MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Elementare und analytische Zahlentheorie

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Die Tagung fand unter der Leitung von Herrn Prof. Dr. H.-E. Richert (Ulm), Herrn Prof. Dr. W. Schwarz (Frankfurt) und Herrn Prof. Dr. E. Wirsing (Ulm) statt.

Im Mittelpunkt des Interesses standen aktuelle Fragen aus der Zahlentheorie, die überwiegend mit analytischen Methoden behandelt wurden.

In 36 anspruchsvollen, auf hohem Niveau stehenden Vorträgen wurde über Fortschritte in der Primzahltheorie, in der additiven und multiplikativen Zahlentheorie, auf dem Gebiet der Siebmethoden und über die neuere Entwicklung bei der Behandlung der Riemannschen Zeta - Funktion berichtet.

Während der Vorträge und insbesondere bei der von Herrn Prof. Dr. P. Erdös geleiteten Problem - Session wurden viele ungelöste Fragen angesprochen. Möglicherweise ist die Lösung des einen oder anderen dieser Probleme Gegenstand eines Vortrages schon bei der nächsten Tagung über elementare und analytische Zahlentheorie.

Das Vortragsprogramm wurde ergänzt durch persönliche Gespräche und fruchtbare Diskussionen der aus 16 Ländern angereisten 46 Teilnehmer.

Wieder ermöglichte es die vorbildliche Organisation des Instituts, daß dieser Gedankenaustausch in so harmonischer Atmosphäre stattfinden konnte. Unser besonderer Dank gilt dem Institutsleiter, Herrn Prof. Dr. M. Barner, und dem Personal, das wesentlich zum guten Gelingen der Tagung beigetragen hat.

K. ALLADI

Multiplicative Functions and Brun's Sieve

Let g be a strongly multiplicative function and $g_y(n) = \prod_{p|n, p<y} g(p)$. For $\mathcal{A} \subset \mathbb{Z}^+$ let $\mathcal{A}(x) = \mathcal{A} \cap [1,x]$. We think of the sum $S_g(\mathcal{A}(x), y) = \sum_{n \in \mathcal{A}(x)} g_y(n)$ as a "generalized sieve problem" because it satisfies the Buchstab identity

$$S_g(\mathcal{A}(x), y) = S_g(\mathcal{A}(x), y_1) + \sum_{y \le p < y_1} g^*(p) S_g(\mathcal{A}_p(x), p),$$

where $g^*(n) = \prod_{p|n}(1 - g(p))$ is the dual of g. When $0 \le g \le 1$, the sum S_g can be given an interpretation in the classical sense. Here $0 \le g^* \le 1$, and g^* keeps track of the amount of sieving done. This case has interesting applications to Probabilistic Number Theory, and the combinatorial sieve can be used to estimate S_g. Even when $0 \le g \le 1$ does not hold, sieve methods can be used to estimate S_g. In particular when $g^*(p) = 2$, note that $g(n) = (-1)^{\nu(n)}$. So by means of the 'pure sieve' it can be shown that

$$\sum_{n \in \mathcal{A}(x)} (-1)^{\nu_y(n)} = o(|\mathcal{A}(x)|), \quad x, y \to \infty, \quad y \le |\mathcal{A}(x)|^{c/\log\log x}$$

for a large class of sets $\mathcal{A}$. In other words $\nu_y(n)$ is uniformly distributed modulo 2 for $n \in \mathcal{A}(x)$. (Here $\nu_y(n) = \sum_{p|n, p<y} 1$.)

G.E. ANDREWS

The Rogers - Ramanujan Identities

This talk was devoted to a discussion of a number of topics arising from Ramanujans work, especially his 'Lost Notebook'. First the series

$$1 + \sum_{n=1}^{\infty} \frac{q^{n(n+1)/2}}{(1+q)(1+q^2)\cdot\ldots\cdot(1+q^n)} = \sum_{n=0}^{\infty} R(n) q^n$$

was discussed. A recent theorem of D.Hickerson, F.J.Dyson and me asserts that $R(n)$ is almost always $= 0$ and that for any integer m, the equation $R(n) = m$ has infinitely many solutions in n. Next we presented a motivated proof of the Rogers - Ramanujan identities (joint with R.J.Baxter) which will soon appear in the American Mathematical Monthly. Finally we discussed the polynomials

$$D_k(n) = \sum_{\lambda=-\infty}^{\infty} (-1)^{\lambda} q^{\lambda(5\lambda+1)/2} \begin{bmatrix} n \\ [\frac{n-k\lambda}{2}] \end{bmatrix}$$

where $\begin{bmatrix} A \\ B \end{bmatrix} = \frac{(1-q^A)(1-q^{A-1})\cdot\ldots\cdot(1-q^{A-B+1})}{(1-q^B)(1-q^{B-1})\cdot\ldots\cdot(1-q)}$ and $[x]$ is the largest integer $\le x$.

Schur's 2nd proof of the Rogers - Ramanujan identities relies on $D_5(n)$. Bressoud (1980) uses $D_4(n)$ and Watson's proof (1929) can be modified to be an assertion about $D_2(n)$. Are there any nice results about $D_k(n)$ for other k which imply the Rogers - Ramanujan identities ? In particular what about $D_3(n)$?

P.T. BATEMAN

Some Bizarre Questions and Theorems about Squares and Triangular Numbers

The triangular numbers are the numbers $j(j+1)/2$, $j = 1,2,\ldots$ and the squares are the numbers k^2, $k = 1,2,\ldots$. Recently I have been asked several questions about these two sequences which seemed somewhat unusual to me. (Perhaps "bizarre" is too strong a term.) Some of these questions were easy to answer, others were more difficult and others seemed impossible to me. The following is one of the impossibly difficult ones. Let S be the set of positive integers n for which there is a square equal to the sum of the squares of exactly n consecutive positive integers. For example, $11 \in S$ since $77^2 = 18^2 + 19^2 + \ldots + 28^2$. If $N(x)$ is the counting function of the set S, it is easy to show that $\sqrt{x} \ll N(x) \ll \frac{x}{\log x}$. What is the exact order of the function $N(x)$. Several other questions were also discussed.

H. DELANGE

The integers n for which $\Omega(n)$ is large

If p is a prime we denote by $\nu_p(n)$ the exponent of p in the factorization of the positive integer n. So $\Omega(n) = \sum_p \nu_p(n)$. We study the distribution of the values of $\nu_p(n)$, p fixed, among the integers $n \le x$ for which $\Omega(n) = k$. We restrict ourselves to the case $k \ge (2+\delta)\,\text{loglog}\, x$, where δ is a fixed positive number. Let $N(x,k)$ be the number of n's $\le x$ for which $\Omega(n) = k$. Let $y = x/2^k$.
Then among others the following theorems were shown:

Theorem 1. Given $A > 0$, we have uniformly for $-\infty < t \le A$:

$$\frac{1}{N(x,k)} \# \{ n \le x,\ \Omega(n) = k \text{ and } \nu_2(n) \le k - 2\,\text{loglog}\, y + t\sqrt{2\,\text{loglog}\, y}\, \}$$

$$= G(t) + O\left(\frac{1}{\sqrt{\text{loglog}\, y}}\right), \text{ where } G(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{t} e^{-u^2/2}\, du .$$

Theorem 2. Let $p_1, \ldots, p_s$ be distinct odd primes. We have uniformly for $\alpha_1 + \ldots + \alpha_s \le \delta'\,\text{loglog}\, y$, where $0 < \delta' < \delta$,

$$\frac{1}{N(x,k)} \# \{ n \le x,\ \Omega(n) = k \text{ and } \nu_{p_j}(n) = \alpha_j \text{ for } 1 \le j \le s \}$$

$$= \left(1 + O\left(\frac{1}{\text{loglog}\, y}\right)\right) \prod_{j=1}^{s} \left(\frac{2}{p_j}\right)^{\alpha_j} \left(1 - \frac{2}{p_j}\right).$$

If we suppose that $k \le \lambda \frac{\log x}{\log 2}$ with $\lambda < 1$, in the above theorems y may be replaced by x.

E. FOUVRY

Exponential sums for monomials

We present results obtained in collaboration with H. Iwaniec in the problem of bounding multidimensional sums of the type

$$\sum_{m_1 \sim M_1} \cdots \sum_{m_j \sim M_j} c(m_1,\ldots,m_j)\; e\left(x \frac{m_1^{\alpha_1} \cdots m_j^{\alpha_j}}{M_1^{\alpha_1} \cdots M_j^{\alpha_j}} \right).$$

The main tool is a lemma of Bombieri and Iwaniec (Double large sieve inequality) which reduces the bounding of such a sum to a spacing problem of points. When combined with other classical techniques of exponential sum theory (Weyl's shift, Poisson summation formula, ...) we prove several theorems according to the size and the number of variables. The most important one is the following:

Theorem. Let $\alpha \neq 0,1$ and $H,M,N,x \geq 1$. Let $\chi(h)$ be an additive character $\chi(h) = e(\xi h)$ ($\xi \in \mathbb{R}$) and φ_m, ψ_n be complex numbers with $|\varphi_m| \leq 1$, $|\psi_n| \leq 1$.
Then for the sum

$$S_{\chi,\varphi,\psi}(H,M,N) = \sum_{h\sim H} \sum_{m\sim M} \sum_{n\sim N} \chi(h)\,\varphi_m\,\psi_n\; e\left(x\,\frac{h\,n^{-1}m^{\alpha}}{H N^{-1} M^{\alpha}}\right)$$

we have the bound

$$S_{\chi,\varphi,\psi}(H,M,N) \ll (HMN)^{\frac{1}{2}} \Big[(H+N)^{\frac{1}{2}} \Big(x^{\frac{1}{8}} H^{-\frac{1}{6}} M^{\frac{1}{12}} N^{\frac{1}{6}} + x^{\frac{1}{8}} H^{-\frac{1}{8}} N^{\frac{3}{8}} + N^{\frac{1}{2}} + N^{\frac{1}{4}} M^{\frac{1}{8}} \Big) x^{\frac{1}{8}} + M^{\frac{1}{2}} + x^{-\frac{1}{4}} M^{\frac{1}{2}} N \Big] \log^4(2xHMN).$$

This theorem leads to an improvement in the problem of finding P_2- numbers in short intervals.

J. FRIEDLANDER

Limitations to the equi - distribution of primes (jointly with A. Granville)

We have studied the analogue for arithmetic progressions of the method of H. Maier for disproving the asymptotic formula for primes in short intervals. We are able to get results for progressions with extremely large moduli. The simplest special case is

Theorem 1. Let $N > 1$. There exist arbitrary large reals x, primes $q \leq x \log^{-N} x$ and integers a with $(a,q) = 1$ such that

$$\left| \psi(x;q,a) - \frac{x}{\varphi(q)} \right| \gg_N \frac{x}{\varphi(q)} .$$

This contradicts the conjecture of Montgomery that for $q < x$, $(a,q) = 1$

$$\psi(x;q,a) = \frac{x}{\varphi(q)} + O_\varepsilon\left(\left(\frac{x}{q}\right)^{\frac{1}{2}+\varepsilon} \log x \right) .$$

We are able to show that even on the average over q the asymptotic formula may fail. Typical examples of our results here are

Theorem 2. Let $N > 1$. There exist arbitrarily large values of x and corresponding values for a for which, if $Q = x \log^{-N} x$, we have

$$\sum_{\substack{Q < q \leq 2Q \\ (q,a)=1}} \left| \psi(x;q,a) - \frac{x}{\varphi(q)} \right| \gg_N \frac{x}{\log\log x} \qquad \text{and}$$

$$\sum_{Q < q \leq 2Q} \max_{(a,q)=1} \left| \psi(x;q,a) - \frac{x}{\varphi(q)} \right| \gg_N x,$$

the latter being optimal. Thus the Bombieri - Vinogradov estimate does not in general hold in this range, contradicting the strong form ($Q = x^{1-\varepsilon}$ may still be possible) of the conjecture of Elliott and Halberstam. The range of Q may be further reduced and this will form the subject of a future work by the author jointly with Hildebrand and Maier, who independently have discovered the same circle of ideas.

A. HILDEBRAND

Irregularities in the distribution of primes (joint work with H. Maier)

Let $\psi(x) = \sum_{n \le x} \Lambda(n)$, where Λ is the von Mangoldt function, and define $\Delta(x,y)$ by $\psi(x+y) - \psi(x) = \big(1 + \Delta(x,y)\big)\, y$. Probabilistic considerations suggest that $\Delta(x,y)$ is roughly of order $y^{-1/2}$. We show, however, that for intervals of length $y = \exp\big((\log x)^{o(1)}\big)$ $\Delta(x,y)$ can assume much larger values.

Theorem. Let $\varepsilon > 0$ be fixed. Then for all sufficiently large x and $2 \le y \le \exp\big(A(\log x)^{1/3}\big)$ there exist values $x_\pm \in [x, 2x]$ such that

$$\pm \Delta(x_\pm, y) \ge c(\varepsilon)\, y^{-(1+\varepsilon)\eta},$$

where
$$\eta = \eta(x,y) = \frac{\log \dfrac{\log y}{\log\log x}}{\log \dfrac{\log x}{\log y}},$$

A is an absolute positive constant and $c(\varepsilon)$ is a positive constant depending only on ε.

The above bound is $\gg 1$ if $y = (\log x)^{O(1)}$ and $\gg_\varepsilon y^{-\varepsilon}$ if $y = \exp\big((\log x)^{o(1)}\big)$.

It is inconsistent with the bound predicted by the probabilistic model used by Cramér to formulate his well - known conjecture.

E. HLAWKA

Elementare Zahlentheorie und Gleichverteilung

Es seien $p_1, \dots, p_s$ verschiedene Primzahlen $\equiv 1 \pmod 4$. Es gilt $p_j = \pi_j \overline{\pi}_j$, π_j Primzahl in $\mathbb{Z}(i)$, $\dfrac{\pi_j}{\overline{\pi}_j} = e^{2\varphi_j} = e^{4\pi i \psi_j}$ ($j = 1, \dots, s$). Es wird betrachtet die Folge $k\,(2\psi_1, \dots, 2\psi_s, \frac{1}{2N})$ für $k = 1, \dots, N$. Für ihre Diskrepanz gilt $D_N \le c\, \dfrac{(\log\log N)^s}{\log N}$. Dies wird angewendet auf Gleichverteilung auf dem Einheitskreis und auf der Sphäre S^2 und S^3. Eine Methode von Siegel wird auf die simultane Approximation von Irrationalzahlen angewendet.

M. HUXLEY

Exponential sums and lattice points

The speaker and N. Watt have succeeded in generalising Bombieri and Iwaniec's bound for $\zeta(\frac12 + it)$ to show

$$\sum_{m=M}^{2M} e\Big(T\,F\big(\tfrac{m}{M}\big)\Big) = O\Big(M^{1/2}\, T^{89/560} (\log T)^B\Big) \quad \text{(Watt)},$$

$$\sum_{l=1}^{L} \Big| \sum_{m=M}^{2M} e\Big(T\,F\big(\tfrac{m}{M}, y_l\big)\Big)\Big| = O\Big(E L M^{1/2}\, T^{89/560} (\log T)^B\Big) \quad \text{(Huxley, Watt)},$$

where $y_1, \dots, y_L$ are well-spaced in $[0,1]$ and $E = L^{-13/560}$ + other terms < 1, when certain combinations of derivatives of $F(x)$ or $F(x,y)$ do not vanish.
A corresponding generalization of Iwaniec and Mozzochi's bound for the circle and divisor problems gives: Let C be a simple closed convex curve, three times differentiable, with area A and nonzero curvature. The number of lattice points inside the enlarged curve MC is

$$A M^2 + O\left(A^{7/22} (\log A)^8\right). \quad \text{(Huxley)}$$

This corresponds to $T = M$ above; work progresses on rounding error sums, and lattice points close to (or on) the curve $y = T F(x/M)$ when $T > M$.

A. IVIĆ

The error term in the mean-square formula for the zeta-function

Several results on the function

$$E(T) := \int_0^T |\zeta(\tfrac12 + it)|^2 \, dt - T\left(\log \frac{T}{2\pi} + 2\gamma - 1\right) \quad (\gamma \text{ is Euler's constant})$$

are presented. These include omega results which are analogous to the sharpest known omega results in the divisor problem. They were proved by J.L. Hafner and myself.
In a joint forthcoming work with H. te Riele we discuss t_n, the n-th zero of $E(T) - \pi$.
One has $t_{n+1} - t_n \ll t_n^{1/2}$, while $t_{n+1} - t_n \gg t_n^{1/2} (\log t_n)^{-3/4}$ for infinitely many n.
Numerical calculations (for $t_n \le 10^6$) carried out so far support the conjectures that

$$\limsup_{n\to\infty} \frac{\log(t_{n+1} - t_n)}{\log t_n} = \frac14, \quad \sum_{t_n \le T} (t_{n+1} - t_n)^2 = T^{5/4 + o(1)} \quad (T \to \infty).$$

Also $\max (t_n - t_{n-1})\, t_{n-1}^{-1/4} = 12.3436\ldots$ for $n = 59464$, while the minimum is 0.0002 for $n = 27021$.

M. JUTILA

Mean value estimates for L-functions

The following mean value theorem for Dirichlet L-functions is discussed:
Let $T^{1/2+\varepsilon} \ll T_0 \ll T^{2/3}$ and let $T \le t_1 < \dots < t_R \le 2T$, where $t_{r+1} - t_r \ge T_0$.
Then for any integer $D \ge 1$,

$$(*) \quad \sum_{\chi \bmod D} \sum_{r=1}^{R} \int_{t_r}^{t_r + T_0} |L(\tfrac12 + it, \chi)|^4 \, dt \ll D\left(R T_0 + (RT)^{2/3}\right)(DT)^{\varepsilon}.$$

The case $D = 1$ (i. e. $\zeta(s)$) is due to H. Iwaniec. Two corollaries:

$$\sum_{\chi \bmod D} \int_T^{T + T^{2/3}} |L(\tfrac12 + it, \chi)|^4 \, dt \ll L\, T^{2/3} (DT)^{\varepsilon},$$

$$\sum_{\chi \bmod D} \int_0^T |L(\tfrac12 + it, \chi)|^{12} \, dt \ll D^3 T^2 (DT)^{\varepsilon}.$$

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The first of these seems to be new, while the second was first proved by T. Meurman in 1984, and an analogous result for a single L - function has been obtained by Y. Motohashi. The estimate (*) is an application of a general mean value theorem for exponential sums involving the divisor function $d(n)$.

A. KARACUBA

Waring's problem in several dimensions

The following theorem is a two - dimensional generalization of Waring's problem:
Consider the system of equations

$$x_1^{n-i} y_1^{i} + \dots + x_k^{n-i} y_k^{i} = N_i \quad , i = 0,1,\dots,n ,$$

where N_i are the given positive integers of the same order of growth, $N_o \to +\infty$, x_s, y_s are the unknowns, also positive integers. This system is solvable if $k > c\, n^2 \log n$, and if $k < c_1 n^2$ then there are such N_i's that the system has no solutions. Note that $k \geq 2^n$ is a necessary condition for solvability of the similar Hilbert - Kamke system.

E. KRÄTZEL

Two theorems on four - dimensional divisor problems and three applications

Let a_1, a_2, a_3, a_4 be positive integers with $1 \leq a_1 \leq a_2 \leq a_3 \leq a_4$ and $a = (a_1, a_2, a_3, a_4)$. The divisor function $d(a,n)$ counts the number of ways of expressing n as the product $n = n_1^{a_1} \cdot n_2^{a_2} \cdot n_3^{a_3} \cdot n_4^{a_4}$.
We consider the behaviour of the function

$$D(a;x) = \sum_{n \leq x} d(a,n) \qquad \text{for large } x .$$

$D(a;x)$ can be represented by $D(a;x) = H(a;x) + \Delta(a;x)$
with the main term

$$H(a;x) = \sum_{\nu=1}^{4} \alpha_\nu\, x^{1/a_\nu} \quad , \quad \alpha_\nu = \prod_{\substack{\mu=1 \\ \mu \neq \nu}}^{4} \zeta\left(\frac{a_\mu}{a_\nu}\right) \quad (a_1 < a_2 < a_3 < a_4).$$

The following two theorems are proved by means of the method of three - dimensional exponential sums:

Theorem 1. The estimation $\Delta(a;x) \ll x^{5/2A_4} \log^4 x$ holds under the conditions $15 A_1 \geq 2 A_4$, $3 A_2 \geq A_4$, $5 A_3 \geq 3 A_4$ $(A_\nu = a_1 + \dots + a_\nu)$.

Theorem 2. The estimation $\Delta(a;x) \ll x^{42/17 A_4} \log^4 x$ holds under the conditions $6 A_1 \geq A_4$, $14 A_2 \geq 5 A_4$, $42 A_3 \geq 25 A_4$.

These theorems are applied to three number theoretical problems: The distribution of direct factors of finite Abelian groups, the distribution of powerfull integers of type 4 and the distribution of cube - full integers in short intervals.

M.G. LU

On sums of mixed powers

(I) On the improvement of the pruning techniques on major arcs.

1. Let $R_{b,c}(n)$ denote the number of representations of n as the sum of one square, four cubes, one b-th power and one c-th power of natural numbers. Now we can establish formulas for $R_{4,k}(n)$ ($4 \le k \le 6$) and give lower estimates of the correct order of magnitude for $R_{4,k}(n)$ ($7 \le k \le 14$), $R_{5,j}(n)$ ($5 \le j \le 8$) and $R_{6,6}(n)$.

2. Let $E(N)$ denote the number of positive integers not exceeding N and not being the sum of four cubes, then we can prove that $E(N) \ll N^{131/147}$.

(II) New application of Davenport's method.

Let $\nu(n)$ denote the number of representations of n as the sum of six cubes and two biquadrates of natural numbers. Then for all sufficiently large n

$$\nu(n) \ge \frac{1}{(48)^2}\left(\log\frac{16}{15}\right)^4 \frac{\Gamma\left(\frac{4}{3}\right)^4 \Gamma\left(\frac{5}{4}\right)^2}{\Gamma\left(\frac{5}{2}\right)} \mathfrak{S}(n)\, n^{3/2}$$

where $\mathfrak{S}(n) \gg 1$ is the singular series.

J.L. MAUCLAIRE

Measures associated to arithmetical functions

A class of problems in probabilistic number theory can be solved by measure theoretic methods. The following case is typical:

Let $f : \mathbb{N} \to \mathbb{C}$ be a multiplicative arithmetical function. The hypotheses

(i) $\lim_{x\to\infty} \frac{1}{x} \sum_{n\le x} f(n)$ exists and is not 0 and (ii) $\limsup_{x\to\infty} \frac{1}{x} \sum_{n\le x} |f(n)|^\lambda < +\infty$

for some $\lambda > 1$, are equivalent to the existence of a sequence of integrable functions F_y on a suitable measured space $(E, d\mu)$ such that $F_y \to F$ $d\mu$-a.e. and in $L^\lambda(E, d\mu)$ with $\int_E F\, d\mu \ne 0$. A sketch of the proof has been presented with some comments.

H.L. MONTGOMERY

Cyclotomic partitions (joint work with D. Boyd)

Let $c(n)$ denote the number of cyclotomic polynomials of degree n. Since the q-th irreducible cyclotomic polynomial has degree $\varphi(q)$, it follows that $c(n)$ is the number of ways of writing $n = \sum k_q \varphi(q)$ where the k_q are non-negative integers. By convention, $c(0) = 1$.

For $\operatorname{Re} z > 0$, $P(z) = \sum_{n=0}^{\infty} c(n) e^{-nz} = \prod_{q=1}^{\infty} \left(1 - e^{-\varphi(q) z}\right)^{-1}$.

In order to derive an asymptotic estimate for $c(n)$, it is first necessary to determine the asymptotic behaviour of $P(z)$ as $z \to 0$ in a suitable domain. To this end we note that

$$\log P(z) = \sum_{q=1}^{\infty} \sum_{k=1}^{\infty} \frac{1}{k} e^{-k\varphi(q)z} = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \zeta(s+1) F(s) \Gamma(s) z^{-s} ds$$

where $F(s) = \sum_{q=1}^{\infty} \varphi(q)^{-s} = \prod_p \left(1 + \frac{1}{(p-1)^s} + \frac{1}{(p^2-p)^s} + \dots \right) = \zeta(s) Q(s)$,

where $Q(s) = \prod_p \left(1 + \frac{1}{(p-1)^s} - \frac{1}{p^s} \right)$. We find that $Q(s)$ behaves like s^{-s} near $s=0$, and that if $|s| > \frac{1}{3}$, $\sigma > \frac{-c}{\log \tau}$ then $Q(s)$ is regular and

$$Q(s) \ll \exp\left(c \frac{\tau}{\log \tau}\right).$$

This yields an accurate estimate of $\log P(z)$ which in turn gives the asymptotic estimate

$$c(n) \sim A \, (\log n)^{-\frac{1}{2}} \, n^{-1} \exp(B\sqrt{n}).$$

Similarly we let $c_d(n)$ denote the number of squarefree cyclotomic polynomials of degree n. This is the number of partitions into distinct parts, $k_q = 0$ or 1. We find that

$$c_d(n) \sim A_d \, n^{-3/4} \exp(B_d \sqrt{n}).$$

Y. MOTOHASHI

Zeros of the Riemann zeta function in short intervals of the critical line

Our interest lies in finding lower bounds for the number of zeros of $\zeta(s)$ actually on the critical line. But, our interest lies also in the intermediate results needed in proving such results on zeros of $\zeta(s)$. This is mainly concerned with finding "good" asymptotic formula for $I(T,A) = \int_0^T |\zeta(\frac{1}{2}+it) A(\frac{1}{2}+it)|^2 dt$, where A is an arbitrary Dirichlet polynomial.

There are several ways to attack this problem. One is to appeal to "good" approximate functional equation for $\zeta(s)$. This includes the approaches used by Selberg, Karacuba and Levinson. The other way is the one found by Atkinson. The original method due to Atkinson concerns the mean square of $\zeta(s)$ on the critical line. And to get a similar asymptotic expansion for $I(T,A)$ as Atkinson's one for $\zeta(s)$ we need some modifications of his idea. We describe our modification below in a somewhat generalized way:

Let $f(m,n)$ an arbitrary function. Then Atkinson's dissection argument is:

$$\sum_{m,n>0} f(m,n) = \sum_{m,n>0} \Big(f(m,m+n) + f(m+n,m) \Big) + \sum_{m>0} f(m,m).$$

We modify this as follows: Let a, b be positive, coprime integers. Then we have

$$\sum_{m,n>0} f(m,n) = \Big(\sum_{am>bn} + \sum_{am<bn} + \sum_{am=bn} \Big) f(m,n),$$

where (e.g. the first sum)

$$\sum_{am>bn} f(m,n) = \sum_{\substack{m,n>0 \\ m+bn\equiv 0 \pmod a}} f\left(\frac{m+bn}{a}, n\right).$$

This simple (almost trivial) identity gives several important results on $I(T,A)$ as well as its extension to L - functions and (possibly) on the fourth power moment of $\zeta(s)$.

L. MURATA

On the number of primes p satisfying the condition $a^{p-1} \equiv 1 \pmod{p^2}$

We put for fixed natural number $a \geq 2$

$$L_a(x) = \{ p ; p \text{ is a prime} \leq x,\ a^{p-1} \equiv 1 \pmod{p^2} \},$$

and present three results about $|L_a(x)|$. Two of them are concerned with the tendency of $|L_a(x)|$. More precisely, the first theorem says that the normal order of $|L_a(x)|$ is loglog x, and the second one that the natural density of the pairing (a,b) satisfying $|L_a(x) \cap L_b(x)| = 0$ is equal to $8/\pi^2$. It seems very difficult to obtain a non - trivial upper bound for $|L_a(x)|$ for fixed a. Our third theorem says that under the assumption of the Generalized Riemann Hypothesis, we can get a nontrivial upper bound for

$$\left| \{ p \in L_a(x) ; \text{the index of } (a \bmod p^2) \text{ in } \left(\mathbb{Z}/p^2\mathbb{Z}\right)^{\times} \text{ is "rather large"} \} \right| .$$

The result can be deduced from a generalization of C. Hooley's work on Artin's conjecture for primitive roots. In addition we talked about another application of this generalization on the magnitude of the least prime primitive root mod p, of course under assumption of G.R.H.

J.L. NICOLAS

On the subsets of a partition

We shall say that a partition of n, say $n = n_1 + \dots + n_s$, represents a, if there is a subsum $n_{i_1} + \dots + n_{i_r}$ of the partition whose sum is a. We shall denote by $R(n,a)$ the number of partitions of n which do not represent a. In a paper recently accepted in the Memoires of the French Math. Soc. J. Dixmier has proved that for a fixed a, and $n \to +\infty$, one has

$$R(n,a) \sim p(n) \left(\frac{\pi}{\sqrt{6n}}\right)^{\psi(a)} u(a)$$

where $\psi(a) = \left\lfloor \frac{a}{2} \right\rfloor + 1$ and $u(a)$ is a constant depending on a. In a paper which will appear in Cambridge in a volume dedicated to P. Erdös for his 75[th] birthday, P. Erdös, A. Sarközy and myself prove

Theorem. There exists $\lambda > 0$ such that when $n \to \infty$, uniformly for $1 \leq a \leq \lambda\sqrt{n}$, we have

$$\psi(a) \log\left(\frac{\pi a}{\sqrt{6n}}\right) - 0.8\, \psi(a) + O\left(\frac{a^2}{\sqrt{n}}\right) \leq \log\left(\frac{R(n,a)}{p(n)}\right) \quad \text{and}$$

$$\log\left(\frac{R(n,a)}{p(n)}\right) \leq \psi(a) \log\left(\frac{\pi a}{\sqrt{6n}}\right) + O\left(\frac{1}{\sqrt{n}}\right).$$

In a forthcoming paper we hope to deal with the case $\lambda\sqrt{n} < a \leq \frac{n}{2}$, and to show the dependency of $R(n,a)$ upon the smallest integer which does not divide a.

A.M. ODLYZKO

Lattice points in higherdimensional spheres

Let $N_n(\alpha,\xi)$ denote the number of lattice points in a sphere in $\mathbb{R}^n$ of radius $\alpha\sqrt{n}$ with center at $\xi = (x_1, \dots, x_n)$. The asymptotic behaviour of $N_n(\alpha,\xi)$ for $\alpha > 0$ fixed and $n \to \infty$ displays some surprising features. The average number (averaged on ξ running over the unit cube) of lattice points is equal to the volume of the sphere. It is shown that the maximal number is larger than the average by an exponential factor (in n) and the minimal number is smaller than the average by another exponential factor.

J. PINTZ

Fast generation of primes and Linnik's theorem

Adleman, Pomerance and Rumely found in 1980 an algorithm which decides the primality of n in $(\log n)^{c \log\log\log n}$ steps (with a running time analysis of Odlyzko). The most important problem in this topic asks for an algorithm which works in polynomial time $(\log n)^c$ in the input size $\log n$. (Assuming the Generalized Riemann Hypothesis Miller constructed a primality test with $c \log^5 n$ steps.) Our first theorem answers a problem raised by Adleman, Pomerance and Rumely.

Theorem 1. (J. Pintz, W.L. Steiger and E. Szemerédi) There exists an infinite set $\mathcal{P}$ of primes such that $n \in \mathcal{P}$ can be tested in $c \log^9 n$ steps.

The second result refers to "very fast" generation of primes with a random algorithm.

Theorem 2. (J. Pintz, W.L. Steiger and E. Szemerédi) A k digit prime might be generated in expected time $c\,k^4$.

This follows from the existence of a set $\mathcal{P}$ containing infinitely many primes whose primality might be proved in expected time needed for $3/2$ exponentiations. ($3/2$ can be replaced even by any $c > 1$). These problems are closely connected with Linnik's theorem for which the following statistical version can be proved.

Theorem 3. Let x and $\varepsilon > 0$ be given then

$$\# \left\{ p \le x^{\frac{5}{12} - \varepsilon},\ p \text{ prime},\ \exists\, l \not\equiv 0\,(p) \text{ s.t. } \left| \pi(x,p,l) - \frac{\operatorname{li} x}{p-1} \right| > \varepsilon \frac{\operatorname{li} x}{p-1} \right\} < c(\varepsilon)$$

with an effective constant depending only on ε.

J. POMYKALA

Remainder term in the Rosser – Iwaniec sieve of dimension $x \in (\frac{1}{2}, 1)$

We consider the x – dimensional Rosser – Iwaniec sieve. The following theorem is an extension of Iwaniec's well known result for the linear sieve (1977):

Theorem . Let $0 < \varepsilon < \frac{1}{3}$, $M > 1$, $N > 1$, $\Delta = MN^{\beta - 1}$, where β is the sieving limit. Then we have the following estimates for the sifting function $S(\mathcal{A}, P, z)$:

$$\pm S(\mathcal{A},P,z) \le \pm \begin{cases} F\left(\dfrac{\log \Delta}{\log z}\right) \\ f\left(\dfrac{\log \Delta}{\log z}\right) \end{cases} + E(\varepsilon, \Delta) + R^{\pm}(\mathcal{A}, M, N) \qquad (z \le \Delta^{1/\beta})$$

where $E(\varepsilon, \Delta) \ll \varepsilon + \varepsilon^{-14} (\log \Delta)^{-\frac{1}{3}}$ and the remainder term $R^{\pm}(\mathcal{A}, M, N)$ has the following shape:

$$R^{\pm}(\mathcal{A},M,N) = \sum_{j \le \exp(13\varepsilon^{-9})} \sum_{m \le M} a^{\pm}_{m,j} \sum_{n \le N} b^{\pm}_{n,j}\, r(\mathcal{A}, mn)$$

with $a^{\pm}_{m,j} = a^{\pm}_{m,j}(M,N,\varepsilon)$, $b^{\pm}_{n,j} = b^{\pm}_{n,j}(M,N,\varepsilon)$ s.t. $|a^{\pm}_{m,j}| \le 1$, $|b^{\pm}_{n,j}| \le 1$.

K. RAMACHANDRA

A remark on $\zeta(1 + it)$

A class of results of the following type were proved: Let $T \ge T_0(\varepsilon)$, where $\varepsilon > 0$ is a constant. Let $x = \exp\left(\frac{\log\log T}{\log\log\log T}\right)$. Consider the interval $T \le t \le T + e^x$. Here with the exception of $k = k(\varepsilon)$ intervals for t each of length $\frac{1}{x}$ we have

$|\log \zeta(1+it)| \le 2\varepsilon \log\log T$. We obtain $k(\varepsilon) = 8\left(\left[\frac{1}{\varepsilon^2}\right] + 1\right)$ if we assume the Hadamard - de la Vallée-Poussin zero free region $\sigma > 1 - \frac{c}{\log(|t|+2)}$.

We can reduce $k(\varepsilon)$ if we assume the Vinogradov zero free region

$$\sigma > 1 - \frac{c}{(\log(|t|+2))^{2/3}(\log\log(|t|+100))^{1/3}}.$$

U. RAUSCH

Asymptotic formulae for summatory functions in algebraic number fields

Let K be an algebraic number field. By means of an improved version of Siegel's summation formula several asymptotic relations are obtained which are either new or sharper than the previously known ones, e.g. for the Piltz divisor problem for numbers in K or the number of units $\eta \in K$ satisfying

$$\sum_{p=1}^{r+1} \left(|\eta^{(p)}| / x_p \right)^{\alpha} \le 1,$$

where $\alpha > 0$ is fixed and $x_1, \dots, x_{r+1} > 0$ are variables.

F. ROESLER

The Riemann Hypothesis as an Eigenvalue Problem

The determinant of the matrix $A_N = (a_{m,n})_{2 \le m,n \le N}$, $a_{m,n} = m-1$ if $m \mid n$ and $a_{m,n} = -1$ if $m \nmid n$, is $N! \cdot \sum_{m \le N} \frac{\mu(m)}{m}$ and hence directly connected with the Riemann Hypothesis. Thus the eigenvalues λ of A_N may be of interest. Some estimates are given: (1) $|\lambda| \le N - \frac{1}{N}$ for all eigenvalues λ of A_N. (2) $\lambda_m \in [m, m+1[$, $1 \le m \le N-1$, if the eigenvalues $\lambda_1, \dots, \lambda_{N-1}$ of A_N are indexed appropriately, with at most $2\sqrt{N}$ exceptions. (3) If $\lambda_m \in [m, m+1[$, m near to N, then

$$\lambda_m = m + 1 - \frac{\tau(m+1)}{\zeta_\infty \log N}\left(1 + O\left(\frac{\log\log\log N}{\log\log N}\right)\right);$$

if $\lambda_m \in [m, m+1[$, m near to $N/2$, $m > N/2$, then

$$\lambda_m = m + \frac{\tau(m)}{\zeta_\infty \log N}\left(1 + O\left(\frac{\log\log\log N}{\log\log N}\right)\right),$$

with τ multiplicative, $\tau(p^m) = \prod_{k=1}^{m} (1 - p^{-k})^{-1}$,

$$\zeta_\infty = \prod_{m=2}^{\infty} \zeta(m) = \sum_{m=1}^{\infty} \frac{\tau(m)}{m^2} = 2.294..$$

Assumptions (unfortunately unproved and certainly difficult to show) on the small eigenvalues of A_N or on the values of the characteristic polynomial of A_N near to 0 lead to proofs for the existence of a small zero - free strip inside the critical strip of the Riemann zeta function.

A. SCHINZEL

The greatest prime factor of the value of a polynomial

Following is the result of joint work with P. Erdös: For every polynomial $f \in \mathbb{Z}[x]$ which is not the product of linear factors with integral coefficients, the greatest prime factor of $\prod_{k=1}^{x} f(k)$ exceeds $x \operatorname{expexp}\left(c(f)\left(\operatorname{loglog} x\right)^{\frac{1}{3}}\right)$ for $x > x_0(f)$, where $c(f)$ is a positive constant. This improves Erdös' result of 1951.

H.P. SCHLICKEWEI

An explicit upper bound for the number of solutions of the S - unit equation

Using my p - adic generalization of the quantitative version of W.M. Schmidt's Subspace Theorem the following result can be proved: Let $S = \{p_1, \dots, p_s\}$ be a finite set of primes. An element $x \in \mathbb{Q}^*$ is called an S - unit, if it is only composed of primes in S. Let $n \geq 1$ and suppose that $a_1, \dots, a_{n+1}$ are given nonzero rational numbers. Consider the equation

$$(*) \quad a_1 x_1 + \dots + a_{n+1} x_{n+1} = 0.$$

Theorem. The number of integral solutions $\xi = (x_1, \dots, x_{n+1})$ of $(*)$ such that

(i) each x_i is an S - unit and

(ii) no proper subsum $a_{i_1} x_{i_1} + \dots + a_{i_m} x_{i_m}$ vanishes

is bounded by

$$\left(8(s+1)\right)^{2^{26n+4}(s+1)^6}.$$

Notice that our bound does not depend upon the a_i nor on the particular primes p_i involved. Previously by independent work of Van der Poorten & Schlickewei and of Evertse it had only been known that the number of solutions is finite.

P.G. SCHMIDT

Exponentenpaare und ein Gitterpunktsatz

Sei $\Delta(x)$ das Restglied beim Teilerproblem $\sum_{k^2 m^3 n^6 \leq x} 1$. In Acta Arithmetica 50 (1988) habe ich $\Delta(x) \ll x^{\frac{1}{7} - \frac{2}{7575}}$ veröffentlicht. Der Beweis basiert auf der Exponentenpaarmethode von B.R. Srinivasan zur Abschätzung mehrdimensionaler Exponentialsummen. Da inzwischen starke Zweifel an der Korrektheit der Srinivasan'schen Methode laut geworden sind, skizziere ich einen davon unabhängigen Beweis eines Hilfssatzes, aus dem die (allerdings etwas schwächere) Abschätzung

$$\Delta(x) \ll x^{\frac{1}{7}} (\log x)^{2 + \frac{2}{7}}$$

folgt. Der Hilfssatz lautet: Ist (α, β, γ) eine Permutation von $(2, 3, 6)$, $\psi(t) = t - [t] - \frac{1}{2}$ für $t \in \mathbb{R}$, $x > 2$, $0 < \xi \leq x$, $N > 0$ und M durch $M^{\alpha+\beta} N^{\gamma} = \xi$ definiert, so gilt mit absoluter $\ll$ - Konstanten

$$\sum_{\substack{\xi/2 < m^{\alpha+\beta} n^{\gamma} \leq x \\ N < n \leq 2N,\ m > n}} \psi\left(\sqrt[\alpha]{\frac{x}{m^{\beta} n^{\gamma}}} \right) \ll \left\{ x^2 \xi^{\alpha-2} \right\}^{\frac{1}{7\alpha}} \cdot \left\{ \left(\frac{N}{M}\right)^{6-\gamma} \log^2 x \right\}^{\frac{1}{7}} \leq \left\{ x \log^2 x \right\}^{\frac{1}{7}},$$

und das = - Zeichen steht genau dann, wenn $(\alpha,\beta,\gamma) = (2,3,6)$.

S. SRINIVASAN

Some order functions of groups

In this talk I report on joint works with (I) M. Ram Murty and (II) Mrs. M.J. Narlikar:

(I) Let $a(n)$ be the number of non isomorphic groups of order n. Then

$$\mu^2(n)\cdot a(n) = O\left(\frac{n}{(\log n)^{A \logloglog n}} \right)$$

with some constant $A > 0$. Also, up to the value of A, this is best possible.

(II) Let $c'(x)$ be the number of $n \leq x$ s.t. every group of order n is abelian but there exists a non - cyclic group of order n. Then

$$c'(x) \sim e^{-\gamma} \frac{x}{\loglog x\ (\logloglog x)^2} \quad \text{as } x \to \infty.$$

J. SZMIDT

The summation formula for Dirichlet series with cubic Gauss sums

We shall consider the cubic Gauss sums

$$g(c) = \sum_{\substack{d \in \mathbb{Z}[\omega] \\ d \equiv 1 \bmod 3}} (d \mid c)_3 \; e\left(\frac{d}{c}\right)$$

where $\omega = e^{\frac{2\pi i}{3}}$, $(\,.\mid .\,)_3$ is the cubic residue symbol in $Q(\omega)$ and $e(z) = e^{2\pi(z+\bar{z})}$.

We consider Dirichlet series

$$\psi(s,n) = \sum_{\substack{c \in \mathbb{Z}[\omega] \\ c \equiv 1 \bmod 3}} g(c)\ (Nc)^{-s} \left(\frac{\bar{c}}{|c|}\right)^n, \quad s \in \mathbb{C},\ \mathrm{Re}\, s > \frac{3}{2},\ n \in \mathbb{Z}.$$

The theory developed by T. Kubata and S.J. Patterson gives the meromorphic continuation and functional equation of these series. Using the residue theorem and the Mellin transform we can derive the summation formula

$$\sum_{\substack{c \in \mathbb{Z}[\omega] \\ c \equiv 1 \bmod 3}} g(c)\ f(Nc) = \sum_{\substack{c \in \mathbb{Z}[\omega] \\ c \equiv 1 \bmod 3}} g(c) \int_0^{\infty} I_f(Nc,x)\, dx$$

where f is a test function and $I_f(Nc,x)$ is an integral transform of f involving the generalized Bessel function. There is an open problem to get the estimation

$$\psi(1+it,n) \ll_{n,\varepsilon} |t|^{\frac{2}{3}+\varepsilon}.$$

G. TENENBAUM

Integers free of large prime factors in arithmetic progressions

In this joint work with E. Fouvry we investigate the distribution in residue classes of the set $S(x,y) = \{ n \le x ; P(n) \le y \}$ where $P(n)$ denotes the largest prime factor of n, with the convention that $P(1) = 1$. The study is motivated by the growing importance of $S(x,y)$ in all branches of analytic number theory as well as by the recent drastic improvements upon classical estimates concerning this set. We give analogues to the Siegel - Walfisz and to the Bombieri - Vinogradov theorems. A crucial feature of the results is that the main terms are expressed as smooth functions. This enables us to obtain an application related to a recent work of Balog, Friedlander and Pintz and which essentially states that, on the average, the integers with a large prime factor are as well distributed in arithmetic progressions as if the Generalized Riemann Hypothesis were true.

R.F. TICHY

Weak uniform distribution of linear recurring sequences (joint work with G. Turnwald)

Let $(u_n)_{n \ge 0}$ be a linear recurring sequence (l.r.s.) of order t with integral coefficients and integral initial values. Let m be a positive integer ≥ 2. (u_n) is called weakly uniformly distributed mod m (WUD mod m), if

$$\lim_{N \to \infty} \frac{\operatorname{card} \{ 0 \le n \le N ;\ u_n \equiv r \bmod m \}}{\operatorname{card} \{ 0 \le n \le N ;\ u_n \text{ is invertible mod } m \}} = \frac{1}{\varphi(m)}$$

for all invertible residue classes r mod m , φ denoting Euler's function. In the case of inhomogeneous first order l.r.s. $u_{n+1} = a u_n + b$ necessary and sufficient conditions for (u_n) to be WUD mod m are proved. It turns out that if (u_n) is WUD mod p^2 (p prime) then it is WUD mod p^k ($\forall k = 1,2,\ldots$). Partial results are also established for higher order l.r.s. Most of the results can be extended to algebraic number fields.

V. TURÁN - SÓS

On additive number - theoretic problems

Let $A(n)$ be the counting function of the sequence (a_i), $a_i \in \mathbb{Z}^+$, and let

$$R_1(n) = \sum_{a_i + a_j = n} 1 .$$

The Erdös - Fuchs theorem and generalizations of it state in a quantitative form that the representation function R_1 globally can not behave too regularly. With Erdös and Sárközy we investigated some local properties of the representation functions (like monotonicity or boundedness of $| R_1(n+1) - R_1(n) |$).

We prove e.g.: R_1 is monotone for $n > n_0$ iff $A(n) = n + O(1)$. However,

$$R_2(n) = \sum_{a_i + a_j = n,\ i < j} 1$$

can be monotone also in some non trivial cases (when $A(n) < n - c n^{\frac{1}{3}}$), but cannot be monotone if $A(n) = o(n / \log n)$.

For

$$R_3(n) = \sum_{a_i + a_j = n,\ i \le j} 1$$

we have some partial results, but we conjecture that also R_3 is bounded for $n > n_o$ iff $A(n) = n + O(1)$. The boundedness of $|R_i(n+1) - R_i(n)|$ depends on the number of blocks in $\mathcal{A} = \{a_i\}$ defined by $B(n) = \# \{ a \le n, a \in \mathcal{A}, a-1 \notin \mathcal{A} \}$. E.g. we can prove that $\lim_{n\to\infty} B(n) n^{-\frac{1}{2}} = 0$ implies $\sup_n |R(n+1) - R(n)| = \infty$ and that there exist sequences with $\limsup_{n\to\infty} B(n) n^{-\frac{1}{2}} > 0$ and $\sup_n |R(n+1) - R(n)| < \infty$.

R.C. VAUGHAN

A new iterative method in Waring's problem

An account was given of recent work on Waring's problem. Let $G(k)$ denote the least s such that every sufficiently large natural number is the sum of at most s k th powers. For small values of k we now know $G(k) \le H(k)$ where $H(k)$ is given by

k	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
H(k)	19	29	41	57	75	93	109	125	141	156	171	187	202	217	232	248

For large k we now know that

$$G(k) < 2k \left(\log k + \log\log k + 1 + \log 2 + O\left(\frac{\log\log k}{\log k} \right) \right).$$

An associated problem is the estimation of

$$R_{k,s}(n) = \operatorname{card}\{ (x_1, \dots x_s) ; x_1^k + \dots + x_s^k = n \}$$

and we can now show that

$$R_{k,s}(n) \gg n^{\frac{s}{k}-1} \mathfrak{S}(n) \qquad (s \ge H(k),\ k \ge 4)$$

where $\mathfrak{S}(n)$ is the usual singular series. We can also show that

$$R_{3,7}(n) \gg n^{\frac{4}{3}}.$$

R. WARLIMONT

On a problem posed by I.Z. Ruzsa

We prove this: Let x be a natural number. Let $\mathcal{A}(x)$ denote the collection of all subsets $A \subset \{1, \dots, x\}$ with the property

$$\sum_{a\in A} \left(\left[\frac{x}{a} \right] + 1 \right) \ge x .$$

Then

$$\min_{A \in \mathcal{A}(x)} \sum_{a \in A} \frac{1}{a} = \log \frac{2^5\, 3^6}{23^3} + O\left(x^{-\frac{1}{2}} \right).$$

D. WOLKE

On prime twins

Let $\psi_2(x,2k) = \sum_{2k<n\le x} \Lambda(n)\Lambda(n-2k) \quad (k \in \mathbb{N},\ x > 2k)$,

$$M(x,2k) = 2(x-2k) \prod_{p\mid k,\, p>2} \frac{p-1}{p-2} \cdot \prod_{p>2} \left(1 - \frac{1}{(p-1)^2}\right)$$

and $E(x,2k) = \psi_2(x,2k) - M(x,2k)$.

The following "almost all" results are discussed:

1. $E(y,2k) = O\left(y(\log y)^{-A}\right)$ for all $k \le \frac{x}{2}$ with at most $O\left(x(\log x)^{-B}\right)$ exceptions and for all $y \in \left[x, x^{8/5-\varepsilon}\right]$ ($A,B,\varepsilon > 0$, arbitrary).

2. Assume the Generalized Riemann Hypothesis. Then for $k \le k_o = (\log x)^{178}$ with at most $O\left(\frac{k_o}{\log x}\right)$ exceptions, we have

$$\psi_2(x,2k) = M(x,2k) + O\left(\frac{x}{\log x}\right).$$

Roughly speaking this means that for "almost all" k's there are "almost infinitely many" prime pairs with difference $2k$.

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