokes equations

A result of Beirão da Veiga already showed that a stationary solution to the Dirichlet boundary value problem for the one-dimensional compressible Navier-Stokes equations does exist if and only if a suitable compatibility condition between the pressure and the external force field is satisfied.
In the non-stationary case we prove that a solution for which the logarithm of the density is bounded up to infinity can exist only if the same compatibility condition holds.
The method of the proof consists in proving that each nonstationary solution must asymptotically converge to the stationary solution.

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W. VARNHORN:

On decay properties of the Stokes equations in exterior domains

Let $\Omega \subset \mathbb{R}^3$ be an exterior domain with a smooth compact boundary $\partial\Omega$. In Ω we consider the stationary Stokes equations in the general form

$$-\Delta u + \nabla p = f, \quad \operatorname{div} u = g, \quad u|_{\partial\Omega} = \phi, \quad u(x) \underset{|x|\to\infty}{\to} u_\infty(x).$$

Here we assume $u_\infty(x) = a + Ax$ with constants $a \in \mathbb{R}^3$, and $A = (a_{ij})$ $(i,j = 1,2,3)$ such that $\sum_{i=1}^{3} a_{ii} = 0$. We proof a decay result of the following type: If f and g fulfill a decay condition for large $|x|$ which is formulated in terms of certain weight functions, then we can conclude a similar condition also for the solution u,p of the Stokes system above. The method rests on an a priori estimate given by Solonnikov in 1977.

W. VELTE:

On optimal constants in some inequalities

Für einige Ungleichungen zwischen äquivalenten Normen, die u.a. in der Theorie der Navier-Stokes'schen Gleichungen eine Rolle spielen, wird die Frage optimaler Konstanten erörtert. Zunächst werden bekannte Resultate referiert, die sich auf Probleme im $\mathbb{R}^2$ beziehen (darunter Resultate von HORGAN und PAYNE über eine Ungleichung von K.O. FRIEDRICHS sowie die zweite KORN'sche Ungleichung). Es wird aufgezeigt, daß es zu einigen dieser Resultate auch ein Analogon für Probleme im $\mathbb{R}^3$ gibt. In einem Sonderfall wird das Spektrum einer zugeordneten Eigenwertaufgabe im Detail studiert.

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W. VON WAHL :

Necessary and sufficient conditions for the solvability of the equations rot u = f and div u = g with u vanishing on the boundary

The single equations

(1) $\quad \operatorname{rot} u = f \quad$ with $\quad u|\partial\Omega = 0$ and

(2) $\quad \operatorname{div} u = g \quad$ with $\quad u|\partial\Omega = 0$

are considered on a bounded or unbounded domain $G \subset \mathbb{R}^3$. We give necessary and sufficient conditions on f and g resp. such that the corresponding problems have a solution with

$\|\nabla u\|_{L^p(G)} \leq c\, \|f\|_{L^p(G)}$ or $\|\nabla u\|_{L^p(G)} \leq c \cdot \|g\|_{L^p(G)}$.

The main tools of the proof are the inhomogeneous Dirichlet- and Neumann-problem resp. for harmonic vector fields; These problems are well understood by means of potential theory. For (1) the conditions are

$$\operatorname{div} f = 0, \quad (n,f) = 0 \text{ on } \partial\Omega \ (n = \text{outer normal}),$$

$$f \perp Z(G) \quad , \quad Z = \{ z \mid \operatorname{div} z = 0, \ \operatorname{rot} z = 0, \ (n,z) = 0 \} .$$

The latter is a space of dimension N = first Betti number of G. For (2) the conditions are $\int_G g\, h\, dx = 0$, where h runs through the null-space of the Neumann-problem $\Delta u = 0$, $\frac{\partial u}{\partial n} = 0$ on G.

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M. WIEGNER:

On L_p-stability of strong solutions to the Navier-Stokes equations on $\mathbb{R}^n$

Let u be a weak solution of the Navier-Stokes equation on $\mathbb{R}^n$ with $u(o) = a \in L_2 \cap L_p$, which fulfills additionally for some $p > n \geq 3$ $u \in L_s(\mathbb{R}^+, L_p)$, $\frac{2}{s} + \frac{n}{p} \leq 1$. Then $u \in L_\infty(\mathbb{R}^+, L_p)$; u is in fact a strong solution and the following theorem holds:

If $u_0(t) = e^{-t\Delta}a$, the solution of the heat equation, satisfies the inequality $\| u_0(t) \|_2 \leq C(1+t)^{-\alpha/2}$ $(\alpha \neq 1)$, then

$$\| u(t) - u_0(t) \|_p \leq C(1+t)^{-(d/2 + n/2(1/2 - 1/p))}$$

with $d = n/2 + 1 - 2 \max \{1 - \alpha, 0\}$, implying especially $\| u(t) \|_p \leq C(1+t)^{-(\bar{\alpha}/2 + n/2(1/2 - 1/p))}$ with $\bar{\alpha} = \min \{\alpha, n/2 + 1\}$.

As a consequence, we can prove the stability result: If $u = u_1$ as above with $a = a_1$, then for $a_2 \in L_2 \cap L_p$ with $\| a_1 - a_2 \|_p$ small, there exists a global strong solution u_2 with $u_2(0) = a_2$ and with the similar decay property.

Berichterstatter: W. Borchers

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