MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Mathematische Logik

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Die Tagung fand unter der Leitung von Herrn W.Felscher (Tübingen), Herrn H.Schwichtenberg (München) und Herrn A.S.Troelstra (Amsterdam) statt. Im Mittelpunkt des Interesses standen Fragen aus dem ganzen Gebiet der Mathematischen Logik mit Schwerpunkten in der Beweistheorie, Rekursionstheorie, Konstruktiven Methoden und Anwendungen in der Informatik.

VORTRAGSAUSZÜGE

G.ASSER: *Über eine Anwendung des Löwenheim-Skolem-Theorems.*
Durch eine einfache Anwendung des Löwenheim-Skolem-Theorems kann man z.B. die folgenden bekannten Sätze der allgemeinen Topologie erhalten (vgl. I.BANDLOW ZML 35):

1. Für jeden T_2-Raum X ist $card(X) \leq 2^{\ell(X)\chi(X)}$; dabei ist $\ell(X)$ die Lindelöf-Zahl von X und $\chi(X)$ der Charakter von X (ARCHANGELSKI 1969).

2. Für jeden T_1-Raum X ist $card(X) \leq 2^{\Psi(X)}$; dabei ist $\Psi(X)$ der Pseudocharakter von X (GRYZLOV 1969).

3. Es sei τ eine überabzählbare reguläre Kardinalzahl und $\lambda << \tau$. Ist X ein T_1-Raum, so dass jede offene Überdeckung von X einer Mächtigkeit $<\tau$ eine Teilüberdeckung einer Mächtigkeit $<\lambda$ besitzt, und ist $\mathfrak{B}$ Pseudobasis von X, für die $Y = \{x \in X : card\{V \in \mathfrak{B} : x \in V\} < \tau\}$ dicht in X ist, so ist $card(\mathfrak{B}) < \tau$ (MISCENKO 1962 für $\tau = \omega_1$ und $\lambda = \omega$; ARCHANGELSKI und PROIZVOLOV 1966 für überabzählbares regulares τ und $\lambda = \omega$).

E.BÖRGER: *First Order Description of Some Programming Constructs And Complexity Questions.* We consider two classes of programs: a) pure PROLOG-programs with binary clauses and besides individual constants no function symbols, b) ALGOL-60 programs consisting only of procedure declarations and procedure calls (in particular, with no arithmetic expressions or assignement of conditional statements). For a) we consider consistency and for b) formal reachability of procedures in a program, problems which have been shown to be complete for the complexity class PSPACE. We show that in such completeness proofs arbitrary polynomial space bounded computations of a Turing machine program can be expressed in a natural way by just one first order formula which - depending on the specific interpretation - represents both the consistency problem for programs of class a) and the formal reachability problem for procedures in programs of class b).

W.BUCHHOLZ: *On Notation Systems for Infinite Derivations.*
We introduce a primitive recursive notation system for wellfounded derivations of ω-arithmetic, in the same way as one usually defines an ordinalnotation system as a system of terms generated from constants for some specific ordinals by function symbols for certain ordinal functions. Instead of the class On of all ordinals we consider the class Z^∞ of all wellfounded derivations of ω-arithmetic (with unrestricted ω-rule). As constants we take the derivations of the finitary system Z of classical first order arithmetic, which canonically can be viewed as notations for specific elements of Z^∞. Instead of ordinal functions we take the operations occurring in the familiar cut-elimination procedure for Z^∞. This approach provides a significant technical and conceptual improvement in the area of applications of cut-elimination. For example we do not need any coding of infinite trees by indices for (primitive) recursive functions, but the only things to be coded are finite strings of symbols, such as formulas, finite derivations, etc. As a by-product we obtain a very concise "algebraic" descriptionof of Minc's continuous cut-elimination operator for locally correct (but not necessarily wellfounded) proof-figures of ω-arithmetic.

W.A.CARNIELLI: *Making Mathematical Sense of Contradictions.*
We discuss the interest and the meaning of making the notion of contradiction a valid object of mathematical anylysis. In particular, we show that certain paraconsistent-type logic systems can be constructed to deal with inconsistencies in knowledge-based systems in computers.

D.van DALEN: *On Identity.*
The study of the first-order theory of intuitionistic equality has yielded a number of results on the decidability of various theories and on the axiomatization of the equality fragment of intuitionistic mathematical theories (apartness, order, group theory, etc. - Statman+v.Dalen, Smorynski). Some key question in this area are still open: axiomatize the equality fragment of $Th(\mathbb{R})$, $Th(\mathbb{R}^e)$, the lambda calculus. The latter is particularly elusive, apart from some negative facts hardly anything is known (the equality is not even stable, i.e. $\neg\neg(x=y)\to x=y$ does not hold). The principles discovered so far are not very specific (e.g. Visser's principle $\forall x(\forall y(T(x,y)\vee\varphi(y))\to\varphi(x))$, where $T(x,y)$ formalizes "$\{x_1...x_n\}\cap\{y_1...y_n\}$ is inhabited" (the sets 'touch'). Unfortunately this formula also holds in $\mathbb{R}$ and a large number of structures).

Sofar no theory is known with "trivial" equality, i.e. with equality fragment being just the equality theory plus, maybe, cardinality conditions. There is one result:

Theorem. $\mathbf{EQ}$ *is complete for all subfamilies of* $\mathcal{P}(\mathbb{N})$.

Conjecture. $\mathbf{EQ}^\infty$ = *second-order equality fragment of* HAS, *where* $\mathbf{EQ}^\infty = \mathbf{EQ}+\{(\exists x_1...x_n)(\bigwedge_{i<j})(x_i\neq x_j)\text{: } n\in\omega\}$.

Fact: $\mathbb{R}^S$ (the singleton reals) have a stable equality, but not a 2-stable equality; in fact SEQ^2 for $\mathbb{R}^S$ implies $\neg\neg\forall\varphi(\neg\varphi\vee\neg\neg\varphi)$.

Finally, little is known about the model theory of $\mathbf{EQ}$. We showed (intuitionistically) that $\mathbb{N}^{\mathbb{N}}\prec\mathbb{R}$, $2^{\mathbb{N}}\prec\mathbb{N}^{\mathbb{N}}$, $\mathbb{R}\prec\mathbb{R}^2$, $\mathbb{R}-\{0\}\prec\mathbb{R}$, $(-\infty,0]\cup[0,\infty)\prec\mathbb{R}$. Notice that $(\mathbb{R}-\{0\})\cup\{0\}\not\equiv\mathbb{R}$.

M.DEUTSCH: *Zur Reduktionstheorie des Entscheidungsproblems.*
Als weitere Verschärfung des bekannten Reduktionssatzes von Gurevic wird bewiesen:

Es sei M der kleinste Mengenbereich mit $\emptyset\in M$ und $(x\in M\wedge y\in M)\to(\{x\}\in M\wedge x\cup y\in M)$; $0:=\emptyset$, $n+1:=\{n\}$; $x\in^* M{:}\Leftrightarrow x\in y\vee(x=y\wedge x\neq 1)$. Es sei E die zweistellige Prädikatenvariable von $\forall\exists\forall\exists^\infty(0,1)$.

Theorem: Zu jeder Formel α der 1 Stufe kann man ein β aus $\forall\exists\forall\exists^\infty(0,1)$ ausgeben mit

(1) α ist erfüllbar $\Leftrightarrow$ β ist erfüllbar $\Leftrightarrow$ β ist auf einer transitiven Teilmenge von E durch $\in^*$ erfullbar,

(2) α ist endlich erfüllbar $\Leftrightarrow$ β ist endlich erfüllbar $\Leftrightarrow$ β ist auf einer endlichen transitiven Teilmenge von E durch $\in^*$ erfullbar.

J.DILLER: *Normal Deductions and Cut-Free Sequent Derivations.*
Natural deductions correspond 1-1 to sequent derivations. This correspondence does not hold for normal deductions in the sense of Pravitz, and Gentzen cut-free sequent derivations. We study the situation for logic and theories that extend minimal E-logic. Following Unterhalt, we take as permutative redex any E-inference whose major premise is the concluison of ($\vee/\exists$ E)-inference. Normal deductions then have an inductive characterization. Its inductive steps give rize to a (marked) cut-free sequent calculus. We show that the inductive constructions of normal deductions correspond canonically and 1-1 (modulo α-congruence) to marked cut-free sequent derivations. If the so-called crude discharge convention is observed, this correspondence extends canonically, but not 1-1, to (standard) cut-free sequent derivations.

H.D.DONDER: *Families of Almost Disjoint Functions.*
Two functions f,g: $\kappa \longrightarrow V$ are almost disjoint iff $|\{\alpha<\kappa\colon f(\alpha)=g(\alpha)\}|<\kappa$. We discuss the consistency strength of some combinatorial principles which assert the existence of large families of almost disjoint functions.

A.G.DRAGALIN: *Constructive Methods in (Classical) Model Theory.*
Model theory and proof theory in classical (and intuitionistic) logic are connected by famous Gödel's completeness theorem (for both Beth-Kripke-like models in intuitionistic case). The traditional versions of these results are essentially nonconstructive, so it causes a nonconstructive and noncomputational character of many model-theoretic results. Nowaday much more constructive methods was worked out for constructing (classical Boolean-valued and intuitionistic) models corresponding to axiomatic theories. We apply them for explicit constructing models for (classical) non-standard analysis and getting some proof-theoretical conservation results.

J.I.GIRARD: *Last Developments in Linear Logic.*
This is a mathematical semantics of proofs valid for 2^{nd} order λ-calculus - or linear logic (LL). A $\mathbb{C}^*$-algebra of finite operators, $\mathbb{A}^*$, is introduced. $\mathbb{A}^*$ basically internalises the isometries $\mathbb{H}\oplus\mathbb{H} \longrightarrow \mathbb{H}$ and $\mathbb{H}\otimes\mathbb{H} \longrightarrow \mathbb{H}$. To each proof Π in LL is associated a pair (Π^0,σ) of y^*. It is shown that

THM1: $\sigma\Pi^0$ is nilpotent.

THM2: Under general hypothesis $EX(\Pi^0,\sigma) := (1-\sigma^2)u\frac{1}{1-\sigma u}(1-\sigma^2) = \Pi'^0$,

where Π' is the normal form of Π.

The general gain is that this approach only involves local and finite operations, and therefore produces a theoretical parallel computer.

L.GORDEEV: *On Consistency and Cut Elimination.*
I consider the familiar weak rule free extension by two (symmetrical) abstraction rules of purely $(=,\in)$-variant of Gentzen's intuitionistic sequent calculus. Abstraction terms are built up from variables (possibly sorted) as $\{x \mid \alpha_i(x)\}$, $i=0,\dots,n$, and are closed under substitutions, where $\alpha_1,\dots,\alpha_n$ are stratified (in Quine's sense) formulae which include parameter x. I define the appropriate realizability-translation in the sequent calculus of the familiar Girard-style strong normalizability proof in Type Theory. Using Jensen's treatment of (Quine's) NF minus Extensionality, by similar techniques strong normalizability for the latter theory reduces to a combinatorial statement provable well within ZF. However, similar approach in the presence of the ω-rule for abstraction terms would certainly assume the truth of the Domain Completion Axiom (DCA): $(\forall x)PAT(x)$, for $PAT(x)\Leftrightarrow$"x *is (the value of) a proper abstraction term*". I ask under which assumptions $(\bigwedge_{i\leq n})(\exists y)(\forall x)(x\in y\Leftrightarrow\alpha_i(x))$ plus DCA is consistent. I conjecture that the following condition is sufficient: "*Let* $\mathfrak{a}$ *be* PAT, *and let* $(Qx)\varphi(x)$ *be any subformula-occurrence in* $\mathfrak{a}$ $(Q=\forall,\exists)$. *Let* $\mathfrak{a}',\mathfrak{a}''$ *be obtained from* $\mathfrak{a}$ *by substituting* $(Qx)(\varphi(x)\wedge PAT(x))$, *resp.* $(Qx)(\varphi(x)\vee\neg PAT(x))$ *for* $(Qx)\varphi(x)$. *Then* $\mathfrak{a}'$, $\mathfrak{a}''$ *are* PAT *modulo provability in classical Predicate Calculus.*" This conjecture implies the consistency of NF.

P.HAJEK: *Approximate Reasoning in Artificial Intelligence.*
Two approaches to analysis of many-valued truth-functional rule based expert systems were presented:
(1) Algebraic approach, based on the theory of ordered abelian groups and stressing comparative aspects (common work with J.Valdes).
(2) Probabilistic approach characterized by the following:
(i) contributions of rules are (transformed) log-linear coefficients of the underlying probability, not just conditional probabilities,
(ii) to keep the set of rules reasonably small, graphical probabilistic models (of Lauritzen, Spiegelhalter and others) are used,
(iii) ro decide whether results of a concrete run of an expert system are probabilistically sound, collapsibility of log-linear models is applied.
Reference: P.Hajek,"Towards a probabilistic analysis of MYCIN-like expert systems", in: COMPSTAT 88, Physica-Verlag, p.117-121.

H.R.JERWELL: *The Geometry of Ordinals.*

A presentation of some of the most important objects of Π^1_2-logic - denotationssystems, dilators, dendroids, β-proofs, dendra, gerbes - and relationships between them. In particular we consider the problem of separations of variables.

H.KOTLARSKI: *Iterated Omega Logic.*
The results presented below are due to Zygmunt Ratajczyk and myself.
Given m, let Σ_m-PA(S) denote the theory PA+"S is a full satisfaction class"+induction for Σ_m formulas in $L_{PA}\cup\{S\}$. Our aim is to describe the strength of this theory, i.e. what are the L_{PA}-consequences of Σ_m-PA(S). Define the iterations of ω-logic in the following manner. We define $\Gamma^\alpha{}_n$, T^α by simultaneous induction on $\alpha<\varepsilon_0$. Let $T^0(\varphi)$ be $PA\vdash\varphi$. Given T^α define $\Gamma^\alpha{}_n$. $\Gamma^\alpha{}_0(\varphi)$ is $T^\alpha\vdash\varphi$. $\Gamma^\alpha{}_{n+0.5}(\varphi)$ is "φ is of the form $\eta\vee\forall z\psi(z)$ and $\forall z\Gamma^\alpha{}_n(\eta\vee\psi(z))$"; $\Gamma^\alpha{}_{n+1}(\varphi)$ is $(T^\alpha\cup\Gamma^\alpha{}_{n+0.5})\vdash\varphi$. Let $T^{\alpha+1}=T^\alpha\cup\{\Gamma^\alpha{}_n(0=1):n\}$. For λ limit we put $T^\lambda=\bigcup_{\alpha<\lambda}T^\alpha$.
THM1 Σ_m-PA(S) has the same arithmetical consequences as $PA+\{\neg\Gamma^\alpha{}_n(0=1):\alpha=\omega_m(k):k\}$.
THM2 Σ_m-PA(S) has the same arithmetical consequences as $PA+\{TI(\varepsilon_\alpha):\alpha=\omega_m(k):k\}$.
Both theorems are obtained by model-heoretic means (using some ideas due to Pudlak).

M.van LAMBALGEN: *Random Sequences and Generalized Quantifiers.*
We present a faithful axiomatisation of von Mises' ideas on random sequences using Friedman's logic for the measure quantifier "almost for all".

Y.N.MOSCHOVAKIS: *Concurrency Modelling.*
The paper described a model for concurrent computation which is based on a game-theoretical representation of each processes's "perception" of the system and incorporates both <u>fair merge</u> and <u>full recursion</u>. The most interesting mathematical contribution is the establishment of several results which "justify" the chosen interpretation of recursion in a non-deterministic setting; chief amonf there is "the Scott-Bekic rule" which reduces nested to simultaneous recursion.

M.OKADA: *Proof Theory and Term Rewriting Theory.*
The purpose of this talk is to provide a bridge between proof theory in logic and term rewriting theory in computer science. It is shown that proof theoretic tools are very useful for analyzing two basic attributes of term rewriting systems, the termination property and the Church-Rosser property. In the first part of this talk we give the relationship among proof theoretic ordinals in logic, Kruskal-Friedman tree-embeddings in graph theory, and the ordering structures used in termination proof for term rewriting systems and for the Knuth-Bendix completion procedure in term rewriting theory. In the second part we utilize the proof-normalization technique in proof theory to analyze Church-Rosser property and completion procedure for term rewriting systems. In particular, we emphasize the correspondence between the paradigm of traditional proof theory and the paradigm of unconditional (classical) term rewriting theory, and next show how the same paradigm can be applied to the theory of conditional rewriting.

H.PFEIFFER: *A Theorem on Labelled Trees and the Limits of its Provability.*
Let τ be an ordinal of the second number class such that to every limit ordinal $\leq \tau$ a fundamental sequence is associated. We define a set $\mathfrak{T}_\tau$ of finite labelled trees, labelled by ordinals out of τ, introduce a relation $\leq_T$ on $\mathfrak{T}_\tau$ such that $\langle \mathfrak{T}_\tau, \leq_T \rangle$ can be shown to be a well-quasi-ordering. This in a certain sense generalises the theorem of KRUSKAL. The relation $\leq_T$ is no embedding but a structure $\langle \mathfrak{T}_\tau, \leq_B \rangle$ can be given which is isomorphic to $\langle \mathfrak{T}_\tau, \leq_T \rangle$ so that $\leq_B$ is an embedding with gap condition. To show the limits of provability of the theorem, that $\langle \mathfrak{T}_\tau, \leq_T \rangle$ is a well-quasi-ordering, a subset D of $\mathfrak{T}_\tau$ is mapped by some function φ onto the system $\overline{\Theta}(\tau)$ of ordinal notations of W.Buchholz, such that $\forall T,T' \in D(T \leq T'$ implies $\varphi(T) \leq \varphi(T'))$. Following H.Friedman and S.Simpson, we have proved that the theorem cannot be proved in a system of $<\tau$-iterated generalized inductive definitions. L.Gordeev gained similar results.

W.POHLERS: *Proof Theory and Infinity Axioms.*
Any closure condition satisfied by the set universe - e.g. being closed under arbitrary functions - is reflected at some cardinal $\aleph$. The requirement that $\aleph$ exists is called an infinite axiom. In the example $\aleph$ would be a regular cardinal which is closed under all thinkable functions. The restriction of the class of functions under which $\aleph$ is required to be closed to those functions which, relative to $\aleph$, are reasonably definable leads to the recursively regular ordinals of definability theory. There are also 'definable counterparts' of other cardinals. We obtain a further restriction when we require that $\aleph$ has to be closed under all functions which - in some sense - are provable relative to $\aleph$. This leads to the notion of the proof theoretic counterpart of a cardinal. We gave an explication of the term 'in some sense provable relative to $\aleph$'. In our program we try to answer the now obvious question:

What are proof theoretic counterparts of large cardinals.

As a paradigm for this kind of research we sketched the treatment of the following infinity axioms:"There exists an infinite ordinal", and "There exists an uncountable regular ordinal". The strongest axiom which until today has been theoretically analyzed is:
"There exists a weakly inaccessible ordinal".

P.PUDLAK: *On Complexity of Proofs.*
We define proof systems G_i for quantified propositional formulas and prove a relation between systems G_i and subtheories S_2^{i+1} of bounded arithmetic S_2. This enables us to reduce the open problem of finite axiomatizability of bounded arithmetic to the problem of polynomial simulation between the systems G_i. We derive some further corollaries of this relationship.

M.RATHJEN: *Systems of 2nd Order Arithmetic.*

In terms of proof-theoretic strength there is a large gap between Δ^1_2-CA and $(\Delta^1_2$-CA)+(BI). To analyse this gap, I provide a variety of induction-, comprehension-, and choice principles. Especially an ordinal analysis for the theories of Π^1_1-transfinite recursion and transfinite Σ^1_2-dependent-choices is carried out. Reference: Jäger,G.& Pohlers,W.:"Eine beweistheoretische Untersuchung von $(\Delta^1_2$-CA)+(BI) und verwandter Systeme". Bayerische Akademie der Wissenschaften, Math.-Naturwissensch. Klasse: Sitzungsberichte (1982).

W.RAUTENBERG: *Common Properties of Local Connectives.*
Let V be a variety of semigroups. The main result is that the propositional consequence $\vdash_V := \bigcap\{\vdash_{(A\ d)} \mid A \in V, d \in A\}$ is finitely based (i.e., axiomatizable by finitely many finitary sequential rules) iff V is finitely based in the equational sense. We apply this among others in order to construct a rule base for the common rules of all four proper semigroup connectives $\wedge, \vee, \Leftrightarrow, +$ (either-or). Here is a base, where we omit writing the connectives and x.yz means x(yz) etc.

p;q;pr/qr (ternary rule)

p.qr/pq.r	(p.qr)s/(pq.r)s
pq/qp	pq.r/qp.r
pp.p/p	pppq/pq

p/ppp.

The ternary rule cannot be replaced by at most binary rules.

J.C.SHEPERDSON: *Language and Equality Theory in Logic Programming.*
The underlying language of symbols used in logic programming is usually taken to be the one consisting of the symbols occurring in the program or the query, but sometimes infinitely many function symbols are included. As far as PROLOG or SLDNF-resolution are concerned it makes no difference, since these operate entirely within the former language. But it does make a difference to the Clark completion of a program, and to 'constructive negation', a recently proposed extension of negation as failure. The difference is transmitted via the equality theory. We consider the effect of the underlying language on the Clark completion, the equality theory, constructive negation and on 3-valued semantics.

W.SCHÖNFELD: *PROLOG-ähnliche Suchverfahren für die volle Prädikatenlogik.*
PROLOG is a proof search procedure for Horn clauses. Since it can be implemented very efficiently, it has found its way into real computer applications. The efficiency strongly depends on backtracking: If the interpreter recognizes that the actual branch in the proof search tree cannot be completed to a proof, this branch is removed up to the next alternative. We show that this principle can be extended tu full first-order logic without too much costs, contradicting a commonly accepted opinion that PROLOG is a wise compromise between expressive power and efficiency. Our method is based on the representation of knowledge bases by bipartite directed graphs and their unfolding to alternating (i.e. and-or-) trees. The underlying logical formalism is tableau calculus.

A.S.TROELSTRA: *Remarks on Intuitionism and Philosophy of Mathematics.*
The title describes the contents. Subjects discussed:
1. absolute and relative certainty
2. mathematics as the science of idealized structures
3. the role of language
4. formalization and the evidence for for axioms
5. examples of informal rigour and concept analysis
6. actualism, platonism and intuitionism, a comparison
7. certainty.

A.VISSER: *The Logic of Interpretability.*
The research on which I report in my talk is done with Dick de Jongh, Craig Smorynski and Frank Veltman. We consider a propositional language with modal operators $\Box$, and $\rhd$. $\Box$ stands for provability in a theory T into which Q can be interpreted. $A \rhd B$ means: T+B is relatively interpretable in T+A. (Theories in this language were first studied by Hajek and Svejdar). We ask ourselves which principles of this language are valid for all arithmetical interpretations in a given theory T. Specifically we look at three modal theories ILW, ILM, and ILP. We show the soundness of these theories for interpretations in respectively 'reasonable' arithmetics, essentially reflexive arithmetics and finitely axiomatizable arithmetics. We demonstrate the use of the modal theories and discuss two different kinds of Kripke semantics. Finally an arithmetical completeness proof for ILP is presented for the special case of interpretations in GB and in ACA_0. This proof is due to C.Smorynski and A.Visser. The proof uses a characterization by H.Friedman of $A \rhd B$ over GB (ACA_0) for A,B in the language of ZF (PA). It is an adaptation of Carlson's proof of arithmetical completeness for the combined provability logic of ZF and PA.

H.VOLGER: *Logik und die Theorie der deduktiven Datenbanken.*
A deductive database is determined by a set of axioms for a theory T which has a term model which is generic. A model $\underline{M}$ is generic if $\underline{M} \models \varepsilon$ implies $T \vdash \varepsilon$ for any sentence ε in $\exists \lambda At$. Moreover, any consistent extension T' of T by new facts has again this property. The following result of Malcev explains why deductive databases are universal Horn theories: A theory T is universal Horn iff T and all its consistent extensions by new facts admit generic resp. initial models iff Mod(T) is closed under substructures and non-empty products. If disjunctive infirmation is admitted in deductive databases one has to consired sets of generic term models rather than one generic model. Using a decomposition into connected components one obtains the following analog of Malcev's result: A theory T and all its consistent extensions by new facts admit in each connected component a term model which is initial iff Mod(T) is closed under substructures and pullbacks. The syntactical characterization of these theories is not known. Both results are obtained as special case of a result for theories T for which Mod(t) is closed under equalizers.

S.S.WAINER: *An Old Theorem of Tait Revised.*
Tait (1961) proved that nested recursion over a well ordering α can be reduced to unrested recursion over ω^{α}, provided α satisfies certain "standardness" criteria. This is a result about recursive programm transformation, and the talk attempted to review and generalize it from his point of view. A simple motivating example is as follows: - for the sake of "space efficiency", one can transform the recursion **I.** $g(o,y) = y,\ g(x+1,y) = f(g(x,y))$ into a while loop **II.** *while* $x \neq y$ *do* $x := x-1;\ y := f(y)$ *od*. The cost of transformation can be measured in terms of the "complexity" of their respective termination proofs: **I** uses Σ_1-Induction whereas **II** uses Π_2-Induction. Each while loop corresponds to a tail-recursion over a certain tree. Using tree-ordinals to measure the structure and complexity of well founded trees, we have the following general form of Taits Theorem:

$$\text{Recursion}(A) = \text{SPACE}(H[\omega^A]) = \text{While}(\omega^A)$$

for "suitable" sets A of tree ordinals, where H is the so-called Hardy Hierarchy.

Berichterstatter: L.Gordeev, Tübingen

Tagungsteilnehmer

Prof. Dr. G. Asser
Ludwig-Jahn-Str. 6

DDR-2200 Greifswald

Prof. Dr. H.G. Carstens
Fakultät für Mathematik
der Universität Bielefeld
Postfach 8640

4800 Bielefeld 1

Prof. Dr. E. Börger
Dipartimento di Informatica
Universita di Pisa
Corso Italia, 40

I-56100 Pisa

Prof. Dr. D. van Dalen
Mathematisch Instituut
Rijksuniversiteit te Utrecht
P. O. Box 80.010

NL-3508 TA Utrecht

Prof. Dr. W. Buchholz
Mathematisches Institut
der Universität München
Theresienstr. 39

8000 München 2

Prof. Dr. M. Deutsch
Fachbereich 3
Mathematik und Informatik
der Universität Bremen
Bibliothekstr., PF 33 04 40

2800 Bremen 33

Prof. Dr. A. Cantini
Dipt. di Filosofia
Universita degli Studi di Firenze
Via Bolognese 52

I-50139 Firenze

Dr. K.-H. Diener
Mathematisches Institut
der Universität Köln
Weyertal 86-90

5000 Köln 41

Dr. W. Carnielli
c/o Prof. Dr. J. Diller
Insitutt f. Mathematische Logik und
Grundlagenforschung
Einsteinstr. 62

4400 Münster

Prof. Dr. J. Diller
Institut für Mathematische
Logik und Grundlagenforschung
der Universität Münster
Einsteinstr. 62

4400 Münster

Prof. Dr. H. D. Donder
Institut für Mathematik II
der Freien Universität Berlin
Arnimallee 3

1000 Berlin 33

Dr. L. Gordeev
Mathematisches Institut
der Universität Tübingen
Auf der Morgenstelle 10

7400 Tübingen 1

Prof. Dr. A. Dragalin
Institute of Mathematics
Lajos Kossuth University
Pf. 12

H-4010 Debrecen

Prof. Dr. P. Hajek
Institute of Mathematics of the
CSAV (Academy of Sciences)
Zitna 25

115 67 Praha 1
CZECHOSLOVAKIA

Prof. Dr. H.-D. Ebbinghaus
Abteilung für Math.Logik und
Grundlagenforsch.der Mathematik
der Universität Freiburg
Albertstr. 23b

7800 Freiburg

Prof. Dr. G. Jäger
Institut für Informatik
ETH-Zürich
ETH-Zentrum

CH-8092 Zürich

Prof. Dr. W. Felscher
Mathematisches Institut
der Universität Tübingen
Auf der Morgenstelle 10

7400 Tübingen 1

Prof. Dr. H. R. Jervell
HF-Data
University of Oslo
P. O. Box 1102 - Blindern

N-0316 Oslo 3

Prof. Dr. J. Y. Girard
16 bis rue Sibuet

F-75012 Paris

Prof. Dr. H. Kotlarski
ul. Wolska 40 m 24

01-187 Warszawa
POLAND

Dr. M. van Lambalgen
Mathematisch Instituut
Fakulteit Wiskunde en Informatica
Universiteit van Amsterdam
Roetersstraat 15

NL-1018 WB Amsterdam

Prof. Dr. H. Pfeiffer
Institut für Mathematik
der Universität Hannover
Welfengarten 1

3000 Hannover 1

Prof. Dr. H. Luckhardt
Fachbereich Mathematik
der Universität Frankfurt
Robert-Mayer-Str. 6-10

6000 Frankfurt 1

Prof. Dr. W. Pohlers
Institut für Mathematische
Logik und Grundlagenforschung
der Universität Münster
Einsteinstr. 62

4400 Münster

Prof. Dr. Y. N. Moschovakis
Dept. of Mathematics
University of California
405 Hilgard Avenue

Los Angeles , CA 90024
USA

Prof. Dr. K. Potthoff
Fach Logik und Wissenschaftslehre
am Philosophischen Seminar
der Universität Kiel
Ludewig-Meyn-Str. 4

2300 Kiel 1

Prof. Dr. M. Okada
Department of Computer Science
Concordia University
1455 de Maisonneuve Blvd. West

Montreal Quebec H3G 1M8
CANADA

Prof. Dr. P. Pudlak
Institute of Mathematics of the
CSAV (Academy of Sciences)
Zitna 25

115 67 Praha 1
CZECHOSLOVAKIA

Dr. P. Päppinghaus
Ahornring 67

8028 Taufkirchen

M. Rathjen
c/o Prof. Dr. W. Pohlers
Institut f. Mathematische Logik u.
Grundlagenforschung d. Universität
Einsteinstr. 62

4400 Münster

Prof. Dr. W. Rautenberg
Institut für Mathematik II
der Freien Universität Berlin
Arnimallee 3

1000 Berlin 33

Prof. Dr. A. S. Troelstra
Mathematisch Instituut
Fakulteit Wiskunde en Informatica
Universiteit van Amsterdam
Roetersstraat 15

NL-1018 WB Amsterdam

Dr. W. Schönfeld
IBM Deutschland GmbH
Wissensch. Zentrum Heidelberg
Tiergartenstraße 15

6900 Heidelberg

Prof. Dr. A. Visser
Filosofische Faculteit
Postbus 80103

NL-3508 TC Utrecht

Prof. Dr. H. Schwichtenberg
Mathematisches Institut
der Universität München
Theresienstr. 39

8000 München 2

Prof. Dr. H. Volger
Fakultät für Mathematik
und Informatik
Universität Passau
Innstr. 27, PF 2540

8390 Passau

Prof. Dr. J. C. Shepherdson
School of Mathematics
University of Bristol
University Walk

GB- Bristol , BS8 1TW

Dr. S. S. Wainer
School of Mathematics
University of Leeds

GB- Leeds , LS2 9JT