MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 45/1988

Stochastische Analysis

23.10. - 29.10. 1988

Die Tagung fand unter der Leitung von J. M. Bismut (Orsay) und H. Föllmer (Bonn) statt. In 37 Vorträgen und mehreren abendlichen Arbeitsgruppen ergab sich ein breites Spektrum von aktuellen Themen der stochastischen Analysis, unter anderem mit den folgenden Schwerpunkten:

- Malliavin-Kalkül und antizipative stochastische Integration
- asymptotische Entwicklung von Wiener-Funktionalen und insbesondere von Wärmeleitungskernen
- hypoelliptische Diffusionen und ihre Potentialtheorie
- grosse Abweichungen
- unendlich-dimensionale Diffusionen und insbesondere Superprozesse
- stochastische Mechanik.

Insgesamt wurde deutlich, dass sich die Wechselwirkungen zwischen Wahrscheinlichkeitstheorie, Analysis, Geometrie und mathematischer Physik weiter intensivieren.

Erst während der Tagung erfuhren die meisten Teilnehmer vom Tode von

Michel Métivier

am 10. Oktober 1988. Viele von uns standen ihm persönlich sehr nahe und wurden durch seine menschliche Wärme und seine Begeisterungsfähigkeit bestärkt; alle empfinden schmerzlich den Verlust für das Gebiet der Stochastischen Analysis. Es wurde beschlossen, diese Tagung seinem Andenken zu widmen.

Vortragsauszüge

Sergio Albeverio (with R. Høegh-Krohn, K. Iwata):

Covariant markovian random fields over $\mathbf{R}^4$

The construction of invariant (i.e. homogeneous = stationary with respect to the Euclidean group on $\mathbf{R}^d$) global Markov scalar random fields has been achieved, besides the Gaussian case (free Markov field) (and the infinitely divisible case) by constructing additive functionals of the free Markov field (a recent result of Høegh-Krohn, Zegarlinski and myself concerns the "φ_2^4-model"). The difficulty of getting such fields for $d > 2$ makes it tempting to look for non scalar fields (like the electromagnetic or other gauge fields). In this lecture we discussed a construction of vector fields with Markovian properties over $\mathbf{R}^4$. They are gotten as solutions of a system of stochastic first order p.d.e. with constant coefficients and source (i.h.s.) a generalized random field of infinitely divisible type (Gaussian white noise plus generalized Poisson part). The system of equations is best formulated using the identification of $\mathbf{R}^4$ with the algebraic field of quaternions, as $\partial A = F$, where F is the source and ∂ is a natural first order quaternionic differential operator with unit coefficients (the quaternionic version of $\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}$) . By suitable choice of the source the solution A is a vector field, covariant (in law) under the proper Euclidean group over $\mathbf{R}^4$. A necessary and sufficient condition for reflection invariance with respect to 3-dimensional hyperplanes in $\mathbf{R}^4$ is also found. If the source is Gaussian white noise the fields are (Euclidean) free electromagnetic potential fields and have the global Markov and reflection positivity properties. Fields with (generalized) Poisson source can be used as approximation of those with Gaussian source and have better support properties, so that additive functionals of these can be studied.

Ludwig Arnold (joint work with Luiz San Martin, Campinas, Brazil)):

A multiplicative ergodic theorem for rotation numbers

Given a vector field X on a Riemannian manifold M of dimension at least 2 whose flow leaves a probability measure μ invariant. The multiplicative ergodic theorem of Oseledec (1968) tells us that μ-a.s. every tangent vector possesses a Lyapunov exponent (exponential growth rate) which is equal to one of finitely many basic exponents corresponding to X and μ.
We prove that in the case of a simple Lyapunov spectrum (which is generic) every tangent plane μ-a.s. possesses a rotation number which is equal to one of finitely

many basic rotation numbers corresponding to X and μ. Rotation in a plane is measured as the time average of the infinitesimal changes of the angle between a frame moved by the linearized flow and the same frame parallel-translated by a (canonical) connection.

Dominique Bakry:

Complex interpolation for diffusion semigroups

For a general Markov symmetric semigroup, E. Stein showed that

$$\|P_{te^{i\alpha}} f\|_p \leq \|f\|_p$$

for

$$p \in \left[\frac{1}{1-\frac{|\alpha|}{17}}, \frac{\pi}{|\alpha|}\right].$$

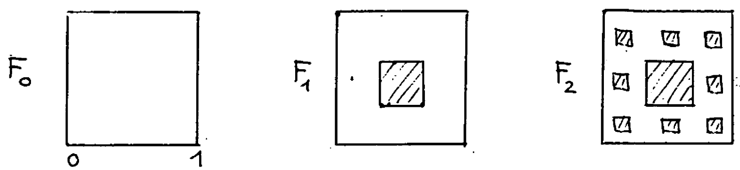
We show that for diffusion semigroups, we may get the same result for

$$p \in \left[\frac{2}{1+\cos\alpha}, \frac{2}{1-\cos\alpha}\right].$$

Martin Barlow:

Diffusions on the Sierpinski carpet

The Sierpinski Carpet (SC) is a "fractal" subset of $\mathbf{R}^2$, formed like this:



$F = \bigcap F_n$ is the SC. Let W^n be Brownian motion on F_n, with orthogonal reflection at the boundaries. We show that there exist constants α_n such that, if $X_t^n = W_{\alpha_n t}^n$, then (X^n) is tight. Let X^{n_k} be a convergent subsequence, and $X = \lim X^{n_k}$.

Theorem. (1) X is an F-valued diffusion. (2) X is μ-symmetric, where μ is Hausdorff $x^{\log 8/\log 3}$ measure on F. (3) X has jointly continuous λ-potential kernel densities $g_\lambda(x, y)$.

Gérard Ben Arous:

Short time asymptotics of degenerate heat kernels

If $X_0, X_1, \dots, X_m$ are smooth vector fields on $\mathbf{R}^d$, such that the commutators of $X_1, \dots, X_m$ span $\mathbf{R}^d$ at each point (Hörmander's condition), let $p_t(x,y)$ be the heat kernel associated to the operator

$$L = \frac{1}{2} \sum_{i=1}^{m} X_i^2 + X_0 \, .$$

The asymptotic behaviour of $p_t(x,y)$ for small t is given by

1) $\lim t \log p_t(x,y) = -\frac{1}{2} d^2(x,y)$ (this result is due to Léandre) where $d(x,y)$ is the sub-Riemannian distance associated to$X_1, \dots, X_m$.

2) $p_t(x,y) = \dfrac{1}{\sqrt{t}^d} e^{-d^2(x,y)/2t} \left(\sum_0^N c_k(x,y) t^k + O(t^{NM}) \right) \qquad \forall N \geq 0$

If (x,y) are not in the cut-locus,

3) $p_t(x,x) = \dfrac{1}{\sqrt{t}^{a(x)}} \left(\sum_0^N c_k(x) t^k + O(t^{NM}) \right) \qquad \forall N \geq 0$

where $a(x)$ is an integer defined by the geometry of the commutators of $X_1, \dots, X_m$ at x (if the drift X_0 is of the form

$$X_0 = \sum_{i=1}^{m} f_i X_i + \sum_{1 \leq i,j \leq m} f_{ij} [X_i, X_j]$$

where f_i, f_{ij} are smooth). This result has been obtained independently by Léandre and myself.

I have presented here a joint work with Léandre about the influence of the drift X_0 on the behaviour of $p_t(x,x)$ for small t. The important point is that the drift can induce an exponential decay of $p_t(x,x)$ instead of the usual blow-up. This is the case if, for instance, $X_0(x)$ is contained in the space generated by the $X_i(x)$ and $[X_i, X_j](x)$. But it may also happen even if $X_0(x)$ is in $span\big(X_i(x), \, 1 \leq i \leq m\big)$ as shown by an example.

Philippe Biane:

Asymptotics of additive functionals of several Brownian motions

Let f be an integrable function on $\mathbf{R}^d$ and $B^1, \dots, B^k$ k independent Brownian motions with values in $\mathbf{R}^d$. We study the long time asymptotics of additive functionals

$$\int_0^t \dots \int_0^t f(B^1_{s_1} + \dots + B^k_{s_k})\, ds_1 \dots ds_k .$$

If $d = 2k - 1$,

$$\frac{1}{\sqrt{n}} \int_0^{nt} \dots \int_0^{nt} f(B^1_{s_1} + \dots + B^k_{s_k})\, ds_1 \dots ds_k$$

converges in law towards $\int f \cdot \alpha(0; t, \dots, t)$ where α is the local time of the process $B^1_{s_1} + \dots + B^k_{s_k}$. If $d = 2k$,

$$\frac{1}{\log n} \int_0^{nt} \dots \int_0^{nt} f(B^1_{s_1} + \dots + B^k_{s_k})\, ds_1 \dots ds_k$$

converges in law towards $\int f \cdot \Gamma_k(t)$ where $\Gamma_k(t)$ is an independent increments process, such that for each t, $\Gamma_k(t)$ has a Gamma-law of parameters $\frac{1}{2^k - 1}$ and t. We have also second order results for the case where $\int f = 0$: In dimension $2k-1$,

$$\frac{1}{n^{\frac{1}{4}}} \int_0^{nt} \dots \int_0^{nt} f(B^1_{s_1} + \dots + B^k_{s_k})\, ds_1 \dots ds_k$$

converges in law towards $\beta\big(f, \alpha(0; t, \dots, t)\big)$ where β is a Gaussian process of covariance

$$E\big[\beta(f,t)\beta(g,s)\big] = \left(- \int f(x) g(y) |x - y|\, dx dy \right) s \wedge t$$

which is independent of α. A similar result holds when $d = 2k$, the process Γ_k replacing $\alpha(0; t, \dots, t)$.

Erwin Bolthausen:

A central limit theorem for directed polymers in random environments

The model is the following: $\xi(0) = 0, \xi(1), \xi(2), \dots$ is an ordinary symmetric random walk on $\mathbf{Z}^d$. $X(t, x)$, $t \in \mathbf{N}$, $x \in \mathbf{Z}^d$, are i.i.d. random variables (the random environment) which are strictly positive, satisfy $EX = 1$, $\operatorname{var}(X) < \infty$ and are also independent of ξ. Let

$$x_t = \prod_{j=1}^{n} X\big(j, \xi(j)\big) \qquad , \qquad t \in \mathbf{N}.$$

The problem is to determine asymptotic properties ($t \to \infty$) of the following random probability measure on $\mathbf{Z}^d$:

$$\mu_t(A) = E\left(1_{\xi_t \in A} x_t \,\middle|\, X\right) \Big/ E\left(x_t \,\middle|\, X\right) \qquad , \qquad A \subset \mathbf{Z}^d .$$

J. Imbrie and T. Spencer proved (J. Stat. Phys., to appear) that if $d \geq 3$ and if the disorder (i.e. $\operatorname{var}(X)$) is small, then the mean-square displacement under μ_t is almost surely asymptotically the same as for the ordinary random walk. Their proof is based on cluster expansion techniques. A very simple proof of a slightly better result (including a central limit theorem) is presented using the martingale limit theorem.

Jean-Dominique Deuschel (joint work with D. Stroock):

Large deviations for certain stochastic integrals

Given a diffusion process $\{u_t, t \in \mathbf{R}_+\}$ on a compact manifold of the form

$$\begin{aligned} du_t &= \sum_1^d X_k(u_t) \circ d\beta_t^k + X_0(u_t)\, dt \\ u_0 &= u \end{aligned}$$

where $X_0, \ldots, X_d$ are smooth vector fields satisfying a weak Hörmander condition and a control condition, let $\{y_t, t \in \mathbf{R}_+\}$ be the real-valued process

$$\begin{aligned} dy_t &= \sum_1^d Y_k(u_t) \circ d\beta_t^k + Y_0(u_t)\, dt \\ y_0 &= 0 . \end{aligned}$$

We derive a large deviation principle for

$$y(T) = \left\{ \frac{1}{T} y_{Tt}, t \in [0,1] \right\}$$

as $T \to \infty$ and identify the corresponding rate function on $C\big([0,1]; \mathbf{R}\big)$. The results apply to the large deviations of the Lyapunov exponent of some homogeneous system.

Markus Dozzi:

On the local time of the multiparameter Wiener process

Let $(W_t, t \in \mathbf{R}_+^p)$ be the p-parameter Wiener process with values in $\mathbf{R}^d$. We consider the asymptotic behaviour of

$$X_t = \int_{[0,t]} f(W_s)\, ds$$

as $t \uparrow \infty$. For $d = 1$ we show that $|t|^{-\frac{1}{2}} X_t$ converges in law as $t \uparrow \infty$. For $d = 2$ (resp. $d \geq 3$) we show that $(\log |t|)^{-p} X_t$ (resp. $(\log |t|)^{-(p-1)} X_t$) converges a.s. as $t \uparrow \infty$ along the diagonal. This can be applied

(a) to describe the behaviour of the local time $L(t,x)$ of W as $x \to 0$ ($x \in \mathbf{R}^d$, $t \in \mathbf{R}_+^p, d < 2p$), and

(b) to prove an ergodic property of X_t.

Eugene Dynkin:

Superprocesses

We say that a transition function $\mathcal{P}_t(\mu, d\nu)$ in the space $\mathcal{M}(E)$ of measures is a supertransition function over a transition function $p_t(x, dy)$ in E if

$$\int \mathcal{P}_t(\mu, d\nu)\nu(B) = \int \mu(dx) p_t(x, B)\,.$$

This implies: if M is an equilibrium measure for $\mathcal{P}$, then

$$m(B) = \int M(d\nu)\nu(B)$$

is an invariant measure for p. We call m the projection of M. For a class of supertransition functions introduced by S. Watanabe in 1968, a lifting operation can be introduced which associates, with every p-invariant measure m, a $\mathcal{P}$-equilibrium M_m such that

$$\int e^{-\langle f,\nu\rangle} M_m(d\nu) = \lim_{t\to\infty} \int \mathcal{P}_t(m, d\nu) e^{-\langle f,\nu\rangle}$$

for all $f \in L_+^1(m)$ (here $\langle f, \nu\rangle = \int f\, d\nu$). Put $m \in C$ if $M_m\{0\} = 1$ and put $m \in D$ if the projection of M_m coincides with m. The mapping $m \mapsto M_m$ is a 1-1 mapping from D onto a subset $\widetilde{\mathcal{M}}$ of the set $\mathcal{M}$ of all extremal $\mathcal{P}$-equilibria

(the inverse mapping coincides with the projection). The entire set $\mathcal{M}$ can be described by considering the space $\mathcal{E}$ of all entrance laws κ for p and evaluating the ergodic decomposition of $\mathcal{E}$ relative to the flow $(\vartheta_u\kappa)_t = \kappa_{t-u}$.
Somewhat dual to this construction is a mapping which associates a harmonic character $F(\mu) = e^{-\langle f,\mu\rangle}$ with every positive p-harmonic function. The general theory is illustrated on the examples of super Brownian motions on Euclidean and hyperbolic spaces.

Ernst Eberlein:

Strong approximation of continuous-time stochastic processes

Given two sequences of stochastic processes $\left(X^n(t)\right)_{t\geq 0}$ and $\left(Y^n(t)\right)_{t\geq 0}$ we study conditions under which the processes can be modified such that

$$\|X^n(t) - Y^n(t)\|_{t_n} \leq \varepsilon_n \qquad a.s.$$

Here $(\varepsilon_n)_{n\geq 0}$ and $(t_n)_{n\geq 0}$ describe a nonincreasing resp. nondecreasing sequence of real numbers and the norm is the supremum norm on $[0, t_n]$. Roughly three assumptions turn out to be necessary: a maximal inequality, a tail estimate and an estimate on the conditional characteristic functions of the increments. Two different approaches are discussed. Furthermore we state assumptions which allow the almost sure approximation of a semimartingale by a continuous process with independent increments. The results are mainly motivated by weak convergence results for semimartingales on $D[0,\infty)$ due to Liptser and Shiryayev.

Hans-Jürgen Engelbert (with W. Schmidt):

On a Stochastic Equation Involving Local Time

We consider the stochastic equation in one dimension

$$(1) \qquad X_t = X_0 + \int_0^t b(X_s)dW_s + \int_{\mathbf{R}} L^X(t,y)v(dy)$$

where b is a Borel function, v is a set function which is a finite signed measure on every bounded set, and L^X is the right (resp. left, symmetric) local time of the unknown process X. A solution $(X, \mathbf{F})$ is a continuous semimartingale up to the explosion time such that there is a Brownian motion $(W, \mathbf{F})$ for which (1) is satisfied up to the explosion time. We compare this equation with stochastic equations with usual drift term $\int_0^t \widetilde{a}(X_s)d\langle X\rangle_s$ or $\int_0^t a(X_s)ds$ for Borel functions $\widetilde{a}$

and a. Then we recall the necessary and sufficient conditions for (weak) existence and uniqueness (in law). We introduce the notion of fundamental solution which spends no time in the zeros of b up to the first entry time in

$$E_b = \{x \in \mathbf{R} : b^{-2} \text{ is not integrable in every open } G \ni x\} .$$

The fundamental solution is unique (in law). Every solution to equation (1) can be constructed from a fundamental solution by time delay in the zeros of b. We then give a generalized Hölder condition (H) which implies the pathwise uniqueness of the fundamental solution. Together with the existence condition $E_b \subseteq \{b = 0\}$, from (H) we obtain the existence of a pathwise unique strong fundamental solution. Also we discuss a generalized Nakao condition which leads to analogous results. Finally, we state a condition for pathwise uniqueness and strong existence which includes both the generalized Hölder condition and the generalized Nakao condition.

Reinhard Höpfner:

On convergence of Martingales associated to continuous-time Markov chains

We consider martingales M (e.g. compensated counting processes) arising in continuous -time Markov chains with countable state space. Suitably normed and scaled ($M^n = M(\cdot n)/a_n$), they converge weakly either to Brownian motion (this covers the ergodic case) or to Brownian motion evaluated at level-crossing times of an independent stable subordinator with index $\alpha, 0 < \alpha < 1$ (in case where the tails of the lifecycle length distribution of Z vary regularly at ∞ with index α). We briefly point out an application to Statistics (LAMN for statistical models where Z is allowed to be recurrent null).

Peter Imkeller:

Die Integratoreigenschaft der Variationsprozesse des stochastischen Kalküls von Zweiparameter-Martingalen

Jedes quadratintegrierbare Zweiparameter-Martingal M hat eine reguläre Version und lässt sich eindeutig in drei Sprungteile und einen stetigen Teil zerlegen, die paarweise orthogonal sind. Die Sprungteile können gut approximiert werden durch einfache Martingale, die durch Kompensation von Sprüngen auf f.s. endlichen zufälligen Mengen entstehen. Mit Hilfe dieses Resultats kann man zeigen, dass alle relevanten Variationsprozesse im stochastischen Kalkül von M regulär sind.

Martingalungleichungen, die aus den Burkholder-Davis-Gundy'schen hergeleitet werden, und eine einfache Ungleichung, die es erlaubt, iterierte quadratische Variationen zu kontrollieren, führen zu einem Beweis der stochastischen Integratoreigenschaft dieser Prozesse. Damit könnte die Grundlage für eine Charakterisierung von "martingalähnlichen" Mehrparameter-Prozessen gegeben sein.

Ingemar Kaj (with K. Fleischmann):

Large deviations for some measure-valued branching diffusion processes

Representations of the moment generating functional and the free energy functional for some critical measure-valued branching diffusions (like super-Brownian motion) are derived. These involve functions solving certain non-linear partial differential equations.

By analyzing such solutions around parameter values critical for existence, we conclude that appropriately renormalized stable branching diffusions, and also the weighted occupation time processes, have large deviation properties (at least on level 1).

Pawel Kröger:

Comparison results for diffusion processes

We consider diffusion processes on $\mathbf{R}^d$ with differential generators in non-divergence form. In the case $d = 1$ a well known comparison result of Skorokhod implies that for each pair of diffusion processes with differential generators $L_t = \Delta/2 + b(t,\cdot)\frac{\partial}{\partial x}$ and $\widetilde{L}_t = \Delta/2 + \widetilde{b}(t,\cdot)\frac{\partial}{\partial x}$ where $b \leq \widetilde{b}$, the following holds for each increasing function f on $\mathbf{R}$ and all x, t_1, t_2:

$$\int P(t_1, x; t_2, dy) f(y) \leq \int \widetilde{P}(t_1, x; t_2, dy) f(y).$$

To establish more general comparison results we investigate which function cones are invariant under the transition maps of diffusion processes with appropriate generators. In particular, we give necessary and sufficient conditions on the coefficients of the generator of a diffusion process on $\mathbf{R}^d$ for the invariance of the cone of all excessive functions with at most polynomial growth at infinity. More general function cones on $\mathbf{R}^d$ are also considered.

Rémi Léandre, Paul-André Meyer:

Multiple integral expansions for solutions of stochastic differential equations

Consider a SDE with C^∞ coefficients.

$$dX_t^i = \sum_\lambda a_\lambda^i(X_t) d_s B_t^\lambda + a_0^i(X_t)\,dt \qquad \text{in Stratonovich sense} \quad , \qquad X_0 = x$$

It leads to an expansion in Itô integrals

$$h \circ X_t = \sum \int_{s_1<\ldots<s_m<t} F(x, s_1, \lambda_1, \ldots, s_m, \lambda_m, t, h)\, dB_{s_1}^{\lambda_1} \ldots dB_{s_m}^{\lambda_m}$$

and to a (conjectural) expansion in Stratonovich integrals

$$h \circ X_t = \sum \int_{s_1<\ldots<s_m<t} \Phi(x, s_1, \lambda_1, \ldots, s_m, \lambda_m, t, h)\, d_s B_{s_1}^{\lambda_1} \ldots d_s B_{s_m}^{\lambda_m} .$$

The algebraic formalism that allows possibly to pass from Itô to Stratonovich integrals demands regularity from the coefficients, namely existence of traces of all orders. The coefficients F are given explicitly by the Isobe-Sato formula

$$\int P_{s_1}(x, dy_1) A_{\lambda_1} P_{s_2-s_1}(y_1, dy_2) \ldots A_{\lambda_m} P_{t-s_m}(y_m, h)$$

where (P_t) is the transition semigroup, and A_λ is the differential operator $\sum a_\lambda^i D_i$. It is shown that the coefficient depends continuously on $s_1 \le s_2 \ldots \le s_m$. Under Hörmander's conditions it has been shown that

$$\int A_1 p_{s_1}(x, y_1) A_2 p_{s_2-s_1}(y_1, y_2) \ldots A_{m+1} p_{t-s_m}(y_m, y)\, dy_1 \ldots dy_m$$

is continuous, $A_1, \ldots, A_{m+1}$ being smooth differential operators acting on the two variables.

Jean-François Le Gall:

Potential theory and sample path properties for a class of hypoelliptic diffusion processes

We consider a simple class of diffusion processes associated with a hypoelliptic differential operator in $\mathbf{R}^3$. This class includes the so-called Brownian motion on the Heisenberg group $\mathbf{H}_3$ studied in particular by Gaveau. Precise estimates are obtained for the Green function of the process and the capacity of small compact sets. These estimates are applied to various sample path properties such as the volume of a tubular neighborhood of the path, or the Hausdorff measure of the range. In contrast to the case of three-dimensional elliptic diffusions, we show that the process has no double points. The previous results have been obtained in collaboration with M. Chaleyat-Maurel.

Yves Le Jan:

Asymptotic properties of Brownian flows

We consider, for a measure preserving stationary isotropic flow, the asymptotic behaviour of the curvature and divergence induced by the action of the flow on a straight line. The curvature appears to be a positive recurrent diffusion whose invariant measure can be computed explicitly. From that result, we can obtain an integration by parts formula for the longitudinal energy along the unstable directions and construct the associated longitudinal diffusion. The unstable foliations should appear as atoms of the invariant σ-field of the diffusion.

Terry Lyons:

Entropy and the construction of singular measures

Consider the path space $C([0,1],\mathbf{R}^d)$ and the σ -algebra $\mathcal{F}_n$ generated by the marginal values $\omega(k/2^n)$, $\quad k=0,...,2^n \quad$, $\mathcal{F}=\bigvee \mathcal{F}_n$ and Wiener measure W on $\mathcal{F}$. Suppose that for each $n>0$ there is a probability measure Q_n on $\mathcal{F}_n$ such that $Q_n|\mathcal{F}_m = Q_m$ if $m \le n$. Let $\varrho_n = dQ_n/dW_n|\mathcal{F}_n$.

Theorem: (Q_n) extend to Q on $\mathcal{F}$ if

$$e_n = \int \varrho_n \log \varrho_n dW \le c2^{\frac{n}{2+\varepsilon}}$$

for some $\varepsilon, c > 0$. In this case the growth of e_n is directly related to the Hölder continuity of Q a.e. path.

Consider 3-dimensional Brownian motion and

$$V_r = \int\int_{J_r} \delta_0(X_s - X_t)dsdt$$

where

$$J_r = ([0,1]^2 \ \setminus \ \bigcup_1^{2^r}\{s \geq \frac{k}{2^n}, t \leq \frac{k+1}{2^n}\}) \cap \{s \leq t\}$$

and put $W^r = \exp(-V_r)dW / \int \exp(-V_r)dW$. Then the entropy of W^r grows polynomially with respect to Wiener measure.

Paul McGill:

Conformal factorizations of Brownian Motion

This is about Brownian motion B_r in dimension 1. Let $A_t = \int_0^t f(B_s)ds$ be an additive functional. If $f \geq 0$ we can define $\tau_t = A_t^{-1}$ and consider $\widetilde{B}_t = B_{\tau_t}$. This lives on the support of f, and $\widetilde{B}_t$ enters each connected region from the boundary. The theory is well known.

If $f = f^+ - f^-$ where $supp f^\pm = D^\pm$ then A_t is fluctuating and generates two time changes τ_t^+, τ_t^-. We consider $B_t^\pm = B_{\tau_t^\pm}$ and the claim (conjecture?) is that (B_t^+, B_t^-) is a conformal factorization of Brownian motion. As a justification we consider the problem of computing

$$\Pi(x, dy) = P_x[B_0^+ \in dy] .$$

In the case $A_t = \int_0^t (1_{(B_s>0)} - \delta^2 1_{(B_s<0)})\, ds$, $\Pi(x, dy)$ is readily computed. Our observation is that if we compute this (basic) case by running a complex Brownian motion Z_t then we can solve the same problem where B_t has a barrier by putting an analogous barrier on Z_t and making the appropriate modifications. The examples suggest that B^+ is purely imaginary while B^- can be regarded as the real part of a complex Brownian motion.

Carl Mueller:

A Limit Theorem for Stochastic Partial Differential Equations

We consider the equations

(1) $$u_t = u_{xx} + u^\gamma \dot{W}$$

(2) $$u_t = u_{xx} + u + u^\gamma \dot{W}$$

for $u(x,t)$, $x \in [0, \pi]$, $u(0,t) = u(\pi, t) = 0$ and $\gamma > 1$. We find upper and lower bounds for the solutions as $t \to \infty$, In both cases, the solutions decrease to 0. Also, the shape of the solution tends to $\sin x$, in both cases.

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David Nualart :

Some classes of σ-fields on the Wiener space

In a recent work A. S. Ustunel and M. Zakai have studied the relation between the independence of certain classes of sub-σ-fields on the Wiener space and the orthogonality of their tangent spaces. These classes of σ-fields are defined using the stability with respect to some operators like L, L^{-1} or the derivative operator D. We present some characterizations and some relations for these classes of σ-fields. In particular, we show that a σ-field is first-chaos generated if and only if its tangent space is deterministic and the Wiener integrals of the elements of the tangent space are measurable with respect to the σ-field. As an application we show that the σ-field generated by a multiple Itô-Wiener integral and its iterated derivatives is first-chaos generated. These results have been obtained in collaboration with A. S. Ustunel and M. Zakai.

Etienne Pardoux :

Stochastic differential equation with boundary conditions

We study SDEs of the type

$$dX_t = f(X_t)\,dt + \sum_1^k g_i(X_t) \circ dW_t^i \quad , \qquad 0 \le t \le 1$$

$$h(X_0, X_1) = \overline{h}$$

where X_t takes values in $\mathbf{R}^d$, $f, g_1, \ldots, g_k : \mathbf{R}^d \to \mathbf{R}^d$, and $h : \mathbf{R}^{2d} \to \mathbf{R}^d$. The stochastic integral is understood as a "generalized Stratonovich integral" (see Nualart-Pardoux, PTRF 1988). Two classes are considered:

A) $f, g_1, \ldots, g_k, h$ are affine functions. We give necessary and sufficient conditions for existence and uniqueness, study the existence of densities via Malliavin's calculus, and give conditions under which the solution is a Markov process, or a Markov process in the sense of fields.

B) $(g_1 \mid \ldots \mid g_d) = I \quad (k = d)$. Under some monotonicity conditions on f and h, we establish existence and uniqueness via the extended Girsanov theorem due to Kusuoka. In the case $d = 1$, we show that the solution is Markov in the sense of fields if and only if f is affine.

Part A is joint work with D. Ocone, Part B with D. Nualart.

Eckhard Platen:

The macroscopic non-equilibrium dynamics of a wide range exclusion process in random medium

Starting from a microscopic stochastic model, which represents an interacting particle system of the diffusion type, the motion of electrons in a solid material, like that of a semiconductor, is modelled. A macroscopic evolution equation for the density of electrons is obtained via a law of large numbers for the underlying wide range exclusion process in random medium. The special choice of the jump rate of the electron finally allows the description of the electron transport by a continuity equation which represents a nonlinear second order partial differential equation. This continuity equation works also in the case of degenerate semiconductors which was not the case for other widely used approaches.

Jürgen Potthoff (with J. Asch):

Extended Stochastic Integrals in White Noise Calculus

In the white noise calculus of T. Hida and his followers one finds a definition for the multiplication of random variables by white noise $x(t), t \in \mathbf{R}$. It is given as $x(t) \cdot 0(\partial_t^* + \partial_t)$. Here ∂_t acts on φ with Wiener-Itô decomposition given by the sequence of integral kernels $(f^{(n)}, n \in \mathrm{N}_0)$ via $f^{(n)}(\cdot) \to nf^{(n)}(t, \cdot)$ and ∂_t^* is its dual: $f^{(n)} \to \delta_t \otimes f^{(n)}$. The attempt to define the stochastic integral $\int_0^1 \varphi(s)\, dB(s)$ as $\int_0^1 (\partial_s + \partial_s^*)\varphi(s)\, dB(s)$ makes a careful definition of $\partial_s\varphi(s)$ necessary (Kuo & Russek). Itô's integral is reproduced for adapted φ if we use $\partial_{s+}\varphi(s)$. This operation can be defined on suitable (rich) subspaces of the square integrable processes. Therefore we found an extension of Itô's integral to (possibly) anticipatory integrands. It was stated that Itô's lemma holds in its usual form for this extended integral. Stratonovich integrals can be treated using $\frac{1}{2}(\partial_{s+} + \partial_{s-})$ above instead of ∂_{s+}.

Philip Protter:

Stochastic Volterra Equations with Anticipating Coefficients

Stochastic Volterra equations are considered where the coefficients $F(t, s, x)$ are random and adapted to $\mathcal{F}_{s \vee t}$ rather than the customary $\mathcal{F}_{s \wedge t}$. Such an hypothesis leads to stochastic integrals with anticipating integrands. We interpret these as Skorohod integrals, which generalize Itô integrals to the case where the integrand anticipates the future of the Wiener process integrator. The solutions are nevertheless adapted processes and even semimartingales if the coefficients are smooth enough. The semimartingale decomposition is obtained. This is joint work with Etienne Pardoux.

Gunter Ritter:

Stability of Diffusion Processes on Hilbert Space

We consider a stochastic differential equation in Hilbert space H, viz.,

(1). $$d\xi_{x,t} = \sigma(\xi_{x,t})d\beta_t + b(\xi_{x,t})dt$$

Here, β_t is Brownian motion in H with (nuclear) covariance Q, σ and b are continuous functions bounded on bounded sets from H to $L(H)$ and to H, respectively. We stated and discussed the following result which was obtained in collaboration with G. Leha:

Theorem: Suppose there exists a compact, positive definite operator K and a real constant C such that

$$traceK(\sigma(x)-\sigma(y))Q(\sigma^*(x)-\sigma^*(y))+2(K(x-y),b(x)-b(y)) \leq C(K(x-y),x-y).$$

Suppose further that there exists a (Ljapunov) function $V \geq 0$, twice differentiable with uniformly continuous second derivative and such that

$$sup_{\|x\|\geq R} \quad \mathcal{L}V(x) \to -\infty \qquad \text{as } R \to \infty$$

($\mathcal{L}$ is the infinitesimal generator associated with equation (1)).Suppose finally that for each $x \in H$ there exists a solution to (1) starting at x and with infinite lifetime. Then there exists an equilibrium distribution for the transition probability defined by $(\xi_{x,t})_{x\in H,t\geq 0}$.

Michael Röckner (joint work with Sergio Albeverio):

Dirichlet forms and quantum fields

We consider quadratic forms on $L^2(E;\mu)$ of the type

$$\mathcal{E}(u,v) = \int_E \langle \nabla u, \nabla v\rangle_H \, d\mu$$

where u, v are finitely based smooth bounded functions on E, E is an (infinite dimensional) topological vector space, H is a Hilbert space densely and continuously embedded in E and μ is a probability measure on E. We prove a necessary and sufficient condition for the closability of (the components of) $\mathcal{E}$. Then we consider the set $\underline{\underline{\mathcal{E}}}$ of all Dirichlet forms extending $\mathcal{E}$ which have sufficiently many finitely based smooth functions in the domain of their generators ordered w.r.t. the usual order on forms. We prove that the closure $\mathcal{E}^0$ of $\mathcal{E}$ is the minimal and construct explicitly the maximal element of $\underline{\underline{\mathcal{E}}}$. Furthermore, we prove that under mild assumptions on E there exists an (infinite dimensional) diffusion on E associated with $\mathcal{E}^0$. Finally, we describe applications to infinite dimensional Gaussian measures and in particular to measures occurring in quantum field theory.

Gilles Royer:

Simulated annealing of diffusion processes

Let X_t^ε be the Kolmogorov process defined by

$$(1) \qquad dX_t^\varepsilon = \varepsilon \, dB_t - \nabla U(X_t^\varepsilon) \, dt \, ;$$

it is known that the second eigenvalue $\lambda(\varepsilon)$ of the corresponding infinitesimal generator is like $e^{-\Lambda/\varepsilon^2}$ when $\varepsilon \to 0$, where $0 < \Lambda < \infty$. We consider the nonhomogeneous process Z_t obtained by replacing in (1) ε by $\varepsilon(t) = \sqrt{\frac{c}{\log(t)}}$. We prove that for $c > \Lambda$ the law of Z_t for $t \to \infty$ has the same limit as $Z_\varepsilon^{-1} \exp\left(-\frac{2U}{\varepsilon^2}\right)$ when $\varepsilon \to 0$. This is a (slight) improvement of a result by Chiang, Huang, Shen ($c > \frac{3\Lambda}{2}$), which looks like corresponding results for processes with finite state space, and provides a direct link between annealing and asymptotic behaviour of the eigenvalue $\lambda(\varepsilon)$.

P. Salminen:

Some properties of multiplicative martingales of a branching Brownian motion

Let X be a binary branching Brownian motion with creation rate 1. Non-negative solutions of $\frac{1}{2}u'' + u^2 - u = 0$ generate all multiplicative invariant functions of X. It is proved that the corresponding multiplicative (non-trivial) martingales tend a.s. to 0. It is also shown that they are not square integrable and an estimate for the explosion time is given.

Hiroshi Sugita:

Multiple Stratonovich integral and essentially continuous multiple Wiener integral

We consider a topological property of multiple Stratonovich integral. Let us say a Wiener functional is *essentially continuous* if it can be realized as a continuous function defined on a certain abstract Wiener space, or equivalently, if we can find a measurable norm which makes it continuous. Then our result is the following:

A multiple Wiener integral is essentially continuous if and only if it is a finite sum of Hu-Meyer's multiple Stratonovich integrals.

As an application, we can derive a certain support theorem for multiple Wiener integrals.

Alain-Sol Sznitman:

Lifschitz tail and Wiener sausage on hyperbolic spaces

The density of states on hyperbolic space corresponding to a Poisson cloud of obstacles depends on the boundary conditions picked on the frontier of the sequence of balls increasing to the full space. If one picks Dirichlet or Neumann conditions, the behavior near its respective lower edge of the corresponding density of states is very different.

John C. Taylor:

Brownian motion on a non-compact symmetric space: skew product decomposition and polar coordinates

Let G be a connected semisimple Lie group and K a maximal compact subgroup — e.g. $G = SL(3,R)$ and $K = SO(3)$. The asymptotics behaviour of the left invariant Brownian motion on the symmetric space G/K may be studied either in horocyclic or polar coordinates. This was done by Malliavin M.P. & Malliavin P. in 1974.
By sharpening and clarifying their work it can be seen that in the polar case this amounts to studying a diffusion in skew-product form on $K/M \times A^+ \cong X' \subset X = G/K$ — where X' is the set of regular points. Using this decomposition it follows as shown by Malliavin &Malliavin that a.s. as $t \to \infty$, $a^+(t)$ tends to infinity in a special direction and $k(t)M$ converges. This result shows that the exit law from a large ball centered at $0 = K$, while uniformly distributed on the K-orbits, tends

to concentrate on one special orbit (given by the limiting direction of $a^{+}(t)$ as the radius goes to infinity.

Ali S. Ustunel:

Independence on the Wiener Space

In this talk I have given some results from joint work with Moshe Zakai and also partly with David Nualart concerning the characterization of independence. The main results are the following: Two multiple Wiener integrals are independent if and only if their kernel functions are strongly orthogonal (i.e., their first order contraction is zero almost surely). Furthermore, this implies that the smallest L^{-1}-stable (L is the Ornstein-Uhlenbeck operator on the Wiener space) σ-fields with respect to which they are measurable are independent. This gives the characterization of independence of "nice" σ-fields via the orthogonality of their "tangent" spaces. In particular two arbitrary families of multiple Wiener integrals are independent if and only if their elements are pairwise independent. At the end we have shown that the orthogonality of the tangent spaces implies the independence of σ-fields satisfying some weaker conditions.

Shinzo Watanabe:

Asymptotics of generalized Wiener functional integrations

By a generalized Wiener functional integration, we mean an expectation of the form $E[G(\omega)\Phi(\omega)]$ where

(i) $$G \in \mathbf{D}_{\infty} = \bigcap_{k>0} \bigcap_{1<p<\infty} \mathbf{D}_{p,k} \quad \text{and} \quad \Phi \in \mathbf{D}_{-\infty} = \bigcup_{k>0} \bigcup_{1<p<\infty} \mathbf{D}_{p,-k}$$

or

(ii) $$G \in \widetilde{\mathbf{D}}_{\infty} = \bigcap_{k>0} \bigcup_{1<p<\infty} \mathbf{D}_{p,k} \quad \text{and} \quad \Phi \in \widetilde{\mathbf{D}}_{-\infty} = \bigcup_{k>0} \bigcap_{1<p<\infty} \mathbf{D}_{p,-k},$$

$\mathbf{D}_{p,k} := (I + D^{*}D)^{\frac{-k}{2}}(L_p), \quad 1 < p < \infty, \quad k \in R$ being the Sobolev spaces over the Wiener space (W, P). We study the asymptotic evaluation of $E[G(\varepsilon,\omega)\Phi(\varepsilon,\omega)]$ when these functionals depend on a small parameter $\varepsilon > 0$. An example of most interest ist the case $p(\varepsilon^2, x, y) = E[\delta_y(X^{\varepsilon}(1,x,\omega)]$ where $X^{\varepsilon}(t,x,\omega)$ is a solution of a SDE and then $p(t,x,y)$ is the associated heat kernel. Among others, Léandre and Ben Arous established its asymptotic expansion when x and y are *not* in the cut-locus. Here we consider the case of x and y in the cut-locus, especially the case when the set of geodesics connecting x and y forms a nice manifold. An essential point of the proof is to construct an "asymptotic" *partition of unity*. Our method applies to all cases appearing in the nilpotent Lie groups $N_{2,2}$ (Heisenberg group) and $N_{4,2}$.

Ruth J. Williams:

Multidimensional diffusions with reflection in non-smooth domains

There is no extant general theory for diffusions confined to a domain by (oblique) reflection at the boundary when the boundary is not smooth and/or there are discontinuities in the directions of reflection. On the other hand, a variety of applications, e.g., in queuing theory, lead naturally to reflected Brownian motions in domains with non-smooth but simple geometries, e.g. cones and polyhedrons. Examples will be given to illustrate some possible behaviors for such processes, which are different from those in smooth domains. In particular, some criteria for existence and uniqueness, and for determining when the critical unsmooth parts of the boundary are reached, will be given for conical and polyhedral domains. Some stationary distributions for these processes will also be determined.

Moshe Zakai (joint work with D. Nualart):

Multiple Wiener-Itô integrals possessing a continuous extension

Let $F(w)$ be a Wiener functional defined by $F(w) = I_n(f)$ where $I_n(f)$ denotes the multiple Wiener-Itô integral of order n of the symmetric, $L^2([0,1]^n)$ kernel f. We show that a necessary and sufficient condition for the existence of a continuous extension of (i.e. the existence of a function $\varphi(\cdot)$ from the continuous functions on [0,1] which are zero at zero to $\mathbf{R}$, with $\varphi(\cdot)$ continuous in the supremum norm and for which $\varphi(w) = F(w)$ a.s.) is that there exists a multimeasure $\mu\{dt_1, \ldots, dt_n\}$ on $[0,1]^n$ such that $f(t_1, \ldots, t_n) = \mu\{(t_1, 1] \ldots, (t_n, 1]\}$ a.e. Lebesgue. Recall that a multimeasure $\mu\{A_1, \ldots, A_n\}$ is a signed measure in A_i for every fixed $A_1, \ldots, A_{i-1}, A_{i+1}, \ldots, A_n$ for every i and there exist multimeasures which are not measures. It is shown furthermore, if $f(t_1, \ldots, t_n) = \mu\{(t_1, 1], \ldots, (t_n, 1]\}$ then all the traces $f^{(k)}$ exist for $k \leq [n/2]$ and each $f^{(k)}$ induces an $(n-2k)$ multimeasure and then

$$I_n(f) = \sum_{k=0}^{[n/2]} \left(-\frac{1}{2}\right)^k \frac{n!}{k!(n-2k)!} \int\limits_{[0,1]^n} W_{t_1} \ldots W_{t_{n-2k}} \mu^{(k)}(dt_1, \ldots, dt_{n-2k})$$

(therefore each of the integrals in this expression equals the corresponding multiple Stratonovich integral).

J. C. Zambrini:

Euclidean Quantum Mechanics

Euclidean Quantum Mechanics is a new probabilistic approach to (nonrelativistic) Quantum Physics, distinct from the one founded on Feynman-Kac and from Nelson's Stochastic Mechanics. It involves a new class of time reversible diffusion processes, the Bernstein processes and is founded on a forgotten idea of E. Schrödinger. It can be regarded as an alternative (Euclidean) version of Feynman's path integral method in Quantum Physics. We give a review of this approach.

Berichterstatter: Hans Föllmer

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